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NON-PERTURBATIVE QCD EFFECTS AT LARGE MOMENTUM TRANSFERS

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ABSTRACT

We calculate the simplest one-instanton correction to the perturbative QCD prediction for e^+e^- annihilation to hadrons. At high centre-of-mass energies Q we find a contribution to the total cross-section from a simple fermion loop of the form

$$\frac{\Delta R}{R} \underset{Q^2 \rightarrow \infty}{\propto} Q^{-11-N_f/3} \begin{cases} (\ln Q^2)^{6(33-4N_r)/(33-2N_f)} \\ \text{or} \\ (\ln Q^2)^{6(33-4N_f)/(33-2N_f)-1} \end{cases}$$

where N_f is the number of quark flavours. The numerical value of this contribution is $O(1)$ for $Q \sim 1$ to 2 GeV.

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Asymptotic freedom¹⁾ enables QCD perturbation theory to make copious and precise predictions for processes involving large momentum transfers. These include the classic results for the total $e^+e^- \rightarrow \text{hadrons}$ annihilation cross-section²⁾, and for the moments of deep inelastic structure functions³⁾. It may also be that QCD perturbation theory can be used to make reliable predictions for the production of hadron jets at large transverse momenta in e^+e^- annihilation⁴⁾ and elsewhere⁵⁾. However, there are certainly non-perturbative effects in QCD which are believed to be crucial in the infra-red region where the effective coupling constant is large. One such effect is the existence of topologically non-trivial gauge field configurations. The most basic of these is the instanton⁶⁾ which has the smallest action $(8\pi^2/g^2)$ of any configuration with Pontryagin number non-zero. Since asymptotic freedom¹⁾ says $g^2(Q^2) \sim 1/\ln Q^2$ as $Q^2 \rightarrow \infty$, the importance of one-instanton configurations, which is $O(\exp - 8\pi^2/g^2)$, is expected to decrease at large momenta relative to perturbative effects. One might expect that processes with large momentum transfers Q^2 which are sensitive to short distance properties of the theory would only "see" small instantons of sizes $\rho \sim x \sim 1/Q$, and that field configurations which are topologically more complicated would be still less significant at large momenta. One might expect that there is some sort of "non-perturbative perturbation theory" at short distances.

In this paper we start a systematic study of these expectations by calculating the simplest one-instanton (+ anti-instanton) corrections⁷⁾ to the short distance quantity:

$$R \equiv \frac{\sigma(e^+e^- \rightarrow \gamma^* \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \gamma^* \rightarrow \mu^+\mu^-)}$$

at large centre-of-mass energy Q using the diagrams of Fig. 1. We find that if the number of quark flavours $N_f \neq 3$ (modulo 6)^{*)}

$$\left. \begin{aligned} \frac{\Delta R}{R} \approx_{Q^2 \rightarrow \infty} & 6\pi^{\frac{3}{2}}(0.1)(1.3)^{N_f} \left(\frac{33-2N_f}{6}\right)^6 \cos\left(\frac{\pi N_f}{6}\right) \\ & \times \frac{\Gamma\left(\frac{7}{2} + \frac{N_f}{6}\right) \left[\Gamma\left(\frac{11}{2} + \frac{N_f}{6}\right)\right]^2}{\Gamma\left(6 + \frac{N_f}{6}\right)} \\ & \times \left(\frac{\prod_{i=1}^{N_f} \bar{m}_i}{N_f!}\right) \left(\frac{Q}{\Lambda}\right)^{-11 - \frac{N_f}{3}} (\ln Q^2)^6 \left(\frac{33-4N_f}{33-2N_f}\right) \end{aligned} \right\} \quad (1)$$

*) There is a slightly different formula for $N_f = 3$ (modulo 6): see Eq. (18).

where Λ is the scale appearing in the asymptotically free coupling constant¹⁾

$$\frac{g^2(Q^2/\Lambda^2)}{4\pi} \underset{Q^2 \rightarrow \infty}{\approx} \frac{12\pi}{(33-2N_f)\ln(Q^2/\Lambda^2)} \quad (2)$$

and the $\bar{m}_i$ normalize the masses of the N_f quarks with $(\text{mass})^2 \ll Q^2$ ⁸⁾:

$$m_i(Q^2/\Lambda^2) \approx \bar{m}_i \left(\ln Q^2/\Lambda^2 \right)^{-\frac{12}{33-2N_f}} \quad (3)$$

