Ref.TH.2509-CERN

A SIMPLE ALGORITHM FOR QCD JETS

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ABSTRACT

A proposal is made, giving multiparticle (parton) spectra inside quark and gluon jets in terms of the tree diagrams of an effective field theory. The resulting "jet calculus" satisfies many consistency checks. A first sample of applications is given.

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In this short note we wish to present the essential points and easiest applications of a new and simple algorithm for dealing with multiparticle spectra inside QCD jets. Full details and further applications will be given elsewhere¹⁾.

We first give the proposal and attempt to justify it. We then present several non-trivial consistency checks of our "jet calculus" rules. Finally, we turn to some easy applications in QCD.

1. JET CALCULUS RULES AND THEIR JUSTIFICATION

We start by recalling results which, by now, are widely believed to hold for single-particle^{*)} spectra inside a jet:

i) On the basis of leading log calculations²⁾³⁾ one finds that single-particle inclusive cross-sections, when normalized to the jet-inclusive cross-section, are process independent, so that a fragmentation function $D^{i,a}(x, Q^2)$ for the process $i(p) \rightarrow a(xp) + \text{anything}$ can be defined.

ii) Such a fragmentation function, or better its moments^{**)}

$$D_n^{i,a}(Q^2) = \int_0^1 dx x^n D^{i,a}(x, Q^2) \quad (1)$$

satisfy, at the leading log level, an evolution equation similar to the one obeyed by deep inelastic structure functions, i.e.,

$$\frac{d}{d \log Q^2/\mu^2} D_n^{i,a}(Q^2/\mu^2) = \frac{\alpha_s(Q^2/\mu^2)}{2\pi} A_n^{ab} D_n^{i,b}(Q^2/\mu^2) \quad (2)$$

Here $\alpha_s(Q^2/\mu^2) = \alpha_s [1 + b \alpha_s \log Q^2/\mu^2]^{-1}$ is the one-loop QCD running coupling constant ($12\pi b = 11N_c - 2N_f$ for N_c colours and N_f flavours) and αA_n are the $O(\alpha)$ anomalous dimension matrices. A_n^{ab} can be seen as moments of parton decay probabilities, as first pointed out by Altarelli and Parisi⁴⁾:

$$A_n^{ba} = \int_0^1 dx x^n P(a \rightarrow b(x) + c(1-x)) \quad (3)$$

*) Throughout this paper we shall deal just with spectra of elementary QCD quanta (quarks and gluons) leaving to Ref. 1) the discussion of how they can be translated into hadronic spectra.

***) Our definition differs slightly from the usual one having x^{n-1} .

with⁴⁾:

$$\begin{aligned}
 P(q \rightarrow q+g) &= P(\bar{q} \rightarrow \bar{q}+g) = C_F \left(\frac{1+x^2}{1-x} \right)_+ ; \\
 P(q \rightarrow g+q) &= P(\bar{q} \rightarrow g+\bar{q}) = C_F \frac{1+(1-x)^2}{x} ; \\
 P(g \rightarrow q+\bar{q}) &= \frac{N_f}{2} [x^2 + (1-x)^2] ; \\
 P(g \rightarrow g+g) &= 2C_A \left[\frac{1-x}{x} + \left(\frac{x}{1-x} \right)_+ + x(1-x) - \frac{1}{12} \delta(x-1) \right] - \\
 &\quad - \frac{N_f}{3} \delta(x-1)
 \end{aligned} \tag{4}$$

where $C_F = (N_c^2 - 1)/2N_c$, $C_A = N_c$ and the $(f)_+$ distribution is defined by

$$\int_0^1 (f(x))_+ g(x) dx = \int_0^1 f(x) [g(x) - g(1)] dx$$

iii) The solution of Eq. (2) is simply:

