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HOW QUARKS THAT ARE STRANGE
BUILD THEIR REGGE EXCHANGE

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ABSTRACT

The scattering of strange mesons can produce resonance states both with and without hidden strangeness. Making use of the very recent analyses of K^-p data at 4.2 GeV, we conduct a definitive test of the hypothesis, suggested by duality diagrams, that resonances made of non-strange quarks build the $f'-\varphi$ exchange in the crossed channel. For this, in fact to succeed the π' is determined to be close to an SU_3 singlet and have an equal amount of strange and non-strange quarks just as required by the quadratic version of the Gell-Mann-Okubo mass formula.

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1. INTRODUCTION: QUARK LINES AND DUALITY

Quark diagrams, together with two-component duality, have been a cornerstone of strong interaction phenomenology for quite some time; yet there still arise novel situations where these concepts can be tested and applied, not just in the new particle processes but also in the more familiar physics of old hadrons. A detailed test of these ideas in a situation only made possible as a result of new high statistics experiments is the purpose of this paper.

Our belief that resonance dynamics in the direct channel are dual to Regge exchange in the crossed channel is conveniently summarized by quark diagrams ¹⁾.

Considering only the dominant connected graphs, we picture meson-meson scattering as producing resonance states each of which are made, like the external mesons themselves, of a quark-antiquark pair. At high energies when such resonances are many and overlapping, this increasingly complicated process is seen simply in the crossed channel as the exchange of the corresponding Reggeon consisting also of a quark-antiquark pair. Moreover, even at lower energies when the produced resonances are far from numerous and well-separated, local duality implies that the resonance cross-sections are still described by this same Regge exchange in an average sense defined by finite energy sum rules.

While the equivalence of resonance and Regge descriptions is natural in such a picture of flowing quark lines, the physical situation is somewhat more complicated. This is because, in general, only a part of the physical amplitude is to be identified with the contribution of the quark exchange graphs: the s channel cross-section is not all resonances, but background too, and the t channel description requires not just Reggeons with quark exchanges, but also a pomeron term involving no quark or quantum number transfer. While diagrammatically this separation into two components - resonance and Regge, background and pomeron - is clear ²⁾, in practice with real experimental data it is not so simple; nevertheless such a separation is absolutely essential if we are to test the consequences of this concept of duality. We shall concentrate exclusively on the resonance-Regge component after this separation for reasons that will become clear.

Now if SU_3 symmetry were exact, there would be seemingly little difference between the $\pi^+\pi^-$ and K^+K^- channels, for example, and the resonance and Regge exchanges in each process would be trivially related.

However, the fact that the kaon is more massive than the pion is interpreted as being due to the λ quark being much heavier than either the p or n quarks. This not only separates the π and K , ρ and K^* , f and K^{**} masses, but, most importantly in the present context, splits those trajectories with no λ quarks from those with only strange quarks. Thus the f and f' trajectories, and similarly the ω and ϕ , are separated by roughly half a unit in intercept. This makes the presence of Regge exchanges formed from $\lambda\bar{\lambda}$ quarks immediately distinguishable from those made of $q\bar{q}$ ($q = p$ or n from now on) simply from the energy dependence of such processes.

This has important implications, as first emphasized by Inami and Miettinen ³⁾, for both the s and t channel descriptions of the scattering of strange mesons. In such a process, resonances both of the $\lambda\bar{\lambda}$ type and of the $q\bar{q}$ type can be produced, where duality diagrams imply that those made of $\lambda\bar{\lambda}$ quarks are dual to the standard $q\bar{q}$ Regge exchanges - the f - ϕ A_2 - ρ - while mesons with non-strange constituents are dual to the lower-lying Regge exchange of $\lambda\bar{\lambda}$ quarks - the f' - ω trajectory ^{*}).

Inami and Miettinen ³⁾ set about testing this proposition of two types of Regge exchange contributing to strange meson scattering, in which strangeness annihilation is possible, by looking at inclusive processes at large missing mass, where many overlapping resonances are supposedly produced. To see how such a test arises, consider the inclusive reactions $ap \rightarrow \lambda X$, with $a = p, \bar{p}, \pi^-, K^+$, in the target fragmentation region. These processes may then be regarded in the missing mass channel as the beam particle scattering off the $p\bar{\lambda}$ system, which is just a reggeized strange meson ^{**}). Each of these is then described in the large M^2 limit by pomeron exchange either alone for $a = p, K^+$, because of exchange degeneracy, or together with f - ω , A_2 - ρ exchanges for $a = \bar{p}, \pi^-$.

Now the couplings of these processes in the triple Regge region can all be related to each other and to the corresponding total cross-sections (σ_{ap}) by factorization, in agreement with experiment. With these couplings fixed, we are in a position to determine the contributions of both the pomeron and f - ω , A_2 - ρ exchanges to the related reaction $K^-p \rightarrow \lambda X$. However, this process, unlike the others we have discussed has a missing

*) Such a distinction would also arise in the scattering of charmed mesons, beautiful or truthful mesons, etc.

