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ABSTRACT

The CERN School of Physics is meant to give young experimental physicists an introduction to the theoretical aspects of recent advances in elementary particle physics. This report contains four sets of lectures dealing with: high energy particle interactions with nuclei; gauge fields; high energy neutrino reactions; quarks in high energy interactions and a summary of a course given on: the QCD catechism - an introduction to the perturbative aspects of quantum chromodynamics.
PREFACE

The notes which are now lying before you represent the Proceedings of the 1978 CERN School of Physics which was held near Zeist, the Netherlands, and which was organized in cooperation with the Dutch Physical Society. The number of students attending the School was 64, most of them from CERN Member States.

The main topics were gauge theories, hadron structure, high energy neutrino reactions and quantum chromodynamics. Shorter lecture series were devoted to high energy particle interactions in nuclei and the physics of new detectors. These lectures were interspersed with talks on specialized topics. For the first time in the history of the CERN Schools of Physics poster sessions were held in which students had the opportunity to display their own research work.

To my mind the 1978 CERN School of Physics has succeeded in attaining its two main goals: to convey important new information and insight to its students and to be the instrument of making new bonds of cooperation and friendship which transcend national boundaries and which, I hope, will be enduring.

I take the opportunity to express my sincere gratitude to all participants: lecturers, discussion leaders, students and my fellow members of the organizing committee, who have contributed so much to the success of the School. Financial support from the Dutch Ministry of Science and Education is gratefully acknowledged.

Finally a word of thanks to the organizing secretary, Ann Caton, whose patience and perseverance was of decisive value for the School and for the assembly of the final notes in the form of these Proceedings.

C. Dullemond
Chairman of the Organizing Committee
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HIGH ENERGY PARTICLE INTERACTIONS WITH NUCLEI

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1. Introduction

The recent interest in multiparticle production processes on nuclei was triggered by re-discovering their "enigmatic simplicity" which has been known to cosmic ray physicists for over 20 years: the mean multiplicity and angular distributions of relativistic secondaries produced on nuclei do not differ markedly from what emerges from p-p collisions. This reluctance of nuclei to multiply particles generated on one nucleon of the target nucleus is illustrated in Fig. 1. It shows the relative multiplicities

\[
\bar{R} = \frac{\text{average no of relativistic particles produced on nucleus}}{\text{average no of relativistic particles produced on proton}} = \frac{\bar{n}_{\text{nucleus}}}{\bar{n}_{\text{hydrogen}}}
\]

for particle production in nuclear emulsion as measured in experiments and calculated from intra-nuclear cascading. \( \bar{R} \) measures the ability of the target nucleus to multiply particles produced on one nucleon. When \( \bar{R} = 1 \) there is no multiplication. We see that while cascading processes at very high energies would lead to considerable multiplication what we, in fact, see is a very weak "response" of the nucleus.

What Fig. 1 tells us is that the space-time evolution of hadron final states cannot be trivially described by cascading in the target. Processes of cascading are very well known in physics from the electromagnetic electron-positron cascades generated by very high energy electrons emitting bremsstrahlung in condensed media. There, one usually assumes that this bremsstrahlung (which subsequently produces electron-positron pairs which, in turn, radiate photons etc) is a sum of radiations from consecutive collisions of the fast electron with atoms of the target. Indeed, such a picture is, in general, very well borne out by experiment with some small but significant exceptions which were first predicted by Landau and Pomeranchuk and later confirmed experimentally. When the electron is very fast and while producing a photon, can undergo more than one virtual collision there is a damping of production. So, there we have an example from a class of phenomena where early stages of evolution of produced systems take place over large longitudinal distances and the observed deviations from cascading signal us the interactions of something else than the physical, well-defined, particles.

How much the mechanism of damping of production in hadron-nucleus interactions is similar to the Landau-Pomeranchuk mechanism is an open question. There is, however, a commonly accepted point of view that the role of large distances and long time intervals is of primary importance in description of high energy interactions. At present the most often used picture of the space-time evolution of produced hadron final states is a hybrid picture of multi-peripheral model, Regge-model, parton-model and "soft" field theory. A representative selection of these strongly overlapping models one can find in refs. In all of them the longitudinal distance over which production takes place is given by a time dilated characteristic time (we take \( \bar{v} = c = 1 \)):

\[
\tau = \frac{E}{m} \tau_0
\]

where \( E \) is the energy of the produced particle, \( m \) its mass. Thus very high energy particles are produced over distances which may become as large as the radii of

It is interesting to note that Landau himself introduced these concepts into hadronic interactions through a relativistic hydrodynamic model, very different from the mechanism he and Pomeranchuk proposed at about the same time for high energy electromagnetic interactions in condensed media.
nuclear targets. This leads, in general, to reduction of multiplicity of produced particles.

Does the internal structure of interacting hadrons play an important role in production? In other words can we learn something about the internal structure of hadrons from hadron-nucleus interactions which would be more than we can get from analyzing hadron-nucleon collisions? This is not clear. Most of the experimental data on hadron-nucleus collisions are on processes with low average $p_L$ of produced particles (see e.g. the review\textsuperscript{12}). Consequently one might expect that such processes are not very sensitive to e.g. quark structure of hadrons and that in interpreting them one might get away with just a general notion of relevant longitudinal distances (1,2) without explicitly involving internal structure of hadrons. In fact, we shall argue that this is possible \textsuperscript{13,14}. On the other hand the data is still far from being detailed\textsuperscript{12} and, with further progress of experiments, one may expect seeing some nontrivial signals of quark parton structure even in the low-$p_L$ production. The first steps to relate measurable quantities with quark structure in low-$p_L$ production processes in hadron-nucleus collisions look quite encouraging \textsuperscript{15,16}.

One can hardly avoid discussing quark (parton) structure when data on large-$p_L$ production is analyzed\textsuperscript{17,18,19}. This is at present THE METHOD to understand hadron-nucleon large-$p_L$ and lepton-nucleon deep inelastic processes, thus one should accept it as a starting point of large-$p_L$ and deep inelastic hadron-nucleus and lepton-nucleus processes. Here, it is reasonable to accept that one or more constituents are being knocked out of a nucleon in the target nucleus and fragment into hadrons while traversing nucleus. Actually there are possible experiments which may look directly at the process of quarks cascading in nuclei. These are multihadron production processes in deep inelastic electron-nucleus (muon) - nucleus and neutrino interactions.