$$D_n^{i,a}(Y) = \exp(A_n Y)_{ab} D_n^{i,b}(0) \tag{5}$$

where $Y = (2\pi b)^{-1} \log(1 + \alpha b \log Q^2/\mu^2)$ in the natural QCD evolution parameter (imaginary time, rapidity) with $Y = 0$ at $Q^2 = \mu^2$. Going to a conjugate variable E (energy, angular momentum) by

$$\bar{D}(E) = \int_0^\infty dY e^{-YE} D(Y) \tag{6}$$

one has simply

$$\bar{D}_n^{i,a}(E) = (E - A_n)_{ab}^{-1} D_n^{i,b}(0) \equiv \langle a | (E - A_n)^{-1} | i \rangle \tag{7}$$

where we have gone over to a vector-operator notation and $|i\rangle$ is given by $|i\rangle = \sum_b D_n^{i,b}(0) |b\rangle$.

Our proposal for an m particle spectrum inside a single jet can now be stated very simply. Consider, as a generalization of $\bar{D}_n^{i,a}(E)$, the quantity:

$$\bar{D}_{\{n_j\}}^{i, \{a_j\}}(E) = \int_0^\infty dY e^{-YE} \int_{j=1}^m \prod dx_j x_j^{n_j} \Theta(1 - \sum x_j) \frac{d\sigma(i \rightarrow \{a_j\} + X)}{\sigma_{tot} \cdot dx_1 \dots dx_j \dots dx_m} \quad (8)$$

Our rules for computing $\bar{D}_{\{n_j\}}^{i, \{a_j\}}(E)$, to be justified below, are:

- a) Consider all the tree diagrams for an effective ϕ^3 theory, in which energy flows from the initial single particle i to the final set $\{a_j, n_j\}$ (e.g., Fig. 1a).
- b) For each tree write the contribution like in old-fashioned perturbation theory as a product of propagators and vertices and sum over all inequivalent trees and time orderings. There is conservation of moment at each vertex so that all internal lines have a well defined moment in terms of the set $\{n_j\}$ (see again for example, the tree of Fig. 1a);
- c) The rules for propagators and vertices, depicted in Figs. 1b and 1c, are as follows:

- c.1) For a p particle intermediate state, write a factor $(E - E_{int})^{-1}$, where $E_{int} = \sum_{k=1}^p A_{n_k}$ and A_n is given in Eq. (3). Each A_{n_k} acts independently on the indices of the k^{th} line.
- c.2) For a vertex corresponding to $a(p) \rightarrow b(n) + c(n')$, $p = n + n'$, write a factor $P_{n, n'}^{bc, a}$ where, in analogy with Eq. (3):

$$P_{n, n'}^{bc, a} = \int_0^1 dx x^n (1-x)^{n'} \hat{P}(a \rightarrow b(x) + c(1-x)) \quad (9)$$

and $\hat{P}$ is just $P(a \rightarrow b + c)$, where terms proportional to $\delta(1-x)$ are omitted.

We now proceed to offer some justification for these simple rules [see Ref. 1) for more details]. Our proposal for a two-particle distribution reduces to the contribution of a single tree and reads:

$$\bar{D}_{n, m}^{i, ab}(E) = \langle a, b | (E - A_n - A_m)^{-1} | a', b' \rangle P_{n, m}^{a' b', i'} \langle i' | (E - A_{n+m})^{-1} | i \rangle \quad (10)$$

to be compared to Eq. (7). Going back to the variable Y one easily gets

$$D_{n,m}^{i,ab}(Y) = \sum_{\ell, \ell_1, \ell_2=0}^{\infty} \frac{(\ell_1 + \ell_2)!}{\ell_1! \ell_2! (\ell + \ell_1 + \ell_2 + 1)!} Y^{\ell + \ell_1 + \ell_2 + 1} \times$$

$$\times \langle a | A_n^{\ell_1} | a' \rangle \langle b | A_m^{\ell_2} | b' \rangle P_{n,m}^{a'b,i'} \langle i' | A_{n+m}^{\ell} | i \rangle \quad (11)$$

whereas:

$$D_n^{i,a}(Y) = \sum_{\ell=0}^{\infty} Y^{\ell} / \ell! \langle a | A_n^{\ell} | i \rangle \quad (12)$$

