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QCD PREDICTIONS FOR HADRONIC FINAL STATES IN e^+e^- ANNIHILATION

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In this paper we defined the pointing vector (a measure of the angular distribution of energy flux) by $p \, d\sigma/d(\cos^2 \theta)$, where θ is the angle in the event plane.

One would like to compare our jet predictions with a "phase space" no-jet model where $d\sigma/d\Omega$ or $d\sigma/d\theta$ is constant. The phase space model, when phrased in terms of our original pointing vector would yield

$$P = p \frac{d\sigma}{d(\cos^2 \theta)} \sim |\sin \theta \cos \theta|^{-1}$$

This formula has spikes at $\theta = 0, 90, 180$ and 270° that are purely kinematical. Thus, for the phase space model a modified definition of the pointing vector is more sensible

$$P' \equiv p \frac{d\sigma}{d\theta}$$

But for the jet model the original pointing vector has no singularities at any θ so that the modified pointing vector P' has kinematical zeros at the cardinal points:

$$P' \propto |\sin \theta \cos \theta| P$$

However, when the jet model predictions for the modified pointing vector P' are smeared over an experimental uncertainty Δ of a few degrees in the determination of the jet axis, the kinematical zeros wash out. Thus the modified pointing vector P' will be a disk in the phase space model and will again resemble a propellor in the jet model.

The jet-finding strategy outlined in our paper can be followed equally well using this refined definition of the pointing vector, for other processes as well as e^+e^- annihilations. Figures 10 and 14 of our paper show QCD predictions for

the pointing vector in the e^+e^- continuum and on an-onium resonance. The unsmeared figures 10, 14 (c), (d) and (e) exhibit $p \, d\sigma/d(\cos^2 \theta)$. The figures 10 14 (b), (f), (g) and (h) which show the energy distribution smeared by non-perturbative effects can be regarded equally well as predictions for the improved pointing vector P' . Distributions for the pointing vector P' are indistinguishable from figures 10, 14 (b), (f), (g) and (h) when the experimental "jitter" Δ is taken into account.

We take this opportunity to draw to the reader's attention two recent related papers which we should have included in our references:

C.L. Basham, L.S. Brown, S.D. Ellis and S.T. Love, Washington University
Preprint RLO-1388-746 (1977).

G. Parisi, Ecole Normale Supérieure, Paris, Preprint LPTENS 78/9 (1978).

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A B S T R A C T

We defend the applicability of asymptotically free QCD perturbation theory to the description of multi-jet hadronic final states and investigate its predictions for the process $e^+e^- \rightarrow \text{hadrons}$ both in the hadron "continuum" and in heavy QQ resonances. We describe the distribution of the hadron energy flow in terms of variables (thrust, sphericity, acoplanarity) chosen to eliminate the sensitivity of the data and of the QCD predictions both to non-perturbative effects related to confinement, and to infra-red divergences which arise in perturbation theory. We present arguments that such analyses should be considered as genuine tests of perturbative QCD.

We propose definitions of two- and three-jet final states which can be used to exploit maximally the perturbative QCD predictions. We show that particularly strong tests of the gluonic degrees of freedom are provided by jet distributions in the hadronic decays of heavy QQ resonances and by the jet-photon angular correlations in sequential decays through even charge conjugation states. Predictions for the hadron energy distributions near new flavour thresholds are also discussed.

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is infra-red finite in perturbation theory ⁸⁾, but also that this expansion is uncontrollably non-uniform near new quark thresholds and resonances ⁹⁾. The cross-section at, say, the mass of the J/ψ is clearly sensitive to both infra-red and non-perturbative effects that bind the charmed quarks together. Yet, contact between experiment and the Euclidean perturbative calculation can be made via a dispersion relation, analytically extending the data to the negative Q^2 axis ¹⁰⁾. The smearing involved in the dispersion relation thus gets rid of the non-perturbative and infra-red sensitivity of the prediction. Poggio et al. ¹¹⁾ have argued that it should be possible to smear the data in a more "local" fashion than the one involved in going from timelike to spacelike Q^2 . Let Δ^2 be a "width" larger than the typical mass difference (non calculable in perturbative QCD) between prominent resonances of a given "onium" family, say the charmonium resonances. Consider the smoothed cross-section ¹¹⁾

$$\sigma(\Delta, Q^2) = \frac{\Delta}{\pi} \int_0^\infty \frac{\sigma_{\text{EXP}}(Q'^2) dQ'^2}{(Q^2 - Q'^2)^2 + \Delta^2} \quad (1.1)$$

This amounts to calculating via a dispersion relation the vacuum polarization at complex Q^2 ($\text{Im } Q^2 = \Delta$), where the perturbation expansion is not sensitive to the non-perturbative effects. Thus the local sensitivity of the data to these effects has also been tamed in $\sigma(\Delta, Q^2)$. The comparison of the smeared data, Eq. (1.1), and asymptotically free perturbation theory should therefore be justified, much as the comparison in the deep Euclidean regime ¹¹⁾. This necessity to smear the data to tame its infra-red and/or non-perturbative sensitivity and compare it to the predictions of perturbative QCD is also found in other processes. The electroproduction resonances must also be smeared over suitable Bjorken x intervals in the structure function, before contact between theory and experiment is made ¹²⁾. Not surprisingly, in discussing tests of perturbative QCD in e^+e^- annihilation, we will again encounter the necessity of smoothing the data in some fashion to eliminate its sensitivity to both infra-red and non-perturbative effects.

The criteria a) and b) given above suggest that ^{4),6),13)-15)} hadron jet cross-sections in e^+e^- annihilation, electroproduction and elsewhere may be calculable in QCD. For example, the diagram of Fig. 1a seems to be trying to be two jets, while some of those in Fig. 1b might be expected ¹³⁾ to give rise to three jets. What we must do is search for definitions of jets, or other variables in e^+e^- annihilation, which are infra-red finite, make perturbative sense, and should survive non-perturbative effects.

1. - INTRODUCTION

One of the challenges in modern high-energy physics is having a theory - quantum chromodynamics (QCD) - which one may believe to be correct, but from which one knows how to calculate only a few observable predictions. The asymptotic freedom of QCD enables one to calculate deep Euclidean Green's functions in renormalization group improved perturbation theory in terms of a logarithmically decreasing running coupling constant ²⁾. Unfortunately, Green's functions in deep Euclidean limits are not observable, and it is necessary to find real observables which may plausibly be calculated in asymptotically free perturbation theory. The most dramatic problem is that in QCD perturbation theory quarks and gluons get out ³⁾, whereas so far they appear confined in the real world. This implies that non-perturbative effects play an essential rôle in the formation of final states. Since we do not know how to deal with these effects, perturbative QCD predictions are sensible only for observables which are insensitive to them ⁴⁾. In addition, perturbative QCD is subject to infra-red divergences analogous to those which arise in QED. We thus distinguish two effects to be considered in formulating valid tests of perturbative QCD ; the former we refer to as "non-perturbative effects" and we use the term "infra-red effects" for the low momentum divergences occurring in perturbation theory.

The infra-red problem has been solved in QED ⁵⁾ where it is known that the infra-red singularities in individual diagrams such as those of Fig. 1b are cancelled if one considers cross-sections with suitable energy and angle cut-offs which may be related to practical experimental resolutions. Sterman and Weinberg ⁶⁾ have noted that the same cancellation occurs in lowest order QCD, and conjectured that an analogous proof of perturbative infra-red finiteness can be made for suitably cut-off cross-sections. Accepting this conjecture, a criterion for QCD predictions emerges ^{4),6)} ; quantities should be regarded as predictable with present artillery if : a) they are finite in QCD perturbation theory and the perturbation series is sufficiently convergent, and b) non-perturbative effects are not obviously dominant.

To make this philosophy more explicit, let us consider $e^+e^- \rightarrow$ hadrons. At Euclidean momenta, the hadronic vacuum polarization is clearly infra-red finite in perturbation theory and low orders are a self-consistent approximation ⁷⁾. We hope this happy situation is not prejudiced by non-perturbative effects, and believe the asymptotically free calculations at Euclidean momenta. At timelike momenta we know that $\sigma_{\text{tot}}(e^+e^- \rightarrow \text{hadrons})$

Specifically, energy and angular cut-offs must be chosen not only to yield an infra-red finite and convergent perturbative expansion as in QED, but also to erase the sensitivity to non-perturbative effects. These manifest themselves phenomenologically ¹⁶⁾ as jets of hadrons with an exponentially cut-off p_T , $\exp(-4p_T^2 \text{ GeV}^{-2})$ or $\exp(-6p_T \text{ GeV}^{-1})$, as opposed to liberated quarks and gluons. The non-zero $\langle p_T \rangle$ no doubt reflects the fact that quarks materialize as real particles with inverse radii of order $R^{-1} \sim \langle p_T \rangle$. This effect cannot be derived in perturbative QCD; we accept it as an empirical fact as is done in the parton models ¹⁷⁾. QCD predictions for the energy flow in a certain "jet" direction must therefore be smeared over an angular resolution sufficiently large to eliminate both infra-red effects, $\ln \delta \alpha_s(Q^2)/\pi \ll 1$, and non-perturbative effects, $\delta > \langle p_T \rangle/Q$. The second requirement is analogous to smearing over the "width" Δ in Eq. (1.1). The perturbative QCD predictions for jet cross-sections smeared in this way should provide reliable tests of the theory.

The most dramatically non-perturbative QCD effect in e^+e^- annihilation is the formation of "onium" resonances. We argue, however, that the properties of (suitably smeared) jets in their decay products are predictable in QCD perturbation theory. The argument is based in the conjecture that in QCD, as in QED, the non-perturbative and infra-red sensitivity of the decay matrix element can be factorized in a functional of the bound state wave function. The normalized decay distributions of jet energies and angles are then insensitive to infra-red and non-perturbative effects, and amenable to calculation in QCD perturbation theory.

Experimental indications for the dominance of "two-jet" final states have been found in e^+e^- collisions ¹⁶⁾ with centre-of-mass energies $Q = 5$ to 7 GeV. Experimentalists measured the sphericity ¹⁸⁾ of an event

$$\hat{S} = \frac{3}{2} \min \frac{\sum_i |\vec{p}_T^i|^2}{\sum_i |\vec{p}_i|^2} \quad (1.2)$$

where the sum runs over all observed particles and the p_T are transverse to a "jet" axis which is chosen to minimize $\hat{S}$. The observed distribution $d\sigma/d\hat{S}$ agrees with a two-jet model with finite $\langle p_T \rangle = O(300) \text{ MeV}$. The distribution of the jet axis relative to the e^+e^- axis is found to be $\sim 1 + \cos^2\theta$, as in leading order QCE (Fig. 1a). If this fixed $\langle p_T \rangle$ model continued to apply at very high energies, one would expect final states

with very highly collimated hadron jets. QCD perturbation theory, through the contribution of processes such as those of Fig. 1b, suggests that this naïve picture will not hold. However, it is not possible to compute $d\sigma/d\hat{S}$ reliably in asymptotically free perturbation theory, because the sphericity $\hat{S}$ acquires infra-red singularities. Various authors ^{19),20)} have proposed variables for measuring the "jettiness" of e^+e^- events which are arguably infra-red insensitive and so have distributions that are calculable in QCD perturbation theory. We discuss such variables in detail in Section 2. They include the maximum directed momentum ²⁰⁾, which we call "thrust"

$$T = 2 \max_i \frac{\sum_j p_{\parallel}^i}{\sum_i |\vec{p}_i|} \quad (1.3)$$

where the sum in the denominator runs over all observed particles and the sum $\sum$ in the numerator runs over all particles in a hemisphere. The momenta $p_{\parallel}^i$ are parallel to a "jet" axis, normal to the plane defining the hemispheres, and chosen to maximize T . Another example is the "sphericity" ¹⁹⁾

$$S = \left(\frac{4}{\pi}\right)^2 \min_i \left(\frac{\sum_j |\vec{p}_j^i|}{\sum_i |\vec{p}_i|} \right)^2 \quad (1.4)$$

which differs from sphericity in that the momenta are summed linearly instead of quadratically. As we shall discuss in detail, a possible strategy for finding "non-trivial" QCD jets like those discussed above is to measure $d\sigma/dT$ or $d\sigma/dS$ experimentally and compare them with two-jet expectations. If there is a tail of events with S or $1 - T$ large, as expected from QCD perturbation theory ^{12),19),20)}, then one may pick out these events and examine them for evidence of three-jet structure. One may look at the distribution of emitted energy-momentum as a function of angle relative to the thrust or sphericity axis (the "pointing vector"), or choose a suitable reference frame in which to search for second and third jet axes relative to which the hadronic p_{out}^i are limited. Through second order in asymptotically free perturbation theory each event is coplanar. It will be useful to have an infra-red safe measure of the acoplanarity, for which we propose

$$A = 4 \min \left(\frac{\sum_i |\vec{p}_{\text{out}}^i|}{\sum_i |\vec{p}_i|} \right)^2 \quad (1.5)$$

where the sum runs over all particles in an event, and p_{out}^i is measured perpendicular to a plane chosen to minimize A .

