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RELATING HARD QCD PROCESSES THROUGH UNIVERSALITY
OF MASS SINGULARITIES

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A B S T R A C T

Hard QCD processes involving final jets are studied and compared by means of a simple approach to mass singularities. This is based on the Lee-Nauenberg-Kinoshita theorem and on a rather subtle use of gauge invariance in hard collinear gluon bremsstrahlung. One-loop results are easily derived for processes involving any number of initial quarks and/or currents. The method greatly simplifies the computation of higher order loops and our results, to be presented elsewhere, allow us to conclude that the crucial features encountered at the one-loop level will persist. We are thus able to relate different hard processes and to show that suitable ratios of cross-sections, being free from mass singularities, can be computed perturbatively, as usually assumed in QCD inspired parton models. We are also able to relate our universal leading mass singularities to leading scaling violations and to extend therefore the results of the operator product expansion method to processes outside the range of the light cone analysis. Some delicate points caused by confinement related singularities (e.g., narrow resonance poles) are also discussed.

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1. INTRODUCTION

The hope that QCD can represent a bona fide theory of strong interactions is nowadays widely spread. Of course, we know that this will remain only a hope until the day when we shall be able to deal qualitatively and quantitatively with quark confinement and have therefore a real handle on physical processes which involve hadrons and not quarks and gluons.

Nevertheless, if confinement is an infra-red phenomenon, it could happen that hard inclusive processes exhibiting a smooth infra-red limit could be free from confinement nuisances and therefore calculable in QCD by a perturbation expansion in the running coupling constant. This attitude leads to new stringent tests of QCD by enhancing its predictive power.

An interesting step in this direction has recently been undertaken by Sterman and Weinberg¹⁾. They noticed that, in $e^+e^- \rightarrow$ hadrons, total as well as multi-jet inclusive cross-sections are free from infra-red and mass singularities. Therefore, on the basis of the aforementioned attitude, these cross-sections can be computed perturbatively in terms of $\alpha_s(Q^2)$, the running coupling constant.

On the other hand, Politzer has noticed²⁾ that in other processes, such as deep inelastic lepton scattering or lepton pair production, mass singularities are present but their effect can be absorbed, at least in lowest order, in the proton structure function.

In the present paper we want to push still further this point of view towards the conclusion that in every hard process in which the produced hadrons have a jet-like structure, mass singularities are generated universally, and in a factorized way, by the initial particles so that they can be singled out and the remaining factors can be calculated perturbatively.

More specifically, we first study, at the one-loop level, jet inclusive processes induced by actual quarks of mass m .

In Section 2, by using well-known techniques^{3),4)} we identify the terms giving rise to $\log Q^2/m^2$ singularities as those arising from either initial or final hard gluons collinear to the incoming quarks. This already suggests that mass singularities are related only to the initial quarks, but still does not allow to reach such a conclusion. Indeed, when calculating squares of amplitudes, i.e., cross-sections, the gluon emitted (absorbed) by the quark collinear with it can be absorbed (emitted) everywhere else and still produce leading divergences. Fortunately, and thanks to gauge invariance, we are able to find (Section 3) a method

that reduces the aforementioned "interference" terms to a treatment of the diagonal case in which the gluon is again absorbed or emitted by the same initial leg. This shows explicitly that for every quark generated process the coefficient of $\log Q^2/m^2$ is the Born cross-section, in which the momentum p_i of each incoming quark is rescaled by x_i , times a universal function of $1 - x_i$, the fractions of the momenta carried away by the collinear gluon for each incoming quark. We then show that our calculations agree with $O(\alpha_s)$ scaling violation as predicted by operator product expansion techniques in deep inelastic scattering.

We can argue that the above-mentioned universality will persist to all orders in α_s for the leading $\log Q^2/m^2$ terms. Our gauge invariance trick greatly simplifies multiloop calculations whose details will be presented elsewhere.

Having isolated in a universal way mass singularities, we are able to find suitable ratios of cross-sections which are regular in m and to which we can apply perturbation theory in $\alpha_s(Q^2)$. Or, in other words, we can reduce all non-perturbative effects on quark generated processes to the quark structure function.

In Section 4 we apply the method to several processes such as multijet states in deep inelastic electron scattering on quarks, massive lepton pair production and large p_T in $q\bar{q}$ or $q\bar{q}$ scattering. Our results show universality of scaling breaking effects independently of the sign of Q^2 . This is due to the simple structure of singularities in perturbative QCD. However, if the confinement dynamics (essentially non-perturbative) should give rise as we all hope to bound states and resonances, any perturbative result as ours, could not be trusted for Q^2 too close to the real positive axis. This implies that perturbative results should be considered as averages⁵⁾. We discuss, however, how for $e^+e^- \rightarrow$ hadrons, the average is over cross-sections while in the Drell-Yan process the averaging is on amplitudes and therefore cannot faithfully represent a smoothing of actual cross-sections in a region of well-separated peaks. It could at most be correct where cross-sections are smooth or possibly represent a smooth background in a region with rapid fluctuations.

In Section 5 we show that renormalization group equations give a connection between leading $\log m^2$ and leading $\log Q^2$ scaling violations. Indeed, in our approach, leading mass singularities are universally related to structure functions whose leading $\log Q^2$ behaviour is controlled by light cone singularities. We

show the correspondence between these two approaches and check it explicitly by comparing the single-loop calculations.

Of course, physical processes are not initiated by free quarks. As usual something must be argued about quark distributions in a single hadron, property controlled by the quark confinement aspect of QCD. By making the usual assumption of softness of wave functions, we argue in Section 6 that our factorization and universality results should not be invalidated. In particular, it is easy to show with our method that mass singularities arising from a gluon collinear to one incoming hadron is unaffected by the presence of spectators arising from a different hadron. This is essential for our factorization so that again suitable ratios of cross-sections can be calculated perturbatively or, alternatively, that all non-perturbative effects can be absorbed into hadron structure functions.