In terms of QCD diagrams, Eq. (12) can be seen as the contribution of the dressed planar ladders of Fig. 2a. These give³⁾ the leading log behaviour of single particle spectra in physical gauges (e.g., in an axial gauge). Picking up now a second particle b in one of the rungs of Fig. 2a, and recalling that they are dressed, we end up with the graphs of Fig. 2b, which are nicely symmetric under $a \leftrightarrow b$. Equation (11) then follows. The weight factor of each triple ladder comes from the ordering of the small angles over which we integrate to obtain the leading logs^{*}). Our definition of $P_{n,m}^{bc,a}$ and conservation of the moment at vertices is provided by conservation of longitudinal momentum. It is arguing this way that we have derived (and partly guessed) Eq. (11) and then obtained the simple form (10). Generalization of the above argument to m particle spectra has been done for $m \leq 4$ explicitly and, for all m , by induction. Even more interesting, we can verify several non-trivial consistency checks, to which we now turn.

2. CONSISTENCY CHECKS

We now give three consistency checks:

a) Momentum sum rules. These constraints, which are essentially a check of unitarity⁵⁾, read as follows in terms of our moments:

$$\sum_{a_{k+1}} D_{n_1 \dots n_k, l}^{i, a_1 \dots a_k a_{k+1}} = D_{n_1 \dots n_k}^{i, a_1 \dots a_k} \sum_{j=1}^k D_{n_1 \dots n_{j-1}, n_{j+1} \dots n_k}^{i, a_1 \dots a_k} \quad (13)$$

where $\sum_{a_{k+1}}$ means a sum over the species of the $(k+1)^{\text{th}}$ particle. The general proof of Eq. (13) is simple and elegant. One uses momentum conservation of $P(a \rightarrow b + c)$ and the definition of $P_{n,m}$ to obtain:

*) It is such ordering which provides the $1/\ell!$ weight of Eq. (12).

$$\sum_b A_i^{ba} = 0 ; P_{n,m+1} + P_{n+1,m} = P_{n,m} \quad (14)$$

$$P_{n,1}^{ab,i} = A_n^{ai} - A_{n+1}^{ai} \quad (15)$$

The right-hand side of Eq. (15) is the difference of inverse propagators typical of a Ward identity. One finds, for the case $k = 1$ of Eq. (13),

$$\sum_b \bar{D}_{n,1}^{i,ab} = \sum_b \langle a,b | \frac{1}{E-A_n} P_{n,1} \frac{1}{E-A_{n+1}} | i \rangle = \langle a | \frac{1}{E-A_n} - \frac{1}{E-A_{n+1}} | i \rangle \quad (16)$$

which is the desired result. For $k = 2$ there are three tree diagrams giving, from Eq. (15), six contributions to the left-hand side of Eq. (13). Three of them give the right-hand side, and the other three cancel by Eq. (14). Generalization to all k has been done, both directly and by induction [see Ref. 1) for more details].

b) Internal quantum number sum rules. If the parton i initiating the jet is a quark, the whole jet has to carry the internal quantum numbers of i . Such a conservation law gives for the moments:

$$D_{0,n,\dots,n_k}^{i,i a_1 \dots a_k} = D_{n,\dots,n_k}^{i,a_1 \dots a_k} - \sum_{j=1}^k D_{n,\dots,n_k}^{i,a_1 \dots a_j a_{j+1} \dots a_k} \int a_{j,i} \quad (17)$$

