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REMARKS ON THE ENHANCEMENT OF $F^+ \rightarrow \pi^+ e^+ e^-$

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A B S T R A C T

The rare decay mode $F^+ \rightarrow \pi^+ e^+ e^-$, induced by the non-leptonic weak Hamiltonian, will probe electromagnetic form factors at high $e^+ e^-$ masses. The resulting "matrix element" enhancement may make the reaction observable, while dynamical enhancement is predicted to be unimportant.

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The success of QCD methods in the description of non-leptonic strange particle decays ¹⁾⁻³⁾, including the induced neutral current process $K^+ \rightarrow \pi^+ e^+ e^-$ ^{4),5)}, makes a comparison between theory and data in charmed particle decays the more interesting. For heavy particles, having energetic decays, the creation and annihilation of quark pairs by hard gluons is expected to be suppressed due to the falling strength of the strong interaction. A description of the decays in terms of simple recombination diagrams between the quarks ⁶⁾⁻⁸⁾ should then be an increasingly good approximation. The predictions, taking into consideration gluon corrections at the weak vertex, are reportedly ⁸⁾ qualitatively consistent with preliminary data ⁹⁾ on D decays, while for the F meson mainly its existence and its mass is established so far ¹⁰⁾.

Even expectedly rare reactions like $F^+ \rightarrow \pi^+ e^+ e^-$ should be checked when we fix our picture of c quark non-leptonic decays. In this article it is argued that an enhancement mostly due to the electromagnetic form factor of the pion may add more practical importance to that reaction than naively expected, and its observation would in any case enhance our knowledge of the applicability of generalized PCAC.

The Cabibbo dominant c decay amplitude is derived from ^{6),7)}

$$\mathcal{L}_w^{\text{eff}} = 2\sqrt{2} G \left\{ \frac{1}{2}(c_{84} + c_{20}) \bar{u}_L^i \gamma_\mu d_L^i \bar{s}_L^k \gamma^\mu c_L^k + \frac{1}{2}(c_{84} - c_{20}) \bar{s}_L^i \gamma_\mu d_L^i \bar{u}_L^k \gamma^\mu c_L^k \right\} \quad (1)$$

which includes the left-handed quark fields $q_L^i = \frac{1}{2}(1 - \gamma_5)q^i$, i being a colour index, and the coefficients c_{84} , c_{20} which embody renormalization effects in the $\underline{84}$ and $\underline{20}$ representations of SU(4) due to hard gluon exchange ¹⁾. Estimates of c_{84} and c_{20} by different groups are very similar ^{6),7)}, and anyhow the results will not depend critically on the explicit choice, which here is the set of Cabibbo and Maiani ⁷⁾,

$$c_{20} \simeq 2.15, \quad c_{84} \simeq 0.68 \quad (2)$$

who assume the value $\alpha(m_c^2) = 0.7$ for the value of the running quark-gluon fine structure constant evaluated at the c quark mass, consistent with the amount of scaling violation in deep inelastic processes ¹¹⁾. The interaction (1) does not give rise to GIM mechanism diagrams or diagrams where one current

creates a virtual vector meson while the other is taken between the initial and final pseudoscalar states, neither to diagrams due to the gluon induced SVZ currents ^{2),5)}, which all have to be considered in the analogous $K^+ \rightarrow \pi^+ e^+ e^-$ case ^{4),5),12)}. It might be added that such diagrams will instead contribute to the Cabibbo suppressed reactions $F^+ \rightarrow K^+ e^+ e^-$, $D^+ \rightarrow \pi^+ e^+ e^-$ and $D^0 \rightarrow \pi^0 e^+ e^-$.

