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ELECTROMAGNETIC DETECTOR FOR GRAVITATIONAL WAVES

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ABSTRACT

We analyze the mode of operation of a two-level parametric electromagnetic detector for gravitational waves which is tunable and potentially more sensitive than the mechanical antennas currently considered.

In this letter we analyze the principle of operation of the two-level electromagnetic detector for gravitational waves (g.w.) suggested by Pegoraro, Picasso and Radicati [1]. A similar detector, which uses both the resonance with the electromagnetic field and with the mechanical structure of the detector has been proposed by Caves [2]. We will show that the principle which underlies these detectors is analogous to the one used in parametric processes and in particular in frequency converters [3]. This analogy will help to clarify why the cross section of these electromagnetic detectors can be larger than the one of the mechanical antennas [4]. We will also discuss the limitations, which the available technology sets on the lowest g.w. flux measurable with our detector.

Both ours and Caves' detector consist of an electromagnetic resonator, with two levels whose frequencies ω_s and ω_a are both much larger than the frequency Ω of the g.w. and satisfy the resonance condition, $\omega_a - \omega_s = \Omega$. In our scheme the two levels are obtained by coupling two identical high frequency resonators. We shall call ω_s the frequency of the level symmetrical in the fields of two resonators and ω_a that of the antisymmetrical one. The g.w., through its coupling to the electromagnetic energy, can induce a transition between the two levels provided their angular momenta along the direction z of the g.w., differ by 2. This can be achieved by putting the two resonators at right angles (fig. 1).

It has been shown in [1] that, in the reference frame where the detector's walls are at rest, the energy transfer between the two levels can be entirely attributed to the time dependence of the anisotropic dielectric tensor ϵ and of the magnetic permeability tensor μ induced in vacuum by the g.w. In [1] it has been shown that the two tensors are equal. In the limit when the walls can be considered at rest in the free falling system, $d\epsilon/dt = -dg_{\perp}/dt \sim \Omega h$. Here, $g_{\perp}$ are the components in the x - y plane (perpendicular to the direction of the g.w.) of the metric tensor in the transverse traceless gauge [5]. The dimensionless amplitude h of the g.w., is given in terms of the energy flux ϕ and of the gravitational constant G : $h = \Omega^{-1} \sqrt{32\pi G\phi/c^3}$.

The rate of change of the electromagnetic energy W in the detector under the action of the g.w. can be derived from Poynting's theorem. If we neglect dissipative terms, we have^(*)

$$\dot{W} = \text{tr} (\dot{\epsilon} W \Pi), \quad (1)$$

where the dot represents the derivative with respect to time. Here $W \Pi$ is the integral over the detector's volume of the traceless part of the electromagnetic stress tensor in the x-y plane. Its components are

$$\begin{aligned} \Pi_{xx} &= -\Pi_{yy} = \frac{1}{4\pi W} \int d^3x \left(E_x D_x - E_y D_y + H_x B_x - H_y B_y \right) \\ \Pi_{xy} &= \Pi_{yx} = \frac{1}{8\pi W} \int d^3x \left(E_x D_y + E_y D_x + H_x B_y + H_y B_x \right). \end{aligned} \quad (2)$$

The fields $\underline{D}$ and $\underline{B}$ differ from $\underline{E}$ and $\underline{H}$ by terms of order h , which are due to the g.w., so that Π contains in general terms of order zero and one in h . As a consequence of the spin 2 of the g.w., only the traceless part, $W \Pi$, of the stress tensor contributes to the power exchange. Since Π changes sign, like $\dot{\epsilon}$, under a $\pi/2$ rotation around the z-axis, we shall call it the quadrupole moment of the electromagnetic field.

In a two-level detector, the tensor Π contains several frequencies and the dominant contribution to $\dot{W}$ comes from the component $\Pi(\Omega)$, bilinear in the fields of the two levels at the frequency Ω of the g.w. It has been shown in [1] that in order to maximize $\Pi(\Omega)$, at constant energy W , the two resonators must be at right angles. Indeed, $\Pi(\Omega)$ changes sign when the two resonators are interchanged^(**). Since Π changes sign under a $\pi/2$ rotation around the z-axis, the maximum value of $\Pi(\Omega)$ obtains when the two operations, interchange of the two resonators and $\pi/2$ rotation, coincide.

(*) Eq. (1) has been derived in [1] to order h . The same relation holds to order h^2 when one of the two levels of the detector, ω_a say, is empty. In addition, we have assumed that the extension in the z-direction of the detector is small compared to the wave length of the g.w.

(**) This happens because the fields of the two levels are the symmetrical and antisymmetrical combinations of the fields of the two cavities (fig. 1).

