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DYNAMICAL AND BOSE-EINSTEIN CORRELATIONS OF CENTRALLY PRODUCED
PION PAIRS IN HADRON-HADRON COLLISIONS^{x)}

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ABSTRACT

We calculate dynamical and Bose-Einstein inclusive correlations of pion pairs (as functions of the invariant pair mass $M_{\pi\pi}$) from the decay of independently produced clusters, whose finite size is explicitly taken into account. We show that recent pp bubble chamber data from FNAL on the correlation functions $C_2(M_{\pi\pi})$ for centrally produced $\pi^+\pi^-$ and $\pi^-\pi^-$ pairs can be understood within this picture. Some implications for Hanbury-Brown Twiss type analyses are discussed.

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1. INTRODUCTION

Experiments have revealed strong short-range correlations in rapidity among centrally produced particle pairs at energies $E_{lab} \gtrsim 100 \text{ GeV}$ ¹⁾. These rapidity correlations can be understood as being due to the intermediate production of clusters which subsequently decay into the observed final-state hadrons²⁾. In rapidity space the most sensitive observable quantity to determine cluster properties from experiment is the strength of the short-range correlation, which is related to the ratio $\langle n(n-1) \rangle_c / \langle n \rangle_c^2$ of the first two moments of the cluster decay multiplicity distribution (averaged over the produced cluster spectrum). Assuming independent emission of clusters, one arrives at an average cluster mass $\bar{M}_c \approx 1.3 \text{ GeV}$ and an average charge multiplicity $\langle n_{CH} \rangle_c \approx 2$ ³⁾. These numbers are consistent with other final-state densities observed, such as gap distributions, zone characteristics, semi-inclusive correlations as well as charge-transfer correlations measured in rapidity space³⁾.

These small numbers and the very fact that high statistics bubble chamber experiments at $E_{lab} \lesssim 20 \text{ GeV}$ have revealed the importance of inclusive resonance production⁴⁾ suggest to use the invariant mass of the pair, $M_{\pi\pi}^2 = (p_1 + p_2)^2$, rather than its relative rapidity, $\Delta y = y_1 - y_2$, as variable to plot the correlation c_2 against; while the shape of $c_2(\Delta y)$ is only determined by the (assumed) isotropy of cluster decay⁵⁾, the shape of $C_2(M_{\pi\pi})$ would be expected to exhibit additional information as, e.g., resonance production^{6), 7)}.

Besides these dynamical effects due to the intermediate cluster production, one expects identical pions emerging from hadron collisions to show Bose-Einstein (BE) correlations^{*)}. In fact it has been repeatedly suggested to utilize this analogue of the Hanbury-Brown Twiss effect to determine the size of the pion emitting source in hadron collisions experimentally⁹⁾. Here again, the variable $M_{\pi\pi}$ is much more suitable than Δy , because there is a one-to-one correspondence between the BE symmetry point $\vec{p}_1 = \vec{p}_2$ and the threshold mass $M_{\pi\pi}^0 = 2m$ (m being the pion mass). Although the BE effect is clearly established experimentally in azimuthal correlations¹⁰⁾ and in the Kopylov variables $\vec{q}^2 = \vec{p}_1 - \vec{p}_2$ ¹¹⁾ as well as in $q^2 = 4m^2 - M_{\pi\pi}^2$ ¹²⁾, the procedures to actually extract the interaction volume from data are plagued by the problem of how to disentangle background,

*) The implications of BE effects in the uncorrelated jet model have been investigated in Ref. 8).

(i.e., dynamical) from BE short-range correlations^{10),11),12)}. Moreover, the situation is confused by the expectation, that abundant production of resonances of width $\Gamma \lesssim 100$ MeV would result in a tremendously narrow BE correlation width, $\Delta M_{\pi\pi} \approx 10$ MeV^{13),14)}, which is below present experimental resolution.