In this paper we try to extract the maximal QCD predictions for $e^+e^- \rightarrow \gamma^* \rightarrow \text{hadrons}$ in the lowest non-trivial order of perturbation theory, within the stated philosophy ^{4),5)} that quantities for which asymptotically free perturbation theory is a consistent expansion scheme are indeed valid predictions, as long as the appropriate smearing is performed.

The layout of the paper is as follows. In Section 2 we first advance plausibility arguments for the calculability of multi-jet cross-sections in QCD perturbation theory, and then discuss suitable choices of infra-red insensitive variables whose distributions should be reliably calculable. We then calculate the distributions in the infra-red safe variables (1.3), (1.4), (1.5) in second order QCD perturbation theory away from new heavy quark-antiquark resonances. Two-jet cross-sections dominate in this continuum (see Fig. 2). We calculate $d\sigma/dT$ (Fig. 3) and $d\sigma/dS$ (Fig. 4) from the diagram of Fig. 1b which gives planar events. We also estimate the non-perturbative contribution to $d\sigma/dT$, $d\sigma/dS$ and $\langle A \rangle$ from a simple two-jet model with finite $\langle p_T \rangle$, essentially to learn how to eliminate the sensitivity of QCD predictions to the details of non-perturbative effects. We also study events produced by new heavy quark pairs just above threshold. These events are characterized by large $\langle 1-T \rangle$, $\langle S \rangle$ and $\langle A \rangle$ which suggest useful triggers for searching for new quark events. In addition, we propose possible tests of the three-jet structure suggested by the diagram of Fig. 1b. In Section 3 we investigate jets in the decay of heavy quark-antiquark vector meson resonances of which $T(9.4)$ is presumably an example. As pointed out by other authors ²²⁾⁻²⁵⁾, planar events are expected to dominate if gluons have spin 1 as in QCD. We calculate the associated cross-sections $d\sigma/dT$ (Fig. 5) and $d\sigma/dS$ (Fig. 6) and compare with the background from $T \rightarrow \gamma^* \rightarrow q\bar{q}$ two-jet events. Figures 7a and 7b show the structures we expect in $\langle 1-T \rangle$ and $\langle S \rangle$ as the centre-of-mass energy Q passes narrow resonances and a heavy quark threshold. Resonance cascade decays such as $T' \rightarrow \gamma + (2^{++}, 1^{++}, 0^{++})$ are expected in QCD to yield two-jet hadronic final states. We calculate their angular distributions and correlations (see the Table), which serve as tests of the QCD hypotheses $2^{++}, 0^{++}, 0^{-+} \rightarrow 2 \text{ gluons}$, $1^{++} \rightarrow q\bar{q} + \text{soft gluon}$ ²⁶⁾. These distributions also test the spin 1 nature of the gluon, discriminate against $q\bar{q}$ decay models for the intermediate states, and may be useful analyzers of the spins of future even charge conjugation states.

Section 4 summarizes our conclusions.

2. - PREDICTIONS FOR JETS IN THE "CONTINUUM" IN e^+e^- ANNIHILATION

We define as "continuum" any region of Q^2 where resonances and/or new flavour thresholds are not prominent. This section is divided into four parts. In the first we review ^{6),13)} and develop plausibility arguments why asymptotically free perturbation theory may be used to calculate jet cross-sections. In the second part we review ^{19),20)} and develop arguments for the choice of "infra-red-safe" variables and observables whose perturbative calculation is not immediately threatened by infra-red catastrophes or non-perturbative sensitivities. The third and fourth parts of this section are devoted to actual infra-red safe perturbative QCD predictions and to the study of ways to search for three-jet events.

2.1. - Plausibility arguments

Since QCD is a renormalizable field theory, any Green's function (or quantity defined in terms of it) will, at least when evaluated in perturbation theory, satisfy a renormalization group equation. The standard solution of the renormalization group equation in terms of a running coupling constant α_s and effective quark masses m_f takes care of the ultra-violet logarithms that might in principle have ruined perturbation theory as a useful tool. In practice this may not be enough. Infra-red logarithms may occur in the calculation of certain quantities, perhaps making perturbation theory useless in the limit $m_f \rightarrow 0$ ⁸⁾ or $Q^2 - 4m_f^2 \rightarrow 0$ (close to new flavour thresholds) ⁷⁾. Renormalization group-improved asymptotically free perturbation theory should, however, be a sensible calculational tool at sufficiently large Q^2 for quantities that are free of these "infra-red" singularities.

One example of such a quantity is the total cross-section $\sigma(e^+e^- \rightarrow \gamma^* \rightarrow \text{hadrons})$. To leading non-trivial order in QCD (see Fig. 1), the result is well known ⁷⁾

$$R = \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} \simeq 3 \sum_{i=1}^N Q_i^2 \left(1 + \frac{\alpha_s(Q^2)}{\pi} + \dots \right) \quad (2.1)$$

$$\alpha_s(Q^2) \equiv \frac{g_s^2}{4\pi} \simeq \frac{12\pi}{(33-2N)\ln\frac{Q^2}{\Lambda^2}} \quad (2.2)$$

with N the number of quark flavours. The only dimensionful quantity is Λ which determines the coupling strength at a given Q^2 and is experimentally determined to be $\Lambda \simeq 500 \text{ MeV}$ ¹¹⁾.

There has been a recent flurry of activity ^{6),13)-15),19),20),22)} in calculating hadron jet cross-sections to the same order of perturbation theory. The underlying idea ^{13),27)} is that hard non-collinear gluon bremsstrahlung will result in three jet events, while to this order the rest of the cross-section will consist in (suitably defined) two-jet events. Let $x_i = 2E_i/Q$, $i=1,2$ be the scaled energies of the two quarks in Fig. 1b. Let p_1 (p_2) be the quark (antiquark) momentum and let k be the gluon momentum. The differential cross-section for this process is ^{13),19),20),22)}

$$\frac{1}{\sigma_T} \frac{d^2\sigma}{dx_1 dx_2} \simeq \frac{2\alpha_s(Q^2)}{3\pi} \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)} = \frac{2\alpha_s(Q^2)}{3\pi} \frac{[Q^2 - p_1 \cdot k]^2 + [Q^2 - p_2 \cdot k]^2}{(p_1 \cdot k)(p_2 \cdot k)} \quad (2.3)$$

where σ_T is the total zeroth order cross-section, and the normal to the plane defined by the three final state particles has an angular distribution $\propto (3 - \cos^2\theta)$ relative to the $e^\pm$ beam axes. The infra-red singularity in the integrated version of Eq. (2.3) cancels, exactly as in QED, against the infra-red singularity in the interference between Figs. 1a and 1c. In fact, the perturbative expression for $d\sigma/dx_1 dx_2$ becomes inoperative unless $\alpha_s/(1-x_1)(1-x_2)\pi \ll 1$. The second form of Eq. (2.3), in terms of products of four momenta, makes it clear that the infra-red problem is intimately related to the problem of dealing with a Green's function that is not deep "Minkowskian" [we could call the process deep Minkowskian if $p_1 \cdot p_2$, $p_1 \cdot k \sim O(Q^2) \gg \Lambda^2$]. In QED there is a well-known standard technique ⁵⁾ to make sense of a perturbative infra-red sensitive differential cross-section like Eq. (2.3). It has been refined and applied to the lowest non-trivial order in QCD by Sterman and Weinberg ⁶⁾. To this order, the technique amounts to computing the cross-section for all but a fraction $\epsilon \ll 1$ of the energy (this eliminates the infra-red sensitivity to soft gluons, $k \rightarrow 0$) to flow into two half-cones of opening angle $\delta \ll 1$ (to eliminate

the infra-red sensitivity to hard collinear gluons, $p \cdot k \rightarrow 0$). The two-jet cross-section so computed from Eq. (2.3) then contains terms like $\alpha_s \ln \delta \ln \epsilon$, and at any given energy δ and ϵ must be chosen large enough for perturbation theory to make sense to a given fixed order⁶⁾. Proving that this prescription can be extended to give finite multi-jet cross-sections needs a strong version of the Kinoshita-Lee-Nauenberg (KLN) theorem⁵⁾ in QCD perturbation theory and a demonstration that the perturbation theory results are not destroyed by non-perturbative effects.

The latter demonstration is beyond present theoretical lore and we must determine empirically whether the smearing width required to remove non-perturbative sensitivity is small enough to allow for useful tests of perturbative QCD. However, we have a heuristic argument in favour of a strong version of the KLN⁵⁾ theorem.

Consider the cross-section for events in which all but a fraction $\epsilon_0 \ll 1$ of the total energy is distributed among n segments of solid angle $\delta_i \ll 1$, each segment containing a finite fraction of the total energy with fractional energy resolution $\Delta x_i \ll 1$. This is a possible definition of a multi-jet cross-section, and it will obey a renormalization group equation. It will not involve the anomalous dimensions of quark or gluon fields because they are gauge-dependent. It will therefore be expressible as a power series in $\alpha_s(Q^2)$. Furthermore it is a positive semi-definite quantity which is no larger than the total cross-section. But the total cross-section is infra-red finite⁸⁾, implying that the n jet cross-section is also infra-red finite, and hence free of any infra-red singularities. Therefore it should be reliably calculable in asymptotically free perturbation theory. This argument is weak because, while the infra-red finiteness of σ_{tot} has been proved order by order in perturbation theory, the positivity of cross-sections does not hold at finite orders in perturbation theory. Nevertheless it seems very plausible²⁸⁾ that the appropriate KLN theorem exists. The greatest progress towards showing the calculability of jet cross-sections in perturbation theory so far seems to be that reported in Ref. 15).

Our detailed analysis of jet cross-sections will involve the implicit assumption, reminiscent of the parton model¹⁷⁾, that non-perturbative effects are characterized by a fixed finite $\langle p_\perp \rangle \simeq 300$ MeV, so that events with much larger $p_\perp$ are resolvable into multi-jets whose distributions reflect those of the final state quanta of QCD perturbation theory. Since

the QCD perturbative $\langle p_{\perp} \rangle$ of the final state quanta grows like $Q/\ln Q^2$, the non-perturbative $\langle p_{\perp} \rangle$ assumed inherent to each jet becomes asymptotically negligible with respect to the relative $p_{\perp}$ of the jets. However, until we know how to handle non-perturbative effects, a genuine QCD "derivation" of jet cross-sections is not possible ¹⁰⁾.