**) Exactly which meson this is, is an essential point of this paper which we discuss below.

mass channel with zero strangeness in which effectively strange mesons scatter. There, according to the rules, not only the $f-\omega$, $A_2-\rho$ Regge trajectories can be exchanged, but also the lower lying $f'-\phi$ trajectory, as depicted in Fig. 1.

Since it is only in a process like $K^-p \rightarrow \Lambda X$ that this type of component can occur and as the contributions of the other exchanges are "known", Inami and Miettinen ³⁾ looked for evidence in the integrated inclusive cross-section for this $f'-\phi$ exchange. Fortunately, because the cross-sections for K^-p and $K^+p \rightarrow \Lambda X$ (or K^-p and $\pi^-p \rightarrow \Lambda X$) are so sizeably different, this component is immediately apparent. Indeed the integrated cross-section for $K^-p \rightarrow \Lambda X$ behaves just like $s^{-1} \sim s^{\alpha_{\phi}(0)-1}$ (where the missing mass is integrated over ranges from 1 to 5 GeV depending on the energy). Moreover, at lower energies this additional component, only present in $K^-p \rightarrow \Lambda X$, appears to be the dominant contribution.

To check that this $f'-\phi$ exchange component expected by duality diagrams is really generated by the production of resonances made of non-strange quarks requires a much more refined test than this quick glance at the energy dependence of an integrated inclusive cross-section. Firstly we must be able to separate the resonance cross-sections from the background so that the notions of quark exchange really do apply and then be able to identify which of these resonance contributions come from non-strange quarks and which not, so as to specify the $\bar{\lambda}\lambda$ quark exchange component in the t channel. This is the aim of this paper.

The ideal place to attempt such an investigation would be in K^+K^- scattering, but, in practice, since its threshold is at 1 GeV and the separation of resonances with strange and non-strange quarks becomes increasingly difficult above this mass when the resonances overlap more and more, it is more straightforward to look once again at the process $K^-p \rightarrow \Lambda X$ (Fig. 1), in which the lowest mass state is the pion. Over the last few years, the ACNO Collaboration ^{*) 4)} have made an experimental investigation of the channel $K^-p \rightarrow \Lambda M$ at 4.2 GeV for neutral states M with masses up to the kinematic limit of 1.88 GeV. Within this mass range particular particle and resonance states have been isolated. These are the $\pi, \eta, \rho, \omega, \eta', \phi, f, A_2$ and f' . With this analysis completed ⁴⁾⁻⁸⁾ we are in a position

*) ACNO = Amsterdam, CERN, Nijmegen, Oxford Collaboration.

to test whether the non-strange quark content of these states does indeed build the $f' - \varphi$ exchange, while the strange quark content gives rise to a higher-lying trajectory. This we study below in Sections 3, 4 and 5 and give our conclusions in Section 6.

2. STRANGE MESON EXCHANGE: NATURAL AND UNNATURAL

In considering the inclusive processes $ap \rightarrow \Lambda X$ (with $a = K^-$ in particular) we have so far only discussed the Regge exchange dual to the missing mass channel. Now we must turn to the equally important question of what is the strange Regge exchange in the conventional t channel. In this channel, which is $\bar{p}\Lambda$, we can have all of K, K_A, K_B, K^*, K^{**} trajectories exchanged, which are the $S = \pm 1$ analogues of the π, A_1, B, ω, f trajectories, respectively. For convenience, we divide these into natural and unnatural parity exchanges, since by this division the natural exchanges lie on one exchange degenerate trajectory and the unnatural on another^{*)}. We denote the former trio of unnatural parity exchanges by K - the lowest mass state on these trajectories - and the latter natural duo similarly by K^* . In an analogous way, since the f, ω, A_2, ρ trajectories are closely degenerate, we call these collectively the ρ trajectory and the $f' - \varphi$ degenerate trajectories we denote by φ .

On Mueller-Regge analysis the inclusive double differential cross-section is given by

$$\frac{d^2\sigma}{dt dM^2} = \sum_{i,j,k} G_{ijk}(0,t,t) (\alpha' s)^{\alpha_i(t) + \alpha_j(t) - 2} \times (\alpha' M^2)^{\alpha_k(0) - \alpha_i(t) - \alpha_j(t)} \quad (1)$$

where $i, j = K, K^*$ and $k = \rho, \varphi$ in the nomenclature just defined. The coupling G_{ijk} is then really the sum of the appropriate individual reggeon couplings^{**)} . The interference terms $i \neq j$ play no rôle in our discussion so we shall not consider these further.

The important fact about the contributing strange meson trajectories is that having equal slopes

$$\alpha' = (m_{K^{**}}^2 - m_{K^*}^2)^{-1} = 0.82 \text{ GeV}^{-2} \quad (2)$$

) That the Q_A, Q_B do not have the same mass, just like the A_1, B , is of little consequence here. The trajectories K_A, K_B are almost the same when compared with their difference from the K^ trajectory.