In this paper, therefore, we set out to investigate the relative importance of Bose-Einstein and dynamical correlations from cluster decay using the invariant pair mass $M_{\pi\pi}$ as the appropriate variable. As far as dynamics is concerned we need, of course, some theoretical prejudice. We shall adopt the attitude to work in a framework which is as simple as possible. Therefore we recur to the old independent emission model of clusters with mean properties borrowed from "rapidity physics". We differ however from previous authors in the treatment of cluster decay. In a preceding paper we have shown, that neither a factorized form of the two particle density nor the thermodynamical manner of including Bose statistics is appropriate when evaluating the decay of a cluster of small mass such as 1.3 GeV¹⁵⁾. Therefore we shall in this work carefully take account of the lightness of clusters by (painfully) imposing energy momentum conservation, isospin invariance and Bose statistics in cluster decay.

We should mention that this work was motivated by the recent publication⁶⁾ of first high-energy two-particle correlations as functions of $M_{\pi\pi}$ and subsequent interpretations^{6),16)} of these data. The conclusion reached in Ref. 16) was that the cluster model fails in explaining the large correlations observed in the region $M_{\pi\pi} \lesssim m_\rho$. Confirming our earlier preliminary results¹⁷⁾, we shall demonstrate in this paper that this conclusion is false and that the cluster picture can, indeed, accommodate for the inclusive correlation data from the 205 GeV/c pp bubble chamber experiment^{*)} as presented in Ref. 6).

2. THE MODEL

Intuitively the most attractive way to view clusters is to describe them as fireballs à la Hagedorn¹⁸⁾, since his fireballs imply both resonances and their interactions in an average sense. Interactions of resonances are expected to occur due to the observed density of secondaries in rapidity space¹⁹⁾. Or, to put it differently, it is unlikely that established resonances are produced independently; they rather cluster into larger structures, i.e., fireballs.

*) Unfortunately, the results of the ISR experiment published in Ref. 7) are not useful for an independent analysis since they have not been corrected for acceptance biases.

It is important to note that the thermodynamic limit cannot be taken when treating fireballs of mass $M_c = 1.3 \text{ GeV}$ ¹⁵⁾. Therefore one is led to the microcanonical description of fireballs as given by Frautschi and Hagedorn²⁰⁾. Unfortunately, their statistical bootstrap scheme has not yet been solved with Bose statistics and isospin included. Being faced with fairly light clusters, however, one may well accept a phase-space description (à la Fermi) as a good approximation to fireball decay (à la Hagedorn).

So we actually calculate our distributions from the density of states $\tau(Q^2)$ for a finite ideal Bose gas of mass $M_c = \sqrt{Q^2}$. For isospin zero pions, the formula for $\tau(Q^2)$ has been given by Chaichian et al.²¹⁾ in terms of a "cluster expansion"

$$B\tau(Q^2) = \sum_{k=1}^{\infty} \sum_{n_1, \dots, n_k=0}^{\infty} h\{n_i\} \cdot B^k \cdot \Omega^{(k)}(Q^2, \underbrace{m, \dots, m}_{n_1 \text{ times}}, \dots, \underbrace{k m, \dots, k m}_{n_k \text{ times}}), \quad (1)$$

with

$$k = \sum_{r=1}^k n_r, \quad \text{and} \quad h\{n_i\} = \prod_{r=1}^k \frac{e^{-3n_r}}{n_r!}.$$

Here Q stands for the cluster four-momentum, and $\Omega^{(k)}$ is the Lorentz invariant momentum space integral for k particles of indicated masses. B is a constant of dimension $[L^2]$ that one would expect to be of the order

$$B \approx \frac{\pi}{h^2 \cdot m^2} = 4.1 \text{ GeV}^{-2},$$

if $m = 140 \text{ MeV}$.