2. THE LEE-NAUENBERG-KINOSHITA THEOREM FOR HARD QCD PROCESSES

It is clear from the preceding discussion that we need to develop a method for extracting the infra-red divergences from the physical processes we want to consider.

It is fortunate that the classic work of Lee, Nauenberg and Kinoshita³⁾ (LNK) provides a general approach to this aim. Their method applies to infra-red (gluon mass $\rightarrow 0$) and mass (quark mass $m \rightarrow 0$) singularities arising from radiative corrections to a process which is finite at lowest order.

Infra-red singularities are cancelled by summing over final soft gluons. The physical argument of LNK for m singularities is that they must be absent provided that the sum is extended to both final and initial states containing outgoing and incoming hard collinear gluons, respectively. These are the states which become degenerate with those of lowest order in the $m \rightarrow 0$ limit.

In discussing $e^+e^- \rightarrow$ hadrons, the sum over final soft and hard collinear gluons is enough. These states are included in the definitions of final jets of Ref. 1). Therefore multijet cross-sections in e^+e^- are expected to be finite as recently confirmed by Sterman¹⁾ through an explicit calculation. The above summation does not exhaust mass singularities for processes with initial quarks. A sum over initial collinear gluons should also be performed.

As an example, let us consider the process $\gamma^* q \rightarrow$ anything (deep inelastic scattering on a quark). At the one-loop level one has:

$$\begin{aligned} & \sigma(\gamma^* q(p) \rightarrow q(p')) + \sigma(\gamma^* q(p) \rightarrow q(p-k) + g(k))_{k \text{ soft or } \parallel p'}^+ \\ & + \sigma(\gamma^* q(p-k) + g(k) \rightarrow q(p'))_{k \text{ hard and } \not\parallel p} = \text{finite} \end{aligned} \quad (2.1)$$

as depicted in Fig. 1. If this was the class of processes to be computed and summed, we would be in a situation identical to that of $e^+e^- \rightarrow \text{hadrons}$ and no $\log Q^2/m^2$ singularity would be found.

The crucial point to realize, however, is that, in Eq. (2.1), only the first two terms belong to the physical cross-section we want to compute. Indeed, if we consider an incoming massive quark (or a slightly off-shell quark with $p^2 \neq 0$), there is no state degenerate with it. Equation (2.1) thus states that the singularities of our physical cross-section given by diagrams 1a and 1b are, up to a minus sign, those of diagram 1c.

On the other hand, there is a physical distinguishable process that has a $\log Q^2/m^2$ *) singularity which we should also consider in order to calculate total cross-sections. It is the emission of a hard (i.e., $E > \Delta E$) gluon parallel to the incoming quark **) [soft gluons having been already included in Eq. (2.1)]. The final result is therefore the one shown in Fig. 2. Notice that even if each diagram in the right-hand side of Fig. 2 is singular as $\Delta E \rightarrow 0$, the singularity drops out in the difference. The reason of this cancellation is clear: the parameter ΔE represents an arbitrary separation between hard and soft. Since both of them are included in our total cross-section, no ΔE dependence is indeed expected to be left.

*) Such a singularity is again cancelled in the LNK theorem by states with an extra initial gluon which again should not be considered in our calculation.

**) This is a distinguishable process for quark induced reactions because it gives rise to a jet in the forward direction. With hadrons, the process would become indistinguishable due to the presence of spectators.

To summarize, calculation of mass singularities is reduced to computing processes with real hard gluons collinear with the initial quark and either emitted or absorbed. It is immediately recognized that this statement does not depend on the final state being a single quark in the lowest order process. It will remain valid for any hard process in which there is just one initial quark.

The cancellation of infra-red and m divergences written in Eq. (2.1) can be visualized by considering the diagrams of Figs. 3a, b and c. Writing the explicit expressions and using the techniques of Kinoshita³⁾, it is easy to see that infra-red divergences cancel out between the soft contribution of diagrams 3a and 3b. The same diagrams cancel the m divergence arising from k hard and parallel to p' , whereas 3c is essential in order to cancel the m divergence coming in 3a from k hard and parallel to p .

Of course, the above diagrams do not exhaust all possible contributions of $O(\alpha_s)$. The remaining diagrams have gluons attached at both ends to lines carrying the same momentum (either p or p'). These two families of diagrams satisfy by themselves the general LNK theorem. In general, as pointed out first by Nakanishi⁴⁾, the cancellation does occur within diagrams (e.g., 3a, 3b, 3c) which can be obtained by cutting the same vacuum diagrams (Fig. 4a in the example) twice to give both initial and final states (double-cut diagrams). The remaining contributions come from cutting diagrams 4b and 4c and are separately finite.

It is amusing to observe that the same double-cut diagrams also describe (up to some trivial changes of initial and final states) the process $e^+e^- \rightarrow \gamma^* \rightarrow \text{hadrons}$ and $q\bar{q} \rightarrow \gamma^* + X \rightarrow e^+e^- + X$.

The generalization of our result in Fig. 2 to hard processes with several final jets is trivial because, on the right-hand side, only gluons collinear with the initial quark appear explicitly.

It is also a simple matter to extend the above considerations to the case of an arbitrary number of incoming quarks or antiquarks as long as their momenta are well distinct from each other. We shall discuss later the inclusion of initial gluons.

At the one-loop level, $\alpha_s \log Q^2/m^2$, the mass singularities of hard total or jet inclusive processes are given by the square of diagrams with a hard gluon emitted minus that with the hard gluon absorbed, summing incoherently over configurations where the gluon momentum is parallel to that of each initial quark or antiquark. From this point of view the case of $e^+e^- \rightarrow \text{hadrons}$ is just the trivial particular case of no incoming coloured quanta.