By selecting events with small Bjorken scaling variable $\omega$ one is assured that the incident lepton interacts with just one nucleon in the nucleus, consequently multihadron production must result from a cascading quark. Since all this happens inside of nuclear matter, the analysis of such events is of great importance for understanding of quark interactions with hadronic matter. Although the existing experimental data are still very modest \textsuperscript{20,21} attempts of such an analysis have already been made \textsuperscript{20}

2. Examples of data on hadron- and lepton-nucleus interactions

2 (a) Hadron-nucleus interactions

One measures the total multiplicities and various differential multiplicities of produced particles, the total and total production cross sections and inclusive cross sections. These quantities are inter-related. Indeed, let the cross section for production of $n$ particles with momenta $p_1, \ldots, p_n$ be $\sigma_n(p_1, \ldots, p_n)$.

The inclusive cross section for producing one particle with momentum $p_i$ is

\[
\frac{d\sigma}{dz_i dp_i} = \int Y \frac{d^3p_i}{E_i} \frac{d^3p_1}{E_1} \ldots \frac{d^3p_n}{E_n}
\]

where $Y = \frac{1}{z_i} \frac{E_i + p_i}{E_i - p_i}$ is the rapidity. After integrating (2.1) over $d^3p_i$ and dividing by cross section for production of one particle $\sigma = \sum_n \int \sigma_n(p_1, \ldots, p_n) \frac{d^3p_1}{E_1} \ldots \frac{d^3p_n}{E_n}$ one obtains the average density of particles at rapidity $y$

\[
\frac{1}{\sigma} \frac{d\sigma}{dy} = \frac{d\bar{n}}{dy}.
\]

The total multiplicity is therefore

\[
\bar{n} = \int dy \frac{d\bar{n}}{dy}.
\]

- 2 -
When only angular distributions can be measured (e.g. in all experiments with emulsion), instead of rapidity, $y$, the pseudorapidity, $\eta$, is used

\[ \eta = - \log \tan \frac{\theta}{2} . \]

The relation between $y$ and $\eta$ is

\[ y = \frac{1}{2} \left( \frac{m}{p_L} \right)^2 \frac{p_T}{p} + O \left( \left( \frac{m}{p_L} \right)^4 \right) , \]

and it turns out that differences $|y - \eta|$ are for fast secondaries typically $< 0.3$ (for a numerical example see ref 27).

We shall briefly describe a few representative pieces of data which will be discussed in more detail in the next sections. We shall not discuss the data where energies of incident hadrons are lower than $\sim 20$ GeV. While looking at the data we should remember that the targets are composed of many nucleons and there may be recoiling nucleons or groups of nucleons present among secondaries. Although they can be identified reasonably safely in most experiments one should be aware of this small contamination. The average transverse momentum of produced particles $\overline{p_L} = 0.5$ GeV. This fact has been known from cosmic ray experiments for more than 20 years. So, the main bulk of production is low-$p_L$ production. Large-$p_L$ particles constitute just a small fraction of average multiplicities.

Fig. 2 shows the total relative multiplicities as defined by (1.1). The target nuclei (from $\mathcal{C}$ to $\mathcal{U}$) are defined by the average number of collisions $\overline{\nu}$ (\(\nu\)) as defined in the Appendix. $\overline{R}$ appears to be, at least for incident protons and pions of various energies, an approximately universal function well represented by

\[ \overline{R} = \frac{\overline{R}}{\nu} = \frac{1}{2} + \frac{1}{2} \overline{\nu} , \]

which is also shown in Fig. 2. Again we can see that even the heaviest nuclei cannot even triple the hadron-nucleon multiplicity. Note that (2.6) implies that the relative total multiplicity is proportional to the number of hit nucleons $\overline{R} = \frac{1}{2} \overline{W}$ (see the Appendix for the definition of $\overline{W}$). Whether this indeed is the "right" parameter to give total multiplicities one may test in nucleus-nucleus collisions\(^{29}\) where there is no relation between $\nu$ and $W$. The existing data on multiplicity distributions in nucleus-nucleus interactions at $\sim 20$ GeV/nucleon give preference to $\overline{W}$.

The average density of particles produced in 400 proton - emulsion interactions at a given pseudorapidity $\eta$ is shown in Fig. 3\(^{31}\). Also the same density for proton-proton interactions at 400 is sketched there. We can see that the main bulk of multiplication of particles comes from the central and small pseudorapidities (rapidities). At large rapidities (projectile fragmentation region) an opposite effect takes place: there are less particles than in nucleon-nucleon collisions. This is a well established effect and has been seen in emulsion for many years as Fig. 4 shows\(^{32}\).

Data from counter experiments on $\phi$ (here, the rapidities are being measured) are shown in Figs 5(a), (b)\(^{12,27}\). One can see how multiplicity densities vary with $\overline{\nu}$, hence with the atomic number of the target\(^{29}\), Fig. 3b), and with energy, Fig. 5(b). Note that as the energy increases the region of low rapidities (target fragmentation region) does not change, while the high rapidity region (projectile fragmentation region) shifts to higher rapidities. This data is not accurate enough in the projectile fragmentation region to show a deficit of multiplication for large rapidities. This is, however, very clearly seen in Fig. 6\(^{33}\).
First a remark about the parametrization used in Fig. 6. For rapidities of the near-forward and forward regions the inclusive invariant cross sections can be fitted as follows

$$\frac{E \, d\sigma}{d^3p} = c \, \sigma \, \frac{E \, d\sigma}{d^3p},$$  \hspace{1cm} 2.7

where $\sigma$ is the atomic number of the target and the subscript $H$ labels the hydrogen target and $c$ is a constant. Following (2.2), we obtain for the multiplicity densities

$$\frac{d\bar{n}_A}{dy \, dp_L} = \frac{A}{\sigma_H} \, \frac{d\bar{n}_H}{dy \, dp_L},$$  \hspace{1cm} 2.8

Since $\sigma_A = \sigma_H A^{0.69}$ fits reaction cross sections over large range of energies \cite{27}, when one finds $\alpha = \alpha' = 0.69$ the relative multiplicity

$$R(y, p_L) = \frac{d\bar{n}_A}{dy \, dp_L} \frac{d\bar{n}_H}{dy \, dp_L},$$  \hspace{1cm} 2.9

does not depend on $A$. When $\alpha < 0.69$ the ratio $G(y, p_L)$ increases with decreasing $A$ and this means that there is a deficit of multiplicity. Fig. 6 shows such a deficit for $y > 6$. From (2.8) we can see that when $\alpha = 1$ the multiplicity is proportional to $\bar{\nu}$.