The terms in Eq. (1) with $k > 1$ can be thought of as being composed out of one or more BE clusters that are defined as "condensate droplets" consisting of i pions of identical momenta. Thus each contribution to $\tau(Q^2)$ is characterized by the number of times, n_i , a BE cluster appears in it, where $2 \leq i \leq k$. The partial sum corresponding to $k = 1$ is nothing but the Boltzmann statistics expression for τ . The total pion number from the term described by the set

$\{n_i\} \equiv \{n_1, \dots, n_k\}$ is given by $\sum_{r=1}^k n_r$. Therefore, if one wants to include all contributions up to a given pion number N , one is faced with a partition problem. For $N = 16$, e.g., one has 914 such partitions.

The above defined BE clusters should not be confused with the usual notion of clusters as intermediate structures in hadron-hadron collisions. To ensure clarity, we shall call the latter clusters henceforth strong interaction (SI) clusters.

The generalization of Eq. (1) to the case of pions with isospin 1 has been derived by Kripfganz²²⁾. The result is an expansion in BE clusters that are degenerate still in their charge composition: instead of by one number i , the BE cluster is now characterized by a set $\{i\} = (i_+, i_-, i_0)$, the numbers of π^+ , π^- , π^0 it contains, respectively. The resulting number of partitions grows so rapidly with N that the problem exceeds the capacity of a big computer. We observe however, that the coefficients $h\{n_i\}$ given in Ref. 22) are small and even negative, whenever they involve BE clusters that consist of pions of different charges. Therefore, we approximate the problem by retaining only the manifestly positive "diagonal" terms, whose BE clusters contain only equally charged pions. In this case we obtain from Eqs. (22) and (23) of Ref. 22) the following natural generalization of Eq. (1) for the level density of an SI cluster with isospin (I, I_3) :

$$B \cdot \tau_{I_3}^I(Q^2) = \sum_{k=1}^{\infty} \sum_{\{n(i,q)\}} h_{I_3}^I \{n(i,q)\} \cdot B^k \cdot \Omega^{(k)}(Q^2, \underbrace{m_1, \dots, m_{n_1}}_{n_1 \text{ times}}, \underbrace{k_1, \dots, k_{n_1}}_{n_1 \text{ times}}), \quad (2)$$

with

$$n_r' = \sum_{q=\pm, 0} n(r, q), \quad l = \sum_{r=1}^k n_r'.$$

$n(r, q)$ denotes the number of times a BE cluster composed of r pions of charge q appears. The coefficients h depend on the whole set $\{n(i, q)\}$ and read

$$h_{I_3}^I \{n(i,q)\} = P_{\{N_t\}}^{I, I_3} \cdot \prod_{t=+,0} \prod_{r=1}^k \frac{\tau_t^{-3n(r,t)}}{n(r,t)!} \quad (3)$$

$P_{\{N_t\}}^{I, I_3}$ denotes the standard Cerulus coefficients for an isospin (I, I_3) object decaying into the set $\{N_t\} = (N_+, N_-, N_0)$ of N_+, N_-, N_0 pions of charges $+, -, 0$, respectively²³⁾. Here $N_t = \sum_{r=1}^k n(r,t)$ and of course $I_3 = N_+ - N_-$. For $N \leq 16$ and $I = 1, I_3 = 0$ this leads to 9742 terms in the expansion, Eq. (2)!

It is straightforward to obtain the inclusive two-particle density $F_2(p_1, p_2)$ of an SI cluster of isospin (I, I_3) into two pions of charges t, t' :

$$F_{2, t, t'}^{I, I_3}(p_1, p_2) = C_{I_3}^I (Q^2)^{-1} \cdot \sum_{k=2}^{\infty} \sum_{\{n(i,q)\}} h_{I_3}^I \{n(i,q)\} \cdot B^k \cdot$$

$$\cdot \left[\sum_{j, j'=1}^k j^3 j'^3 n(j,t) (n(j',t) - \delta_{jj'} \delta_{tt'}) \Omega^{(k-2)}(\tilde{Q}^2, \dots, \cancel{j}, \dots, \cancel{j'}) \right] \quad (4)$$