3. EXPLICIT COMPUTATIONS AT THE ONE-LOOP LEVEL

A. We now turn to the explicit computation of the one-loop correction to a general hard process involving a single incoming quark. The generalization to many incoming quarks is straightforward and will be discussed below.

We use the basic equation depicted in Fig. 2 trivially modified for an arbitrary multijet final state (see Section 2). We start by considering the gluon-bremsstrahlung cross-section of Fig. 2b. This process exhibits a singularity in m (the quark mass) whenever the final gluon is attached, at least once, to the initial quark leg. Two cases are distinguished (see Fig. 5), depending on whether the gluon is both emitted and reabsorbed by the quark carrying momentum p (Fig. 5a) or not (Figs. 5b and 5c).

We shall define a spin and colour averaged cross-section

$$\frac{1}{N} \text{Tr} \left(\not{p} \sigma(p, \{q_i\}) \right)$$

where $SU(N)$ is taken to be the colour gauge group, p is the incoming quark momentum and $\{q_i\}$ a set of further momenta characterizing either final jets or currents. The trace is meant to be over both colour and Dirac indices. In practice we will always compute forward absorptive amplitudes so that a flux factor proportional to the incoming momentum is tacitly included in σ .

We set to zero the quark mass everywhere except in small denominators. Then in the Feynman gauge, diagram 5a gives:

$$\text{Tr} [\not{p} \sigma_1(p, \{q_i\})] = -\frac{g^2}{(2\pi)^3} \int \frac{d^3 k}{2k_0} \frac{1}{(p \cdot k)^2} \text{Tr} \left[\sum_{i=1}^{N^2-1} \lambda^i \sigma_0(p-k, \{q_i\}) \lambda^i \not{p} \not{k} \not{p} \right] \quad (3.1)$$

where λ^i are the $SU(N)$ generators in the fermion representation and σ_0 is the lowest order cross-section for the process under study. Performing the trivial Dirac algebra, Eq. (3.1) immediately gives:

$$\text{Tr} [\not{p} \sigma_1(p, \{q_i\})] = \frac{\alpha_s}{2\pi^2} C_F \int \frac{d^3 k}{2k_0} \frac{1}{p \cdot k} \text{Tr} [\sigma_0(p-k, \{q_i\}) \not{k}] \quad (3.2)$$

where $\alpha_s = g^2/4\pi$ and C_F is the quark Casimir operator

$$C_F = \frac{N^2 - 1}{2N}$$

The mass singularity (of the type $\log Q^2/m^2$) comes from the vanishing of $p \cdot k$ when k is parallel to p . Having reduced ourselves to a single small denominator, we can set $k = zp$ elsewhere. Trivial angular integrations then give:

$$\text{Tr}[\not{p} \sigma_1(p, \{q, i\})] = \frac{\alpha_s}{2\pi} C_F \log \frac{Q^2}{m^2} \int dz z \text{Tr}[\not{p} \sigma_0(p(1-z), \{q, i\})] \quad (3.3)$$

where Q^2 is the common scale of all large invariant masses in the process. We see that, as expected from the structure of diagram (5a), the result factors out a universal correction in front of the lowest order cross-section with a rescaled initial momentum.

At first sight, the situation looks completely different for the diagrams of Figs. 5b and c, where the parallel gluon couples once to the incoming quark and once into a blob which contains all the details of the process. The blob is meant to remind us that the gluon is attached everywhere except on the external line of momentum p . If the sought universality of mass singularities has to show up some sort of miracle has to occur: this miracle is provided by gauge invariance, which allows us to find an interesting trick relating diagrams (5b and c) to objects having again the topology of Fig. 4a. This trick, which we shall refer to as the Collinear Ward Identity (CWI) consists of two observations: i) Diagrams 5b and c have only one small propagator, hence we can immediately set $k = zp$ in the numerators; then the factor $(p-k)\gamma_\rho$ in the numerator gives $\not{p} \frac{1-z}{z} k_\rho$, where ρ is the gluon Lorentz index. ii) k_ρ is now dotted into the blob and it would give zero (Ward identity) if all possible diagrams were included. Since the ones omitted in the definition of the blob are just those in which the gluon is attached on the quark of momentum p , we get instead:

$$\begin{aligned} k_\rho G'_\rho &= -k_\rho G(p-k, \{q, i\}) \frac{(p-k)_\rho}{(p-k)^2} = -G(p-k, \{q, i\}) \frac{\not{p} k}{(p-k)^2} \\ k_\rho G'_\rho \not{p} &= G(p-k, \{q, i\}) \not{p} \end{aligned} \quad (3.4)$$

where G'_0 is the blob with the external gluon and G is the one without (right and left blobs in Fig. 5b). We then immediately obtain as the contributions of Figs. 5b, and 5c:

$$\text{Tr}[\not{p} \sigma_1(p, \{q, i\})] = \frac{d_s C_F}{\pi} \log \frac{Q^2}{m^2} \int \frac{dz(1-z)}{z} \text{Tr}[\not{p} \sigma_0(p(1-z), \{q, i\})] \quad (3.5)$$

which is again a universal correction to the lowest order process. Notice that, although we have reduced (by the CWI) diagrams 5b and 5c to something like 5a, their resulting z dependence is completely different.

Adding the contributions as in Fig. 5, we get:

$$\text{Tr}[\not{p} \sigma_1(p, \{q, i\})] = \frac{d_s C_F}{2\pi} \log \frac{Q^2}{m^2} \int_{\frac{\Delta E}{E}}^1 dz \left(z + \frac{2(1-z)}{z} \right) \text{Tr}[\not{p} \sigma_0(p(1-z), \{q, i\})] \quad (3.6)$$

where we have indicated the lowest limit of integration in z coming from the restriction to hard gluons.