**) From now on we assume exchange degeneracy only for trajectories and not couplings.

they have intercepts differing by

$$\alpha_{K^*}(0) - \alpha_K(0) = 1 - \alpha'(m_{K^*}^2 - m_K^2) = 0.55 \quad (3)$$

This difference of half a unit in intercept means that the natural and unnatural parity exchanges dominate in different regions of phase space - of s, M^2 . The K^* having the higher intercept dominates as the energy increases at a fixed missing mass, while at a fixed energy the K wins over the K^* as the missing mass increases. Despite these quite different behaviours the data on inclusive Λ production are not so good that an unambiguous separation of natural and unnatural parity exchanges is possible. However, if an effective strange meson trajectory [some $\alpha_i(t) = \alpha_j(t)$ in Eq. (1)] is used in a triple Regge analysis at fixed energy of either $K^-p \rightarrow \Lambda X$ at 14.3 GeV, or $\bar{p}p \rightarrow \Lambda X$ at 5.7 GeV or $pp \rightarrow \Lambda X$ at 69 GeV, each of Paller et al. ⁹⁾, Ganguli and Sadoulet ¹⁰⁾ and Blumenfeld et al. ¹¹⁾, respectively, have found it to lie between the K and K^* trajectories. Is there some way we can in fact learn something about the natural and unnatural parity couplings and hence separate these exchanges ^{*})?

3. THE EXCHANGE BUILT BY RESONANCES WITH NO HIDDEN STRANGENESS

We have two interrelated problems to resolve. The first is: does the production of resonances with non-strange quarks build the $f' - \phi$ trajectory? To answer this, we must investigate the production of low mass states, and to be able to use finite mass sum rule duality ^{**)}, consider all those states with mass within some 1 GeV^2 interval. Then on the average we ask, does

$$\left\langle \frac{d^2\sigma}{dt dM^2} \right\rangle = \sum_{i=K, K^*} g_{ii\phi}(t) \frac{(\alpha' s)^{2\alpha_i(t)-2}}{(\alpha' M^2)^{\alpha_\phi(0)-2\alpha_i(t)}} \quad (4)$$

From this form, it is immediately clear that we can only check that this average is correctly given by Eq. (4), if we know what are the relative weights of the K, K^* trajectories.

^{*}) It is because of the lack of such a separation that a previous analysis ¹²⁾ of these data ⁴⁾ in the whole missing mass region came to an inconclusive result.

^{**)} See, for example, Ref. 13) and references therein.

Fortunately, not only has an analysis by the ACNO collaboration of their K^-p data allowed the low mass resonances to be separated from the background ⁵⁾⁻⁸⁾, but also a helicity decomposition of the vector meson production ⁶⁾ allows the natural and unnatural parity exchanges to be separated for their production. Since the unnatural parity K exchange cannot contribute to the production of the pseudoscalar π, η, η' mesons, we concentrate on the natural parity exchange component. The forward production of helicity one vector mesons couple only to this exchange.

Since it is just the vector mesons that have been analyzed completely in this way, we consider resonance production $K^-p \rightarrow \Lambda M$ up to the ϕ mass. Since $\Delta M^2 = 1/\alpha' \simeq 1 \text{ GeV}^2$ is the standard "duality average", this will hopefully be sufficient. Thus $M = \pi, \eta, \rho(\lambda = +1), \omega(\lambda = +1), \eta'$ and $\phi(\lambda = +1)$ are all states produced by K^* exchange. We are now in a position to check that the non-strange quark components of these states build the $f' - \phi$ trajectory as expected. This means does

$$\frac{d^2\sigma}{dt dM^2} \sim (\alpha' M^2)^{\alpha_{\phi(0)} - 2\alpha_{K^*}(t)} \quad (5)$$

on the average? More precisely, we have ^{*)}

$$\begin{aligned} \Sigma_n(t) &= \alpha' \int_{M_1^2}^{M_2^2} dM^2 (\alpha' M^2)^n \frac{d^2\sigma}{dt dM^2} \\ &= \frac{g_{K^*K^*\phi}(t) (\alpha' s)^{2\alpha_{K^*}(t)-2}}{\alpha_{\phi(0)} - 2\alpha_{K^*}(t) + n + 1} \\ &\quad \times (\alpha' M^2)^{\alpha_{\phi(0)} - 2\alpha_{K^*}(t) + n + 1} \Bigg]_{M^2=M_1^2}^{M^2=M_2^2} \end{aligned} \quad (6)$$

where the integer n must normally be chosen odd or even depending on the crossing properties of the Mueller amplitude $d^2\sigma/dt d(M^2/s)$. However, since here this amplitude represents $K_R^{*+}K^- \rightarrow K_R^{*+}K^-$ scattering in the direct, missing mass, channel, the corresponding u channel is exotic. This

*) The α 's are included to make M^2 dimensionless.

hopefully results in third double spectral function effects being weak and so we expect no fixed poles. We can then conveniently take both $n=0$ and $n=1$. (We do not use $n>1$ because of the increasing dependence on the cut-off, M_2^2 , when higher moments are considered - we shall comment further on this in Section 5.)