2.2 - Infra-red sensitive and insensitive variables

In Section 1 we defined the sphericity variable $\hat{S}$ ¹⁸⁾ which has been used historically ¹⁶⁾ to search for e^+e^- jet axes and to provide a rough description of how "jetty" events are. We also stated that sphericity distributions $d\sigma/d\hat{S}$ or averages $\langle \hat{S}(Q^2) \rangle$ are not reliably calculable in QCD perturbation theory because infra-red singularities do not in general cancel ^{19),20)}. The basic reason is that $\Sigma |p^i|^2$ has a different value for two massless particles with parallel momenta from what it has for one particle with the sum of the two momenta. Squares of momenta not being additive, there is little chance for the infra-red singularities in sphericity to cancel between the square of the diagram in Fig. 1b and the interference between the diagrams in Fig. 1a and Fig. 1c. Farhi ²⁰⁾ and Georgi and Machacek ¹⁹⁾ have proposed the thrust T and spherocity S variables defined in Section 1, which avoid the obvious infra-red catastrophes. Thrust is completely linear in the momenta. Spherocity is not linear for general momenta, but is linear when the particles are moving parallel, which is where the infra-red problems (for $p \neq 0$) might arise. We therefore believe that T and S are infra-red insensitive variables to all orders of perturbation theory. They are also clearly independent though strongly correlated. In general

$$1 \geq T \geq \frac{1}{2} \quad ; \quad 0 \leq S \leq 1 \quad (2.4)$$

where the left-hand inequalities come from two-jet events, the right-hand ones from spherical distributions. For the three quantum final states $(q\bar{q}g, ggg)$ with $x_1 + x_2 + x_3 = 2 : x_i \equiv 2k_i/Q$ which concern us here :

$$\begin{aligned} T &= \max(x_1, x_2, x_3) \\ S &= \frac{64}{\pi^2} (1-x_1)(1-x_2)(1-x_3) / T^2 \end{aligned} \quad (2.5)$$

and we have the bounds

$$1 > T > \frac{2}{3} ; \quad 0 \leq S \leq 0.5404$$

$$\frac{64}{\pi^2} \frac{(1-T)(2T-1)}{T^2} \leq S \leq \frac{16}{\pi^2} (1-T) \quad (2.6)$$

In general the thrust and sphericity axes need not be identical. However, they clearly coincide for three-quantum final states, both aligning with the momentum of the most energetic quantum. Therefore the angle between the thrust and sphericity axes will be $O(\alpha_s/\pi)^2$ + non-perturbative effects. We should also point out that in general neither the thrust nor the sphericity axis will lie in the plane which minimizes the acoplanarity defined in Section 1, both being out of it by an angle given by $O(\alpha_s/\pi)^2$ or non-perturbative effects. It is possible to define variables very similar to thrust, sphericity and acoplanarity for which the corresponding axes automatically coincide ^{*)}. We refrain from introducing them here so as not to encumber the literature with yet another set of definitions. For the final states with up to three QCD quanta ($q\bar{q}g$ or ggg) that we will consider, knowledge of S and T is equivalent to the knowledge of x_1 , x_2 and x_3 , the energies of the quanta. One may thus wonder why we do not directly give results in terms of the latter variables. One reason is that the distributions in S or T due to non-perturbative effects make sense, while the corresponding distributions in x_i , $i=1,2,3$, would be somewhat contrived. Another reason is that for events that are not obviously three-jet-like the measurement of x_i is inefficient.

The linearity in the particle momenta of the acoplanarity A defined in Eq. (1.5) indicate that it should be infra-red safe, like S and T .

^{*)} Let thrust be defined as usual. To first order in QCD events become planar. This suggests the definition of an "oblateness" in terms of the maximum momentum orthogonal to the thrust axis. Oblateness and sphericity coincide to $O(\alpha_s)$ in QCD. The acoplanarity may be defined in terms of the momenta orthogonal to the thrust and oblateness axis. This definition of acoplanarity coincides with ours to $O(\alpha_s^2)$ in QCD.

2.3. - Differential cross-sections for sphericity and thrust

In this section we first give a set of perturbative QCD results for differential and doubly differential cross-sections in sphericity and thrust, as well the average values of these quantities. These results can only be compared with experiment after they are "smoothed" to eliminate their infra-red sensitivity. We discuss this smoothing procedure in the second part of the section. At the end of the section, we briefly discuss the effects of new flavour thresholds on the average values of sphericity, acoplanarity and thrust.

Let σ_{tot} be the total $e^+e^- \rightarrow \gamma^* \rightarrow \text{hadrons}$ cross-section to leading order in QCD. The differential cross-section for the process $e^+e^- \rightarrow q\bar{q}$ is

$$\frac{1}{\sigma_T} \frac{d^2\sigma(q\bar{q})}{dx_1 dx_2} = \delta(1-x_1)\delta(1-x_2) \left(1 + O\left(\frac{\alpha_s}{\pi}\right)\right)$$

and the differential cross-section for $e^+e^- \rightarrow q\bar{q}g$ was already quoted in Eq. (2.3). The transformation from (x_1, x_2) to (S, T) is given by (2.5) so that if $x_1 > x_2 > x_3$

$$dx_1 dx_2 = \frac{\pi^2 T}{64(1-T)(1-\pi^2 S/16(1-T))^{1/2}} dT dS \quad (2.7)$$

and similarly for other orderings of the x_i . The double differential cross-section (2.3) can therefore be written as

$$\begin{aligned} \frac{1}{\sigma_T} \frac{d^2\sigma(q\bar{q}g)}{dT dS} \simeq \frac{2\alpha_s}{3\pi} \left(\frac{\pi^2 T}{64(1-T)(1-\pi^2 S/16(1-T))^{1/2}} \right) \cdot \\ \left[2 \left(\frac{T^2 + x_{2+}^2}{(1-T)(1-x_{2+})} + \frac{T^2 + x_{2-}^2}{(1-T)(1-x_{2-})} \right) + \frac{(2-T-x_{2+})^2 + x_{2+}^2}{(2-T-x_{2+})(1-x_{2+})} + \right. \\ \left. + \frac{(2-T-x_{2-})^2 + x_{2-}^2}{(2-T-x_{2-})(1-x_{2-})} \right] \quad (2.8) \end{aligned}$$

where

$$x_{2\pm} = 1 - \frac{T}{2} \left(1 \pm \sqrt{1 - \pi^2 S / 16(1-T)} \right)$$

To test the sensitivity of the above results to the spin 1 nature of QCD gluons, we note that coloured scalar gluons (S) would yield ¹³⁾

$$\frac{1}{\sigma_T} \frac{d^2\sigma(q\bar{q}S)}{dTdS} \approx \frac{\tilde{\alpha}_S}{3\pi} \frac{x_3^2}{(1-x_1)(1-x_2)} \quad (2.9)$$

corresponding to Eq. (2.3), whereas

$$\frac{1}{\sigma_T} \frac{d^2\sigma(q\bar{q}S)}{dTdS} \approx \frac{\tilde{\alpha}_S}{3\pi} \left(\frac{\pi^2 T}{64(1-T)(1 - \pi^2 S / 16(1-T))^{1/2}} \right) \cdot$$

$$\left[2 \left(\frac{(2-T-x_{2+})^2}{(1-T)(1-x_{2+})} + \frac{(2-T-x_{2-})^2}{(1-T)(1-x_{2-})} \right) + \frac{T^2}{(T+x_{2+}-1)(1-x_{2+})} \right. \\ \left. + \frac{T^2}{(T+x_{2-}-1)(1-x_{2-})} \right] \quad (2.10)$$

corresponds to Eq. (2.8). These differential cross-sections in T and S may be directly comparable with experiment even if Q is not large enough for the three-jet structure suggested by (2.3) and (2.9) to be apparent in the data. However, the double differential cross-sections (2.8) and (2.10) may be too differential to be usefully compared with low statistics data. We therefore note that

$$\frac{1}{\sigma_T} \frac{d\sigma(q\bar{q}g)}{dT} \approx \frac{2\alpha_S}{3\pi} \left[\frac{2(3T^2-3T+2)}{T(1-T)} \ln \frac{2T-1}{1-T} - \frac{3(3T-2)(2-T)}{(1-T)} \right] \quad (2.11)$$

and

$$\frac{1}{\sigma_T} \frac{d\sigma(q\bar{q}S)}{dT} \approx \frac{\tilde{\alpha}_S}{3\pi} \left[2 \ln \frac{2T-1}{1-T} + \frac{(4-3T)(3T-2)}{1-T} \right] \quad (2.12)$$

The corresponding analytical expressions for $d\sigma/dS$ are cumbersome and we do not give them here. At the end of this section we plot and interpret all these cross-sections and discuss the "non-perturbative" effects. Before that, for completeness and comparison with earlier results, we discuss the average values of sphericity and thrust in perturbation theory. The average values of S at fixed T are

$$\langle S(T) \rangle_{q\bar{q}g} \approx \frac{32}{\pi^2} \frac{(3T-2)(1-T)(-3T^3+18T^2-20T+8)}{T \left[2(3T^2-3T+2) \ln \frac{2T-1}{1-T} + 3(3T-2)(T-2)T \right]} \quad (2.13)$$

and

$$\langle S(T) \rangle_{q\bar{q}S} \approx \frac{32}{3\pi^2} \frac{(3T-2)(1-T)(9T^3-36T^2+36T-8)}{T^2 \left[2(1-T) \ln \frac{2T-1}{1-T} + (3T-2)(4-3T) \right]} \quad (2.14)$$

The distributions for these quantities are barely distinguishable, $\langle S(T) \rangle$ being severely constrained by the inequality Eq. (2.6). We have therefore not plotted (2.13) and (2.16).

Finally, the above formulae can be integrated up to yield $\langle 1-T \rangle$ and $\langle S \rangle$, for comparison between vector and scalar gluon results, and with formulae given previously in the literature. We find

$$\langle 1-T \rangle_{q\bar{q}g} \approx \frac{2\alpha_s}{3\pi} \left[-\frac{3}{4} \ln 3 - \frac{1}{18} + 4 \int_{2/3}^1 \frac{dT}{T} \ln \frac{2T-1}{1-T} \right] \approx 1.57 \left(\frac{2\alpha_s}{3\pi} \right) \quad (2.15)$$

which disagrees with the previous calculation²⁰⁾ of this quantity. For comparison, we also find for scalar gluons

$$\langle 1-T \rangle_{q\bar{q}S} \approx \frac{\tilde{\alpha}_s}{3\pi} \left(\frac{1}{18} + \frac{1}{4} \ln 3 \right) \approx 0.165 \left(\frac{2\tilde{\alpha}_s}{3\pi} \right) \quad (2.16)$$

For sphericity, we have

$$\begin{aligned}\langle S \rangle_{q\bar{q}g} &\approx \frac{64}{\pi^2} \frac{\alpha_s}{3\pi} \left(-\frac{229}{9} + 64 \ln \frac{3}{2} \right) \\ &\approx \frac{\alpha_s}{3\pi} (3.28)\end{aligned}\tag{2.17}$$

which is in accord with the previous calculation ¹⁹⁾ of this quantity. For scalar gluons one would obtain :

$$\begin{aligned}\langle S \rangle_{q\bar{q}s} &\approx \frac{\tilde{\alpha}_s}{3\pi} \frac{64}{\pi^2} \left(\frac{59}{9} - 16 \ln \frac{3}{2} \right) \\ &\approx \frac{\tilde{\alpha}_s}{3\pi} (0.442)\end{aligned}\tag{2.18}$$

We might suspect that in QCD, a vector gluon theory, $\alpha_s \approx (0.1 \text{ to } 0.3)$ at soon to be accessible ranges of Q^2 ($10 \text{ GeV}^2 < Q^2 < 1000 \text{ GeV}^2$). However, we would have no idea what value of $\tilde{\alpha}_s$ to expect if strong interactions were described by a scalar gluon theory. It may therefore be useful to quote

$$\frac{\langle 1-T \rangle}{\langle S \rangle} = \begin{cases} 0.75 & \text{for scalar gluons} \\ 0.96 & \text{for vector gluons} \end{cases}\tag{2.19}$$