We find a dominant contribution to R controlled by small instantons of size $\rho \sim 1/Q$ as $Q^2 \rightarrow \infty$, in contrast⁷⁾ to the real part of the hadronic vacuum polarization which "feels" large instantons: $\rho \gg 1/Q$. We should emphasize that we have not attempted to calculate more complicated diagrams (see Fig. 2 for example) in background instanton fields. Nor have we evaluated fermion loops in more complicated field configurations. Any or all of these may make contributions to R which are bigger than (1) at large Q^2 . However, obtaining analytic expressions for them and evaluating them is a formidable task, and we think it is interesting to isolate and evaluate the contribution (1); its smallness is a necessary (though perhaps not sufficient) condition for the applicability of a perturbative QCD analysis of $\sigma(e^+e^- \rightarrow \gamma^* \rightarrow \text{hadrons})$.

To calculate non-perturbative QCD effects we adopt the usual⁹⁾ WKB philosophy of summing over classical extremum solutions weighted by the exponentials of their actions, and doing "conventional" perturbation theory about each classical solution. This procedure is meaningful if the perturbation theory around each classical solution is renormalizable, and if the sum over classical solutions is finite. We have assumed that the first of these criteria can be satisfied, as has recently been formally demonstrated¹⁰⁾, and address ourselves to the sum over classical solutions. In QCD these are known¹¹⁾ to be multi-instanton configurations, whose actions may be of order 1 when the instanton size parameters ρ_i are large. Thus multi-instanton configurations would not always be minor perturbations, but it may still be possible to treat a number of physically interesting problems using a dilute gas approximation^{7),9)}. We hope, on physical grounds, that some processes involving large momenta Q should only be affected by small, calculable, instantons: $\rho \sim 1/Q$. Rather than plumb the gory depths of multi-instanton effects, we will evaluate processes calculated in one-instanton background fields. When we get answers which are finite and only sensitive to small instantons $\rho \sim 1/Q$, we will believe in "non-perturbative perturbation theory"

and infer that configurations with many large instantons or other more complicated field configurations are relatively unimportant as $Q \rightarrow \infty$. However, we should emphasize at this stage that we are not (yet?) able to estimate the non-perturbative effects of field configurations more complicated than one-instanton, and so cannot exclude the possibility that some such effects may dominate over the one-instanton contribution (1).

We start⁷⁾ with the simplest deep inelastic process: $e^+ e^- \rightarrow \gamma^* \rightarrow \text{hadrons}$. The leading asymptotic freedom prediction $R = 3 \sum_i Q_i^2$ is zeroth order in QCD perturbation theory (Fig. 1a) and evaluating the leading (?) non-perturbative correction (Fig. 1b) does not involve us in perturbation theory around an instanton¹⁰⁾, which we believe to be well-behaved (asymptotically free). Just as the basic fermion graph of Fig. 1a can be evaluated

$$\Pi_{\alpha\beta}^0(x) = -T_r \left\{ \gamma_\alpha S^0\left(\frac{x}{2}, -\frac{x}{2}\right) \gamma_\beta S^0\left(-\frac{x}{2}, \frac{x}{2}\right) \right\} \quad (4)$$

where $S^0(x, y)$ is the free fermion propagator in zero field, we see that the same diagram calculated in the background instanton field of Fig. 1b is

$$-T_r \left\{ \gamma_\alpha S^1\left(\frac{x}{2}, -\frac{x}{2}; z, \rho\right) \gamma_\beta S^1\left(-\frac{x}{2}, \frac{x}{2}; z, \rho\right) \right\} \quad (5)$$

where $S^1(x, y, z, \rho)$ is the fermion propagator¹²⁾ in the field of a background instanton centred at z_μ and of size ρ . These variables must be integrated with the known⁹⁾ weight

$$\int d^4z \int d\rho d(e) \quad (6)$$

where the instanton function $d(\rho)$ has⁹⁾ the following form for small quark masses $m_i \ll 1/\rho$ if a Pauli-Villars regularization scheme⁹⁾ is used:

$$d(e) = \frac{0.1}{\rho^5} \left(\prod_{i=1}^{N_f} m_i \right) (1.3 \rho)^{N_f} \left[\frac{8\pi^2}{g^2(\rho)} \right]^6 \exp \left[-\frac{8\pi^2}{g^2(\rho)} \right] \quad (7)$$