In Fig. 6 the dependence on $p_L$ is integrated over. It turns out that although $\alpha(y, p_L)$ is a complicated function of $p_L$ it has, for smaller values of $y$, a tendency to increase with $p_L$. In fact this effect is quite spectacular as it was shown in refs. \cite{17,18,19}. Fig. 7 demonstrates that $A$-dependence is indeed, as in (2.7) a power law dependence. The experiment\cite{18} integrated over $y$, dependence so only the $p_L$ dependence of $\alpha'$ was determined for pions, kaons, protons and antiprotons as shown in Fig. 8. The same phenomenon was also seen in production of massive dihadron states \cite{19}, Fig. 9. What is surprising in these results is that $\alpha > 1$ for large $p_L$. For low $p_L$ one observes only $\alpha < 1$. Eq. (2.7) provides us with the following picture provided $c \approx 1$ which is approximately the case: Each nucleon is a source of particles ejected into the element $d^3p$ of the momentum space. When $\alpha = 1$ the beams of produced particles on each nucleon just add up to give the hadron-nucleus inclusive cross section. When $\alpha < 1$ the beams from each nucleon get attenuated by the surrounding nuclear matter. Now then, the interesting thing is that Fig. 8 tells us that for large enough $p_L$ the beams from each nucleon are also fed from some other channels to produce a beam from the target nucleus which is amplified by the presence of the surrounding nuclear matter.

**2(b) Lepton-nucleus interactions**

Data on lepton-nucleus interactions for incident particles above 20 GeV is very modest. The examples given below are presented in a way which is parallel to the hadron-nucleus data we have just discussed.

Fig. 10 shows a plot, analogous to the one of Fig. 6, obtained for hadrons produced on nuclei by 20.5 GeV/c incident electrons \cite{34}. The exponent $\alpha'$ plotted in Fig. 10 is, however, not the same as in Fig. 6: The ratio of the number of single-particle inclusive hadrons detected /gram/cm$^2$/electron to the analogous number for deuterium was measured for $\Lambda$, $D$, $\Xi$, $C$, $C$, $J\bar{p}$

$$R_{A,D}(y, p_L) = \left( \frac{d\bar{n}_A}{d\bar{n}_D} \right),$$  \hspace{1cm} 2.10

and then fitted to a power of $A$

$$R_{A,D} = \left( \frac{A}{2} \right)^{\alpha''},$$  \hspace{1cm} 2.11
Since $\alpha' < 0$ (see Fig. 10) this means that we see an attenuation in the number of forward hadrons electroproduced from nuclei. This is qualitatively the same phenomenon as observed in ref. 33 and shown in Fig. 6.

Fig. 11 shows the inclusive single particle density of hadrons produced in 150 GeV $\mu^-$-emulsion interactions as a function of pseudorapidity \( \eta \) (an analogue of Fig. 3). Since the average energy of the virtual photon is \( \sim 61.7 \text{ GeV} \), Fig. 11 shows also single particle inclusive densities for the interactions: proton-emulsion at 67 GeV and pion-emulsion at 60 GeV. We can see that there are significant differences at the central region of pseudorapidities.

In the experiment of ref. 20, the energies and angles of the outgoing muon were measured. Therefore each event was characterized by a definite value of the dimensionless Bjorken scaling variable

\[
\omega = \frac{2(E-E')}{Q^2} \geq m \tau_y \approx m \tau_y
\]

where \( E - E' \) is the muon energy loss, \( Q^2 \) -muon four momentum transfer squared, \( m \) - nucleon mass. In the high energy limit \( \omega \) is proportional to the lifetime of the virtual photon \( \tau_y \). So, \( \tau_y \) measures the distance over which the virtual photon can interact coherently with the target nucleus:

\[
r \text{(in fermis)} \approx \frac{\omega}{5}
\]

Thus, for $\omega \lesssim 5$ events, we have the following mechanism of particle production at work: the incident particle interacts with just one nucleon of the target and the products of this interaction, not the incident particle, can collide with the other nucleons of the target and produce more particle. So, at small $\omega$'s, multiplication means cascading, and the nature of such cascading is of primary importance because at these huge momentum transfers and energy losses we presumably knock out bits of protons and see the results of their interactions with the rest of the nucleus.

In experiment of ref. 20 all muon induced jets were divided into five $\omega$-bins and the multiplicities in these bins were compared with multiplicities of muon-hydrogen interactions. The results are shown in Fig. 12. Note that the multiplicities from emulsion are everywhere larger than multiplicities from hydrogen. This means that in the lowest $\omega$ bin we see cascading. There is no significant difference in multiplication of particles at small and large $\omega$'s but this does not prove that the mechanisms of production are the same.

There are also some data available on neutrino and antineutrino multihadron production 21 in the Fermilab 15-foot bubble chamber filled with 64% neon - 36% hydrogen mixture. Thus 96% of the interactions occur on $C^\nu (A \approx 20)$. The results are enigmatically simple: No differences are seen which could distinguish the hadronic states created by pions and neutrinos.

3. Discussion

3(a) Low-$p_t$ processes in hadron-nucleus interactions

Let us start with the calculations of ref. 13 which is representative of the most popular theoretical approach which is a combination of multiperipheral, "soft" field theory (Reggeon field theory) and the parton model 8,9,10, and gives an approximate agreement with the data as shown in Figs 5(a),(b). This approach starts with sophisticated diagrammatic technique whose essential ingredient is the diagram of Fig. 13. It shows a space-time development of the inclusive spectrum of produced hadrons in a collision of fast hadron with strongly interacting target. The initial hadron is assumed to be a multi-peripheral ladder of partons. Since partons are pointlike their cross sections (for dimensional reasons) \( \sim \frac{1}{\Lambda^2} \), hence only slow partons interact. Therefore the interaction of the incident hadron takes some time before it produces slow enough parton to interact with the target. The faster the hadron the more time it takes - from the moment of its pro-
duction - to interact. In fact the formula (1.2) takes just this effect into account. So the graph of Fig. 13 implies the relation (1.2)\(^\text{11}\).