The contribution from collinear gluons in the initial states (Fig. 2c) can be dealt with in exactly the same way. The result is the one obtained above except that the lowest order process now factored out is the one with an incoming quark of momentum p (and not $p - k$) as follows from the definition of degenerate initial states. Adding all contributions, we obtain, at order α_s

$$\begin{aligned} \text{Tr}[\not{p} \sigma(p, \{q, i\})] &= \text{Tr}[\not{p} \sigma_0(p, \{q, i\})] + \frac{d_s C_F}{2\pi} \log \frac{Q^2}{m^2} \cdot \\ &\cdot \int_{\frac{\Delta E}{E}}^1 \frac{dz}{z} (z^2 + 2(1-z)) \text{Tr}[\not{p} (\sigma_0(p(1-z), \{q, i\}) - \sigma_0(p, \{q, i\}))] \end{aligned} \quad (3.7)$$

Notice that, as anticipated in Section 2, the dangerous singularity at $z = 0$ (limit of soft gluons) is cancelled by the difference of cross-sections appearing in the numerator. The final result is independent of ΔE and the integration in z can be extended down to zero. Changing also the variable to $x = 1-z$ we can finally write:

$$\sigma(p, \{q_i\}) = \sigma_0(p, \{q_i\}) + \frac{\alpha_s}{2\pi} C_F \log \frac{Q^2}{m^2} \int_0^1 \frac{dx}{1-x} (1+x^2) [\sigma_0(xp, \{q_i\}) - \sigma_0(p, \{q_i\})] \quad (3.8)$$

where, from now on, we shall for brevity omit writing the $\text{Tr}(\not{p} \dots)$ factor. Equation (3.8) can be rewritten as:

$$\sigma(p, \{q_i\}) = \int_0^1 dx \sigma_0(xp, \{q_i\}) q(x, \frac{Q^2}{m^2}) \quad (3.9)$$

where

$$q(x, \frac{Q^2}{m^2}) = \delta(x-1) + \frac{\alpha_s}{2\pi} C_F \log(\frac{Q^2}{m^2}) \left(\frac{1+x^2}{1-x} \right)_+ + O(\alpha_s^2) \quad (3.10)$$

and the regularized distribution appearing in Eq. (3.10) is defined as

$$\left(\frac{1+x^2}{1-x} \right)_+ \equiv \frac{1+x^2}{1-x} - \delta(1-x) \int_0^1 dy \frac{1+y^2}{1-y} \quad (3.11)$$

Equation (3.9) can be interpreted by saying that the $O(\alpha_s)$ cross-section is obtained by convoluting the lowest order cross-section with the $O(\alpha_s)$ distribution of quarks in a quark $q(x, Q^2/m^2)$.

B. The generalization of the above result to a process involving several initial quarks and antiquarks is straightforward. Due to our results of Section 2, the only diagrams to be considered contain a hard gluon which can now be parallel to either one of the initial particles. As a consequence, the total phase space integral breaks into independent regions to be added incoherently. The fact that each phase space region contributes in a universal way (i.e., independently of the process) is a general consequence of our CWI. The general result is then

$$\begin{aligned} \sigma(\{p_i\}, \{q_i\}) &= \int \prod_{j=1}^n dx_j \sigma_0(\{x_j p_i\}, \{q_i\}) \left\{ \prod_{j=1}^n \delta(x_j - 1) + \right. \\ &\quad \left. + \sum_{j=1}^n \prod_{l \neq j} \delta(x_l - 1) \frac{\alpha_s}{2\pi} C_F \log \frac{Q^2}{m_j^2} \left(\frac{1+x_j^2}{1-x_j} \right)_+ \right\} = \end{aligned} \quad (3.12)$$

$$= \left[\prod_{j=1}^n dx_j q(x_j, \frac{Q^2}{m_j^2}) \right] \sigma_0(\{x_j p_i\}, \{q_i\}) + O(\alpha_s^2)$$

with $q(x_j, Q^2/m_j^2)$ as given in Eqs. (3.10) and (3.11).

The fact that the result is factorized [to $O(\alpha_s)$ additive] in the initial momenta, will be crucial for relating different processes, such as deep inelastic scattering, lepton pair production or large p_T hadronic scattering.

C. We conclude this section by recovering, in the case of deep inelastic scattering, some familiar results and by mentioning some results for quark non-initiated processes.

For $\gamma^* + q \rightarrow \text{anything}$ the "cross-section" σ can be expressed in terms of the more familiar structure functions $F_1(x)$, $F_2(x) = 2xF_1(x)$. One finds, up to uninteresting constants,

$$\text{Tr}[\not{x} \sigma(p, q)] = F_1(x_B)$$

$$\text{Tr}[\not{x} \sigma_0(xp, q)] = \frac{1}{x} F_1^{(0)}\left(\frac{x_B}{x}\right) = \frac{1}{x} \delta\left(\frac{x_B}{x} - 1\right) = \delta(x_B - x) \quad (3.13)$$

where $x_B = Q^2/2p \cdot q \equiv -q^2/2p \cdot q$ and q is the photon momentum. Inserting Eq. (3.13) into Eqs. (3.8) and (3.9), we get:

$$\begin{aligned} F_1(x_B) &= q\left(x_B, \frac{Q^2}{m^2}\right)_{O(\alpha_s)} = \delta(x_B - 1) + \frac{\alpha_s}{2\pi} C_F \log \frac{Q^2}{m^2} \left(\frac{1+x_B^2}{1-x_B} \right)_+ = \\ &= F_1^{(0)}(x_B) + \frac{\alpha_s}{2\pi} C_F \log \frac{Q^2}{m^2} \int_0^1 dx \frac{1+x^2}{1-x} \left[\frac{1}{x} F_1^{(0)}\left(\frac{x_B}{x}\right) - F_1^{(0)}(x_B) \right] \end{aligned} \quad (3.14)$$

Equation (3.14) justifies our preceding identification of $q(x, Q^2/m^2)$ as the quark distribution in a quark.