For $k = 1$ such a constraint can be checked as follows:

$$D_{0,n}^{i,i a} = \sum_{b \neq i} \langle a | (E-A_n)^{-1} | b \rangle \langle b | A_n | i \rangle \langle i | (E-\bar{A}_n)^{-1} | i \rangle \quad (18)$$

where $\bar{A}_n$ is just the matrix element of A_n connecting quark to quark and we have used

$$\bar{A}_0 = 0 \quad \text{and} \quad P_{0,n}^{ib,i} = A_n^{bi}$$

We can then write

$$\sum_{b \neq i} |b\rangle \langle b | A_n | i \rangle = \sum_b |b\rangle \langle b | (A_n - \bar{A}_n) | i \rangle = (A_n - \bar{A}_n) | i \rangle \quad (19)$$

and obtain

$$\bar{D}_{0,n}^{i,a} = \langle a | \frac{1}{E-A_n} | i \rangle - \delta_{ai} \langle i | \frac{1}{E-\bar{A}_n} | i \rangle = \bar{D}_n^{i,a} - \bar{D}_n^{i,a} \delta_{ai} \quad (20)$$

Generalization to $k = 2$ is straightforward, whereas a general proof of Eq. (17) has been obtained by induction (see Ref. 1)).

c) Rederivation of a result of Taylor in $\lambda\phi_6^3$. For six-dimensional ϕ^3 theory, J.C. Taylor has obtained (Ref. 6)), in the leading log approximation, the jet multiplicity generating function

$$G(u, Q^2) = \sum_{m=1}^{\infty} u^m \sigma_m(Q^2) \quad (21)$$

where σ_m is the m quanta exclusive cross-section. It is well known⁷⁾ that such a quantity can be related to the integrated multiparticle inclusive cross-sections, i.e., to the zero moments of our multiparticle spectra. In our framework then, the problem is reduced to computing the sum of all the trees with zero-moment propagators and an extra $(u-1)$ factor at each vertex.

This problem can be solved in various ways; for example, as done by Schwimmer⁸⁾ in a very analogous Reggeon calculus problem (some sign of intercept and coupling), by solving the classical equations of motion of that (one dimensional) field theory. Defining $F(u, Q^2) = G(u, Q^2)/G(1, Q^2) - 1$ one finds the equation:

$$\frac{d}{dY} F(Y) = A_0 F(Y) + \frac{1}{2} P_{0,0} F^2(Y); \quad (22)$$

$$F(0) = (u-1),$$

where, with an interaction Lagrangian $-1/3! \lambda \phi_6^3$ one finds:

$$Y = \frac{4}{3} \log \left(\frac{\bar{\alpha}(m^2/\mu^2)}{\bar{\alpha}(Q^2/\mu^2)} \right); \quad \bar{\alpha}(Q^2/\mu^2) = \alpha \left[1 + \frac{3}{4} \alpha \log Q^2/\mu^2 \right]^{-1}; \quad \alpha = \frac{\lambda^2}{(4\pi)^3}$$

$$P(x) = x(1-x) - \frac{1}{12} \delta(x-1); \quad A_n = \frac{1}{(n+2)(n+3)} - 1/12$$

$$P_{n,m} = B(n+2, m+2); \quad A_0 = \frac{1}{2} P_{0,0} = 1/12 \quad (23)$$

One then solves Eq. (22) and obtains:

$$\frac{G(u, Q^2)}{G(1, Q^2)} = \frac{u}{u + (1-u) \left[\frac{\bar{\alpha}(u^2/\mu^2)}{\bar{\alpha}(Q^2/\mu^2)} \right]^{1/3}} \quad (24)$$

which is Taylor's result [Ref. 6), Eq. (6)].