$$+ 2p_{10} \delta^{(3)}(\vec{p}_1 - \vec{p}_2) \delta_{tt'} \sum_{j=1}^k j^3 (j-1) n(j,t) \Omega^{(k-2)}((Q-jp_1)^2, \dots, \cancel{j}, \dots),$$

with

$$\tilde{Q} = Q - j p_1 - j' p_2.$$

Note that the integral over F_2 is normalized to the second moment over the SI cluster decay multiplicity distribution

$$\int \prod_{i=1}^2 \frac{d^3 p_i}{2p_{10}} F_{2, t, t'}^{I, I_3}(p_1, p_2) = \langle n_t (n_{t'} - \delta_{tt'}) \rangle^{I, I_3} \quad (5)$$

The δ function term in Eq. (4) is the most prominent consequence of Bose-Einstein statistics: it is due to two equally charged pions emanating out of one BE cluster. This contribution will in the following be called "condensation" term.

So far we have been only concerned with the very SI cluster properties. To make contact with experiment, we must formulate the independent emission hypothesis²⁴⁾. According to this hypothesis, the normalized inclusive one- and two-particle densities from a hadron-hadron collision are given in terms of an SI cluster yield function $\tilde{w}(Q^2)$ that is independent on the SI cluster momentum (we suppress isospin for the moment)

$$S_1(p_1) = \int d^4Q \tilde{w}(Q^2) F_1^Q(p_1), \quad (6)$$

$$S_2(M_{\pi\pi}) = \int \prod_{i=1}^2 \pi \frac{d^3 p_i}{2 p_{i0}} \delta(\sqrt{(p_1+p_2)^2} - M_{\pi\pi}) * \\ * \left\{ \int d^4Q \tilde{w}(Q^2) F_2^Q(p_1, p_2) + S_1(p_1) \cdot S_1(p_2) \right\}. \quad (7)$$

F_1^Q, F_2^Q are the inclusive one- and two-particle decay densities of an SI cluster at four-momentum Q . The structure of F_2^Q for $\vec{Q} = \vec{0}$ has been given in Eq. (4). The independent emission hypothesis is reflected in the very structure of the last term in Eq. (7).

Using the convolution

$$S_1 \otimes S_1(M_{\pi\pi}) = \int \prod_{i=1}^2 \pi \frac{d^3 p_i}{2 p_{i0}} \delta(\sqrt{(p_1+p_2)^2} - M_{\pi\pi}) S_1(p_1) S_1(p_2), \quad (8)$$

one defines the inclusive two-particle correlation

$$C_2(M_{\pi\pi}) = S_2(M_{\pi\pi}) - S_1 \otimes S_1(M_{\pi\pi}). \quad (9)$$

Within the independent emission hypothesis, Eq. (7), C_2 is simply given by

$$C_2(M_{\pi\pi}) = \int d^4Q \tilde{w}(Q^2) F_2(Q^2, M_{\pi\pi}), \quad (10)$$

with

$$F_2(Q^2, M_{\pi\pi}) = \int \prod_{i=1}^2 \frac{d^3 p_i}{2 p_{i0}} \delta(\sqrt{(p_1 + p_2)^2} - M_{\pi\pi}) F_2^Q(p_1, p_2). \quad (11)$$