In order to compare with the $O(\alpha_s)$ results of the operator product expansion (OPE) method we consider the moments of F_1 and obtain

$$M_n\left(\frac{Q^2}{m^2}\right) \equiv \int_0^1 dx_B x_B^{n-1} F_1(x_B) \underset{O(\alpha_s)}{=} 1 + \frac{\alpha_s}{2\pi} \log \frac{Q^2}{m^2} A_n \quad (3.15)$$

where

$$A_n \equiv \int_0^1 dx \frac{1+x^2}{1-x} (x^{n-1} - 1) \quad (3.16)$$

Leaving to Section 5 the detailed discussion of the comparison between our method and the OPE approach, we just remark that the constants A_n of Eq. (3.16)

coincide with the α_s coefficients of the anomalous dimension of the operators of lowest twist appearing in Wilson's expansion⁷⁾. We also notice that Eq. (3.15) coincides with the first iteration of the evolution equation of Altarelli and Parisi⁶⁾, for the non-singlet part of the structure functions. Their equation embodies all the leading scaling violations given by the OPE method. In particular, Eq. (3.16) gives the coefficients A_n as the moments of the regularized distribution $[(1+x^2)/(1-x)]_+$. In Ref. 6) this was obtained by imposing the constraints of fermion number conservation in order to fix the coefficient of $\delta(x-1)$ in Eq. (3.11). In our scheme such a contribution, which comes from the diagrams with initial gluons, gives automatically the correct prescription. In Section 5 we shall see how the leading $\log Q^2/m^2$ contributions coming from multiloop processes are expected to provide a complete derivation of the Altarelli-Parisi equation⁶⁾.

D. Until now we have only considered processes which turn a quark (antiquark) into a quark (antiquark). This is all one needs for the non- (flavour) singlet part of the structure function. In general, however, several transitions giving rise to mass singularities can occur, gluon $\rightarrow$ quark, quark $\rightarrow$ gluon, and gluon $\rightarrow$ gluon.

The discussion given above for quark $\rightarrow$ quark can be applied with only trivial modifications to the case of gluon $\rightarrow$ quark (antiquark), as it is obvious from the structure of the relevant diagrams (see Fig. 6 and compare it with Fig. 5). It is enough to replace z with $1/z$ and to multiply the whole expression with the appropriate colour weights [giving a factor $C_F N/(N^2-1) = \frac{1}{2}$]. We obtain, as the analogue of Eq. (3.7)

$$\sigma_{\text{gluon}}(p, \{q_i\}) = \frac{\alpha_s}{4\pi} \log \frac{Q^2}{m^2} \int_0^1 dz [z^2 + (1-z)^2] \cdot [\sigma_0(zp, \{q_i\}) + \sigma_0((1-z)p, \{q_i\})] \quad (3.17)$$

where $\sigma_0(\bar{\sigma}_0)$ is the lowest order cross-section for incoming $q(\bar{q})$.

This can be expressed, in the language of Eq. (3.9), by saying that the probability of finding a quark of momentum zp is a gluon of momentum p is given by

$$P_{g \rightarrow q}(z, \frac{Q^2}{m^2}) = \frac{\alpha_s}{4\pi} \left(\log \frac{Q^2}{m^2} \right) (z^2 + (1-z)^2) \quad (3.18)$$

again in agreement with Ref. 6).

The transitions $q \rightarrow q$, $\bar{q} \rightarrow \bar{q}$ and $g \rightarrow q$ or $\bar{q}$ exhaust those which occur at the one-loop level if the lowest order process is not initiated by gluons, otherwise also $q \rightarrow g$ and $g \rightarrow g$ transitions have to be considered.

A preliminary inspection shows the straightforward applicability of our techniques to these cases, even though the actual calculation is rather different. We have been able to rederive the known results⁶⁾ for specific processes and we are now trying to develop general process-independent techniques analogous to the ones described in this section for the $q \rightarrow q$, $g \rightarrow q$ transitions.

4. APPLICATION TO DIFFERENT HARD PROCESSES AND JUSTIFICATION OF PARTON MODEL RESULTS

A. Before applying the results of Section 3, we have to discuss briefly mass singularities at higher orders. Provided we limit ourselves to leading logs (i.e., leading powers of $\alpha_s \log(Q^2/m^2)$), our techniques can be applied in a straightforward way. It is only at the leading log level, but to all orders, that the CWI of Section 3 can be used to reduce the calculation to one involving only the initial quarks in a way independent of the process.

Multiloop calculations are greatly simplified by our method and universality for leading mass singularities stems in a straightforward way. As we shall see in Section 5, if universality holds, then leading $\log m^2$ divergences are directly related to the leading $\log Q^2$ dependence of deep inelastic structure functions, which is known from the operator product expansion⁷⁾.

The explicit form of the $\alpha^2 \log^2 m^2$ term predicted this way has been checked in QED by the explicit two-loop calculation. These results, together with their extension to QCD, will be discussed in detail elsewhere.

B. Let us now apply this result to multijet processes involving different numbers of initial coloured particles. If instead of calculating jet cross-sections, one specifies a final quark momentum, extra mass singularities arise. By a treatment analogous to the one presented here it is possible to show that such singularities can be reabsorbed into a universal quark fragmentation function.

i) $e^+e^- \rightarrow \text{hadrons}$. This is the simplest case with no initial coloured quanta. For this process we recover the results of Sterman and Weinberg¹⁾, i.e., their perturbative calculation of multijet cross-sections.

ii) Deep inelastic scattering on a quark. This is the case we have extensively discussed in Section 3 and for which we have derived, independently of the number of jets in the final state, Eq. (3.9). For the special case of a three-jet process our results coincide with those of Ref. 8).