3. APPLICATIONS TO QCD JETS

Having gained confidence in the general formalism through the above consistency checks, we now turn to some simple applications in QCD, leaving more complicated (but, perhaps, also more interesting) ones to further work.

a) One-particle spectra and average multiplicities. We derive (occasionally rederive) here some simple results on single particle spectra with special emphasis on the comparison of quark and gluon jets. We start by computing the fraction $x_{q,g}$ of the total momentum carried by quarks and gluons in a quark or gluon jet. Such four quantities are given by:

$$\bar{D}_1^{i,a} = \langle a | \frac{1}{E - A_1} | i \rangle \quad ; \quad i, a = q, \bar{q}, g \quad (25)$$

In QCD A_1 takes the form

$$A_1 = \begin{pmatrix} -\frac{4}{3}C_F & N_f/3 \\ \frac{4}{3}C_F & -N_f/3 \end{pmatrix} \stackrel{N_c=3}{=} \begin{pmatrix} -16/9 & N_f/3 \\ 16/9 & -N_f/3 \end{pmatrix} \quad (26)$$

The largest eigenvalue of A_1 is zero, with eigenvector $(1, 4C_F/N_f)$, giving at large Q^2 :

$$X_q \Big|_{q \text{ or } g \text{ jet}} = 1 - X_g \Big|_{q \text{ or } g \text{ jet}} = \frac{N_f}{4C_F + N_f} \stackrel{N_c=3}{=} \frac{3N_f}{16 + 3N_f} \quad (27)$$

Clearly, we cannot distinguish a quark from a gluon jet by x_q . In order to consider the average momentum carried by a particle of each type, we have to compute the average number of particles $D_0^{i,a}$. Unlike the case of ϕ_6^3 , in QCD such a quantity suffers from infra-red divergences and an absolute prediction would be both unreliable and of little use (because of confinement). We found, however,

that the ratio of multiplicities of a fixed species in a quark and in a gluon jet is free from such divergences and is simply given by

$$\frac{\langle n_q \rangle|_{qjet}}{\langle n_g \rangle|_{gjet}} = \frac{\langle n_q \rangle|_{qjet}}{\langle n_g \rangle|_{gjet}} = \frac{C_F}{C_A} = \frac{1}{2} \left(1 - \frac{1}{N_c^2}\right) \quad (28)$$

This result comes from the fact that the eigenstate of the largest (indeed infrared divergent) eigenvalue of the matrix A_0 projects onto single quark and gluon states in a ratio C_F/C_A . We then conclude that

$$\langle x_q \rangle|_{qjet} = \frac{x_q}{\langle n_q \rangle|_{qjet}} = C_A/C_F \langle x_q \rangle|_{gjet} \quad (29)$$

and the same for $\langle x_g \rangle$. Equation (29) confirms the naïve expectation that quark jets are harder and that gluon jets yield higher multiplicities.

The same conclusion comes from the study of the region $x \sim 1$. This can be done by looking at large moments. For large n , A_n had diagonal matrix elements of order $\log n$ and off-diagonal ones of order $1/n$. Straightforward calculations then yield (see also the first paper of Ref. 3)).

$$\frac{\frac{1}{\sigma} \frac{d\sigma}{dx_q}|_{qjet}}{\frac{1}{\sigma} \frac{d\sigma}{dx_g}|_{gjet}} \underset{x \rightarrow 1}{\sim} \frac{N_f}{2(C_A - C_F)} \frac{(1 - x_q)}{(2C_F \gamma) \log(1 - x_q)} \quad (30)$$

Exactly the same result is found for the ratio of gluon spectra near $x = 1$. Furthermore, the gluon spectrum is softer than the quark spectrum by a factor $(1 - x)$. Since Eqs. (29) and (30) hold both for quarks and for gluons, one is tempted to believe them for hadrons as well.

An amusing interpretation of Eq. (28) is possible in the limit of large N_c, N_f ⁹⁾¹⁰⁾. In this case $C_A/C_F \rightarrow 2$ and the result (28) can be interpreted simply as a gluon producing a "two-sided" jet compared to the "one-sided" quark jet (see Fig. 3). The situation could be similar to the one of a Pomeron jet versus a Reggeon jet (or a SPEAR jet), in which case interesting differences should also show up at the two-particle correlation level, in particular for what concerns Bose-Einstein interferences¹¹⁾.

b) Two-particle spectra and correlations. Work is in progress on trying to extract all the information on two-particle correlations contained in our proposal. Here we limit ourselves to some simple results that can be obtained analytically.