Note that the $\vec{Q}$ dependence in $F_2(Q^2, M_{\pi\pi})$ has dropped out, since F_2 is an invariant function. The d^4Q integration in Eq. (10) is essentially over longitudinal phase space. We thus arrive at the simple form for C_2 :

$$C_2(M_{\pi\pi}) = \int dM_c w(M_c) F_2(M_c^2, M_{\pi\pi}), \quad (12)$$

where

$$w(M_c) = \int d^4Q \delta(\sqrt{Q^2} - M_c) \tilde{w}(Q^2). \quad (13)$$

It is now a trivial matter to include SI cluster isospin and pion charges

$$C_2^{t\bar{t}}(M_{\pi\pi}) = \int dM_c \left\{ \sum_{I, I_3} w^{I, I_3}(M_c) F_2^{I, I_3, t, \bar{t}}(M_c^2, M_{\pi\pi}) \right\}, \quad (14)$$

where t, t' denote the observed pion charges. To leave matters simple, we choose the shape of $w^{I, I_3}(M_c)$ to be independent of (I, I_3) :

$$w^{I, I_3}(M_c) = \alpha_{I, I_3} w(M_c).$$

This leads us to

$$C_2^{t, t'}(M_{\pi\pi}) = \int dM_c w(M_c) \sum_{I, I_3} \alpha_{I, I_3} \mathcal{F}_2^{I, I_3}(M_c^2, M_{\pi\pi}). \quad (15)$$

Here the function $w(M_c)$ contains the cluster production dynamics. In the following we shall choose a simple parametrization for it:

$$w(M_c) = (M_c - M_0)^\beta \exp(-\gamma M_c). \quad (16)$$

We adjust the model to the empirical findings from rapidity space distributions, i.e., we adjust β, γ, M_0 to reproduce $\bar{M}_c \approx 1.3$ GeV. A reasonable choice consists in putting $M_0 = 0.5$ GeV, $\beta = 4$, $\gamma = 6$ GeV⁻¹. Note that this value of γ is also suggested by inclusive p_T spectra. The parameter B in Eq. (4) was taken to be 4 GeV⁻², which yields $\langle n_{ch} \rangle_c = 2.1$ and therefore the reasonable average pion energy in the SI cluster rest system is 413 MeV. The normalizations α_{I, I_3} control the actual numbers of produced SI clusters; they must evidently depend on the primary energy and shall be discussed in the following section.

3. NUMERICAL RESULTS

We first admitted both isoscalar and isovector SI clusters but the correlations turned out to depend little on isospin I . Therefore we shall only present results for isovector SI clusters in the following. The calculations were done on a computer, using a program of Kajantie and Karimäki to evaluate the momentum-space integrals²⁵⁾. We allowed for SI cluster decay multiplicities up to 16.

For our first orientation, we display in Fig. 1 the correlation functions $C_2^{+-}(M_{\pi\pi})$ and $C_2^{--}(M_{\pi\pi})$ due to single SI clusters of various charges and discrete representative mass $M_c = 1.32$ GeV. The quantum number dependence of the shapes

of these distributions demonstrates our previous claim that the thermodynamic limit is too rough an approximation in this context. Note that C_2^{--} has been multiplied by factors of 4 and 16 in Figs. 1a and 1b, respectively.

C_2^{--} is clearly narrower than C_2^{+-} for neutral and positive clusters. The suppression of C_2^{--} can be understood as being mainly due to charge conservation: it needs at least a four-particle state to accommodate two negatives in an $I_3 = 0$ configuration and a five-particle state to get two negatives out of a $I_3 = +1$ configuration. Thus the suppression factors 4 and 16 are consequences of the decay multiplicity distribution of the SI clusters. For negatively charged SI clusters, it is equally probable to find pion pairs of the same and opposite charges. Therefore, the correlations C_2^{+-} and C_2^{--} are fairly much the same. Note that the "condensate" terms have not been added in Fig. 1. Its inclusion would in fact render the integrated correlations from negatively charged SI clusters exactly equal.