with $g^2(\Lambda\rho)$ given by Eq. (2) with $Q \rightarrow 1/\rho$ at small ρ , as shown by the computation⁹⁾ of one-loop corrections to the instanton contribution to the vacuum-to-vacuum amplitude $\langle 0|0 \rangle$. We believe that if full account is taken of these renormalization effects, it will probably emerge that m_i in Eq. (7) should be interpreted as the effective mass at the scale ρ of the instanton: $m_i(\Lambda\rho)$, which is approximated to leading order by Eq. (3) with $Q \rightarrow 1/\rho$ at small ρ ⁸⁾. Thus within the philosophy outlined above, the m_i should be regarded as short distance current algebra masses, rather than⁷⁾ (dynamically generated?) constituent quark masses. To evaluate the one-instanton corrections to (4) we must add the new loop diagram (5) and divide by the vacuum-to-vacuum amplitude

$$\langle 0|0 \rangle = 1 + \int d^4z \int d\rho d(e) \quad (8)$$

which has within the context of our approximation^{7),9)} the effect of subtracting the free propagator loop (4) from (5) to get

$$\pi'_{\alpha\beta}(x) = - \int d^4z \int d\rho d(e) \left\{ \text{Tr}(\gamma_\alpha S'(\frac{x}{z}, -\frac{x}{z}; z, e) \gamma_\beta S'(-\frac{x}{z}, \frac{x}{z}; z, e)) \right. \\ \left. - \text{Tr}(\gamma_\alpha S^0(\frac{x}{z}, -\frac{x}{z}) \gamma_\beta S^0(-\frac{x}{z}, \frac{x}{z})) \right\} \quad (9)$$

The one-anti-instanton contribution is essentially identical to (9), and we will not discuss it explicitly. Nor indeed will we expose in detail the evaluation of (9), because another independent calculation has recently become available⁷⁾ but we will outline a few technical points. The massless fermion propagator S_0^1 in a background instanton field is known^{12)*}, but we want to study massive fermions, albeit with $m \ll 1/\rho$. We therefore make a Laurent expansion in m :

$$S^1(x, y; z, e) = - \frac{\psi_0(x; z, e) \psi_0^\dagger(y; z, e)}{m} + S_0^1(x, y; z, e) \\ + m S_1^1(x, y; z, e) + O(m^2) \quad (10)$$

where ψ_0 is the zero mode of a massless fermion. The $O(1/m^2)$ and $O(1/m)$ terms in the expansion of $\text{Tr}(\gamma_\alpha S^1 \gamma_\beta S^1)$ have zero trace. Terms in $O(m^0)$ may arise either from $\text{Tr}(\gamma_\alpha S_0^1 \gamma_\beta S_0^1)$ or from the combination of zero modes and S_1^1 . Using the representation

$$S_1^1(x, y; z, e) = \int d^4w S_0^1(x, w; z, e) S_0^1(w, y; z, e) \quad (11)$$

*) Ref. 12) discusses propagators in an SU(2) instanton field, but the SU(3) case is trivially related by numerical factors.

the operator realization¹²⁾

$$S'_0(x, y; z, e) = \langle x | \delta \cdot D \frac{1}{-D^2} \left(\frac{1+\gamma_5}{2} \right) + \frac{1}{-D^2} \delta \cdot D \left(\frac{1-\gamma_5}{2} \right) | y \rangle \quad (12)$$

and the completeness of $\int d^4 w |w\rangle \langle w|$ it is easy to check that the only part of S'_1 which contributes to the trace in (9) is

$$\bar{S}'_1(x, y; z, e) \equiv \langle x | \frac{1}{-D^2} \left(\frac{1+\gamma_5}{2} \right) | y \rangle \quad (13)$$

Adding together the $O(m^0)$ contributions to the trace in (9) and summing instantons and anti-instantons we find the gauge invariant expression

$$\begin{aligned} \Pi'_{\alpha\beta}(x) = 2 \int d^4 z \int d\rho d(e) \left\{ \frac{3 \sum_{i=1}^{N_f} Q_i^2}{4\pi^4} \left(\frac{1}{(\rho^2 + (\frac{x}{2} - z)^2)(\rho^2 + (\frac{x}{2} - z)^2)} \right)^2 \rho^2 \right\} \\ \times \left\{ \frac{-8z^2 x_\alpha x_\beta}{x^4} + \frac{8x \cdot z (z_\alpha x_\beta + x_\alpha z_\beta - x \cdot z g_{\alpha\beta})}{x^4} \right. \\ \left. + \frac{2(x^2 g_{\alpha\beta} - x_\alpha x_\beta + 2x_\alpha z_\beta - 2x_\beta z_\alpha)}{x^2} \right\} \end{aligned} \quad (14)$$