The invariant mass of the non-forward multijet system can be related to the momentum xp carried by the quark initiating the hard process by:

$$(xp + q)^2 = M^2 \quad (4.1)$$

which implies

$$x = x_B + y_B \quad (4.2)$$

where

$$x_B = -\frac{q^2}{2p \cdot q}, \quad y_B = \frac{M^2}{2p \cdot q}$$

Using Eq. (3.14) we rewrite Eq. (3.9) as

$$\sigma^{n \text{ jet}}(x_B, y_B, Q^2, \{q_i\}) = \int dx \, \tilde{\sigma}_0^{n \text{ jet}}(x, x_B, y_B, \{q_i\}) F_1(x, \frac{Q^2}{x^2}) \quad (4.3)$$

Because of Eq. (4.2), we also have

$$\tilde{\sigma}_0^{n \text{ jet}} = \delta(x - x_B - y_B) \tilde{\sigma}_0^{n \text{ jet}}(x_B, y_B, \{q_i\}) \quad (4.4)$$

Thus ratios of different multijet cross-sections at the same value of $x_B + y_B$ are independent of $F_1(x, Q^2/m^2)$ and do not contain mass singularities. Hence, within

the philosophy of Ref. 1) and as we shall justify fully in Section 5, such ratios can be computed perturbatively in terms of the running coupling constant $\alpha_s(Q^2)$.

iii) Processes with two initial quarks (antiquarks). We now discuss hard processes initiated by $q\bar{q}$, $q\bar{q}$ or $q\bar{q}$. These include large p_T hadronic processes and massive lepton pair production, both with an arbitrary number of jets in the final state.

As discussed at the one-loop level in Section 3, the mass singularity factorizes to give:

$$\sigma(p_1, p_2, \{q_i\}) = \int_0^1 dx_1 dx_2 \sigma_0(x_1 p_1, x_2 p_2, \{q_i\}) F_1(x_1, \frac{Q^2}{m^2}) F_1(x_2, \frac{Q^2}{m^2}) \quad (4.5)$$

The $\alpha_s \log^2 Q^2/m^2$ term coming from the product of the $O(\alpha_s)$ corrections in each F_1 in Eq. (4.5) has been explicitly checked by a two-loop calculation in which our CWI has been used. This result is non-trivial to obtain by standard techniques.

Once more we can consider a more differential cross-section which will fix x_1 and x_2 in terms of final state variables, defined in analogy with the variables x_B, y_B of Eqs. (4.2). We can then eliminate the factors $F_1(x_1, Q^2/m^2) \cdot F_1(x_2, Q^2/m^2)$ in Eq. (4.5), either from ratios of $q\bar{q}$ (or $q\bar{q}$) initiated multijet cross-sections or from ratios with deep inelastic processes. Again these ratios will be reliably computed by improved perturbation theory. Going to specific examples, our method will cover with the minimum number of final jets, lowest order large p_T reactions such as $q\bar{q} \rightarrow q\bar{q}$, $q\bar{q} \rightarrow q\bar{q}$ and $q\bar{q} \rightarrow g\bar{g}$ and massive lepton pair production $q\bar{q} \rightarrow \gamma^* + X$. For the latter case, our results confirm, on general grounds, the explicit calculations of Politzer²⁾ and justify, with some reservation to be made below, the use of the modified Drell-Yan formula including scaling violations. If we add an extra final jet (large p_T lepton pairs) the result agrees with the ansatz of Ref. 9), if we restrict ourselves to the $q\bar{q}$ initiated hard process (other transitions such as gluon $\rightarrow$ gluon are needed for the general case).

For the case $q\bar{q} \rightarrow q\bar{q}$ at large p_T , the $O(\alpha_s)$ corrections have been explicitly computed by Sachrajda¹⁰⁾. He found universality from a set of miraculous cancellations of unwanted terms which originate naturally from our CWI.

C. We remark now that, in the way we have proceeded, scaling violation and mass singularities come out to be the same for q^2 positive or negative, since, to leading order, $\log(-q^2) \approx \log|q^2|$.

If the dynamics of QCD, however, is such to produce out of quarks and gluons, bound states, resonances and hadronic thresholds as we hope, then the continuation from the deep Euclidean region ($q^2 < 0$) to the real positive axis becomes delicate and clearly perturbation theory (whether improved or not) will not be able to reproduce such structures. In the language of 't Hooft¹¹⁾, although the renormalization group (RG) equation always allows a replacement of α_s with $\alpha_s(Q^2)$ (see also Section 5), singularities in q^2 get reflected into singularities in the α_s plane. For q^2 real and positive these singularities tend to condense near $\alpha_s = 0$ on the real positive axis thus preventing a perturbative expansion in α_s .

One can still hope, following Poggio, Quinn and Weinberg⁵⁾, that perturbation theory can represent an average over the actual (bumpy?) cross-section. For $e^+e^- \rightarrow \text{hadron}$ this attitude can be supported⁵⁾ by computing the vacuum polarization amplitude A at complex q^2 (to avoid the above-mentioned difficulties) and noticing that

$$\frac{1}{2i} [A(q^2 + i\Delta) - A(q^2 - i\Delta)] = \frac{1}{\pi} \int_0^\infty dq'^2 \frac{\text{Im} A(q'^2)}{(q'^2 - q^2)^2 + \Delta^2} = \langle \sigma_T(q^2) \rangle_\Delta \quad (4.6)$$

where $\langle \sigma_T(q^2) \rangle_\Delta$ is the average of σ_T ($e^+e^- \rightarrow \text{hadrons}$) over an interval Δ .

The validity of Eq. (4.6) reflects the fact that the variable q^2 in which we go complex is the one in which the discontinuity must be taken in order to get the cross-section.