Next we compare the relative strength $\langle n_t(n_t - \delta_{tt}) \rangle / \langle n_t \rangle$ per cluster of definite mass M_c as a function of M_c (Fig. 2). The relative ordering of the different curves is again expected from charge conservation and charge symmetry. Since all these mean values show soon a straight line behaviour, the relative magnitudes of C_2^{+-} and C_2^{--} become sensitively dependent on SI cluster mass for both neutral and positively charged SI clusters. More interesting information is contained in Fig. 3, where we plotted the integrals of the F_2 densities, again per SI cluster of given M_c vs M_c , Fig. 3a, and the "condensate" terms contained in these integrals (in per cent) in the case of identical pions, see Fig. 3b. Typically, this contribution is of the order of 20-30% in the region of the observed average SI cluster mass, and it becomes the dominant term in the region of low SI cluster masses. The dependence on the SI cluster charge is appreciable.

After this prelude on cluster properties, let us smear out the SI cluster mass as described in the last Section, Eqs. (15) and (16). We have thus three parameters, α_{1I_3} , to adjust the model to the mean central pion multiplicities $\bar{n}_{\pi^+} = 3.8$ and $\bar{n}_{\pi^-} = 2.7$ measured in the $E_{lab} = 205$ GeV pp bubble chamber experiment⁶⁾ ["central" was defined in Ref. 6) by a cut in Feynman x , $|x| \leq 0.6$]. These numbers suffice to fix two of our parameters, or to put it differently, we can still play (within certain limits) with the number of, say, negative SI clusters. Setting their production rate equal to zero clearly minimizes C_2^{--} . In this case, we obtain $\bar{v}_0 = 1.97$ and $\bar{v}_+ = 1.1$ for the mean number of neutral

and positive SI clusters, respectively. The over-all cluster distribution function $\alpha \cdot w(M_c)$, $\alpha = \sum_{I_3} \alpha_{II_3}$, is plotted in Fig. 4. It corresponds to an average charge multiplicity per average SI cluster of $\langle n_{ch} \rangle_c = 2.12$ and $\bar{M}_c = 1.28$ GeV.

The comparison with the experimental data⁶⁾ is contained in Fig. 5. The full curves shown there are the model predictions without the "condensation" term in Eq. (4). We find that the observed experimental ratio

$$\int dM_{\pi\pi} C_2^{+-}(M_{\pi\pi}) / \int dM_{\pi\pi} C_2^{--}(M_{\pi\pi}) \approx 5$$

is correctly borne out by the model. Furthermore, the strong peaking of $\pi^+\pi^-$ correlations at $M_{\pi\pi} < m_\rho$ is described by the model as well. The $\pi^-\pi^-$ correlation, however, is evidently much larger in the low $M_{\pi\pi}$ region than the full curve predicts. In fact, the width of this curve is only slightly narrower (by about 40 MeV) than for $\pi^+\pi^-$. So we do need the "condensation" terms, to be discussed below. As far as the large $M_{\pi\pi}$ is concerned, on the other hand, we have to remember that our ideal gas approximation to fireball decay becomes poor at large M_c . In fact, the large mass tail of $C_2(M_{\pi\pi})$ is somewhat sensitive to the size of γ in Eq. (16) which is not the case for the proper fireball model¹⁷⁾. Therefore one should not attach too much weight to the large $M_{\pi\pi}$ behaviour of our approximation. In any case, one can easily understand the background under the ρ signal in $C_2^{+-}(M_{\pi\pi})$ as coming from three- and more-pion decay of SI clusters.

The "condensation" term that should be added to the full curve drawn in Fig. 5b for C_2^{--} amounts to $0.17 * \delta(M_{\pi\pi} - 2m)$. In order to visualize this term, we remember that its very δ function character stems from the usual limit of continuous counting of states in the derivation of the level density τ ²¹⁾. With a proper choice of a finite quantization volume, the δ function becomes smeared out to a function $C_{BE}^{--}(M_{\pi\pi})$ with a width σ related to the spatial extension of an SI cluster. Hence we make a properly normalized Gauss-ansatz for it:

$$C_{BE}^{--}(M_{\pi\pi}) = .17 \frac{2}{\sqrt{2\pi}\sigma} \exp(-(M_{\pi\pi} - 2m)^2 / (2\sigma^2)). \quad (17)$$