We set M_1^2 in Eq. (6) to zero, since the pion is the first contributing state, and M_2^2 we define as the average mass squared of the last contributing resonance and the first omitted one that would contribute. Thus we have $M_2^2 = \frac{1}{2} (m_\eta^2 + m_{A_1}^2) = 1.02 \text{ GeV}^2$. With these values

$$\frac{(\alpha' M_2^2) \Sigma_0(t)}{\Sigma_1(t)} = \frac{\alpha_\varphi(0) - 2\alpha_{K^*}(t) + 2}{\alpha_\varphi(0) - 2\alpha_{K^*}(t) + 1} \quad (7)$$

From this ratio at each value of t we can then determine $\alpha_\varphi(0) - 2\alpha_{K^*}(t)$.

Now it is only the non-strange quark component of the resonance cross-sections which supposedly contribute to building $f' - \varphi$ exchange. The π, ρ, ω have only $q\bar{q}$ quarks so they contribute 100% to Σ_n , while the φ resonance production decouples completely. However, the η, η' we know contain both $\lambda\bar{\lambda}$ as well as $q\bar{q}$ ($q=p$ or n) quarks and it is crucial to know how much of each.

While the vector mesons belong to an ideally mixed nonet so that we know the ρ, ω contribute 100% to our sum rule and the φ not at all, the situation for the pseudoscalar mesons is more confused. One thing that is quite certain is that the η, η' do not belong to an ideally mixed nonet with one being pure $\lambda\bar{\lambda}$ and the other $q\bar{q}$. Indeed from the Gell-Mann-Okubo mass formula ¹⁴⁾, it is known that the physical situation is much closer to no singlet-octet mixing with $\eta \simeq \eta_8, \eta' \simeq \eta_1$. Regge schemes for the mixing of trajectories naturally lead to the use of quadratic mass formulae; then the physical η, η' masses in the Gell-Mann-Okubo mass formula give a mixing angle of some -11° (45° away from ideal) where the mixing angle θ is defined by

$$\begin{aligned} \eta &= \eta_8 \cos \theta - \eta_1 \sin \theta \\ \eta' &= \eta_8 \sin \theta + \eta_1 \cos \theta \end{aligned} \quad (8)$$

With such a mixing angle ($\theta \simeq -11^\circ$) the η, η' are roughly equal orthogonal mixtures of $q\bar{q}$ and $\lambda\bar{\lambda}$ quarks ^{*}). When this exact situation occurs [i.e., $\theta = -\frac{1}{2} \sin^{-1}(\frac{1}{3})$], we call this simply "equal mixing". This is favoured by tests of SU_3 relations for differential cross-sections as shown by Martin and Michael ¹⁵⁾. However, dual unitarization schemes ^{16), 17)} suggest that the $q\bar{q}, \lambda\bar{\lambda}$ components of the η, η' are not orthogonal (perhaps because the physical states also contain a $c\bar{c}$ component) and so have different mixing angles. Rosenzweig ¹⁶⁾ finds the fraction of $q\bar{q}$ quarks in the η, η' to be $\cos^2 45^\circ, \sin^2 35^\circ$, respectively (i.e., fractions 0.5, 0.33), while Inami et al. ¹⁷⁾ have very similarly 0.43, 0.33. In contrast, equal mixing gives these proportions to be $\frac{1}{2}, \frac{1}{2}$ and no mixing $\frac{1}{3}, \frac{2}{3}$. Can we in fact distinguish these possibilities?

4. PSEUDOSCALAR MIXING AND THE WAY THE $f' - \phi$ TRAJECTORY IS GENERATED

For the moment, so as to introduce just one parameter, let us assume that the physical η, η' are members of a nonet and so have orthogonal wave functions ^{*)}. It is then convenient for numerical purposes [see Eq. (11) below] to define this parameter to be not the mixing angle θ of Eq. (8) but $\beta = \theta - \theta_{\text{ideal}}$, so that

$$\tan \beta = \frac{\tan \theta + \sqrt{2}}{1 - \sqrt{2} \tan \theta} \quad (9)$$

when $\beta = 0$ for ideal mixing, $\beta = \pi/4$ for equal mixing and $\beta = 54.7^\circ$ for no singlet-octet mixing. We will discuss below what happens if the physical η, η' are not orthogonal states as in the Inami et al. scheme ¹⁷⁾; we shall see that β is, for numerical reasons connected with our choice of cut-off, simply related to just $\theta_{\eta'}$ and not θ_{η} .