One is left to consider contact and pole diagrams from the T product of the electromagnetic current and the weak interaction (1), as for $K^+ \rightarrow \pi^+ e^+ e^-$ ^{5),12)}. The first and second terms in (1) will then give rise to $F^+ \rightarrow \pi^+ e^+ e^-$ and $D^0 \rightarrow \bar{K}^0 e^+ e^-$, respectively. For each reaction also the other term will contribute by a Fierz transformation giving a factor $1/3$ between colour singlets; the influence of strong interactions in the two reactions will then be contained in multiplicative factors χ_+ and χ_- instead of $\frac{1}{2}(c_{84} \pm c_{20})$, where

$$\chi_{\pm} = \frac{1}{2}(2c_{84} \pm c_{20}) \approx \begin{cases} 1.17 \\ -0.26 \end{cases} \quad (3)$$

using (2). Thus $F^+ \rightarrow \pi^+ e^+ e^-$ is only slightly dynamically enhanced, similar to the K^+ decay ⁵⁾, while $D^0 \rightarrow \bar{K}^0 e^+ e^-$ is strongly suppressed by dynamical effects. One may mention that also $D^+ \rightarrow K^+ e^+ e^-$ will presumably have a small dynamical enhancement, but this doubly Cabibbo suppressed reaction is a dubious case for observation.

The differential rate for $F^+ \rightarrow \pi^+ e^+ e^-$ dependent on $e^+ e^-$ invariant mass, neglecting the electrons' rest mass, is then

$$\frac{d\Gamma}{dx} = \frac{\chi_+^2 \alpha^2 G^2 M_F^5 \cos^4 \theta_c}{192 x^5} \lambda^{1/2}(1, \mu^2, x^2) |f_{F\pi}(x^2 M_F^2)|^2 \quad (4)$$

where

$$x = \sqrt{(p_{e^+} + p_{e^-})^2} / M_F, \quad \mu = M_\pi / M_F, \quad \alpha = 1/137,$$

and, as usual $\lambda(a,b,c) = a^2 + b^2 + c^2 - 2ab - 2ac - 2bc$, giving $x_{\max} = 1 - \mu$; $f_{F\pi}(q^2 = (p_{e^+} + p_{e^-})^2)$ is defined by the T matrix element for the decay in the following way, provided $e^+ e^-$ come from a virtual photon,

$$T(F^+ \rightarrow \pi^+ e^+ e^-) = \chi_+ \frac{G}{\sqrt{2}} \frac{\alpha}{s^2} f_{F\pi}(q^2) \cos^2 \theta_c (k+p)_\mu \bar{u} \gamma^\mu v \quad (5)$$

k_α and p_α being the faon (F) and pion four-vectors. The novel feature in this reaction compared to meson decays studied earlier is that the q^2 dependence of the matrix element can in no way be neglected; $(q^2)_{\max}^{\frac{1}{2}}$ is more than twice the ρ mass so that the reaction probes non-trivial features of the mesons' electromagnetic form factors. Defining

$$\widetilde{h}_{F\pi} = \frac{J(\{h_{F\pi}(q^2)\}, M_\pi/M_F)}{J(\{1\}, M_\pi/M_F)}, \quad (6)$$

$$J(\{h_{F\pi}(q^2)\}, \mu) = \int_0^{(1-\mu)^2} dx^2 h_{F\pi}(x^2 M_F^2) \lambda^{1/2}(1, \mu^2, x^2)$$

one has the integrated rate, for $M_F = 2.03 \text{ GeV}^{10}$,

$$\Gamma(F^+ \rightarrow \pi^+ e^+ e^-) = |\widetilde{f}_{F\pi}|^2 \cdot 9.5 \times 10^5 \text{ s}^{-1} \quad (7)$$

which can also be related to the rate for the corresponding $F_{\ell 3}$ decay, assuming the η - η' mixing angle $\phi = \text{Arcsin} \frac{\sqrt{2}-1}{\sqrt{6}}$ and defining $\mu' = M_\eta/M_F$:

$$\begin{aligned} \frac{\Gamma(F^+ \rightarrow \pi^+ e^+ e^-)}{\Gamma(F^+ \rightarrow \eta e^+ e^-)} &= \left(\frac{\chi_+}{\sqrt{2}} \frac{\alpha}{\pi} \cos \theta_c \right)^2 \frac{|\widetilde{f}_{F\pi}|^2}{|\widetilde{f}_{F\eta}|^2} \frac{J(\{1\}, \mu)}{J(\{1\}, \mu')} \\ &= \frac{|\widetilde{f}_{F\pi}|^2}{|\widetilde{f}_{F\eta}|^2} \cdot 1.15 \times 10^{-5} \end{aligned} \quad (8)$$

Defined as in Eq. (8), the $F_{\ell 3}$ form factor $F_{F\eta}(q^2)$ has the normalization $F_{F\eta}(0)=1$ in the $SU(4)$ limit ⁷⁾.