The situation is different in the Drell-Yan process because now q^2 is not the variable in which the discontinuity is to be taken. In this case, going to complex q^2 gives the average of the amplitude (then squared) and not the average of the cross-section. Two illustrations of this point can be given: The first concerns the ratio [presumably measured in $\sigma(\pi^- \text{ carbon} \rightarrow \mu^+\mu^- + X) / \sigma(\pi^+ \text{ carbon} \rightarrow \mu^+\mu^- + X)$]

$$R = \frac{\sigma(\mu\bar{\mu} \rightarrow \mu^+\mu^- X)}{\sigma(d\bar{d} \rightarrow \mu^+\mu^- X)} \quad (4.7)$$

which is predicted to be close to 4 by the DY formula. Yet, on a resonance of definite isospin, R is clearly 1 and the only way to obtain $R \approx 4$ is to average the amplitude over two resonances, of isospin zero and one, falling within the averaging range and coupled as predicted by exchange degeneracy and duality.

The second observation is that, as previously noticed¹²⁾, the famous $1/3$ ($1/N_c$, for N_c colours) factor of the DY formula is not expected to be there in the large N_c limit (narrow resonance limit) at fixed (but large) s and M^2 . In this limit (narrow) vector meson (VM) dominated events, give

$$\sigma(\mu^+\mu^-) = \sigma(\text{VM}) \cdot B, \quad B = \Gamma(\text{VM} \rightarrow \mu^+\mu^-) / \Gamma(\text{VM} \rightarrow \text{all})_{(4.8)}$$

One finds¹²⁾

$$\sigma(\text{VM}) \approx O\left(\frac{1}{N_c^2}\right), \quad B \approx O(N_c^2)$$

and therefore $\sigma(\mu^+\mu^-) = O(1)$ against the DY prediction. The discrepancy can be readily explained by our smoothing argument.

In conclusion, unlike what happens in e^+e^- or even in the Bloom-Gilman duality for deep inelastic scattering, the DY formula does not give directly an average cross-section but rather a smooth background which neglects rapid fluctuations of the cross-section. In the absence of such fluctuations (e.g., strongly overlapping resonances) the DY prediction should become again reliable for the cross-section.

5. RELATION BETWEEN MASS SINGULARITIES AND SCALING VIOLATIONS

We want now to combine our results on mass singularities with the renormalization group (RG) equations and relate them to the scaling violations as obtained from OPE. Cross-sections being related to truncated Green functions satisfy RG equations with the anomalous dimension γ_i of the external currents. The correspondent additional dependence on Q^2/μ^2 can be singled out by defining reduced cross-sections G

$$G\left(\frac{Q^2}{m^2}, \frac{Q^2}{\mu^2}, x_{B_i}, d_s\right) = Q^2 \sigma(Q^2, m^2, \mu^2, x_{B_i}, d_s) \exp \int_{d_s}^{\alpha_s(Q^2)} \frac{\gamma_i(\alpha)}{\beta(\alpha)} d\alpha \quad (5.1)$$

that satisfy RG equations without anomalous dimension. We have indicated with x_{B_i} all scaling variables depending on the initial or final momenta. Then,

$$g\left(\frac{Q^2}{m^2}, \frac{Q^2}{\mu^2}, x_{B_i}, \alpha_s\right) = g\left(1, \frac{Q^2}{m(Q^2)}, x_{B_i}, \alpha_s(Q^2)\right) \quad (5.2)$$

where $\alpha_s(Q^2)$ and $m(Q^2)$ are the running strong coupling constant and mass, respectively. The Q^2 dependence of $m(Q^2)$ appears only at the non-leading log level and can therefore be neglected in our approach. Therefore, Eqs. (5.1) and (5.2) simply state what we have already anticipated in Section 4, i.e., that if we exhibit the dependence of cross-sections in m^2/Q^2 at leading log, the total leading $\log Q^2$ dependence is correctly taken into account by replacing α_s by $\alpha_s(Q^2)$. We see then that apart from the trivial dependence on Q^2 given by the $\alpha_s(Q^2)$ appearing in the Born approximation σ_0 in Eq. (3.12)*), the remaining dependence on both Q^2 and m^2 of whatever hard cross-section, lies in the structure function

$$q = F_i(x_j, \alpha_s(Q^2) \log \frac{Q^2}{m^2})$$

on which we now focalize our attention.

For leading $\log Q^2$ terms we can use the lowest order expressions for $\alpha_s(Q^2)$ and the Callan-Symanzik β function:

$$\alpha_s(Q^2) = \frac{\alpha_s}{1 + b \alpha_s \log \frac{Q^2}{\mu^2}} \quad (5.3)$$

where

$$\beta(\alpha_s) = -\alpha_s^2 b + O(\alpha_s^3) \quad (5.4)$$

*) q_i being a structure function (cf. 3.14) satisfies RG equations without anomalous dimensions. Therefore σ_0 satisfy also RG equations and with the anomalous dimensions of σ . In the physical cases treated in Section 4, the involved current was the electromagnetic one and therefore $\gamma_i = 0$ in (5.1).

From the fact that F_1 depends on $\alpha_s(Q^2) \log Q^2/m^2$ it is easy to find that

$$\frac{\partial F_1}{\partial \log Q^2} = - \frac{\alpha_s(Q^2)}{\alpha_s(\mu^2)} \frac{\partial F_1}{\partial \log \mu^2} \quad (5.5)$$

which clearly relates the evolution of F_1 in $\log m^2$ to that in $\log Q^2$, both at the leading log level. From the fact that our treatment of mass singularities should give the first one while the second one could be inferred from the operator product expansion, we clearly see the interconnection between these two approaches. Let us indeed investigate this in more detail by considering the moments M_n of Eq. (3.15). The standard operator product expansion gives for a target quark

$$M_n \left(\frac{Q^2}{\mu^2}, \frac{Q^2}{\mu^2}, d_s \right) = C_n \left(\frac{Q^2}{\mu^2}, d_s \right) \langle O_n \left(\frac{\mu^2}{\mu^2}, d_s \right) \rangle_q \quad (5.6)$$