^{*)} Here $q\bar{q}$ is a shorthand for a state of non-strange quarks $(p\bar{p} + n\bar{n})/\sqrt{2}$. Although we speak of the process $K^- p \rightarrow \Lambda M$ producing states of $q\bar{q}, \lambda\bar{\lambda}$, it is in fact, as seen in Fig. 1, only possible to produce $p\bar{p}, \lambda\bar{\lambda}$ states and not $n\bar{n}$ states to the lowest order in the diagrams that we are considering. This difference between a $q\bar{q}$ state and a $p\bar{p}$ state just introduces an isospin Clebsch-Gordan of $1/\sqrt{2}$ which appears in all amplitudes equally and hence is of no concern.

^{+) In the trajectory mixing scheme of Ref. 17), the η, η' are only orthogonal and ideally mixed at the planar level, while it is the cylinder corrections that produce the non-orthogonal, non-ideal physical particles.}

The contributions of the η, η' to the finite mass sum rules involving their $q\bar{q}$ components are fractions $\cos^2\beta$ and $\sin^2\beta$, respectively, of their differential cross-sections. We can now evaluate the left-hand sides of the sum rules Σ_0, Σ_1 of Eq. (6) at each value of t . The ACNO group have binned their data^{5),6)} according to the value of $t' = t - t_{\min}$, where for $p_L = 4.2$ GeV $t_{\min}$ varies somewhat from one produced state to another, e.g., for π $t_{\min} = +0.009$, for η' $t_{\min} = -0.046$ GeV². Since Regge dynamics is supposedly a function of t (particularly the trajectory function) we interpolate these values of t' to obtain the differential cross-sections for $\pi, \eta, \rho, \omega, \eta', \phi$ at fixed values of t .

Now if these states really do build the $f' - \phi$ trajectory we expect to find from the ratio of Σ_0, Σ_1 [Eq. (7)] the value of $\alpha_\phi(0) - 2\alpha_{K^*}(t)$ to coincide with that fixed by the appropriate particle masses. Thus at $t=0$ we expect

$$\begin{aligned} \alpha_\phi(0) - 2\alpha_{K^*}(0) &= \alpha'(2m_{K^*}^2 - m_\phi^2) - 1 \\ &= -0.55 \end{aligned} \quad (10)$$

Is this correct?

As an illustration let us look at the result of evaluating the sum rules in the first t bin, $t \in [-0.05, 0]$ GeV². There from the ratio, Eq. (7), and the probabilities $\cos^2\beta$ and $\sin^2\beta$ of finding non-strange quarks in the η, η' we have

$$\alpha_\phi(0) - 2\alpha_{K^*}(t) = a + \frac{\tan^2\beta - 1}{b + c(\tan^2\beta - 1)} \quad (11)$$

where $a = -0.51 \pm 0.08$, $b = 5.0 \pm 1.5$, $c = 2.1 \pm 0.6$ only including experimental uncertainties and not theoretical ones. Conveniently taking into account that this [Eq. (11)] is at $t = -0.025$ GeV² and not $t=0$, we have, assuming canonical slope for this K^* trajectory, the same equation at $t=0$ now with $a = -0.55 \pm 0.08$. It is immediately seen that the physically expected intercept, Eq. (10), is reproduced if $\tan^2\beta = 1$, which corresponds to the equal mixing favoured by the Gell-Mann-Okubo mass formula.

The different mixing schemes produce appreciably different results for the intercept as detailed in Table 1. Data are seen to fix the intercept to lie between -0.9 and 0 depending on the amount of η, η' mixing, with equal mixing giving the physically expected value of -0.55 , and ideal mixing being excluded.

In the topological ¹⁶⁾, trajectory ¹⁷⁾ mixing scheme, or any other in which the η, η' mixing angles are not equal, the parameter β is essentially $\theta_{\eta'}$. The reason is that the contribution of the η to Eqs. (6) and (7) is determined by the quantity $(M_2^2 - rm_\eta^2)$ compared to $(rm_{\eta'}^2 - M_2^2)$ for the η' , where $r = (\Delta\alpha + 2)/(\Delta\alpha + 1)$ with $\Delta\alpha = \alpha_\varphi - 2\alpha_{K^*}$. At small t when $r \sim 3$, the contribution of the η cross-section is then suppressed by a factor of $1/17$ compared to the η' . Thus the exact amount of $q\bar{q}$ in the η is irrelevant at least at small t and it is the η' contribution that is determined. In the formula, Eq. (11), β is then just related to the non-strange component of the η' which is $\sin^2\beta$. With $\Delta\alpha(0) = -0.55$, Eq. (10), we have

$$\theta_{\eta'} \equiv \beta = 46.5^\circ \pm 3.6^\circ \quad (12)$$

for which both topological and trajectory mixing give 35° and no mixing 54.7° . In Fig. 2 we illustrate the different η, η' mixing schemes in the $q\bar{q}, \lambda\bar{\lambda}$ plane together with our result of Eq. (12) for the η' mixing angle.