By current algebra and generalized PCAC ⁵⁾, or equivalently by methods motivated by gauge theory ¹²⁾, $f_{F\pi}(q^2)$ is given by the sum of the diagrams in the Figure, with obvious common factors appearing explicitly in Eq. (4). References 5) and 12) have, however, different results for the pole diagrams and different sign relative to the contact diagrams, which incidentally made little difference in kaonic decay, but which will be seen to give clearly different results in the faonic case. By PCAC, for instance the virtual faon

(see Fig.1a) will be annihilated by the divergence

$$\partial_\mu (\bar{S}_L \gamma^\mu C_L) = -\frac{1}{2} f_F M_F^2 \varphi_F,$$

f_F , M_F and φ_F being the $F_{\ell 2}$ coupling, mass and field of the faon ⁵⁾. Since the faon is strongly off-shell, it has been argued ¹²⁾ that m_F^2 should be replaced by the actual value of the four-momentum squared. Assuming the view of Ref. 12) to be correct, one then calculates $f_{F\pi}(q^2)$ to be

$$\begin{aligned} f_{F\pi}(q^2) &= \frac{4\pi^2 f_F f_\pi}{(M_F^2 - M_\pi^2) q^2} \{ M_\pi^2 (F_F(q^2) - 1) - M_F^2 (F_\pi(q^2) - 1) \} \\ &\approx 4\pi^2 f_F f_\pi \frac{1}{q^2} (F_F(q^2) - 1) \end{aligned} \quad (9)$$

while the result of Ref. 5) corresponds to interchanging F_F and F_π in Eq. (9). These electromagnetic form factors can be expected to be very different at large q^2 . It will be seen that Eq. (9) leads to the less optimistic view of the reaction's observability.

From quark model diagrams additional terms in $f_{K\pi}(q^2)$ arise which were interpreted ¹²⁾ to be due to intermediate resonances instead of K^+ . From the effective angle dependence and parity conservation at the γ emission vertex one can argue that such terms should represent pseudoscalar states, i.e., the radially or angularly excited relatives of K^+ (or π^+). Due to the internal wave functions, excited states are obviously less important than the ground states when coupled to the external meson at the γ vertices. The q^2 dependence of these terms is, moreover, reminiscent of poles in the electromagnetic current, which should really arise from the quark model diagrams. One would then obviously perform double-counting by also introducing electromagnetic form factors in the elastic terms while keeping the extra [inelastic ¹²⁾] ones.

Following Eq. (9), the dominant contributions to the rate will come from $x \simeq M_\rho/M_F$. One can then consider the matrix element of the axial current between a pseudoscalar state and an on-shell vector state, to seek consistency. Fakirov and Stech's parameter-free expressions for the axial form factors ⁷⁾, based on nearest (mono-) pole dominance, reproduce the proportionality of pole diagrams to the squared mass of the intermediate pseudoscalar meson obtained by Ref. 5). [The matrix elements of Ali and Yang ¹³⁾, based on axial form factors without pseudoscalar poles and an asymptotic behaviour in dubious agreement with theoretical ideas ¹⁴⁾, is less valuable for a comparison.]

Thus the version of the present Eq. (9) advocated by Ref. 5), with $f_{F\pi}(q^2)$ dominated by the ϕ pole, has the lion's share of admittedly model-dependent support. Charm meson decays into purely hadronic states ⁷⁾ may help to clarify whether the pole dominance models are reliable for high mass pseudocalars.