In hadron - nucleus collisions where many collisions take place one constructs - from elements shown in Fig. 13 - a multitude of diagrams whose relevance is often difficult to appreciate. Since one cannot calculate all of them and one has no systematic way of constructing approximate solutions, this general approach \(^{22,10}\) forms merely a framework for many more specific models.

Let us go back to the results of ref. \(^{13}\). In order to obtain numerical results for comparison with experiment the sophisticated diagrammatic description has to be supplemented with tree extra inputs: (i) the distribution of the number of collisions, \(P(\nu)\), (ii) the hadron-nucleon rapidity distributions of secondaries, \(d\hat{N}_y(y,E)\), (iii) the distribution of energies among a given number of hadron-nucleon collisions, \(P(E,E_{1},...,E_{\nu})\). In ref. \(^{13}\): (i) is the same as our Eq.(A/30) which is a simple probabilistic expression, (ii) is a fit to the existing experimental data, (iii) is just an arbitrary phenomenological assumption. The extra assumptions (i) - (iii) do not come from any field theoretic or parton model considerations nevertheless they completely determine the rapidity distribution in hadron nucleus collisions presented in Figs 5(a), (b). In fact the hadron - nucleus rapidity distributions of \(^{13}\) are:

\[
\frac{d\hat{N}_y(y,E)}{dy} = \sum_{\nu=1}^{A} P(\nu) \int dE_{1}...dE_{\nu} P(E_{1},E_{2},...,E_{\nu})
\]

\[
\times \sum_{i=1}^{\nu} \frac{d\hat{N}_y(y,E_{i})}{dy}
\]

with

\[
\rho(E_{1},E_{2},...,E_{\nu}) = C(E_{1}) \delta(E_{1} - E_{2}) \left( \prod_{i>2}^{\nu} \rho(E_{i}) \delta(E_{i} - E_{i-1}) \right)
\]

\[
E_{\nu} = \text{energy of the incident hadron}
\]

\[
P(\nu) \text{ the normalized probability of } \nu \text{ collisions (see Appendix)}.
\]

Eq. (3.1) has a simple classical-proba-

bilistic interpretation: the hadron - nucleus rapidity distribution is an averaged sum of all possible hadron-nucleon rapidity distributions. The method of averaging is a purely phenomenological assumption with the restriction imposed that each nucleon of the target nucleus can interact only once. Thus one may wonder whether the calculated curves shown in Figs 5(a), (b) have anything to do with "soft" field theory, multiperipherality or parton model.

The following classical calculation, relevant to the above, has recently been done \(^{23}\) following an old suggestion of ref. \(^{24}\). Let us assume that in a hadron-nucleon collision there exists a "leading particle"\(^{\text{(*)}}\) which carries away a substantial fraction of the energy brought in by the incident particle. This leading particle hits the nucleons while it traverses the target nucleus and at each collision looses a fraction of its energy and produces objects which after some time \(\tau\) (1.2) become hadrons.

\[
E \rightarrow E_{1} \rightarrow E_{2} \rightarrow E_{3}
\]

The rapidity distribution of hadrons from each interaction, \(d\hat{N}_y(y,E)\) we take from experiments on hadron-hydrogen interactions after subtracting the leading particle from all interactions except the last one. For a target nucleus with \(A\) nucleons we have

\[
\frac{d\hat{N}_y(y,E)}{dy} = \sum_{\nu=1}^{A} P(\nu) \sum_{i=1}^{\nu} \frac{d\hat{N}_y(y,E_{i})}{dy}
\]

\[
3.2
\]

\___(*)\ The "leading particle" may consist, in fact, of few fast particles diffractively generated from the incident particle. In the case of incident pions the role of the "leading particle" may be played by e.g. \(3\pi\) systems diffractively produced.
with the same notation as in (2.2). We have still to decide on the sequence of energies (the incident energy \( E_l \) is given): \( E_1, E_2, \ldots \). A reasonable starting point is to use \( E_k = (1-K)E_l \), where \( K \) is the inelasticity coefficient (fraction of energy transferred to secondaries), which has been measured in many cosmic ray experiments and its average found to be \( \approx 0.5 \). Now all elements of (3.2) are specified and Fig. 3 show some of the results of preliminary calculations with \( K = 0.55 \).\(^{23}\) compared with experiment.\(^{27,28}\) The "soft" field theory (Reggeon field theory) can also accommodate this model of the "leading particle cascade" \(^{26}\).

A few comments are in order here. The first is that the predictive power of the "soft" field theory is very low: to get numerical results one has to make phenomenological assumptions and the soft field theory is, at present, just a loose framework for various phenomenological approaches. The second is that the agreement with experiment, especially of ref.\(^{23}\), is very impressive. The third is that if the confrontation of the forthcoming, more accurate, data with the calculations of the kind given in ref.\(^{23}\) remains as impressive as it is today we should draw a pessimistic conclusion: the low-\( p_L \) production processes are well described by a simple superposition of the hadron-nucleon distributions and we are not going to learn from them anything new except that production takes place over large longitudinal distances.\(^{29}\) One should stress however that before reaching such conclusion one must test the models against much more detailed data.