With equal mixing favoured we now proceed to the determination of the trajectory $\alpha_\varphi(0) - 2\alpha_{K^*}(t)$ at other values of t away from the forward direction. Once again the η contributes less than the η' and equal mixing, trajectory mixing and no singlet-octet mixing give rather similar results. In Fig. 3 we plot the value of $\alpha_{K^*}(t) - \frac{1}{2}\alpha_\varphi(0)$ determined in each t bin, assuming equal mixing [in fact β given by Eq. (12)], together with its linear form expected from Eqs. (2) and (10) fixed by the K^*, φ, K^{**}, f' masses.

We see the data reproduces quite well the expected trajectory, its slope and intercept. This constitutes our main result. The fact that our trajectory has a somewhat flatter shape than the canonical is indeed in agreement with the K^*, K^{**} trajectory determined from the energy dependence of hypercharge exchange reactions by Navelet and Stevens ¹⁸⁾ who find a slope of 0.68 GeV^{-2} with intercept 0.35 , as we have here.

The trajectory $\alpha_{K^*}(t) - \frac{1}{2}\alpha_\varphi(0)$ calculated from the differential cross-sections for the production of low mass states in $K^-p \rightarrow \Lambda M$ by K^* exchange are themselves seen to generate a Regge trajectory dual to the $q\bar{q}$ missing mass channel with intercept $\alpha_\varphi(0)$. We have already seen both from Eq. (11) at $t = -0.025 \text{ GeV}^2$ and from Fig. 3 that this intercept agrees with the physical value given by the φ and f' masses, provided we have an equal mixing scheme for the pseudoscalar mesons. To see that the intercept of

this $\lambda\bar{\lambda}$ trajectory really is distinguishable from that of the $q\bar{q}$ trajectory we plot in Fig. 4 the value of $\alpha_\varphi(0)$ calculated from the experimentally determined $\alpha_\varphi(0) - 2\alpha_{K^*}(t)$, setting $\alpha_{K^*}(t) = 1 + \alpha'_{K^*}(t - m_{K^*}^2)$. Taking into account the uncertainty in the K^*, K^{**} intercept due to difference in the slope between $\alpha'_{K^*} = (m_{K^{**}}^2 - m_{K^*}^2)^{-1}$ and $2(m_{K^{***}}^2 - m_{K^*}^2)^{-1}$, the result for $\alpha_\varphi(0)$ is plotted in Fig. 4 for the values given by equal mixing. This is seen to be in agreement with the physical intercept

$$\alpha_\varphi(0) = 1 - m_\varphi^2 / (m_{f'}^2 - m_\varphi^2) = 0.17 \quad (13)$$

and roughly independent of t within errors. If the slope of the K^*, K^{**} trajectory were changed to 0.68 as suggested by Navelet and Stevens¹⁸⁾, for example, the displacement of the point at $t = -0.35 \text{ GeV}^2$ is shown, making $\alpha_\varphi(0)$ much less dependent on the t bin in which it is evaluated. The intercept is seen to be well below $\alpha_\varphi(0) = 0.5$. If instead of equal mixing, which is strongly favoured, we had no singlet-octet mixing or the topological mixing scheme of Refs. 16) and 17), the values of $\alpha_\varphi(0)$ would be raised or lowered by roughly 0.08 from those seen in Fig. 4. For either equal mixing suggested by the Gell-Mann-Okubo mass formula and by earlier SU_3 tests¹⁵⁾ or the dual unitarization schemes^{16), 17)}, the intercept of this $\lambda\bar{\lambda}$ trajectory is well below that of the $q\bar{q}$ trajectories. We really see a $\lambda\bar{\lambda}$ trajectory with intercept given by Eq. (13) generated by the production of $q\bar{q}$ states just as duality diagrams suggest.

5. THE SIZE OF THE COUPLINGS AND THE OTHER NATURAL/UNNATURAL PARITY EXCHANGES

At the same time as determining the trajectory $\alpha_\varphi(0) - 2\alpha_{K^*}(t)$ we calculate the coupling $G_{K^*K^*\varphi}(t)$, defined by Eqs. (1) and (6), which sets the scale for both Σ_0, Σ_1 of Eq. (7). This coupling is really the sum of true triple Regge couplings given by

$$G_{K^*K^*\varphi}(t) = \frac{1}{32\pi} \sum_{\substack{i,j \\ = K^*, K^{**}}} \sum_{k=\varphi, f'} \beta_{N\lambda i}(t) \beta_{N\lambda j}(t) \\ \times \xi_i^*(t) \xi_j(t) g_{ijk}(t) \beta_{KKK}(0) \quad (14)$$

where g_{ijk} is the triple-Regge residue and β_{abR} the particle a - particle b - reggeon R coupling defined by the contribution of reggeon R to the ab total cross-section being $\sigma_{ab}^{\text{tot}} = \beta_{aaR}^{(0)} \beta_{bbR}^{(0)} (\alpha' s)^{\alpha_R(0)-1} \xi_R(t)$. $\xi_R(t)$ is the signature factor for which $\text{Im} \xi = 1$. The coupling, $G_{K^*K^*\varphi}$, is found to be roughly linear in t , in the range we consider, viz $-0.4 \text{ GeV}^2 \leq t \leq 0$, with value

$$G_{K^*K^*\varphi}(t) = (8 - 25t) \text{ GeV}^{-4}$$

with some 5-10% error.