Following Ref. 7),

$$\widetilde{F_{F\pi}}^2 = 1.37 \quad (10)$$

which will be inserted in Eq. (8). Evaluating $\widetilde{f_{F\pi}}^2$, one observes first that a simple ad hoc expression

$$F_F(q^2) = \frac{1}{(1 - \frac{q^2}{(m_V - \frac{1}{2}i\Gamma_V)^2})} , \quad m_V = 0.764 \text{ GeV}, \Gamma_V = 0.119 \text{ GeV} \quad (11)$$

fits at least time-like data on the pion's electromagnetic form factor up to $\sim 1 \text{ GeV}^2$ as good as any of the more elaborate conventional expressions ¹⁵⁾. Using Eq. (11), one has by Eqs. (6) and (9) :

$$\widetilde{f_{F\pi}}^2 = 9.5 \quad (\rho \text{ dominance}) \quad (12)$$

with $f_\pi = 0.120 \text{ GeV}$ and $f_F \cong f_K = 0.145 \text{ GeV}$. One can argue that the actual value of $\widetilde{f_{F\pi}}^2$ may well be a factor ~ 2 larger since one expects ¹⁶⁾ $f_F \gtrsim f_K \gtrsim f_\pi$. The enhancement factor contained in (12), compared to the "linearized" version ^{5), 12)} of (11), is 6.8.

Due to the rather speculative character of the electromagnetic form factor $F_F(q^2)$, one cannot claim to give more than order-of-magnitude estimates for the PCAC result ⁵⁾ obtained by permuting F_F and F_π in Eq. (9). The integral over $(1/q^4)|F_F(q^2)-1|^2$ is dominated by the ϕ pole. Constructing $F_F(q^2)$ as a sum of a ϕ and a ψ term, each given by Eq. (11) multiplied by the $\bar{s}$ and c charges and with the ϕ and ψ masses and total widths substituted for m_V and Γ_V , one obtains

$$\widetilde{f_{F\pi}}^2 \sim 16 \quad (\phi \text{ dominance}) \quad (13)$$

The pole/linear enhancement factor is now ~ 37 . Again, the f_F coupling may make this estimate a factor ~ 2 larger. As discussed, the ϕ pole approach is more plausible than the ρ approach. However, since PCAC is strictly not justified at the F mass, one might leave to experiment to judge which approach to use; the e^+e^- mass should give a clean signature.

By Eqs. (7), (8), (10), (12) and (13), and using the estimate ^{6),7)}
 $\Gamma(F^+ \rightarrow \text{all}) \sim 6 \times 10^{12} \text{s}^{-1}$, the reliability of which is not clear, one has

$$\Gamma_1(F^+ \rightarrow \pi^+ e^+ e^-) = 9.0 \times 10^6 \text{s}^{-1} \quad (14)$$

$$\frac{\Gamma_1(F^+ \rightarrow \pi^+ e^+ e^-)}{\Gamma_1(F^+ \rightarrow \eta e^+ \nu_e)} = 8.0 \times 10^{-5} \quad (15)$$

$$BR_1(F^+ \rightarrow \pi^+ e^+ e^-) \sim 1.5 \times 10^{-6} \quad (16)$$

and

$$\Gamma_\phi(F^+ \rightarrow \pi^+ e^+ e^-) \sim 1.5 \times 10^7 \text{s}^{-1} \quad (17)$$

$$\frac{\Gamma_\phi(F^+ \rightarrow \pi^+ e^+ e^-)}{\Gamma_\phi(F^+ \rightarrow \eta e^+ \nu_e)} \sim 1.4 \times 10^{-4} \quad (18)$$

$$BR_\phi(F^+ \rightarrow \pi^+ e^+ e^-) \sim 2.6 \times 10^{-6} \quad (19)$$

where the subscripts refer to Eqs. (12) and (13), respectively. For the decays $K^+ \rightarrow \pi^+ e^+ e^-$ and $K^+ \rightarrow \pi^0 e^+ \nu_e$ one has roughly 5.4×10^{-6} and 2.6×10^{-7} for the experimental results analogous to (15) [(18)] and (16) [(19)] - the first one being reproduced by theory to within 30% ⁵⁾ irrespectively of which approach is used.

Thus the considerations of pole effects in $F^+ \rightarrow \pi^+ e^+ e^-$ state in a cleaner form some problems which are also present in $K^+ \rightarrow \pi^+ e^+ e^-$. If a combination of the "matrix element" enhancement and advanced experimental technology should make the faonic decay detectable, it might help to clear up these points, while giving more insight into the validity of generalized PCAC.

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FIGURE CAPTION

(a) Pole contributions, and

(b) contact contributions

to $F^+ \rightarrow \pi^+ \gamma$.

FIG.1