I personally hope that these models will fail to reproduce these new, more accurate, data. Let us therefore discuss -------------------

\(^{29}\)One is tempted to apply at this point the principle of "Ocassam's razor": "Beings ought not to be multiplied except out of necessity", and ignore the soft field theory as a tool for description of low-\( p_L \) phenomena.

mechanisms of low-\( p_L \) production which explicitly involve the internal structure of hadrons. A reasonable starting point is to accept that the incident hadron, \( \hat{h} \), is composed of \( N_c \) constituents which one can most naturally identify with constituent quarks. Our constituents are not part of and each carries a finite fraction of the momentum of the incident hadron and is responsible for a finite fraction of the cross section: \( p_c \sim \frac{1}{2} \rho, t_c \sim \frac{1}{4} \tau \) for the proton, \( p_{c*} \sim \frac{1}{2} \rho, t_{c*} \sim \frac{1}{4} \tau_{c*} \) for the pion. Thus instead of parton partons of the "soft" field theory we introduce lumps of hadronic matter (or lumps of partons) as "sources" of particle production. The assumption which enables one to compute many characteristics of various production processes is \( ^{15,16} \) that the contribution to the central rapidity region of one constituent interacting with the target is independent of the target.\(^{32}\)

The probabilistic concepts developed in Appendix for hadron-nucleus interactions can be applied here to compute e.g. the average number of interacting constituents in the incident hadron

\[
W_{hT} = \frac{N_c \sigma_{cT}}{\bar{\sigma}_{hT}},
\]

3.3

where \( \sigma_{cT} \) is the inelastic cross section of the constituent \( c \) with the target \( T \), and \( \bar{\sigma}_{hT} \) is the inelastic cross section of the hadron \( h \) with the target \( T \).

An immediate consequence of our assumption is that any composite system interacting with one constituent produces the same density of particles in the central region.

The density produced particles is therefore proportional to the number of constituents in the projectile colliding with one constituent. This "unit" of particle production may be visualised as a tube cut out from the projectile by one constituent going along a straight line.

\(^{32}\)One can give arguments supporting this assumption \(^{15,16} \) but it will remain a phenomenological step which only successful predictions can justify.
\[ \frac{dR_{HA}(y)}{dy} = \frac{W_{HA}}{W_{HC}} = \frac{N_A \sigma_{CT}}{N_A \sigma_{CC}} \]

\[ = \left( \frac{\sigma_{CT}}{\sigma_{HC}} \right) \frac{\sigma_{CC}}{\sigma_{AC}} \] 3.4

Note that (3.4) is invariant against the transformation \( h \rightarrow T \).

From (3.4) we can get various ratios of multiplicities in the central region of rapidities. Relative (to hydrogen) hadron-nucleus \( A \) multiplicities:

\[ R_{HA}(y) = \frac{dR_{HA}}{dy} \]

\[ = \frac{\sigma_{TA}}{A \sigma_{HA}} = \frac{\sigma_{HA}}{\sigma_{CA}} \] 3.5

where \( \frac{\sigma_{HA}}{\sigma_{CA}} \) is the average number of collisions in \((A, A)\) collisions (see Appendix). Note that (3.5) implies that (in the central region of rapidities) the ratio of pion to proton-nucleus multiplicities is:

\[ \frac{dR_{TA}}{dy} = \frac{\sigma_{CA} \sigma_{PC}}{\sigma_{PA} \sigma_{CC}} = \frac{\sigma_{PA}}{\sigma_{PC}} \]

3.6

and the ratio of their relative (to hydrogen) multiplicities is

\[ \frac{R_{TA}(y)}{R_{PA}(y)} = \frac{\sigma_{CA}}{\sigma_{PA}} = \frac{\sigma_{PC}}{\sigma_{CC}} = \frac{\sigma_{PA}}{\sigma_{PC}} \]

3.7

Since \( \sigma_{AC} \propto \frac{1}{3} \sigma_{AH} \) the ratios (3.6) and (3.7) are, within this approximation, equal. One can estimate (3.6) and (3.7) knowing that at 30-60 GeV \( \sigma_{PC} \approx 21.2 \) mb and \( \sigma_{PA} \approx 32.3 \) mb.

Note that in ref. 15 the ratio (3.6) is given as \( \frac{1}{3} \sigma_{PA} \). This is different from our result although numerically is very close (\( \sigma_{PC} : \sigma_{PC} \approx 2:3 \)).

and they weakly on energy. Also \( \sigma_{TA} \approx 28.5 \times 10^{-7} \) mb and \( \sigma_{PA} \approx 46 \times 10^{-6} \) mb. One gets

\[ \frac{dR_{TA}}{dy} \approx \frac{R_{TA}(y)}{R_{PA}(y)} \approx 1.06 \times 10^{-6} \] 3.8

This ratio, properly averaged for 60 GeV pions and protons interacting with emulsion, is 0.84. The experiment, Fig. 11, for the central region \( 1.99 < \gamma < 2.76 \) gives 0.87. For the same parametrization of cross sections and taking \( \sigma_{CA} = 10 \) mb we obtain

\[ R_{\text{central region}}(\gamma) \approx 1.02 \sigma_{PA} \]

3.9

Fig. 14 compares (3.9) with the 50 GeV data for the same as above central region of rapidities. From (3.4), (2.2) and the fit to \( \sigma_{PA} \) given above one can also obtain the inclusive cross section

\[ \frac{d\sigma_{\text{central region}}}{dy} = 1.21 A \sigma_{PA} \] 3.10

In Fig. 6 we compare it with experiment. Note that the central region at 300 GeV should go no further than \( \gamma < 4 \), hence the region of \( \gamma > 4 \) is outside of the rapidity of (3.10).

From this model of constituent quarks interacting with nucleus one can also compute multiplicities of specified secondary hadrons at \( x \approx \frac{1}{2} \) and \( x \approx \frac{2}{3} \). At \( x \approx \frac{2}{3} \) production of secondary protons is determined by the probabilities of absorbing one quark. At \( x \approx \frac{1}{2} \) production of

\( \bar{x} \) is the Feynman scaling variable \( \bar{x} \approx \frac{p_T}{\sqrt{s}} \) where \( |p_{\text{min}}| \) is the maximum longitudinal momentum which a particle can have in the cms. So, \( \bar{x} \) is the longitudinal momentum in cms scaled into the interval \( (-1, +1) \).
secondary pions and kaons is given by the probabilities of absorbing two quarks and some quark combinatories. The computed yields are found in a reasonable agreement with the data at $\sim 20$ GeV $^{15}$.

We close this section with a few general remarks. From the phenomenological models within the framework of the "soft" field theory $^{13,23}$ we seem to be getting an almost trivial picture of hadron-nucleus interactions at low $p_t$ which reduces to hadron-nucleon collisions: after splitting hadrons into infinitely many pointlike partons we end up with the statement that we do not need any knowledge of the internal structure of hadrons to account for hadron-nucleus interactions. Hence nuclei are uninteresting targets for low-$p_t$ phenomena. However one should check many things before one accepts such conclusion. In particular the yields of various particles $^{28}$ may tell us to what an extent nuclei distort hadron-nucleon production.