This is to be compared at $t=0$ with a typical coupling $\beta_{NNf}^2(0) g_{fff}(0) \beta_{KKf}(0)/16\pi$ which we believe is roughly 150 GeV^{-4} . The somewhat small value of the $G_{K^*K^*\varphi}$ coupling may appear at first rather worrying in view of the fact that at energies with $s < 25 \text{ GeV}^2$, the invariant cross-section integrated over both x and p_T^2 for $K^-p \rightarrow \Lambda X$ is so much larger than for $K^+p \rightarrow \Lambda X$ or $\pi^\pm p \rightarrow \Lambda X$: a difference attributed by Inami and Miettinen ³⁾ to $f' - \varphi$ exchange as the first evidence for the existence of such exchanges in strangeness annihilation processes, and hence a measure of such a coupling. However, since Inami and Miettinen ³⁾ consider cross-sections integrated over missing masses much larger than we have considered here, it is expected that it is the K exchange and not K^* contribution that dominates in such a region [see Section 2, Eq. (1)]. Indeed the present data indicate that f production is, for example, controlled by unnatural and not natural parity exchange in keeping with this expectation. Thus we must conclude that the large contribution seen in $K^-p \rightarrow \Lambda$ attributed to φ exchange must be predominantly due to the $G_{KK\varphi}$ coupling. What can we say about this unnatural parity coupling - unfortunately nothing. This is because, although the ρ, ω, φ with helicities $0, -1$ are produced by unnatural parity exchange, the scalar mesons, the ϵ, S^* , also contribute and these have not been fully analyzed in the present data.

That leaves us with the question: can we test whether the $\lambda \bar{\lambda}$ component of the states produced by natural parity exchange build the $f - \omega - A_2 - \rho$ exchange? Here the states that contribute are the η, η' , now with components $\sin^2 \beta, \cos^2 \beta$ respectively, and the φ with helicity $+1$. But the important question of cut-off in the sum rules arises. In the case we considered in Sections 2.-4. the last included state, the η' , and the first omitted state, the A_1 , have mass² relatively close, so that M_2^2 of Eq. (6) is quite well defined. Here we do not know whether to take the cut-off halfway between the φ and the D or the φ and the E , and the difference is quite large. Independent of the precise mixing scheme, whether equal,

none or topological, we find in each t bin ^{*)} the values of $\alpha_p(0)$ given in Table 2 from the analogue of Eq. (1) knowing the K^* trajectory for two choices of cut-off. We see that the value of $\alpha_p(0)$ is strongly cut-off dependent, though not inconsistent within the large errors with the expected value of 0.5. The reason that the errors are so much bigger here than those shown in Figs. 3 and 4 for α_φ , remembering the experimental quantities have roughly some 5-10% uncertainty, is that, if the ratio $\Sigma_1(t)/\Sigma_0(t)$ of Eq. (7), which determines the trajectories, is in the range of, say 2-3, as it is in the case of φ exchange, $\Delta\alpha = \alpha_\varphi - 2\alpha_{K^*}$ is determined to within 10%. However, if this ratio is closer to one, $\Delta\alpha$ has a very much larger error, as in the case considered in this section. This could, in principle, be remedied by studying the ratio of some higher moment sum rule, Σ_2 for instance, to Σ_0 , but this would make the uncertainty due to the cut-off even larger, so it is a "swings and roundabouts" situation. The uncertainty in the cut-off we take to be measured by $\Delta M^2 = \frac{1}{2}(\mu_2^2 - \mu_1^2)$, where the cut-off is $M^2 = \frac{1}{2}(\mu_1^2 + \mu_2^2)$. This uncertainty is then at least three times larger in the case of $\lambda\bar{\lambda}$ states considered in this section than for $q\bar{q}$ states investigated in Sections 2.-4.

Despite these uncertainties $\alpha_p(0)$ is certainly greater than our calculated values of $\alpha_\varphi(0)$. The coupling $G_{K^*K^*\rho}$ is found to be the same order of magnitude as $G_{K^*K^*\varphi}$ in this t range. This ends our investigation: all that remains is to give our conclusions.

6. COMMENTS AND CONCLUSIONS

Recent analyses, completed within the last month ⁶⁾, of the high statistics K^-p experiment at 4.2 GeV ⁴⁾⁻⁸⁾ allow the set of resonance states produced in $K^-p \rightarrow \Lambda M$ within a one GeV² mass range to be separated from background and to isolate their natural parity K^*, K^{**} exchange contribution. Using these data we are able to show that, in the sense of finite mass sum-rule duality, the non-strange quark content of these resonance states do indeed generate $f' - \varphi$ exchange in the crossed channel and that the strange quark component build a high-lying singularity compatible with $f - \omega - A_2 - \rho$ exchange. This analysis succeeds, just as simple quark diagrams and two component duality would suggest, provided only that the π' has an almost equal mixture of strange and non-strange quarks. This mixture is in agreement with the quadratic version of the Gell-Mann-Okubo mass formula ¹⁴⁾ and with various SU_3 tests of Martin and Michael ¹⁵⁾.