The perturbative QCD predictions for differential cross-sections cannot be directly compared with the data, because of infra-red and non-perturbative effects. For the sake of definiteness, let us phrase the discussion in terms of $d\sigma/dT$. The prediction to leading order in QCD ($e^+e^- \rightarrow q\bar{q}$) is $d\sigma/dT \sim \delta(1-T)$. To the next order the T distribution acquires a tail Eq. (2.13). Much as in QED, the $T \rightarrow 1$ infra-red singularity of Eq. (2.13) cancels against the infra-red singularity in $e^+e^- \rightarrow q\bar{q}$ to $O(\alpha_s)$. But the perturbative result Eq. (2.13) is still unreliable for $T \rightarrow 1$, the effective expansion parameter being large in this limit. The technique to give a meaning to the perturbative calculation

is well known from QED. It suffices to average the results over resolution "bins" of width $\Delta T(\text{pert.})$ large enough for the perturbative result Eq. (2.11) to be justifiable for $T > \Delta T(\text{pert.}) \sim \alpha_s/\pi \sim (\log Q)^{-1}$. The analogous procedure in QCD may or may not be enough, depending on whether or not infra-red properties of QCD are analogous to the ones of QED. There is at least one aspect of QCD - confinement - that is not shared by QED. To learn (hopefully) how to eliminate the sensitivity of the perturbative QCD calculation to these non-perturbative effects we must at present invoke experiment to some extent. The point is that even the quarks of the leading $e^+e^- \rightarrow q\bar{q}$ amplitude come out as jets of particles with non-zero p_T ($T < 1$, $S > 0$). Quark confinement thus naturally limits the "angular resolution" of "quark detectors". Thus, to give a meaning to the perturbative QCD calculation, we must also average the results over a resolution $\Delta T(\text{non-pert.})$ larger than the uncertainty induced by the process that converts the QCD quanta into real colourless particles. To account for this process we study the $d\sigma/dT$ and $d\sigma/dS$ distribution in a model where the quarks of the leading $e^+e^- \rightarrow q\bar{q}$ amplitude turn into jets of particles with an exponentially damped p_T distribution, and a logarithmically rising multiplicity $n(Q)$. This smoothes the leading $\delta(1-T)$ or $\delta(S)$ prediction into a $d\sigma/dT$ or $d\sigma/dS$ distribution with characteristic widths :

$$\begin{aligned}\Delta T(\text{Non-Pert.}) &\approx \frac{1}{2} \langle n(Q) \rangle \frac{\langle p_T \rangle}{Q} \sim \frac{\ln Q}{Q} \\ \Delta S(\text{Non-Pert.}) &\approx \left(\frac{4}{\pi}\right)^2 \frac{\langle p_T \rangle^2}{Q^2} \langle n(Q) \rangle^2 \sim \frac{\ln^2 Q}{Q^2}\end{aligned}\tag{2.20}$$

As Q increases the widths become narrower and narrower and the perturbative QCD tail of Eq. (2.11) emerges from under the "background" of $S > 0$, $T < 1$ "non-perturbative" events. As we discuss in detail below, the results are to be interpreted as tests of perturbative QCD only when averaged over bins larger than both $\Delta T(\text{pert.})$ (to have a sensible perturbation theory) and $\Delta T(\text{non-pert.})$ (to eliminate the sensitivity to the details of non-perturbative effects). The second of these "averaging" procedures is analogous to the average over "onium" resonances discussed around Eq. (1.1) for the annihilation cross-section.

The "non-perturbative" jet model we have used in presenting results for $d\sigma/dT$ and $d\sigma/dS$ is the following. The inclusive distribution for hadrons in a leading order $q\bar{q}$ jet is assumed to be of the form ¹⁶⁾

$$\frac{d\sigma}{dz dp_T^2} \sim f(z) \exp\{-4p_T^2 \text{GeV}^{-2}\} ; \quad z = \frac{E_h}{E_{\text{jet}}} \quad (2.21)$$

where $f(z)$ was chosen to fit the SLAC inclusive hadron data ¹⁶⁾.

Our results for $d\sigma/dT$ and $d\sigma/dS$ at different e^+e^- c.m.s. energies are presented in Figs. 3 and 4, respectively. The curves labelled $(q\bar{q})_{\text{NP}}$ correspond to the "non-perturbative" jet model just discussed. The curves labelled $q\bar{q}g$ are the perturbative QCD prediction. To be interpreted as tests of perturbative QCD, the results shown in Fig. 3 are to be "binned" over the larger of a non-perturbative width $(\Delta S)_{\text{NP}}$ and an infra-red perturbative width $(\Delta S)_{\text{QCD}}$. These widths are Q^2 dependent and shown in the figure. At $Q \lesssim 18$ GeV, $(\Delta S)_{\text{NP}} > (\Delta S)_{\text{QCD}}$. Above $Q \simeq 18$ GeV it is the other way around. Similar considerations apply to $(\Delta T)_{\text{NP}}$ and $(\Delta T)_{\text{QCD}}$, see Fig. 4. The figures show that for $Q \gtrsim 10$ GeV the QCD "tail" will be observable. In a scalar gluon theory the perturbative results are similar in shape to the ones shown in these figures and their normalization is arbitrary. Thus these differential cross-sections are not good tests of the $J^P = 1^-$ nature of the gluons.

We have not worked out the QCD predictions for acoplanarity distributions. For this, we have a better excuse than their considerable technical complication. We estimate the non-perturbative background to the average acoplanarity to be

$$\langle A(\text{Non-Pert}) \rangle \approx \frac{16}{\pi^2} \langle n(Q) \rangle^2 \frac{\langle p_T^2 \rangle}{Q^2} \quad (2.22)$$

This is expected to be larger than the $O(\alpha_s^2)$ perturbative acoplanarity for foreseeable values of e^+e^- colliding beam energies.

We now turn to the discussion of new flavour thresholds. Another possible source of events with $T < 1$ or $S > 0$ is heavy quark-antiquark pair production just above threshold. Very close to threshold one would expect such events to be almost spherical ²⁹⁾, because the heavy $Q\bar{Q}$ pair would give heavy mesons $(H, \bar{H})$ essentially at rest, each of which, if sufficiently heavy, is expected to decay weakly into three-jet final states :

$$\begin{array}{l}
 e^+e^- \rightarrow Q \bar{Q} \\
 \quad \quad \quad \swarrow \searrow \\
 \quad \quad \quad \bar{H} + n\pi, \gamma \\
 \quad \quad \quad H + n\pi, \gamma \\
 \\
 H = Q\bar{q} \rightarrow q\bar{q}q\bar{q}
 \end{array}
 \tag{2.23}$$

At foreseeable energies, such a sequence of events would probably yield nearly spherical final states, with the six jets not easily discernible. Thus, close to a new flavour threshold

$$\langle T \rangle_{Q\bar{Q}} \sim \frac{1}{2} , \quad \langle S \rangle_{Q\bar{Q}} \sim 1 , \quad \langle A \rangle_{Q\bar{Q}} \sim 1
 \tag{2.24}$$

Above threshold the Q and $\bar{Q}$, and hence H and $\bar{H}$, will have non-zero momentum in the embryo jet direction. But the large $\langle p_T \rangle$ of hadrons due to the weak decay and transverse to the embryo jet axis should be largely unchanged. We therefore expect

$$\langle S \rangle_{Q\bar{Q}} \approx \frac{4m_Q^2}{Q^2} , \quad \langle A \rangle_{Q\bar{Q}} \approx \frac{4m_Q^2}{Q^2}
 \tag{2.25}$$

These expectations were borne out by calculations in a simplified model in which the heavy quarks decayed into three quark jets with energy distributions given by a four-fermion (V-A) interaction. Discussion of $\langle 1 - T \rangle$ is more delicate because of the longitudinal boost required to get from threshold to a moving quark. However, we anticipate behaviour analogous to (2.24)

$$\langle T \rangle_{Q\bar{Q}} \sim 1 - \frac{2m_Q^2}{Q^2}
 \tag{2.26}$$

The expectations (2.24) and (2.25) are plotted in Figs. 7 and 8 together with the continuum predictions of this section and the resonance predictions of Section 3. Notice that the search for jumps in $\langle 1 - T \rangle$, $\langle S \rangle$ and $\langle A \rangle$ as functions of Q may be useful in locating new flavour thresholds. The "signal to noise" ratio for a threshold is larger in $\langle A \rangle$, say, than in σ_{tot} .

2.4. - Looking for three-jet events : the pointing vector *) and the jet boost

We define as "two-jet" events at a given Q^2 those for which $(1-T)$ and S are of the order of magnitude of the corresponding "non-perturbative" average values quoted in Eqs. (2.20)

$$\begin{aligned} 1-T &\sim \Delta T (\text{Non-Pert.}) \\ S &\sim \Delta S (\text{Non-Pert.}) \end{aligned} \quad (2.27)$$

A way to eliminate these "trivial" events is to "trigger" on large $1-T$ or S relative to the above. Because the perturbative $O(\alpha_s/\pi)^2$ acoplanarity will only compete with the "non-perturbative" acoplanarity Eq. (2.22) at enormous values of Q^2 , we expect most events in the continuum to be "planar" in the sense of having $A \sim \langle A \rangle (\text{non-pert.})$. A meaningful question is then whether the distribution of hadron momentum in the event plane for events that are not two-jet events is more or less uniform or whether the hadron momenta tend to cluster so as to form three or more jet events. QCD suggests 4),6),13),14) that the latter be the case, i.e., events with low thrust should look like propellers rather than frisbees, balls, or two-jet events. We now discuss two ways of analyzing this prediction.

The pointing vector

Define the jet axis $\vec{e}$ maximizing thrust as indicated in Fig. 8. The plane normal to $\vec{e}$ divides momentum space into two hemispheres, one of which will contain a smaller amount of $\sum |\vec{p}_T^i|$. We will define as $\theta=0$ the $\vec{e}$ direction in that hemisphere as indicated in Fig. 9. Find also the plane containing $\vec{e}$ with respect to which the perpendicular momentum is minimized as indicated in Fig. 8. The angle θ shown in Fig. 9 is to be measured in this "event plane" (which in practice will coincide or nearly coincide with the acoplanarity plane, see the end of Section 2.2). To eliminate the $\theta \rightarrow -\theta$ ambiguity in the event plane we proceed as follows. Let $\vec{p}_P^i$ be the projection of a particle momentum on the event plane. We distinguish the $90^\circ < \theta < 180^\circ$ from the $180^\circ < \theta < 270^\circ$ quadrant by defining the first quadrant as that which contains more $|\sum \vec{p}_P^i|$, as indicated in Fig. 9. Thus, the $+\vec{e}$ hemisphere will contain the hardest jet-candidate, the $90^\circ < \theta < 180^\circ$ quadrant

*) We point out that our pointing vector is not Poynting's Poynting vector.

will contain the next to hardest jet-candidate and the remaining quadrant will contain the softest jet-candidate. The question is the extent to which these jet-candidates are jets and we proceed to discuss it.

Suppose one has a set of events at fixed Q . Superimpose them in such a way that their event planes, their $\theta = 0^\circ$ and clockwise θ directions coincide. In practice we will be interested in events with T fixed in some range. Define the single particle "pointing vector"

$$P(Q, T, \theta) = P \frac{d\sigma}{dT d\cos^2\theta} \quad (2.28)$$

where the right-hand side is to be measured for all particles in the set of oriented events. This is like measuring the power emitted by an antenna as a function of angle. Because of the quantum nature of the antenna at hand, we first had to orient all antenna "bips".

We proceed to discuss the pointing vector in QCD perturbation theory. The fact that P is linear in momentum gives us confidence that no infra-red sensitivity will emerge in perturbation theory. Events with T sufficiently smaller than unity consist, to leading order (α_s), of a $q\bar{q}$ pair and a hard gluon. Fixing T and the associated direction of the hardest quantum still leaves an angular range over which the other two quanta may emerge. Massless particle kinematics constrains the intermediate energy quantum to emerge in the range

$$90^\circ < \theta < 180^\circ$$

$$\frac{T}{2-T} < |\cos\theta| < \frac{4T-2-T^2}{T^2} \quad (2.29)$$

and the softest quantum to the range

$$180^\circ < \theta < 270^\circ$$

$$\frac{1-T}{T} < |\cos\theta| < \frac{T}{2-T} \quad (2.30)$$

The pointing vector within these ranges can be computed from Eq. (2.8) by a simple change of variables. It is shown in Figs. 10c, 10d and 10e, for the choices $T=0.9, 0.8, 0.7$, respectively. The figures are polar plots with the pointing vector shown as a vector of length $P(Q,T,\theta)$ pointing in the θ direction. The $O(\alpha_s)$ perturbative QCD pointing vector is Q independent.