When the constituent quarks are the sources of production one expects these yields to be distorted and one can predict what they should be (for the first attempt see ref. $^{15}$). The discussion given above gives some idea how the constituent quark description of low-$p_t$ production can be implemented. It certainly has more predictive power than the "soft" field theory and thus is more attractive.

3(b) Large-$p_t$ processes in hadron-nucleus interactions

First let us point out that the anti-shadowing effect shown in Fig. 8, hence the stronger than $\sim A$ dependence of the inclusive cross section at large $p_t$, is also reasonably well represented by the formula $^{18}$

$$E \frac{d\sigma_i}{dp} = a_i(p_t) A^{\nu_i} + b_i(p_t) A + c_i(p_t) A^{\nu_i},$$

where $i$ specifies the outgoing particle and $a$, $b$ and $c$ are positive. Eq. (3.11) not only can mimick the standard $A^{\nu(p_t)}$ dependence fitted in experiment but it also suggests that the effect of antishadowing comes from incoherent multiple collisions within nucleus: We identify the first term with the contribution of "soft" (low-$p_t$) processes which are slow and take place over long distances of the order of nuclear diameter or more, so the particles are not produced inside the nucleus but, more likely, outside. The coefficient $a_i(p_t)$ is a rapidly decreasing function of $p_t$. Hence forth we shall discuss only processes at large $p_t$, thus neglecting the first term in (3.11). For large $p_t$'s the production processes become fast, the products of the first interaction are created deep inside of nucleus and can interact again before leaving nucleus.

Let us discuss the mechanisms of the stronger than $\sim A$ dependence of production of large-$p_t$ objects in one and two interactions:

1. The incoming particle I produces at 1 a large-$p_t$ object $S$ which leaves the nucleus.
2. I produces at 1 a large-$p_t$ object $S$ which subsequently, through another large $-(p_t+p_t')$ scattering at 2 emerges as a large-$p_t$ object $S$.
3. I produces at 1 a large-$p_t$ object $S$ which subsequently converts into a large-$p_t$ object $S$ through a small-$\Delta p_t$ process at 2.

To estimate dependence on $A$ we compute the probabilities of these mechanisms: $p(i) \sim A^{\nu_1}$, because there are $A$ nucleons in the target, $p(ii)$ and $p(iii) \sim A^{\nu_2}$,
because once the interaction inside the nucleus takes place the average number of nucleons on the path of produced object is proportional to the nuclear radius, hence to $A^{1/3}$. Note that any extra internal collision brings another factor $A^{1/3}$. So, single, double, triple etc interactions contribute terms with $\sim A$, $\sim A^{4/3}$, $\sim A^{5/3}$ etc dependences, respectively. Note also that the signs of all (i) - (iii) additive processes are positive because (ii) and (iii) add to the beam of $S(p_L)$ or (i), draining the other beams: $S(p'_L)$ and $Q(p_L)$. Our description implies that if we included an attenuation of $S(p_L)$ in (i) we would have to subtract particles from the $S(p_L)$ beam and have dependence weaker than $\sim A$ of the production in (i). We shall not consider here these attenuation corrections. Adding (i) + (ii) or (i) + (iii) or (i) + (ii) + (iii) we get (3.11). The problem remains however of calculating the coefficients and fitting the data. As far as I know, nobody obtained a complete and convincing explanation of the results of refs 17,18 so far. The most striking feature of the data is that the antishadowing depends very strongly on the outgoing, detected, particle:

$$\alpha_{n+}(large\ p_L) \approx 1.1, \ \alpha_{p,p}(large\ p_L) \approx 1.4.$$ 

This suggests that the quantum numbers of the objects taking part in large-$p_L$ production are relevant. For instance it is difficult to imagine only real particles taking part in the process: The differences between different particles are too small. When we take the other extreme and accept that quarks are the objects S and Q of (i) - (iii) and higher order processes, we do indeed get large differences between different outgoing particles. Suppose the object $q$ is a quark. Production of large $-p_L p$-ion is achieved by combining this quark, at 2, with an antiquark and we get

$$\frac{E d\sigma}{d^3 p} = a_{n+} A + b_n A^{4/3}.$$ 

On the other hand, to produce proton a quark needs two interactions (two quarks), at 2 and 3 (not marked in (iii)). Thus

$$\frac{E d\sigma}{d^3 p} = a_p A + b_p A^{5/3}.$$ 

This means that $\alpha^{effective}_{n+} < \alpha^{effective}_p$ which is indeed the case. This approach, however, runs also into serious troubles when we try to describe production of strange particles. Perhaps the objects S, Q etc, are multi-quark objects but not physical particles? The problem is still open. We shall come back to it when we discuss deep inelastic lepton-nucleus interactions. In refs 36-39 one can find discussion of various aspects of large-production on nuclei.

3(c) High energy lepton interactions with nuclei

Multihadron production by high energy leptons opens new possibilities of investigating short time development of high energy interactions. In many respects lepton-nucleus multihadron production looks similar to hadron-nucleus production. For instance the attenuation of the forward electroproduced hadrons at 20 GeV 34, Fig. 10, is qualitatively similar to the forward production in neutron-nucleus interactions at $\sim 300$ GeV 33, Fig. 6. Unfortunately, at low-$p_L$, there is still little to compare or to analyze. Let us therefore make a few general remarks.

As we saw in the case of hadron-nucleus interactions the average number of collisions, $\bar{v}$, of the incident particle is an important quantity which characterizes multiparticle production. $\bar{v}$ has a well defined probabilistic interpretation for nondiffractive hadronic collisions (see the Appendix). This is, in general, not so for leptonic interactions. We prefer, therefore, to consider a quantity $C$, called shadow henceforth, which is always well defined and which is uniquely
related to $\bar{v}$ when the number of collisions does have a probabilistic interpretation.