Naturally one would expect $\sigma = m^2 = 0.02 \text{ GeV}^2$. The corresponding prediction for the $\pi^+ \pi^-$ correlations is shown as a dashed curve in Fig. 5b. For the purpose of illustration, we have also included in this figure the cases $\sigma = 0.01 \text{ GeV}^2$ and $\sigma = 0.03 \text{ GeV}^2$. We see that the "condensation" terms add appreciably but not sufficiently to the low $M_{\pi\pi}$ peaking of C_2^{--} . Roughly speaking the SI cluster decay accounts for 80% of the observed low mass enhancement. One might be tempted to increase the number of neutral clusters in order to better reproduce the peak in C_2^{--} . In Fig. 6, we display the situation for the (arbitrary) choice $\bar{v}_0 = \bar{v}_-$. As expected, the C_2^{--} correlation does show a stronger enhancement for small $M_{\pi\pi}$ than previously found; but the general behaviour of the $\pi^+ \pi^-$ correlation is now significantly above the data.

As a grain of salt, the data indicate a dip structure at $M_{\pi\pi} = 700 \text{ MeV}$, which, if it persisted, would be a serious problem for such simple SI cluster emission models as ours.

To give an impression of the importance of energy-momentum conservation in SI cluster decay, we have included in Fig. 6a the results of the most simple-minded thermodynamical ansatz

$$F_2(p_1, p_2) \sim \left\{ \exp(E_1/kT) - 1 \right\}^{-1} \left\{ \exp(E_2/kT) - 1 \right\}^{-1}, \quad (18)$$

arbitrarily normalized, and for the two temperatures $T = 120 \text{ MeV}$ and $T = 180 \text{ MeV}$.

4. DISCUSSION AND SUMMARY

We have demonstrated that the main features of present inclusive correlation data for $C_2^{+-}(M_{\pi\pi})$ and $C_2^{--}(M_{\pi\pi})$ can be naturally understood and economically described in a conventional independent emission model for fireballs of isospin one and average properties known from rapidity space investigations. The dynamical correlations for $\pi^+ \pi^-$ and $\pi^- \pi^-$ pairs are predicted to be very similar in shape, with $\pi^- \pi^-$ suppressed by about a factor of 5.5. This result suggests to use the experimental $C_2^{+-}(M_{\pi\pi})$, normalized to match the experimental $C_2^{--}(M_{\pi\pi})$ for $M \gtrsim 700 \text{ MeV}$ as dynamical background when analyzing data to extract the Hanbury-Brown Twiss effect.

The "condensation" terms from SI cluster decay amount to about 30% of the observed $\pi^-\pi^-$ correlations for pair masses below 450 MeV. They seemingly need some augmentation by BE correlations between equal pions out of different clusters. Such correlations have been evaluated in resonance approximations by Thomas¹³⁾ and Grassberger¹⁴⁾. Inserting decay widths and presently known production rates of established resonances they arrive at effects which are concentrated within the mass region $280 \text{ MeV} \lesssim M_{\pi\pi} \lesssim 300 \text{ MeV}$. Taking clusters instead, we could expect a broadening of this region. However, the evaluation of these effects within our present statistical framework would need a number of additional assumptions on details of the cluster emission mechanism (such as long-range correlations and over-all energy momentum conservation) and should only be pursued in connection with semi-inclusive data^{*)}. Therefore, as far as the Bose-Einstein effect is concerned, our present treatment is complementary to Refs. 13) and 14)^{**)}.

To make further progress in the understanding of the short-range correlation phenomena, one evidently needs:

- semi-inclusive correlations as functions of the invariant masses of two and more particles (including neutrals) to observe higher resonances directly and test production mechanisms more severely;
- higher statistics data of good mass resolution to see details of resonance interference and to settle the above-mentioned dip problem in $C_2^{\pi\pi}(M_{\pi\pi})$;
- relevant ISR data to see possible energy dependences of correlation functions $C_2(M_{\pi\pi})$.