How much of the three-jet structure of perturbative QCD may survive non-perturbative effects is a Q dependent question. To investigate it, we have converted the QCD quanta into jets with use of the limited transverse momentum model discussed in previous sections. For gluons we have used a model wherein the gluon jet multiplicity rises asymptotically to $9/4$ of the quark-jet multiplicity. The puritanic QCD view would then be to interpret our results as predictions with an angular binning larger than the non-perturbative jet width. We have chosen to present our results at a "low" energy for which the QCD jet structure will only begin to be observable: $Q=18$ GeV. The differential $d\sigma/dT$ cross-section at this energy is shown in Fig. 10a, where one can see that for $T<0.9$ the radiative QCD tail is dominant, but the corresponding relative cross-section is not large. As Q increases much more of the perturbative QCD tail emerges from under the "background" of leading order $q\bar{q}$ events. The pointing vector for these "background" events at $Q=18$ GeV, concentrated around $T=0.97$, is shown in Fig. 10b. As T decreases to 0.9 the pointing vector shows some acollinearity between the leading jets, the background contribution is still large, see Fig. 10f. By $T=0.8$ the background has disappeared and three-jet structure becomes visible (see Fig. 10g). At $T=0.7$ the cross-section is very small but the jets are very evident (see Fig. 10h). Should our expectations for the pointing vector be borne out by experiment, one could eventually decide to measure, not only T but also the corresponding scaled momentum of the particles in the ranges $60^\circ < \theta < 180^\circ$, $180^\circ < \theta < 300^\circ$. Identification of the x values as belonging to QCD quanta would allow direct verification of Eq. (2.8) for T sufficiently smaller than unity.

The jet boost

The above analysis could give strong circumstantial evidence for the expected three-jet structures, but it would not reveal whether the hadrons group themselves into jets event-by-event. To see this, suppose we have chosen events with $1-T$ and/or S large, as discussed above. The pointing vector will tell whether particles in the hemisphere $\theta < \pi/2$ have small p_T . If they do, we may ask whether the particles with $\theta > \pi/2$ come out event-by-event as two back-to-back small p_T jets in their centre-of-mass system. To get to this reference frame, one needs to boost the momenta by an amount ζ :

$$\sinh \zeta = \frac{T}{2\sqrt{1-T}} \quad , \quad \cosh \zeta = \frac{2-T}{2\sqrt{1-T}} \quad (2.31)$$

Since the two fundamental quanta not responsible for the fastest jet have a total energy $Q(1-T/2)$ and an invariant mass $Q\sqrt{1-T}$. In this reference frame the two remaining jets should emerge with finite p_T relative to axes inclined at an angle $\tilde{\theta}$:

$$\sin \tilde{\theta} = \frac{x_2 \sin \theta}{1-T/2} = \frac{\pi \sqrt{S}}{2(2-T)} \quad (2.32)$$

The angular distribution $d\sigma/d(\sin \tilde{\theta})$ for a given value of T can therefore be obtained from (2.8) or (2.10) by the substitution

$$S = \frac{4(2-T)^2}{\pi^2 \sin^2 \tilde{\theta}} \quad , \quad dS = \frac{8(2-T)^2}{\pi^2 \sin^3 \tilde{\theta}} d(\sin \tilde{\theta}) \quad (2.33)$$

Verification of the existence, after removal of the fastest jet, of finite p_T secondary jets with the angular distribution specified by (2.8), (2.33) would complete the vindication to order α_s/π of the QCD jet predictions whose plausibility was argued in Section 2.1.

3. - QCD PREDICTIONS FOR HEAVY "ONIUM" RESONANCE DECAYS

We define as "onia" the bound states of a heavy quark and its antiquark, such as charmonia ⁷⁾ and presumably the corresponding states of the newly discovered T family ³⁰⁾. This section is divided into three parts. In the first we review ^{7), 22)-26)} and develop plausibility arguments for using asymptotically free perturbation theory to calculate jet cross-sections in onium decays. In the second part we present for $J^{PC} = 1^{--}$ onium decays the distributions and mean values of the infra-red safe variables introduced in Sections 1 and 2. In the third part we discuss the angular distributions and correlations of jets in the decays of even charge conjugation onia ($J^{PC} = 2^{++}, 1^{++}, 0^{++}, 0^{-+}$), and the possibilities for determining the spin of the gluon and/or the onia themselves.

3.1. - Plausibility arguments for QCD calculations on resonance

Habit has replaced rigour in the QCD ansätze for the decays of bound states of heavy quarks. The J/ψ is thought ⁷⁾ to decay dominantly into three gluons as indicated in Fig. 11a, its $J = 0$ and 2 partners dominantly into two gluons as indicated in Fig. 12, while the 1^{++} decays are more complicated ³¹⁾: see Fig. 13. The very small width of the J/ψ is thus "explained" in QCD as a consequence of the smallness of α_s^3 ($Q^2 = m_{J/\psi}^2$). Now one wishes to go one step further and analyze in terms of "gluon jets" the hadronic final states ²²⁾⁻²⁶⁾ in the decays of the T and hypothetical heavier meson families. More than ever we need a justification for a positronium-like treatment of these decays. In order to offer some plausibility arguments, we first start with a QED reminder ²¹⁾.

Charge conjugation invariance implies that orthopositronium ($J^{PC} = 1^{--}$) mainly decays into three photons, while parapositronium ($J^{PC} = 0^{-+}$) mainly decays into two photons. To leading order in α , the corresponding decay rates and photon momentum distributions are factorizable into an e^+e^- "flux" - the square of the wave function at the origin $|\psi(0)|^2$ - times the perturbatively calculated cross-section $|\mathcal{M}|^2$ for e^+e^- annihilation at rest. The calculation is sensitive to non-perturbative infra-red effects only to the extent that $\psi(0)$ comes from a solution of the Coulomb binding problem which "knows about" the theory at distances much larger than $1/m_e$. On the other hand the annihilation amplitude $\mathcal{M}$ is only sensitive to short distances of order $1/m_e$.

The situation is more complicated at the next order in α ²¹⁾, but the decay rates for ortho- and parapositronium can still be written as sums of terms factorizable into an infra-red sensitive and a short-distance piece. The orthopositronium decay rate in $O(\alpha^4)$ is the sum of a piece $\propto |\psi(0)|^2$ and another piece containing the square of Bethe-Salpeter wave-function for relativistic momenta. Each of these is infra-red-sensitive in the sense described above, but each is multiplied by the relevant cross-section for e^+e^- annihilation which is a short-distance $O(1/m_e)$ phenomenon. Individual diagrams contributing to the $|\psi(0)|^2$ term are infra-red divergent, but their sum is infra-red finite as one would expect. To the order explored so far in QED, all the long-distance sensitivity of the positronium calculations is confined in the wave function $\psi(r)$. Normalized quantities like $1/T (d^2T/dx_1 dx_2)$, where the x_i are scaled photon momenta in orthopositronium decay, are infra-red insensitive and feel the theory only at short distances $O(1/m_e)$.

It is natural to conjecture that the decay rate of an "onium" state in QCD can also be written as a sum of terms, each of which is factorizable into an infra-red sensitive $|\psi|^2$ piece times an annihilation rate sensitive only to short distances $O(1/m_e)$. Just as in QED, the total decay rate should be infra-red finite, but the cancellation of infra-red singularities should already reveal in $O(\alpha_s^4)$ features which do not appear in orthopositronium decay in $O(\alpha^4)$. The non-Abelian gluon coupling allows orthonium to decay into four gluon final states as indicated in Fig. 11b. If the final state four-jet cross-section is defined with cut-offs analogous to the δ and ϵ introduced in Section 2 to discuss three-jet final states in the continuum, then it is clearly infra-red finite. Therefore the three-jet cross-section will also be finite to order α_s^4 , but this will in general involve a cancellation between virtual gluon corrections (Fig. 11c) to three-gluon final states and final states with four gluons (Fig. 11b) where two are collinear or one is soft. Therefore QCD perturbation theory on resonance imposes a similar constraint on the definition ⁵⁾ of a gluon "jet" as was found in the continuum. We also emphasize that the onium jet predictions depend on non-perturbative contributions to the p_T of hadrons in gluon jets being in some sense small, and that as in the continuum any angular definition of a jet should be ample enough to accommodate the non-perturbative p_T spread. For gluon jets, and for the reasons discussed in Section 2.3, we will assume a jet multiplicity roughly twice as large as for quark jets.

With these strictures in mind, we expect the leading order prediction for the scaled rate of ortho-onium (say T) decay into three gluons ^{22)-25),32)}

$$\frac{1}{\Gamma_{3g}} \frac{d^2\Gamma}{dx_1 dx_2} = \frac{1}{\pi^2 - 9} \left\{ \frac{(1-x_1)^2}{(x_2 x_3)^2} + \frac{(1-x_2)^2}{(x_1 x_3)^2} + \frac{(1-x_3)^2}{(x_1 x_2)^2} \right\} \quad (3.1)$$

to be testable to the extent that $\alpha_s(Q^2=m^2)/\pi \ll 1$. The normal to the event plane should have an angular distribution $\propto (3 - \cos^2\theta)$ ²⁴⁾ if the beams are unpolarized ³³⁾. Unfortunately, the J/ψ may be too light to test (3.1) : the gluon momenta are $O(1 \text{ GeV})$ and in high multiplicity decays the non-perturbative spread of the corresponding jets will suffice to erase all jet evidence. On the other hand, low multiplicity decays of the J/ψ are plentiful - but events with $N=3$, say, are somewhat statistically biased for a test of a three-jet picture. We illustrate our predictions with particles of the T family, but even cleaner tests will be possible if heavier quark-antiquark bound states are found.

3.2. - 3S_1 onium decays ($J^{PC} = 1^{--}$)

In the e^+e^- annihilation continuum one expects " $q\bar{q}$ " final states (Fig. 1a) to predominate, and hence

$$\langle 1-T \rangle = O\left(\frac{\alpha_s}{\pi}\right), \quad \langle S \rangle = O\left(\frac{\alpha_s}{\pi}\right), \quad \langle A \rangle = O\left(\frac{\alpha_s}{\pi}\right)^2 \quad (3.2)$$

whereas the three-gluon decays of 3S_1 onia such as the $J/\psi, \psi', T, T', \dots$ should give

$$\langle 1-T \rangle = O(1), \quad \langle S \rangle = O(1), \quad \langle A \rangle = O\left(\frac{\alpha_s}{\pi}\right) \quad (3.3)$$

as indicated in Fig. 11a. The double differential cross-section in T and S can be calculated from the gluon energy spectra ³²⁾ (3.1). We find

$$\frac{1}{\Gamma_{3g}} \frac{d\Gamma_{3g}}{dT dS} \simeq \frac{3\pi^2}{32(\pi^2-9)} \frac{T}{(1-T)[1-\pi^2 S/16(1-T)]}^{\frac{1}{2}} \cdot$$

$$\left[\left(\frac{(1-T)^2}{x_{2+}^2 (T+x_{2+}-1)^2} + \frac{(1-x_{2+})^2}{T^2 (T+x_{2+}-1)^2} + \frac{(T+x_{2+}-1)^2}{T^2 x_{2+}^2} \right) + (x_{2+} \leftrightarrow x_{2-}) \right] \quad (3.4)$$

where $x_{2\pm}$ have been defined in (2.8). The corresponding single differential cross-sections are

$$\frac{1}{\Gamma_{3g}} \frac{d\Gamma_{3g}}{dT} \simeq \frac{3}{\pi^2-9} \left[\frac{4(1-T)}{T^2(2-T)^3} (5T^2-12T+8) \ln \frac{2-2T}{T} + 2 \frac{(3T-2)(2-T^2)}{T^3(2-T)^2} \right] \quad (3.5)$$

which is plotted in Fig. 5, and labelled $T \rightarrow ggg$, and $(1/\Gamma_{3g})(d\Gamma_{3g}/dS)$ which is analytically cumbersome and plotted in Fig. 6 with the same label. These (energy-independent) distributions are very different from those in the continuum, see Figs. 3a and 4a ; for higher Q values the difference is much more dramatic. The average sphericity for different values of T is easily calculated to be

$$\langle S(T) \rangle_{3g} \simeq \frac{128}{\pi^2} \left\{ \frac{(1-T)(3T-2)}{6T^5(2-T)^2} [(2-T)^2(-3-14T+33T^2-15T^3)-3T^2(1-T)^2] \right.$$

$$+ \frac{(1-T)^3}{T^4(2-T)^3} [-(2-2T+T^2)T^2-(4-T)(2-T)^3] \ln \frac{2-2T}{T} \left. \right\} \cdot$$

$$\left\{ \frac{4(1-T)}{T^2(2-T)^3} (5T^2-12T+8) \ln \frac{2-2T}{T} + \frac{2(3T-2)(2-T^2)}{T^3(2-T)^2} \right\}^{-1} \quad (3.6)$$

Finally, we have, for the average thrust on resonance

$$\langle T \rangle_{3g} \approx \frac{3}{\pi^2 - 9} \left[6 \ln \frac{2}{3} - \frac{3}{2} + \frac{4\pi^2}{3} + 20 \int_0^1 dz \frac{\ln z}{2+z} \right] \approx 0.889 \quad (3.7)$$

and for the average sphericity on resonance

$$\begin{aligned} \langle S \rangle_{3g} \approx & \frac{384}{\pi^2(\pi^2-9)} \left[\frac{8277}{576} + \frac{51}{2} \ln \frac{2}{3} + \frac{61}{16} \left(\ln \frac{2}{3} \right)^2 \right. \\ & \left. + \frac{61}{8} \int_{2/3}^1 \frac{dz}{z} \ln(2-2z) + \frac{13}{8} \int_{2/3}^1 \frac{dz}{2-z} \ln(2-2z) - \frac{13}{8} \int_{2/3}^1 dz \frac{\ln z}{2-z} \right] \approx 0.134 \end{aligned} \quad (3.8)$$

which are plotted together with continuum predictions in Figs. 7 and 8, respectively.