*) Note $t_{\min} = -0.057$ for φ production, so we cannot evaluate the sum rules in the first t bin, $t > -0.05$ GeV².

This separation of strange meson scattering into two components depending on whether $q\bar{q}$ or $\lambda\bar{\lambda}$ states are produced in the s channel is seen from the data to be equivalent to the separation into two components depending on whether $\lambda\bar{\lambda}$ or $q\bar{q}$ states are exchanged in the t channel. However, the production of a $\lambda\bar{\lambda}$ state is not synonymous with decay into $K\bar{K}$ channels, as proposed by Inami and Miettinen ³⁾, since states such as the η, η' below $K\bar{K}$ threshold make important contributions to $\lambda\bar{\lambda}$ production in the low mass region. This is one of several reasons for the failure of an earlier analysis ¹²⁾ of this problem in a related context. It is only because this high statistics data allows a clear separation of resonance states produced by natural and unnatural parity exchange that such a detailed test of duality diagrams is feasible and found to work exceedingly well.

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Table 1

The dependence of the intercept difference
 $\alpha_\varphi(0) - 2\alpha_{K^*}(0)$
on the pseudoscalar mixing scheme

Mixing scheme	Mixing angles	$\alpha_\varphi(0) - 2\alpha_{K^*}(0)$
Ideal	$\beta = 0$	-0.89 ± 0.01
Topological	$\theta_\eta = 45^\circ, \theta_{\eta'} = 35^\circ$	-0.64 ± 0.03
Trajectory	$\theta_\eta = 49^\circ, \theta_{\eta'} = 35^\circ$	-0.64 ± 0.03
Equal	$\beta = 45^\circ$	-0.55 ± 0.08
No	$\beta = 54.7^\circ$	-0.40 ± 0.07
Extreme	$\beta = 90^\circ$	-0.05 ± 0.14

Table 2

The values of $\alpha_\rho(0)$ computed
in each t bin for different cut-offs
in the finite mass sum rules

$-t \text{ GeV}^2$	$\alpha_\rho(0)$	
	$M_2^2 = 1.35 \text{ GeV}^2$	$M_2^2 = 1.53 \text{ GeV}^2$
[0.05, 0.1]	1.0 ± 0.3	0.65 ± 0.2
[0.1, 0.15]	1.2 ± 0.4	0.75 ± 0.25
[0.15, 0.2]	1.5 ± 0.7	0.9 ± 0.4
[0.2, 0.3]	1.7 ± 1.1	0.95 ± 0.6
[0.3, 0.4]	1.6 ± 1.1	0.6 ± 0.6

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* * *

FIGURE CAPTIONS

- Fig. 1: The two Mueller-Regge quark diagrams contributing to resonance production in the process $K^- p \rightarrow \Lambda X$, in which states made of $p\bar{p}$ and $\lambda\bar{\lambda}$ quarks can be produced.
- Fig. 2: The figure shows the η, η' vectors in the $q\bar{q} = (p\bar{p} + n\bar{n})/\sqrt{2}$, $\lambda\bar{\lambda}$ plane for the different mixing schemes, together with our result of Eq. (12) for the value of β shown by the hatched cone. I = ideal, E = equal, T = topological or trajectory mixing and η_1, η_8 are the singlet and octet states. η'_T is the η' in both the schemes of Refs. 16) and 17), while η_T is just that of Inami et al. 17); the η in the Rosenzweig scheme 16) coincides with η_E .
- Fig. 3: The trajectory $\alpha_{K^*}(t) - \frac{1}{2}\alpha_\phi(0)$ determined from the sum rules Σ_0, Σ_1 is shown in each t bin assuming equal mixing (i.e., $\beta = \pi/4$). The dashed line is the expected K^* trajectory with intercept lowered by $-\frac{1}{2}\alpha_\phi(0)$ and with slope of 0.82 GeV^2 fixed by the K^*, K^{**} masses.
- Fig. 4: The value of $\alpha_\phi(0)$ computed in each t bin from $\alpha_\phi(0) - 2\alpha_{K^*}(t)$ determined here assuming the K^* trajectory is known, as described in the text. The dotted point in the bin $0.3 \leq -t \leq 0.4 \text{ GeV}^2$ is the result if the K^* trajectory is flattened to the scattering region slope of 0.68 GeV^2 of Ref. 18). The dashed line denotes the expected value of the $f' - \phi$ intercept given by Eq. (13).

Fig. 1

Fig. 2

Fig. 3

Fig. 4