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*) The method used in Ref. 26) for investigating local strangeness conservation in pp annihilation might then be useful.

***) Recently, Giovannini and Veneziano²⁷⁾ suggested to eliminate correlations from within clusters (named jets in their paper) by forming certain combinations of differently charged pion pair distributions. Unfortunately, though, they assume their jets to decay independently of quantum numbers (like charge), which certainly does not hold for short-range correlation phenomena.

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FIGURE CAPTIONS

Fig. 1 : Correlation functions $C_2^{+-}(M_{\pi\pi})$ (dotted curves) and $C_2^{--}(M_{\pi\pi})$ (full curves) from single isovector SI clusters of mass $M_c = 1.32$ GeV and

- a) zero charge
- b) positive charge
- c) negative charge.

The correlations C_2^{--} have been scaled up by factors 4 and 16 in cases (a) and (b), respectively.

Fig. 2 : The correlation strength $\langle n_t(n_t, -\delta_{tt'}) \rangle / \langle n_t \rangle$ from a single SI cluster as a function of its mass M_c .

Full curve: $t = +, t' = -, \text{ neutral clusters.}$
Dashed curve: $t = +, t' = -, \text{ non-neutral clusters.}$
and: $t = -, t' = -, \text{ negatively charged clusters.}$
Dashed-dotted curve: $t = -, t' = -, \text{ neutral clusters.}$
Dotted curve: $t = -, t' = -, \text{ positively charged clusters.}$

Fig. 3 : a) The integral

$$\int \prod_{i=1}^2 \frac{d^3 p_i}{2p_{i0}} F_2(p_1, p_2)$$

from single cluster decay plotted vs SI cluster mass M_c . The meaning of the various distributions is the same as in Fig. 2.

b) The percentage of the "condensation" term [see text after Eq. (5)] contained in the curves of Fig. 3a) in the case of $\pi^+ \pi^-$ distributions:

- (---) negatively charged,
- (.....) positively charged,
- (-.-.-) neutral SI clusters.

Fig. 4 : SI cluster distribution assumed to be produced during collision [see Eq. (16)]. It corresponds to $\alpha = 4$, $\beta = \text{GeV}^{-1}$, $M_0 = 500$ MeV. Average cluster mass is $\bar{M}_c = 1.28$ GeV. Mean SI cluster numbers: $\bar{v}_- = 0$, $\bar{v}_+ = 1.1$, $\bar{v}_0 = 2$.

Fig. 5 : Inclusive correlations $C_2^{+-}(M_{\pi\pi})$ and $C_2^{--}(M_{\pi\pi})$ calculated from the SI cluster distribution shown in Fig. 4 (assuming no production of negative SI clusters). The full curve in Fig. 5b does not contain the "condensation" term. The latter has been added with various widths [see Eq. (17)]:

(---) $\sigma = 0.1 \text{ GeV}^2$,

(-·-·-·-) $\sigma = 0.02 \text{ GeV}^2$,

(·····) $\sigma = 0.03 \text{ GeV}^2$.

Data are taken from Ref. 6).

Fig. 6 : a) Inclusive correlation $C_2^{+-}(M_{\pi\pi})$.

Full curve: prediction from SI cluster decay, with equal amounts of neutral and negative clusters, $\bar{v}_0 = \bar{v}_- = 0.6$, $\bar{v}_+ = 1.7$.

Dashed (dotted) curve: calculated shapes from simple thermodynamical ansatz Eq. (18) for $T = 120$ (180) MeV.

b) Inclusive correlation $C_2^{--}(M_{\pi\pi})$ calculated from SI cluster decay as in Fig. 6a. The full curve does not contain the "condensation" term. The dashed-dotted curve included the "condensation" term with $\sigma = 0.02 \text{ GeV}^2$.

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5

Fig.6