For any process $\bar{v}$ whose cross sections are measured on proton, $\sigma_p(\bar{v})$, and a nucleus, $\sigma_A(\bar{v})$, the shadow is defined as follows

$$C_{\bar{v}} = \frac{\sigma_A(\bar{v}) - \sigma_p(\bar{v})}{\sigma_p(\bar{v})}, \quad C_{\bar{v}} \leq 1.$$  \hspace{1cm} 3.12

When $\bar{v}$ is a nondiffractive production process and the incident particle is a hadron,

$$C = \frac{\bar{v} - l}{\bar{v}}.$$  \hspace{1cm} 3.13

Examples of shadows for hadron-nucleus interactions:

I. 200 GeV protons on $^1\text{H}^{12}$, $\bar{v}$ -total nondiffractive production, $C = \frac{l - l}{l} \approx 0.75$.

II. ~300 GeV neutrons on $^1\text{H}^{33}$, $\bar{v}$ -inclusive production at $y \approx 8$, $C = \frac{l - A}{A} \approx 0.9$.

III. Large-$p_T$ production by protons on $^1\text{H}^{18}$, $\bar{v}$ -inclusive production at $p_T \approx 6$ GeV. Outgoing protons $C = \frac{l - A}{A}$, outgoing pions $C \approx -0.52$.

So, attenuation is characterized by positive $C$ (shadowing) while large-$p_T$ production by negative $C$ (antishadowing).

There are measurements available of $\sigma_A(\bar{v})$ and $\sigma_p(\bar{v})$ when $\bar{v}$ is the inelastic electron (muon) scattering but at comparatively low energies ($\approx 10$ GeV).

Throughout the whole investigated region of $Q^2$ and $\omega$, $C$ was found small but with both signs. \cite{40}. These experiments show that $C$ is a very relevant characteristic of inelastic reactions, and they force us to revise the standard vector meson dominance model. \cite{41}. Unfortunately, at very high energies the shadows for inelastically scattered leptons were not measured.

Let us discuss briefly multiparticle production associated with photoproduction (or electro- or muonproduction) of vector mesons and with low shadows. \cite{42}. When a vector meson of mass $m_\nu$ is produced by the incident photon (real or virtual) of energy $E$, the localization of production is given by the inverse of the longitudinal momentum transfer

$$\Delta_\nu^{-1} \approx \frac{E}{m_\nu^2}.$$  \hspace{1cm} 3.14

So, for light mesons e.g. $\rho (m_\rho \approx 0.7$ GeV) and $E = 200$ GeV, $\Delta_\rho^{-1} \approx 80$ fm while for very heavy meson e.g. $\Omega (m_\Omega \approx 9$ GeV) and $E = 200$ GeV, $\Delta_\Omega^{-1} \approx 0.5$ fm. Moreover the $\rho$-meson cross section is of the order of the $\pi$ -meson cross section but $\Omega$ -meson cross section is less than 1 mb. Consequently one should expect at high enough energies of incident photons with $\rho$ -trigger a production of particles very much as in $\pi$ -meson collisions and the shadow, $C$, of the incident photon to be approximately equal to that of pion.

Instead, the photons with $\Omega$ -trigger should have $C \approx 0$ and the production process should provide us with a very important piece of information on interaction of secondaries. Indeed, in this case, the number of collisions of $\Omega$ is expected to be close to 1, therefore any increase of multiplicity relative to the multiplicity of particles produced on hydrogen targets must be attributed to the existence of cascading of produced particles in the target nucleus. Note that in the models of low-$p_T$ particle production discussed above \cite{13,14} such cascading is neglected altogether and yet there is a very good agreement with data.

Production of particles associated with inelastic $\epsilon (\mu)$ -nucleus scattering can also be of considerable importance for understanding short-time development of production. Fig. 11 shows that, although the pseudorapidity distributions of multiplicities are similar for incident virtual photons, protons and pions of equivalent energies, there are also some relevant differences. For instance in the central region of pseudorapidities we have

$$\frac{d\tilde{n}^{(\mu)}}{d\eta} < \frac{d\tilde{n}^{(p)}}{d\eta} < \frac{d\tilde{n}^{(\pi)}}{d\eta}.$$  \hspace{1cm} 3.15

Since the large and small pseudorapidity regions differ little we also have...
\[ \bar{R}_A^{(n)} < \bar{R}_A^{(p)} < \bar{R}_A^{(\pi)} \quad \text{and} \quad \bar{R}_A^{(n)} < \bar{R}_A^{(p)} < \bar{R}_A^{(\pi)}. \]

3.16

An attempt to explain these inequalities for pions and protons was discussed above. It is an interesting question how and why the muoproduction fits into (3.15) and (3.16).

The evidence of cascading shown in Fig. 12 is of great interest because at small \( \omega \)'s we may be knocking out of nucleons their fragments (quarks? groups of quarks?) which then interact with the rest of the nucleus. To say something more about these interactions one would have to do a better experiment which would measure relative multiplicities for fixed \( \omega < \delta \) as a function of increasing energy loss of lepton \( (E - E') \). Then we could say whether e.g. the ejected system fragments into hadrons, as in the case of deep inelastic production on a free nucleon, which then cascade, or whether the nuclear matter surrounding the hit nucleon interferes with fragmentation. If indeed the ejected system has to fragment before interacting, the relative multiplicity should approach unity as \( E - E' \) increases because of time dilation effect which would push fragmentation outside of the target nucleus. As we can see an improvement of the measurements shown in Fig. 12 could greatly help in understanding of large-production results of refs 18,19. In fact it would be very interesting to repeat a Cronin-type 17 of experiment with incident leptons. Then, the analysis of the results would be simpler because the process of production would be initiated by a point-like incident particle.

This concludes this strongly biased outline of high energy particle interactions with nuclei.

4. Conclusions

1. Hadron induced low-\( p_L \) processes give evidence for a delay in production of particles which takes place along large, of the order of magnitude of nuclear diameters or more, longitudinal distances. It is not clear whether such processes will tell us something interesting about the internal structure of hadrons. More accurate and detailed data which are forthcoming may clarify the situation.

2. Clearly, hadron induced large-\( p_L \) processes are more sensitive to the internal hadronic structure. Understanding of the existing data is still far from satisfactory and much work is to be done. Important help may come from deep inelastic lepton-nucleus interactions.

3. Lepton induced productions of hadrons on nuclei are still in their infancy. Thus it is fitting to cherish great expectations for them. Indeed, various possibilities of productions associated with experimentally controlled leptonic kinematics may help penetrate mysteries of short-time development of production processes.