In fact, in QCD by no means all hadronic final states at masses of 3S_1 onia will proceed via three gluons. There will be $\bar{q}q$ final states arising both from the photonic decays $^3S_1 \rightarrow \gamma^* \rightarrow \bar{q}q$ and from non-resonant e^+e^- annihilation. The second of these "backgrounds" becomes bothersome if the e^+e^- beam energy resolution is too large as compared with the width of the 3S_1 resonance. To study these backgrounds, let us define :

$$b = \frac{\Gamma(^3S_1 \rightarrow \gamma^* \rightarrow e^+e^-)}{\Gamma(^3S_1 \rightarrow 3g)} = \frac{\Gamma_{ee}}{\Gamma_{3g}} = \frac{81\pi\alpha^2 e_q^2}{10(\pi^2-9)\alpha_s^3} \quad (3.9)$$

where e_q is the charge of the quark. Then

$$\frac{\Gamma(^3S_1 \rightarrow \gamma^* \rightarrow q\bar{q})}{\Gamma_{3g}} = \frac{\Gamma_{q\bar{q}}}{\Gamma_{3g}} = R b$$

where

$$R = 3 \sum_{Q'(m_{Q'} < m_Q)} e_{Q'}^2$$

is the hadronic annihilation rate off resonance. [We neglect QCD radiative corrections to R , consistent with the neglect of $O(\alpha_s^4)$ corrections to direct onia decay.] If we assume a Gaussian energy resolution function with full width at half maximum $\Delta E \gg \Gamma(^3S_1)$, the signal to background ratio at the peak is

$$\frac{\sigma(^3S_1 \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \gamma^* \rightarrow \text{hadrons})} \equiv \frac{\sigma_R}{\sigma_{NR}} = 2 \sqrt{\frac{\ln 2}{\pi}} \frac{\Gamma_{ee}}{\Delta E} \frac{\Gamma_h}{\Gamma} \frac{9\pi}{2\alpha^2 R} \quad (3.10)$$

where

$$\Gamma_h = \Gamma_{3g} (1 + Rb)$$

is the full hadronic width, and

$$\Gamma \equiv \Gamma(^3S_1) = \Gamma_{3g} [1 + (N_L + R)b]$$

is the total onium width with N_L the number of charged lepton decay channels. Then we get a relative $\bar{q}q$ "contamination" of

$$\begin{aligned} r &= \frac{\sigma(q\bar{q})}{\sigma(3g)} \Big|_{Q=m(^3S_1)} = \frac{\Gamma_{q\bar{q}}}{\Gamma_{3g}} + \frac{\Gamma_h}{\Gamma_{3g}} \frac{\sigma_{NR}}{\sigma_R} \\ &= Rb + R \frac{\alpha^2}{9\sqrt{\pi \ln 2}} \frac{\Delta E}{\Gamma_{ee}} [1 + (N_L + R)b] \end{aligned} \quad (3.11)$$

As examples of the dilution effects of (3.11), we find from Eqs. (2.17) and (3.7) that on resonance

$$\langle 1-T \rangle_{^3S_1} = \left[0.111 + (1.57) \frac{2\alpha_s}{3\pi} r \right] \frac{1}{1+r} \quad (3.12a)$$

and from (2.17) and (3.8),

$$\langle S \rangle_{^3S_1} = \left[0.134 + (3.28) \frac{\alpha_s}{3\pi} r \right] \frac{1}{1+r} \quad (3.12b)$$

At the mass of the $T(9.4)$ we expect

$$\alpha_s(m_\pi^2) = 0.15 \text{ to } 0.26$$

where the lower value is determined from charmonium analysis and the higher one from deep inelastic scattering data. This gives

$$b = \begin{cases} 0.05 \text{ to } 0.01, & e_Q = -1/3 \\ 0.21 \text{ to } 0.04, & e_Q = 2/3 \end{cases}$$

Taking

$$R = \frac{10}{3}, \quad N_L = 3, \quad \Delta E = 10 \text{ MeV}, \quad \Gamma_{ee} = e_Q^2 (6 \text{ keV})^{34})$$

we find that

$$r = \begin{cases} 0.43 \text{ to } 0.25, & e_Q = -1/3 \\ 0.82 \text{ to } 0.20, & e_Q = 2/3 \end{cases}$$

The electronic width is in fact very model-dependent³⁵⁾. Increasing it by a factor two gives instead

$$r = \begin{cases} 0.30 \text{ to } 0.14, & e_Q = -1/3 \\ 0.76 \text{ to } 0.16, & e_Q = 2/3 \end{cases}$$

In plotting Figs. 5, 6 and 14, we have optimistically taken $r=1/6$; when the appropriate data are available the $\bar{q}q$ normalization must be scaled accordingly. However, these figures indicate that evidence for a three-gluon contribution may become apparent at the T mass with a sufficiently detailed analysis, while the mean values (3.12) will probably not show significant deviations from their values off resonance unless the beam energy resolution is very good.

Figures 7a and 7b show the quantities $\langle 1-T \rangle$ and $\langle S \rangle$ plotted as one passes through the possible sequence of narrow³⁵⁾ resonances below a new heavy quark threshold, under the assumptions

$$\Lambda = 500 \text{ MeV}$$

$$e_Q = -1/3 \tag{3.13}$$

We see that narrow resonances show up as bumps in $\langle 1 - T \rangle$ and $\langle S \rangle$ as well as in the total e^+e^- cross-section. They should also show up to some extent in $\langle A \rangle$, which in perturbation theory is $O(\alpha_s/\pi)^2$ off resonance, and $O(\alpha_s/\pi)$ on resonance.

The decays of the T offer a test ground for the analysis of jet structure in terms of the pointing vector introduced in Section 2.4. The pointing vector for the "background" events $e^+e^- \rightarrow \gamma \rightarrow q\bar{q}$ via T decay or the continuum is shown in Fig. 14b. Again we have assumed a $q\bar{q}$ to three-gluon ratio of $1/6$, which may be overoptimistic if the resolution to width ratio is too large. The differential cross-section $d\sigma/dT$ at $Q=m(T)$ is shown again in Fig. 14a for quick reference. Figures 14c,d,e show the perturbative QCD pointing vector, for $T=0.9, 0.8$ and 0.7 , respectively. Figures 14f,g,h show the corresponding pointing vector for real particles, obtained from the previous one in a fixed $\langle p_T \rangle$ model where gluon jets have about twice the multiplicity of quark jets. At $T=0.9$ (Fig. 14f) the background is important and there is no dramatic evidence for more than two jets. At $T=0.8$ (Fig. 14g), the leading jets become acollinear and a third jet tries to make itself observable. By $T=0.7$, the three-jet structure is observable (see Fig. 14h) but $d\sigma/dT$ has diminished considerably (see Fig. 14a). Jets would be much more evident if our safe assumption of multiplicities much larger for gluons than for quarks turned out to be a considerable overestimate. Again, and as usual, the predictions are to be interpreted as tests of perturbative QCD only when averaged over angular widths larger than the non-perturbative widths of the jets.

3.3. - Hadronic decays of P wave and pseudoscalar states

In this section we discuss the characteristics of processes of the type

$$e^+e^- \rightarrow \gamma' \rightarrow \gamma + P_T \quad \begin{matrix} \searrow \\ \text{hadrons} \end{matrix} \quad (3.14)$$

where P_T is a P wave member of the T family²⁸⁾. Although we are phrasing the discussion in terms of the T family³⁰⁾, the results are of course equally applicable to any heavy quark family of bound states and the accuracy of our predictions is expected to increase with increasing quark mass. To the lowest relevant order in QCD their hadronic decays are

determined by the diagrams 7), 29) of Figs. 12 and 13. The two gluon decays of the 0^{++} and 2^{++} states provide particularly simple laboratories for testing QCD. Here we will only be interested in the direction of the gluon jets, which may be defined in the usual infra-red-safe manner by measuring their thrust axis. These gluons will be so hard that we expect the practical determination of the jet axis to be very efficient. We have where possible refrained from making infra-red-sensitive statements, but we must point out that the comparison of particle content, multiplicities and transverse momenta for these allegedly gluonic jets with the same information for $e^+e^- \rightarrow q\bar{q}$ jets at the same energy will be most interesting.

While the two-body decay of the 0^{++} state is necessarily isotropic and serves only to test two-jet dominance of the final state, the 2^{++} decay distribution is sensitive to the gluon spin, as has also been discussed by Krammer and Krasemann²⁶⁾. Conversely, to the extent that QCD predictions will be borne out, the photon and jet angular distributions serve to test spin-parity assignments for the decaying states. The simplest example is the 0^{++} where in addition to an isotropic decay one predicts the familiar $1 + \cos^2\theta_{e\gamma}$ distributions for the photon relative to the beam axis in the process (3.14). Identical predictions hold for the cascade processes

$$e^+e^- \rightarrow \gamma \rightarrow \gamma + 0_r^{-+} \xrightarrow{\text{hadrons}} \quad (3.15)$$

where 0_r^{-+} denotes a pseudoscalar state for which the hadronic decays are also expected to be dominated by the two-gluon diagram of Fig. 12. The distributions for 2^{++} decay will be discussed in detail below. While our results coincide with those of Krammer and Krasemann²⁶⁾, we shall derive them in a slightly different way which displays explicitly the independence of the results on the details of the wave functions, and is more easily generalized to the case of the 1^{++} decay.

The 1^{++} decay is potentially interesting because it contains in lowest order a contribution from the three-gluon vertex of Fig. 13c which otherwise can be studied only by looking for four-jet events which appear only in order α_s corrections on a 1^{--} resonance and α_s^2 for the continuum. However, it has been argued by Barbieri et al.³¹⁾ that the decay is dominated by the diagram of Fig. 13a which diverges logarithmically in the weak binding limit ($\Delta E \ll M$) and therefore is enhanced by a factor

$O(\ln(M/\Delta E))$ relative to 13b and 13c, for which the corresponding divergences cancel. Averaging over the gluon jet direction, one obtains a prediction for the quark jet angular distribution which is again independent of the details of the wave function and distinctly different from non-QCD hypotheses such as a direct $P - \bar{q}q$ coupling. However, the relative enhancement of this diagram is sensitive to the binding energy and therefore the prediction is subject to caution. Nevertheless if detailed analysis of the 1^{++} hadronic decay turns out to be experimentally feasible, it may provide an interesting laboratory for testing QCD and/or probing onium wave functions.