Integrating over the instanton position z we find^{*)} that

$$\Pi'_{\alpha\alpha}(x) = \int d\rho d(e) \frac{36}{\pi^2} \frac{\rho^4}{x^2} \frac{\partial}{\partial x^2} \left[\frac{1}{x^2} \frac{1}{\sqrt{1+4\rho^2/x^2}} \ln \left(\frac{\sqrt{1+4\rho^2/x^2} + 1}{\sqrt{1+4\rho^2/x^2} - 1} \right) \right] \quad (15)$$

The Fourier transform is^{*)}

$$\tilde{\Pi}'_{\alpha\alpha}(Q) = \int d\rho d(e) \left[-\frac{12}{Q^2} + 36\rho^2 \int_0^1 du K_2 \left(\frac{2\rho Q}{\sqrt{1-u^2}} \right) \right] \quad (16)$$

*) These expressions are now agreed by the authors of Ref. 7).

We note^{*)} that the expression in brackets in Eq. (16) is regular at $Q^2 = 0$ since

$$K_2(2x) \xrightarrow{x \rightarrow 0} \frac{1}{2x^2} + \text{constant.}$$

Since a negative integer power of Q^2 has no imaginary part except at the origin, and since the singularity at the origin is cancelled, we need only consider the imaginary part of the second term in Eq. (16) for $Q^2 \neq 0$. To exhibit its Q dependence we have to scale ρ^2 as $1/Q^2$ which shows that the imaginary part of $\tilde{\Pi}'_{\alpha\alpha}$ is only sensitive to small instantons. In the leading contribution to $\tilde{\Pi}'_{\alpha\alpha}$ at large Q^2 the ρ and u integrals of Eq. (16) are proportional to the finite integrals

$$\begin{aligned} & \int_0^\infty d\sigma \sigma^{8+N_f/3} K_2(\sigma) \int_0^1 du (1-u^2)^{\frac{7}{2}+N_f/6} \\ &= \frac{2^{6+N_f/3} \Gamma\left(\frac{7}{2}+\frac{N_f}{6}\right) \left[\Gamma\left(\frac{11}{2}+\frac{N_f}{6}\right)\right]^2 \Gamma\left(\frac{1}{2}\right)}{\Gamma\left(6+\frac{N_f}{6}\right)} \end{aligned} \quad (17)$$

So far we have worked in Euclidean space, but we now assume that the naïve continuation may be made to Minkowski space at time-like Q^2 to evaluate the absorptive part. Putting all the factors together, we find that for $N_f \neq 3$ (modulo 6) the contribution dominating by $O(\ln Q^2)$ at large Q^2 in the absorptive part comes from the powers of Q^2 , and has the form (1). For $N_f = 3$ (modulo 6) the absorptive part comes from the logarithmic factors, and we have

$$\begin{aligned} \frac{\Delta R}{R} &\underset{Q^2 \rightarrow \infty}{\approx} 36 \pi^{\frac{5}{2}} (0.1) (1.3)^{N_f} \left(\frac{33-2N_f}{6}\right)^6 \left(\frac{33-4N_f}{33-2N_f}\right) \\ &\times \frac{\Gamma\left(\frac{7}{2}+\frac{N_f}{6}\right) \left[\Gamma\left(\frac{11}{2}+\frac{N_f}{6}\right)\right]^2}{\Gamma\left(6+\frac{N_f}{6}\right)} \\ &\times \left(\frac{\prod_{i=1}^{N_f} \bar{m}_i}{\prod^{N_f}}\right) \left(\frac{Q}{\Lambda}\right)^{-11-\frac{N_f}{3}} (\ln Q^2)^{6\left(\frac{33-4N_f}{33-2N_f}\right)-1} \end{aligned} \quad (18)$$

*) This property was pointed out to us by J. Finjord.

We see from Eqs. (1) and (18) that the dominant corrections to R from the fermion loop in a one-instanton field are negligible as $Q \rightarrow \infty$. This, and the fact that our answer for the absorptive part (but not the real part⁷⁾) is sensitive only to small instantons $\rho \sim 1/Q$, is consistent with the "non-perturbative perturbation theory" prescription of ignoring configurations with many large instantons. However, we cannot yet demonstrate that they do not dominate over the one-instanton results (1) and (18).