In [1] we have discussed the operation of the detector when both levels are occupied in which case $\Pi(\Omega)$ is of order zero in $\hbar$. In this letter we shall instead assume that the upper level ω_a is initially empty so that $\Pi(\Omega)$ has no term of order zero in $\hbar$. After a time t , the g.w. induces in this level an electromagnetic field whose amplitude is $A_a \sim \hbar t \omega_a A_s$, where A_s is the amplitude of the lower, symmetrical level. The induced quadrupole moment $\Pi(\Omega)$ is therefore $\Pi(\Omega) \sim \hbar t \omega_a$ and the power absorbed by the detector is $\dot{W} \sim \hbar^2 \omega_a^2 t \Omega W_s$.

Our detector can be operated as a frequency converter if instead of measuring the total power absorbed by the detector, one measures the power transferred to the upper level. In this way one gains a factor ω_a/Ω . Indeed, if we assume that the total number of photons $N = N_a + N_s$ is conserved, we have (*)

$$\dot{W} = \dot{W}_a + \dot{W}_s = \omega_a \dot{N}_a + \omega_s \dot{N}_s = (\omega_a - \omega_s) \dot{N}_a. \quad (3)$$

The power transferred to the upper level is therefore

$$\dot{W}_a = (\Omega^{-1} \omega_a) \dot{W} \sim (\Omega^{-1} \omega_a) \hbar^2 (\omega_a t) \Omega W_s. \quad (4)$$

The amplification factor ω_a/Ω is characteristic of the parametric frequency converters. These devices are, like our detector, electromagnetic resonators with two levels ω_1, ω_2 coupled by a time dependent element (e.g. a capacity, the equivalent of our time dependent ε), modulated at a frequency $\Delta\omega = \omega_2 - \omega_1$, which causes a periodic energy transfer between the two levels. If the capacity is of the form $C = C_0 (1 + \eta \cos \Delta\omega t)$, the power $\dot{W}_2$ in the upper level, ω_2 say, is given by

$$\dot{W}_2 = \frac{1}{4} \eta \sqrt{\omega_1 \omega_2} \sin \left(\frac{1}{2} \eta \sqrt{\omega_1 \omega_2} t \right) W_0, \quad (5)$$

where W_0 is the average energy in the system.

(*) Eq. (3) is the equivalent of the Manley and Rowe [6] relation used in the theory of frequency converters [3].

The equivalent of η and $\sqrt{\omega_1 \omega_2}$ are in our case h and $\sqrt{\omega_s \omega_a}$, so that, for all cases of interest, $\sin \frac{1}{2} h \sqrt{\omega_s \omega_a} t \simeq \frac{1}{2} h \sqrt{\omega_s \omega_a} t \simeq \frac{1}{2} h \omega_a t$, and eq. (5) reduces to eq. (4). In contrast to ordinary frequency converters, the two levels of our resonator differ not only in their frequencies, but also in their angular momenta, since the signal, the g.w. whose frequency we want to convert, carries both energy and angular momentum.

To compare our electromagnetic detector to the mechanical ones, we introduce the cross section Σ defined as the ratio of the measured power to the incident energy flux

$$\Sigma = \Sigma^0 \times f(\gamma/\Gamma), \quad (6)$$

where Σ^0 is the resonant cross section. The factor $f(\gamma/\Gamma)$ reduces to one in the monochromatic limit when the duration $1/\Gamma$ of the g.w. packet is much longer than the detector decay time $1/\gamma$. In the opposite limit (short burst limit, i.e. $1/\Gamma \ll 1/\gamma$), $f(\gamma/\Gamma) \approx \gamma/\Gamma \ll 1$. The resonant cross section for our electromagnetic detector, obtained from (4), is

$$\Sigma_{em}^0 = 32\pi(GW_s)^{-1} (\omega_a \Omega^{-1}) Q_{em}, \quad (7)$$

where $Q_{em} = \omega_a/\gamma_{em}$ is the Q-value of the resonator. For a mechanical quadrupole antenna of mass M and length L , the resonant cross section is [4]

$$\Sigma_{mech}^0 = 16\pi(GML^2\Omega^2)^{-1} Q_{mech}, \quad (8)$$

where $Q_{mech} = \Omega/\gamma_{mech}$. Equations (7) and (8) have been written in such a way as to display the correspondence between the various terms. In particular the energy corresponding to W_s , is not the whole mass but only the elastic energy $ML^2\Omega^2$. The amplification factor ω_a/Ω reduces to one in the mechanical case, where the total power absorbed by the antenna coincides with the power transferred to the upper level, the lower level having zero frequency.