In the following discussion we assume unpolarized beams ; the corresponding distribution for transversally polarized beams can be obtained using Bjorken's trick³³⁾. We use the notation $\theta_{e\gamma}$, θ_{ej} and $\theta_{\gamma j}$ to denote the angles between the electron and photon, electron and hadronic jet, and photon and jet, respectively.

2^{++} decay

We shall work in the dipole approximation for radiative decay of the T' which means we keep only terms linear in the photon energy. Then there is only one invariant amplitude which can contribute

$$\mathcal{M}(\Upsilon'(P) \rightarrow 2^{++} + \gamma(k)) \propto \eta^\mu T_{\mu\nu} F^{\nu\rho} P_\rho \quad (3.16)$$

where η is the T' polarization vector, F is the electromagnetic field tensor, T is the spin-2 polarization tensor satisfying the conditions

$$\begin{aligned} T_{\mu\nu} &= T_{\nu\mu}, \quad T^\mu{}_\mu = 0, \\ P^\mu T_{\mu\nu} &= P^\nu T_{\mu\nu} = 0 \end{aligned} \quad (3.17)$$

and $P = P_{T'} = P_{2^{++}} + O(k)$. We write the matrix element for the 2^{++} decay in the form :

$$\mathcal{M}(2^{++} \rightarrow X) = T_{\mu\nu} \mathcal{M}^{\mu\nu}$$

In the lab. frame, which in our approximation coincides with the 2^{++} rest frame, the final state distribution takes the form :

$$\begin{aligned}
 W \propto & \left| (p \cdot M \cdot k) - \frac{1}{3} (p \cdot k) \text{Tr} M \right|^2 + p^2 k^2 \left(\text{Tr} |M|^2 - \frac{1}{9} |\text{Tr} M|^2 \right) \\
 & - p^2 (k |M|^2 k) - k^2 (p |M|^2 p) + \\
 & \text{Re} \frac{2}{3} (\text{Tr} M) [p^2 (k M^* k) + k^2 (p M^* p)],
 \end{aligned} \tag{3.18}$$

where M is a 3×3 matrix, $M_{ij} = \mathcal{M}_{ij}$, and the vectors p and k represent the three-momenta of the e and γ , respectively.

There are two independent colour gauge invariant decay amplitudes which correspond to helicity 0 and 2; in the rest frame one has

$$M_{ij}^{\alpha_1 \alpha_2} = A g_i g_j / q^2 \varepsilon^{\alpha_1} \cdot \varepsilon^{\alpha_2} + B (\varepsilon_i^{\alpha_1} \varepsilon_j^{\alpha_2} + \varepsilon_j^{\alpha_1} \varepsilon_i^{\alpha_2}) \tag{3.19}$$

where ε^α are gluon polarization vectors and g is a gluon three-momentum. In the non-relativistic bound state approximation, the relevant combination of amplitudes is determined by calculation of the diagrams of Fig. 12 and retaining the terms linear in the quark three-momentum. This gives the non-relativistic approximation to the P wave bound state. Contraction with the tensor $T^{\mu\nu}$ automatically projects out the 2^{++} state. The relevant term in the Feynman amplitude for Fig. 12 is ³⁶⁾:

$$(\varepsilon_i^{\alpha_1} \varepsilon_j^{\alpha_2} + \varepsilon_j^{\alpha_1} \varepsilon_i^{\alpha_2} + \frac{g_i g_j}{2q^2}) \bar{v}(q) \bar{q}_i \gamma_j u(q') \tag{3.20}$$

Because the quark and gluon variables factorize in this approximation, the prediction for angular distributions is independent of the details of the wave function. The result (3.20), which corresponds to a pure helicity-2 amplitude, gives $B = 2A$ in Eq. (3.19). Inserting this result in Eq. (3.18) and summing over gluon polarizations, we obtain the distribution

$$W(e^+ e^- \rightarrow \gamma' \rightarrow 2^{++}(\rightarrow jj) + \gamma) \propto (1 + \cos^2 \theta_{ej})(1 + \cos^2 \theta_{\gamma j}) \tag{3.21}$$

To test how specific is the QCD prediction that gluons have spin one, we compare this distribution with the ones one would obtain in hypothetical theories where the gluons are scalar or the 2^{++} state directly couples to a quark-antiquark pair. For scalar gluons there is only one invariant coupling :

$$\mathcal{M}(2^{++} \rightarrow \text{scalar gluons}) = T_{\mu\nu} g^\mu g^\nu,$$

$$M_{ij} = g_i g_j.$$

This leads to a distribution

$$\begin{aligned} W_{\text{scalar gluons}} \propto & 4 + \cos^2 \theta_{e\gamma} - 3(\cos^2 \theta_{j\gamma} + \cos^2 \theta_{ej}) \\ & - 6 \cos \theta_{ej} \cos \theta_{e\gamma} \cos \theta_{j\gamma} + \cos^2 \theta_{ej} \cos^2 \theta_{j\gamma}. \end{aligned} \quad (3.22)$$

For a direct coupling of the 2^{++} state to quarks, there are two independent decay amplitudes :

$$\begin{aligned} \text{helicity } 0 : & \quad m_{\mu\nu} = q_\mu q_\nu A_0 \\ \text{helicity } 1 : & \quad m_{\mu\nu} = (q_\mu \gamma_\nu + \gamma_\mu q_\nu) A_1 \end{aligned}$$

where q is the relative momentum of the quarks. Summing the corresponding expression for (3.18) over quark spins gives a distribution of the form

$$\begin{aligned} W_{\text{quarks}} \propto & A_0^2 W_{\text{scalar gluons}} \\ & + A_1^2 (1 - 2 \cos^2 \theta_{ej} \cos^2 \theta_{j\gamma} + \cos \theta_{ej} \cos \theta_{j\gamma} \cos \theta_{e\gamma}). \end{aligned} \quad (3.23)$$

The qualitative difference among the different decay hypotheses is most transparent in the two "particle" correlations which are given in the Table. In particular, even though a direct coupling to quarks does not give a unique prediction, the jet angular dependence is bounded :

$$\begin{aligned} \frac{1}{\Gamma} \frac{d\Gamma}{d\cos\theta} &= (1 + \alpha_j \cos^2 \theta), \quad \theta = \theta_{ej} \text{ or } \theta_{j\gamma}, \\ -3/5 &< \alpha_j < -1/3, \end{aligned}$$

for either quarks or scalar gluons. Thus the QCD prediction $\alpha_j = +1$ is clearly distinguishable. In Fig. 15 we compare the distribution in $\cos\theta_{\gamma j}$ (or equivalently $\cos\theta_{ej}$) for different hypotheses and different values $\cos\theta_{e\gamma}$. Notice how sensitive the results are to the QCD assumption that gluons have spin 1.

1^{++} decay

We proceed as before by first writing down the invariant amplitude for T' decay, which is uniquely determined in the dipole approximation :

$$\mathcal{M}(T' \rightarrow 1^{++} + \gamma) \propto \eta_\mu(T') \eta_\nu(1^{++}) \tilde{F}^{\mu\nu}, \quad (3.24a)$$

where the η 's are polarization vectors and $\tilde{F}$ is the dual electromagnetic field tensor. Then if the 1^{++} decay matrix element is

$$\mathcal{M}(1^{++} \rightarrow X) \propto \eta_\mu(1^{++}) M^\mu \quad (3.24b)$$

the final state distribution is given by

$$W \propto \epsilon_{ijk} \epsilon_{i'j'k'} M_k M_{k'}^* (k^2 \delta_{ii'} - k_i k_{i'}) (p_j p_{j'} - \delta_{jj'} p^2) \quad (3.25)$$

where p and k are again the electron and the photon three-momenta.

A difficulty arises, however, when we attempt to extract a QCD prediction ³¹⁾ from the graph of Fig. 13a. If we keep the quarks on their mass shell the amplitude diverges as $1/k_g$, where k_g is the gluon momentum. This violates the QCD analogue to Low's theorem which states that the zero photon energy limit of a radiative transition, $A \rightarrow B + \gamma$, is uniquely determined from the on-shell, non-radiative transition $A \rightarrow B$. Since the $1^{++} \rightarrow \bar{q}q$ transition via a single gluon is forbidden (by charge conjugation as well as by colour) the amplitude for $1^{++} \rightarrow \bar{q}q + \text{gluon}$ must in fact vanish for $k_g \rightarrow 0$. If one takes into account the binding energy which forces the quarks slightly off shell the amplitude no longer diverges

$(\mathcal{M} \rightarrow (k_g + \Delta E)^{-1})$, but the Born term by itself is no longer gauge invariant. There must be an additional contribution to the amplitude which restores gauge invariance. This term is not uniquely specified and lies outside our simple approximation. We could adopt one of the two points of view. Suppose the bound quarks are sufficiently massive that we believe the Coulomb-like potential is completely dominant over the confining potential, so that we may take the positronium analogy literally. Then we would argue that corrections to the contribution of Fig. 13a are of order α_s as illustrated in Fig. 16. Since the leading diagram of Fig. 13a violates gauge invariance only to $O(\Delta E) = O(\alpha_s M)$, our procedure is self-consistent if we restore gauge invariance by adding only corrections of the same order. Then the amplitude is uniquely specified up to terms of order α_s which are explicitly gauge invariant. The latter will be negligible in comparison with the dominant term. A safer procedure would be to say that, given our ignorance of the bound state potential, our approximation is valid only for configurations where each external gluon carries a momentum large compared with the binding energy - which may then be neglected. However, for the T family we expect $\Delta E \sim 1$ GeV. Since the average energy per particle in a three-body final state is 3 GeV, if we restrict ourselves to configurations with $k_g \gg 1$ GeV, we will no longer be in a region where the diagram of Fig. 13a is enhanced relative to the others. Since it is not clear how well the P_T states can be studied experimentally, it may not be worth while to do a complete analysis at present. We shall consider the first point of view as a plausible possibility ; this gives dominance of quasi-two-jet events with a characteristic angular distribution.

Enforcing gauge invariance in the way specified above yields an amplitude proportional to $k_g(k_g + \Delta E)^{-2}$ for $k_g \rightarrow 0$. If we keep only terms of lowest order in k_g^{-1} , the rest frame amplitude necessarily reduces to the form

$$\vec{M} \cdot \vec{\eta}(1^{++}) \propto \frac{k_g}{(k_g + \Delta E)^2} \left\{ \vec{\eta}(1^{++}) \cdot (\vec{\epsilon}_1 \times \vec{\epsilon}_2) + O\left(\frac{k_g}{M}\right) \right\} \quad (3.26)$$

where ϵ_i are the gluon polarization vectors. (The virtual gluon polarization vector is effectively transverse because it decays into massless quarks and is at rest in the decay frame in this approximation.) The form (3.26) is equivalent to the matrix element for $1^{++} \rightarrow 1^{--} + \gamma$. Inserting (3.26) in (3.25), summing over gluon polarization and integrating over the direction of the on-shell gluon gives the distribution

$$W(e^+e^- \rightarrow \gamma' \rightarrow 1^{++}(\rightarrow \bar{q}qg) + \gamma) \propto$$

$$2 - \cos^2 \theta_{\gamma e} + \cos \theta_{\gamma e} \cos \theta_{ej} \cos \theta_{\gamma j} , \quad (3.27)$$

where here the jet direction refers to a quark jet. We may note in passing that in this picture the branching ratio for the 1^{++} state to decay into charmed particles would be $\sim 1/4$, whereas states decaying via gluons would be expected to yield very few charmed particles in the final state.