The numerical evaluation of (1) and (18) is not without interest. To estimate it we use $\Lambda = 0.3$ to 0.7 GeV and the procedure of Georgi and Politzer⁸⁾ to estimate the quark masses¹³⁾ as follows:

$$N_f = 3 : \quad \bar{m}_s : m_s(Q^2 = 1 \text{ GeV}^2) = 0.5 \text{ GeV} \quad (19a)$$

$$\bar{m}_d = \frac{1}{20} \bar{m}_s, \quad \bar{m}_u = \frac{1}{1.8} \bar{m}_d$$

$$N_f = 4 : \quad \text{as above, with } \bar{m}_c : \bar{m}_c(Q^2 = 10 \text{ GeV}^2) = 1.5 \text{ GeV} \quad (19b)$$

These result in the following coefficients multiplying the (Q/Λ) factors in Eqs. (1) and (18)

$$N_f = 3 : \quad \text{coefficient} \approx 10^5 \text{ to } 10^6, \quad (20a)$$

$$N_f = 4 : \quad \text{coefficient} \approx -(4 \times 10^4 \text{ to } 10^6) \quad (20b)$$

While these factors are enormous, they can conveniently be reabsorbed into the mass which scales Q :

$$N_f = 3 : \quad \frac{\Delta R}{R} \approx \left(\frac{Q}{1 \text{ to } 2 \text{ GeV}} \right)^{-12} (\ln Q^2)^{3.67} \quad (21a)$$

$$N_f = 4 : \quad \frac{\Delta R}{R} \approx - \left(\frac{Q}{1 \text{ to } 1.6 \text{ GeV}} \right)^{-12 \frac{1}{3}} (\ln Q^2)^{4.08} \quad (21b)$$

We close with the following comments:

- A) The non-perturbative contribution (21) to $\sigma(e^+e^- \rightarrow \text{hadrons})$ is negligible at high Q^2 , but appears like a brick wall as Q^2 is decreased. Its numerical value suggests that this wall is located between $Q = 1$ and 2 GeV, and that somewhere in this region (which is known to contain hadronic resonances) the conventional perturbative QCD analysis of $\sigma(e^+e^- \rightarrow \text{hadrons})$ should become inapplicable. However, other non-perturbative effects could restrict still further the use of QCD perturbation theory.
- B) Comparing with the analysis of Ref. 7), we see that the two terms in Eq. (16) for the real part of $\tilde{\Pi}'_{\alpha\alpha}(Q)$ both become comparable to the perturbative contribution at similar momenta $O(1.5)$ GeV. As we have found that $\text{Im } \tilde{\Pi}'_{\mu\mu}$ vanishes rapidly as $Q^2 \rightarrow \infty$, simple application of dispersion relations tells us that $\text{Re } \tilde{\Pi}'_{\mu\mu} \rightarrow \text{const}/Q^2$ for $Q^2 \rightarrow \infty$, with the constant determined by the integral over the imaginary part.
- C) The principal uncertainties in evaluating the latter come from the choice of renormalization prescription and of the appropriate scale parameter Λ . It is known that in other applications the higher order corrections in QCD perturbation theory are smaller if the Pauli-Villars renormalization scheme used here is adopted. Higher order effects are potentially more significant in these non-perturbative calculations than in conventional calculations. Also the experimental uncertainty in Λ is very crucial here, as seen directly from Eqs. (1) and (18).
- D) It is not yet possible to estimate the scale where perturbative QCD effects become dominant in other processes, but this question is under further study.

ACKNOWLEDGEMENTS

We thank V. Baluni, R. Crewther, G. Parisi, J.C. Polkinghorne, A. Schwimmer and A. Ukawa for useful discussions and N. Andrei and D.J. Gross for instructive exchanges of views. One of us (J.E.) also thanks the particle physics group in the D.A.M.T.P. Cambridge for their kind hospitality while this work was started.

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FIGURE CAPTIONS

Fig. 1 : Lowest order quark loop contributions to $e^+e^- \rightarrow \gamma^* \rightarrow \text{hadrons}$ (a) in the absence of instantons, and (b) in the presence of an instanton.

Fig. 2 : Next to lowest order contributions to $e^+e^- \rightarrow \gamma^* \rightarrow \text{hadrons}$ in perturbation theory (a) in the absence of instantons, and (b) in the presence of an instanton.

(a)

(b)

FIG.1

(a)

(b)

FIG.2