i) We start by a check of factorization when one particle is fast (finite x) and the other is slow ($x \sim 0$). This we do by considering the correlation

$$C_{n\epsilon}^{i,ia} \equiv \bar{D}_{n\epsilon}^{i,ia} - \bar{D}_n^{i,i} \cdot \bar{D}_\epsilon^{i,a} \quad (31)$$

where $\epsilon \ll 1$ and we have taken the fast particle to be the incoming quark line. One has

$$\begin{aligned} \bar{D}_{n\epsilon}^{i,ia} &= \langle i, a | (E - A_\epsilon - \bar{A}_n)^{-1} | i, b \rangle P_{\epsilon n}^{bi,i} \langle i | (E - \bar{A}_{n+\epsilon})^{-1} | i \rangle \\ \bar{D}_n^{i,i} \cdot \bar{D}_\epsilon^{i,a} &= \langle i, a | (E - A_\epsilon - \bar{A}_n)^{-1} | i, i \rangle \end{aligned} \quad (32)$$

Since A_ϵ has a positive eigenvalue for $\epsilon \ll 1$, the pole at $E = A_\epsilon + \bar{A}_n$ dominates. Also:

$$P_{\epsilon n}^{bi,i} = \int dx x^\epsilon (1-x)^n \hat{P}(q \rightarrow q+q) = A_\epsilon^{bi} + O(\epsilon) \quad (33)$$

One then finds that the leading term (as $Y \rightarrow \infty$) in $C_{n\epsilon}^{i,ia}$ goes to zero as $\epsilon \rightarrow 0$.

ii) Let us now consider $D_{n,m}^{i,ab}$ for both n and $m \ll 1$. This calculation should give us some information on the quantity $\langle n_a n_b \rangle$ (or $\langle n_a (n_a - 1) \rangle$ for $a = b$). Unlike the case of $\lambda \phi_6^3$, where all the moments of the multiplicity distribution were computed by an explicit sum over trees, here life is not so simple for two reasons. First, the limit $n, m \rightarrow 0$ is singular (infra-red divergence) and, second, one obtains a set of coupled differential equations which, in general, cannot be solved analytically. We have thus computed directly $D_{\epsilon\epsilon}^{i,ab}$ for $\epsilon \ll 1$. One finds, of course, that the pole at $E = 2A_\epsilon$ dominates, hence $\langle n(n-1) \rangle_\epsilon \sim \langle n \rangle_\epsilon^2$. The constant of proportionality is larger than one [in ϕ_6^3 one had $\langle n(n-1) \rangle = 2(\langle n \rangle^2 - \langle n \rangle)$] indicating long-range correlations perhaps within a KNO scaling¹²⁾ framework. It is also interesting to compare quark and gluon jets. For the ratio, again, infra-red divergences cancel out and one finds:

$$\lim_{\epsilon \rightarrow 0} \frac{\langle n_g(n_g-1) \rangle_\epsilon |_{gjet}}{\langle n_g(n_g-1) \rangle_\epsilon |_{gjet}} = \frac{1}{4} \frac{C_F}{C_A} + \frac{3}{4} \left(\frac{C_F}{C_A} \right)^2 \Big|_{N_c=3} = 7/27 \quad (34)$$

The same ratio is obtained for $\langle n_g(n_g - 1) \rangle_\epsilon$ and $\langle n_g n_q \rangle_\epsilon$.

iii) Finally, we would like to study correlations among two fast particles (which is experimentally the most accessible case). In particular, one would like to see if there is, indeed, some evidence that gluon jets, unlike quark jets, have the double multiperipheral structure of Fig. 3b with weaker dynamical correlations. This presumably cannot be done analytically, because it involves inverting back to momentum a double transform. All we can say so far is that we have considered $D_{nm}^{i,ab}$ for both n and $m \gg 1$ and that the results appear to be consistent with behaviours of the type $(1 - x_1 - x_2)^{k(Y)}$ for $x_1 + x_2 \rightarrow 1$ and of the type $(1 - x_1)^{k_1(Y)} (1 - x_1 - x_2)^{k_2(Y)}$ for $1 - x_1 \ll 1$ and $1 - x_1 - x_2 \ll 1 - x_1$.