The large Q^2 dependence of the coefficients C_n is given in terms of their anomalous dimension γ_n . At the leading log level

$$C_n \left(\frac{Q^2}{\mu^2}, d_s \right) \xrightarrow{\text{leading log } \frac{Q^2}{\mu^2}} C_n(1,0) \left(\frac{\alpha_s(Q^2)}{\alpha_s(\mu^2)} \right)^{-A_n/2\pi b} \quad (5.7)$$

where $\alpha_s(Q^2)$ and b are given by (5.3) and (5.4) and A_n is given by

$$\gamma_n = \frac{\alpha_s}{2\pi} A_n + O(\alpha_s^2) \quad (5.8)$$

But the Q^2 dependence of C_n determines that of M_n , so that

$$M_n \underset{\log Q^2 \rightarrow \infty}{\sim} \left(1 + d_s b \log \frac{Q^2}{\mu^2} \right)^{A_n/2\pi b} \quad (5.9)$$

or, equivalently,

$$\frac{\partial M_n}{\partial \log Q^2} = \frac{A_n}{2\pi} \frac{d_s M_n}{1 + d_s b \log \frac{Q^2}{\mu^2}} = \gamma_n(d_s(Q^2)) M_n \quad (5.10)$$

Therefore from (5.5)

$$\frac{\partial M_n}{\partial \log m^2} = -\frac{A_n}{2\pi} \frac{d_s M_n}{1 + d_s b \log \frac{m^2}{\mu^2}} = -\gamma_n(d_s(m^2)) M_n \quad (5.11)$$

which implies

$$M_n \underset{\log m \rightarrow \infty}{\propto} (1 + d_s b \log \frac{m^2}{\mu^2})^{-A_n/2\pi b} = \left(\frac{d_s(m^2)}{d_s} \right)^{A_n/2\pi b} \quad (5.12)$$

We see therefore that if a method, as ours, gives the leading log m expansion of M_n , we expect by general OPE arguments to reproduce the power expansion (5.12) characteristic of an anomalous dimension $-\gamma_n$. Conversely, it is a trivial matter to show that if our leading $\alpha_s \log Q^2/m^2$ expansion resums so to give

$$M_n(d_s \log \frac{Q^2}{m^2}) = M_n(0) \left(1 - b d_s \log \frac{Q^2}{m^2} \right)^{-A_n/2\pi b} \quad (5.13)$$

then

$$M_n\left(\frac{Q^2}{\mu^2}, \frac{m^2}{\mu^2}, d_s\right) = M_n(0) \left(\frac{1 + b d_s \log \frac{Q^2/\mu^2}{m^2/\mu^2}}{1 + b d_s \log \frac{m^2}{\mu^2}} \right)^{A_n/2\pi b} \quad (5.14)$$

which establishes for M_n the properties (factorization and anomalous dimension behaviour) implied by OPE. We see therefore the interesting fact that leading mass singularities represent an alternative way of studying short-distance behaviour as implied by light cone properties. This correlation between infra-red and ultra-violet properties is perhaps not surprising in asymptotically free theories.

Moreover, the universality of mass singularities allows to draw conclusions on processes (as done in Section 4) to which light-cone dominance -- and therefore OPE -- cannot be applied.

6. CONCLUSIONS

Up to now we have considered processes initiated by actual quarks. Quarks not being free particles in our experiment, we must discuss which are the implications of our results for processes containing hadrons in the initial states. If a quark wave function could be conveniently defined such that quarks far from mass-shell are strongly damped, then we would be able to extend our results by convoluting them with that wave function. Of course, diagrams involving spectators will appear. For instance a gluon emitted parallel to an active quark belonging to an initial hadron could be reabsorbed by a spectator of another initial hadron. This seems at first sight to ruin our universality. Nevertheless, it appears that these processes are just what is needed for our CWI to reproduce the universal results.

Another attitude at our disposal is not to attempt the calculation of the actual hadron induced process, but to try to reabsorb all unmeasurable quark mass terms into an unknown structure function. This is the attitude of Ref. 2) for lowest order Drell-Yan processes. By using the factorization discussed in Section 5 of the Q^2 and m^2 dependences in the universal factors, one obtains in this way a structure function for every initial hadron which depends only on the hadron involved but not on the hard process in which it enters. Therefore, we expect our results of Section 4 connecting different hard processes to hold also for processes involving initial hadrons.

Keeping in mind the subtle relevant points related to the confinement dynamics such as the problem arising from narrow resonances (cf. Section 4), and hoping that others such as soft reinteraction phenomena are harmless, we justify from QCD and for all hard processes involving final jets improved parton model results. These consist of perturbative calculation of the hard process in the running coupling constant folded into universal parton distributions containing scaling violations. This implies that jet phenomenology is a crucial testing ground for QCD.

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FIGURE CAPTIONS

- Fig. 1 : Graphical form of LNK theorem for the total deep inelastic cross-section on a quark. Soft bremsstrahlung is included in (b); the hard gluon is parallel to p' in (b) and to p in (c).
- Fig. 2 : The singular part of the one-loop deep inelastic cross-section after use of the LNK theorem. In (b),(c) gluons are hard and parallel to p .
- Fig. 3 : Three contributions to the deep inelastic cross-section among which infra-red and mass divergences are cancelled.
- Fig. 4 : (a) the double cut diagram relevant to the processes of Fig. 3; (b) and (c) double cut diagrams for the remaining contributions.
- Fig. 5 : Diagrams contributing to mass singularities at the one-loop level. The blob with an incoming (outgoing) gluon in b and c is meant to exclude the coupling of the gluon to the quark of momentum p .
- Fig. 6 : The analogue of diagrams 5a,b,c for the gluon $\rightarrow$ quark transition.

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5

Fig 6