For purposes of comparison we take as a non-QCD hypothesis the decay of the η^{+-} into one scalar and one pseudoscalar gluon

$$\mathcal{M}(1^{+-} \rightarrow 0^- + 0^+) \propto \vec{q}(1^{++}) \cdot \vec{g}$$

where $\vec{g}$ is the 0^+ direction. This gives

$$W(1^{++} \rightarrow \text{spinless gluons}) \propto 1 - \cos^2 \theta_{e\gamma}$$

$$+ 2 \cos \theta_{e\gamma} \cos \theta_{ej} \cos \theta_{\gamma j} . \quad (3.28)$$

Perhaps a more realistic alternative to QCD would be a direct coupling to light quarks. If we neglect the final state quark masses there is only one amplitude which can contribute, since for zero helicity the spin and orbital angular momentum co-ordinates along the decay axis vanish. The state $|L, L_z\rangle = |S, S_z\rangle = |1, 0\rangle$ has no component $J=1$. The only allowed decay amplitude is then

$$\mathcal{M} = \eta_{\mu}(1^{++}) \bar{\psi} \gamma_{\mu} \gamma_5 \psi ,$$

giving an angular distribution identical to that for 1^{--} decay into massless fermions :

$$W(1^{++}, \text{direct quark coupling}) \propto$$

$$1 - \cos \theta_{j\gamma} \cos \theta_{je} \cos \theta_{\gamma e} \quad (3.29)$$

Comparison of the different hypotheses for the two "particle" correlations are shown in the Table, and the distributions in $\theta_{j\gamma}$ (or equivalently θ_{je}) are shown in Fig. 17 for different values of $\theta_{e\gamma}$. We must emphasize, however, that QCD predicts only an approximate two-jet structure. The analysis proposed here should only be used for events with low sphericity and high thrust and is expected to be applicable only if the majority of the events in 1^{++} hadronic decay are of this type. Should this type of event turn out not to dominate, a more detailed study of 1^{++} final states might be warranted, as it offers the possibility of isolating the three-gluon vertex seen in Fig. 13c.

We conclude that the study of photon and hadronic jet angular distributions in the radiative decays of $\Upsilon'(1^{--})$ and $\Upsilon''(1^{--})$ (?) may be a very nice infra-red insensitive tool for checking the spin-parity assignments of the $(2^{++}, 1^{++}$ and $0)$ states, and to "measure" gluon spin. The tests are sufficiently clear-cut that they may not require prohibitive statistics.

4. - CONCLUSIONS

At present QCD is the only realistic candidate for a theory of hadrons ; yet, its fundamental quanta are not the hadrons we see. We have presented and elaborated upon arguments 4),6),13)-15) why, in spite of the above, some perturbative calculations in terms of quarks and gluons can be used as serious tests of QCD. In the present state of the QCD lore, the danger to be avoided is the sensitivity of the predictions to incompletely understood non-perturbative effects. Perturbative infra-red sensitivities are tamed as in QED 6). We argue that the device to avert the non-perturbative evil in the calculation of jet cross-sections is to average the jets over their non-perturbative width. This width is related to the process that confines colour in colourless hadrons of radius R and allegedly produces a fast falling transverse momentum distribution with $\langle p_T \rangle \sim O(R^{-1})$. Thus we propose to trade a complicated multiparticle jet for an over-all directed momentum 19),20), to be attributed to the primordial QCD quantum that is allegedly the mother of the jet. We have argued that differential cross-sections for the distributions and magnitude of these directed momenta - jet cross-sections - are neat tests of perturbative QCD.

In a sense, the leading (zeroth) order perturbative QCD prediction for jet cross-sections in e^+e^- annihilation has already been successfully tested¹⁶⁾. QCD is the only known realistic field theory for which it makes sense to say that $e^+e^- \rightarrow \gamma^* \rightarrow \text{quark-antiquark}$ is the leading order contribution to the total cross-section²⁹⁾, and that the perturbative approximation improves as Q^2 gets large. When quarks are traded for jets and the jet momentum and axis is specified by averaging the jet over its infrared width, the observed distribution of the jet axis agrees with QCD. All we are saying is that we can argue that similar and more detailed predictions concerning higher orders of perturbation theory will be testable with existing or soon to be constructed colliding beam machines. At energies $Q \gtrsim 10$ GeV, three-jet events will allegedly be detectable and serve to "discover the gluon". We have proposed detailed ways to implement the QCD predictions for these multi-jet cross-sections. Several other authors have discussed jet properties in QCD and phrased them in terms of "broad jets": jets that do not narrow up with increasing energy as fast as one would expect in a model with a fixed "non-perturbative" $\langle p_T \rangle$. To leading order in QCD, the broadening is due to events where a hard non-collinear gluon is emitted. We have described a more detailed analysis wherein the most spectacular among such events are analyzed as giving rise to two "narrow" jets, rather than one broad jet.

The study of a jet structure of the final hadrons will be most interesting in the decays of very heavy onium resonances²²⁾⁻²⁵⁾, of which the T ³⁰⁾ may be the first example. The number of jets, their momentum distribution and their correlations with γ rays and the e^+e^- axis²⁶⁾ will provide sensitive tests of the $J^P = 1^-$ quantum numbers of QCD gluons. We have argued that even on resonance, where the non-perturbative features of QCD are most prominent, predictions can be made that are tests of perturbative QCD. On onium resonances we have only considered predictions to leading order in QCD. Should these predictions turn out to be right, the analysis of their corrections to higher order in QCD will offer a laboratory where the typically non-Abelian (i.e., three-gluon) couplings of QCD will be testable. Conversely, if these expectations are borne out, analysis of hadronic final states in terms of thrust, sphericity and jets will provide an ideal tool for distinguishing between super heavy onia and the intermediate vector bosons of weak interactions; their hadronic decays will be dominated by three-jet and two-jet configurations, respectively.

We have said very little about jets just above new flavour thresholds, other than pointing out that the "new physics" events will not be jetlike. Jumps in acoplanarity may be an efficient device to detect new thresholds and to sieve the new physics events.

We have not discussed the charge or other quantum number of jets. The reason ^{6),19)} is that these are no doubt infra-red-sensitive and non-perturbative effect-sensitive observables. They are infra-red-sensitive because one may "softly" lose large amounts of charge away from the jet direction (one may not softly lose large amounts of momentum, contrariwise). The non-perturbative sensitivity of jet quantum numbers is made obvious by the example of colour.

Our analysis of jets in e^+e^- annihilation can be generalized to deep inelastic leptonproduction and several authors have worked out aspects of the jet physics in this process. The justification of "infra-red-safety" in lepton pair plus hadron jet production in hadronic collisions or in the high p_T hadronic domain, may be much more difficult. In spite of this, we believe that "jet physics" will develop as an important ingredient of hadron physics of the future. Complicated multiparticle structures as jets, when simplified and studied as single objects, will serve to study the allegedly very simple fundamental short-distance interactions between quarks and gluons.

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Decay process	$\frac{1}{W} \frac{dW}{d \cos \theta_{e\gamma}}$	$\frac{1}{W} \frac{dW}{d \cos \theta_{\gamma j}}$
$\begin{matrix} 0^{++} \\ 0^{-+} \end{matrix} \rightarrow 2 \text{ jets}$	$\frac{3}{4}(1 + \cos^2 \theta_{e\gamma})$	1
$2^{++} \rightarrow \text{vector gluons}$	$\frac{3}{2}(1 - \frac{1}{3} \cos^2 \theta_{e\gamma})$	$\frac{3}{4}(1 + \cos^2 \theta_{\gamma j})$
$2^{++} \rightarrow \text{scalar gluons}$		$\frac{5}{6}(1 - \frac{3}{5} \cos^2 \theta_{\gamma j})$
$2^{++} \rightarrow \bar{q}q$		$\frac{3}{3-\alpha}(1 - \alpha \cos^2 \theta_{\gamma j})$ $\frac{1}{3} < \alpha < \frac{3}{5}$
$1^{++} \rightarrow \bar{q}q + \text{soft gluon } ^*)$	$\frac{3}{2}(1 - \frac{1}{3} \cos^2 \theta_{e\gamma})$	$\frac{21}{22}(1 + \frac{1}{2} \cos^2 \theta_{\gamma j})$
$1^{++} \rightarrow 0^+ + 0^-$		$\frac{3}{4}(1 + \cos^2 \theta_{\gamma j})$
$1^{++} \rightarrow \bar{q}q$		$\frac{9}{8}(1 - \frac{1}{3} \cos^2 \theta_{\gamma j})$
*) Here the jet direction refers to a quark jet.		

TABLE - Two "particle" angular distributions in the processes $e^+e^- \rightarrow 1^{--} \rightarrow \gamma + [(\frac{P}{0^-}) \rightarrow \text{hadrons}]$. In all cases the distributions in $\cos \theta_{ej}$ and $\cos \theta_{\gamma j}$ are identical.

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FIGURE CAPTIONS

- Figure 1 Contributions to $e^+e^- \rightarrow \gamma^* \rightarrow \text{hadrons}$ which are :
- a) zeroth order in QCD perturbation theory,
 - b) second order and possibly yielding three-jet events,
 - c) second order and interfering with the zeroth order graph to give a finite two-jet cross-section.
- Figure 2 Qualitative illustration how acoplanarity and sphericity or thrust may be used to distinguish two-jet, frisbee- and balloon-like events.
- Figure 3 The infra-red safe cross-section $d\sigma/dT$ calculated for the e^+e^- continuum in order α_s/π , indicating hard gluon bremsstrahlung ($\bar{q}qg$) and non-perturbative two-jet contributions $(\bar{q}q)_{NP}$. We have also indicated the non-perturbative (NP) and perturbative infra-red (QCD) widths (see text).
- Figure 4 The infra-red safe cross-section $d\sigma/dS$ calculated for the e^+e^- continuum in order α_s/π , indicating hard gluon bremsstrahlung ($\bar{q}qg$) and non-perturbative two-jet contributions $(\bar{q}q)_{NP}$.
- Figure 5 Analogous to Fig. 3, but for three-gluon decays of a heavy resonance of mass of the $T(9.4)$. The non-perturbative contribution is normalized according to $\sigma(e^+e^- \rightarrow \bar{q}q)/\sigma(e^+e^- \rightarrow 3g) = 1/6$ at $Q = m_{res}$.
- Figure 6 Analogous to Fig. 4, but for three-gluon decays of a heavy resonance of mass of the $T(9.4)$ as in Fig. 5.
- Figure 7 a) Calculations of $\langle 1 - T \rangle$ for different centre-of-mass energies, including the continuum, narrow resonances and a heavy quark threshold, and
b) analogous calculations for $\langle S \rangle$.
- Figure 8 The thrust axis $\vec{e}$ and the plane orthogonal to it, indicating the specification of the event plane.
- Figure 9 The event plane, indicating the specification of θ .

- Figure 10 Pointing vector cross-sections for $q\bar{q}g$ jets and background $q\bar{q}$ jets with or without smoothing for infra-red effects. See text.
- Figure 11 Contributions to 1^{--} onium decay :
a) from three gluons in lowest order,
b) a sample four-gluon final state, and
c) a radiative correction to a three-gluon final state.
- Figure 12 Lowest order hadronic decays of 2^{++} , 0^{++} , 0^{-+} states.
- Figure 13 Lowest order hadronic decays of a 1^{++} state.
- Figure 14 Pointing vector cross-sections for the Γ three-gluon jets and continuum $q\bar{q}$ jets with or without the smoothing for infra-red effects, and assuming a $\bar{q}q$ to three-gluon ratio of $1/6$ (see text).
- Figure 15 Photon-jet angular correlations for 2^{++} decay for different values of the electron-photon angle.
a) $2^{++} \rightarrow$ vector gluons via the diagram of Fig. 12,
b) $2^{++} \rightarrow$ scalar gluons or quarks in a helicity zero state,
c) $2^{++} \rightarrow$ quarks with helicity 1.
- Figure 16 Higher order contributions to 1^{++} hadronic decays which could cancel the gauge variant piece of Fig. 13a.
- Figure 17 Photon-jet angular correlations for 1^{++} decay for different values of the electron-photon angle.
a) $1^{++} \rightarrow q\bar{q} + \text{soft gluon}$ ($\theta_{\gamma j}$ is the angle of a quark jet and the photon),
b) $1^{++} \rightarrow$ spin-zero gluons,
c) $1^{++} \rightarrow \bar{q}q$ via a direct coupling.

FIG. 1

FIG.2

FIG. 4

FIG. 3

FIG. 5

FIG.6

FIG. 7a

FIG. 7b

FIG. 8

FIG. 9

FIG. 10

(a)

(b)

(c)

FIG. 11

FIG. 12

FIG. 13

FIG. 14

Jet angular distributions : 2^{**} decays

FIG. 15

FIG.16

Jet angular distributions: 1^{++} decays

FIG. 17