ACKNOWLEDGEMENTS

We are happy to acknowledge interesting discussions with D. Amati, M. Ciafaloni and G. Parisi. Special thanks are due to A. Schwimmer who has provided us with most stimulating criticism. One of us (K.K.) wishes to thank the Theoretical Physics Division of CERN for the very warm hospitality enjoyed during this work.

REFERENCES

- 1) K. Konishi, A. Ukawa and G. Veneziano, in preparation.
- 2) For one-loop results see, for specific processes, H.D. Politzer, Nuclear Phys. B129 (1977) 301; C.T. Sachrajda, Phys. Letters 73B (1978) 185 and CERN TH.2459 (1978). K.H. Craig and C.H. Llewellyn Smith, Phys. Letters 72B (1978) 349, and, for a general process, D. Amati, R. Petronzio and G. Veneziano, CERN preprint TH.2470 (1978).
- 3) For the generalization to n loops:
In QCD: Yu.L. Dokshitzer, D.I. Dyakonov and S.I. Troyan, Proc. XIII Winter School of LNPI, Leningrad (1978), vol. I, p. 3. D. Amati, R. Petronzio and G. Veneziano, in preparation; C.H. Llewellyn Smith, private communication; J. Frenkel, M.J. Sheiler and J.C. Taylor, Oxford preprint 25/78 (1978); H. Georgi, private communication.
In ϕ_6^3 : P.M. Fishbane and T.M. Yan, private communication.
- 4) G. Altarelli and G. Parisi, Nuclear Phys. B126 (1977) 298.
G. Parisi, Proc. 11th Rencontre de Moriond (1976), ed. J. Tran Thanh Van.
- 5) G. Veneziano, Phys. Letters 36B (1971) 397 and Phys. Rev. Letters 28 (1972) 578.
- 6) J.C. Taylor, Phys. Letters 73B (1978) 85.
- 7) A.H. Mueller, Phys. Rev. D4 (1971) 150.
- 8) A. Schwimmer, Nuclear Phys. B94 (1975) 445.
- 9) G. 't Hooft, Nuclear Phys. B72 (1974) 461.
- 10) G. Veneziano, Nuclear Phys. B117 (1976) 519.
- 11) A. Giovannini and G. Veneziano, Nuclear Phys. B130 (1977) 61.
- 12) Z. Koba, H.B. Nielsen and P. Olesen, Nuclear Phys. B40 (1972) 317.

FIGURE CAPTIONS

Fig. 1 : Feynman rules of our "jet calculus":

- a) Example of a tree for a five-particle spectrum;
- b) Rule for intermediate states;
- c) Rule for the vertex.

Fig. 2 : a) Diagrams for a single-particle spectrum $i \rightarrow a + x$.
b) Diagrams for a two-particle spectrum $i \rightarrow a + b + x$.
Solid lines are dressed quark or gluon propagators joined at dressed vertices.

Fig. 3 : Processes contributing to:

- a) quark jet;
- b) gluon jet in the large N limit, in the graphical notation of Refs. 9) and 10).

$$\begin{matrix} b_1 & n_1 & a_1 \\ b_2 & n_2 & a_2 \\ \vdots & \vdots & \vdots \\ b_p & n_p & a_p \end{matrix} = \left(\frac{1}{E - \sum_{k=1}^p A_{n_k}} \right) b_k a_k$$

(b)

$$\begin{matrix} & & b \ n \\ n+n' & \nearrow & \\ a & & \\ & \searrow & c \ n' \end{matrix} = P_{n,n'}^{bc,a}$$

(c)

FIG.1

FIG. 2

FIG. 3