From eq. (7) it follows that to achieve a high sensitivity, the detector must have a very large Q-value. This requires in particular using superconducting cavities. In addition, the Q-values of the

generator and of the receiver^(*) must be of the same order as the one of the cavity.

For sake of definitness we shall consider a detector made of two cylindrical cavities with length ℓ and radius r such that $\ell = 2r$ oriented as in fig. 1, oscillating in the TE_{011} mode at a frequency $\omega = 8.3 \times 10^9$ (0.3 m/r). In this case, the relation between the maximum energy that can be stored in a superconducting Nb cavity and the frequency of the electromagnetic field is

$$W \omega^3 = K \simeq 10^{31} \text{ Joule (rad s}^{-1}\text{)}^3. \quad (9)$$

In terms of the minimum detectable power transferred by the g.w. to the antisymmetrical level, the minimum observable value of h can thus be written as (see eqs (4) and (6))

$$h_{\min} \sim (\delta P_a / KQ)^{\frac{1}{2}} f(\gamma/\Gamma)^{-\frac{1}{2}} \omega, \quad (10)$$

where $\omega \simeq \omega_a \simeq \omega_s$.

The minimum detectable power δP_a must satisfy two criteria:

- (a) Maximum sensitivity: $\delta P_a \geq \delta P_{\min}$, where with present technology $\delta P_{\min} \sim 10^{-22}$ Watt for $\omega \sim 10^9 - 10^{10}$ rad s⁻¹ in the band width of 1 Hz [7]. This is two orders of magnitude larger than the quantum limit. This sets a lower limit to the observable h ^(**)

$$h > 3 \times 10^{-23} \left(\frac{0.3 \text{ m}}{r} \right) \left(\frac{10^{12}}{Q} \right)^{\frac{1}{2}} \left(\frac{\delta P_a}{10^{-22} \text{ Watt}} \right)^{\frac{1}{2}} f(\gamma/\Gamma)^{-\frac{1}{2}} \quad (11)$$

- (b) Frequency selectivity: $\delta P_a \geq R \delta P_s$, where R is a number which depends upon the details of the measurement and δP_s is the power at frequency $\omega_s + \Omega = \omega_a$, due to the finite width of the symmetrical level. If we assume a Lorentzian power distribution and a constant power of the lower level we get

$$\delta P_s = (P_s / 4Q^2) (\omega/\Omega)^2 f(\gamma/\Gamma)^{-\frac{1}{2}}. \quad (12)$$

(*) The Q -values of the generator and of the receiver are defined as the ratio $\omega/\Delta\omega$ where $\Delta\omega$ is their band width.

(**) In this relation, we can take Q as essentially independent of ω .

The minimum detectable h that arises from this condition is (with $\Omega = 10^3 \text{ rad s}^{-1}$)

$$h > \frac{\omega}{\Omega} R^{\frac{1}{2}} \frac{1}{2Q^2} f(\gamma/\Gamma)^{-\frac{1}{2}} = 4 \times 10^{-18} R^{\frac{1}{2}} f(\gamma/\Gamma)^{-\frac{1}{2}} \left(\frac{0.3 \text{ m}}{r}\right) \left(\frac{10^{12}}{Q}\right)^2. (13)$$

If the receiver does not discriminate the parity of the field at frequency ω_a , R is of order one. Discrimination of the parity would considerably reduce this value.

Equation (13) shows that the selectivity condition gives at present, and for frequencies in the KHz range, the most stringent limitation on the minimum detectable h . Indeed, for all values of $Q < 1/3 R^{1/3} 10^{16}$ the selectivity criterium dominates over the sensitivity one. We stress that these results are based on the assumption that the frequency distribution remains Lorentzian up to a distance from ω_s greater than 10^4 times the half-width. Furthermore, they are valid under the assumption that the walls of the detector are approximately free falling. The results are however essentially independent of the special choice of the mode of oscillation.

By comparing eq. (13) with the results quoted in [1] it follows that for $Q > 10^{11}$ the mode of operation quadratic in h , is more sensitive than the one proposed in [1]. Furthermore, in the quadratic mode, we can keep the frequency of the two levels constant [8] by feeding energy into the lower level. This is not possible when both levels are occupied. Even more important, the linear mode requires a coherent source of g.w. and a knowledge of its phase. This can, in principle, be obtained in the case of a continuous source, but not with short bursts.

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FIGURE CAPTION

Fig. 1 Sketch of the magnetic field distribution in the symmetrical and in the antisymmetrical level corresponding to the TE_{011} mode. In the right part of the figure, the resonant transition induced by the g.w. is represented.

Fig. 1