Many discussions of the topics of these lectures with Professor Andrzej Bialas are gratefully acknowledged.
Appendix

Probabilistic Calculations of Cross Sections and Averages

Hadron-nucleus reaction cross section at the impact parameter \( \xi \) is

\[
\sigma_r^{(A)}(\xi) = \frac{A}{A} \left| \Psi_A(0,0,\ldots,0) \right|^2 \times \left\{ 1 - \frac{1}{A} \prod_{j=1}^{A} [1 - \sigma_r(\xi_j - \xi)] \right\}^A
\]

\[
\sigma_r^{(A)} = \int d^3 \sigma_r^{(A)}(\xi)
\]

where \( \Psi_A \) is the ground state wave function of the target nucleus, \( \sigma_r(\xi) \) - the dimensionless impact parameter distribution of the hadron-nucleon inelastic cross section normalized as follows

\[
\sigma_r = \int d^3 \sigma_r(\xi)
\]

Approximating \( |\Psi_A(0,0,\ldots,0)|^2 \approx \prod_{x=A}^A \phi(r_x) \)
we have

\[
\sigma_r^{(A)} = \left\{ 1 - \frac{1}{A} \prod_{j=1}^{A} \sigma_r^{(\xi)}(\xi_j - \xi) \right\}^A
\]

where

\[
\sigma_r^{(\xi)} = \int d^3 D(\xi) \sigma_r(\xi - \xi)
\]

\[
D(\xi) = \int d^3 \rho(\xi - \xi), \int d^3 D(\xi) = 1
\]

To see that (3) does indeed originate from probability calculus one can re-write it

\[
\sigma_r^{(A)}(\xi) = \sum_{y=1}^{A} \left( \begin{array}{c} A \\ y \end{array} \right) \left[ \sigma_r^{(\xi)}(\xi) \right]^{y} \left[ 1 - \sigma_r^{(\xi)}(\xi) \right]^{A-y}
\]

where the consecutive terms correspond to exactly one, two, three etc collisions with the target nucleus.

Similarly, for the total inelastic cross section, one gets \( (1 - \sigma_r^{(\xi)} + \sigma_r^{(\xi)})^A \)

\[
\sigma_t^{(A)}(\xi) = \left\{ 1 - \left[ 1 - \sigma_r^{(\xi)}(\xi) \right] \right\}^{A} \]

is the probability that the target is left in the ground state

\[
\sigma_t^{(A)}(\xi) = 1 - \left\{ 1 - \sigma_r^{(\xi)}(\xi) \right\}^{A}
\]

with \( \sigma_r^{(\xi)} = \int d^3 D(\xi) \sigma_r(\xi - \xi) \),

\[
\int d^3 \rho(\xi - \xi) \sigma_r^{(\xi)}(\xi - \xi) = \sigma_r^{(\xi)}\sigma_r^{(\xi)}(\xi - \xi) = \frac{1}{4} \sigma_r^{(\xi)}(\xi - \xi)
\]

for a purely imaginary elastic amplitude. The term \( \sigma_r^{(\xi)}(\xi - \xi) \) takes care of the fact that some interactions leave the target nucleus in its ground state, hence do not contribute to inelastic cross section \( \sigma_t^{(A)}(\xi) \).

Assuming the elastic amplitude to be purely imaginary one can calculate it from the optical theorem

\[
\gamma(\xi) = \left| \gamma(\xi) \right|^2 + \sigma_t^{(A)}(\xi)
\]

Using (7) as an input we get from (8):

\[
\gamma(\xi) = \left\{ 1 - \left[ 1 - \sigma_r^{(\xi)}(\xi) \right] \right\}^{A}
\]

\[
\gamma(\xi) = 1 - \left\{ 1 - \frac{1}{\sigma_r^{(\xi)}(\xi)} \right\}^{A}
\]

This is the well known formula of the Glauber model for the elastic amplitude in the impact parameter representation.

Indeed, it turns out that the Glauber model cross sections are identical with expressions gotten from classical probability calculus. To complete the proof let us recall the formula for quasielastic cross section of the Glauber model

\[
\sigma^{(A)}_{q.e.}(\xi) = \left\{ 1 - \sigma_r^{(\xi)}(\xi) \right\}^{A}
\]

\[
\sigma^{(A)}_{q.e.}(\xi) = \left[ 1 - \sigma_r^{(\xi)}(\xi) \right]^{A} - \left\{ 1 - \sigma_r^{(\xi)}(\xi) + \frac{1}{4} \sigma_r^{(\xi)}(\xi) \right\}^{A}
\]

The reaction cross section is
\[
\sigma_{r, G}^{(A)}(\xi) = \sigma_{m, G}^{(A)}(\xi) - \sigma_{G}^{(A)}\theta_{r, G}(\xi)
\]
\[
= 2\Gamma(\xi)\left[1 - \Gamma(\xi)^2\right] - \sigma_{G}^{(A)}\theta_{r, G}(\xi)
\]
From (10) and (9) we get
\[
\sigma_{r, G}^{(A)}(\xi) = 2 - 2\left[1 - \frac{1}{2}\bar{\sigma}_c(\xi)\right]^A
\]
\[-\left[1 - \frac{1}{2}\bar{\sigma}_c(\xi)\right]^2 - \left[1 - \bar{\sigma}_c(\xi)\right]^A
\]
\[+ \left[\bar{\sigma}_c(\xi) + \frac{1}{2}\bar{\sigma}_c^2(\xi)\right] = 1 - \left[1 - \frac{1}{2}\bar{\sigma}_c(\xi)\right]^2
\]
\[-\left[1 - \bar{\sigma}_c(\xi)\right]^A + \left[1 - \bar{\sigma}_c(\xi) + \frac{1}{4}\bar{\sigma}_c^2(\xi)\right]\]
\[= 1 - \left[1 - \bar{\sigma}_r(\xi)\right]^A.
\]
Which completes the proof.

Since
\[
\sigma_{r, A}^{(A)} = \sigma_c^{(A)} - \sigma_{A}^{(A)} - \sigma_{r}^{(A)}
\]

it follows that also \(\sigma_{r, A}^{(A)}\) is given by the same formula (10) in both approaches. From (8) and (9) we get the expression for the total cross section
\[
\sigma_c^{(A)} = \int d^2\xi \sigma_c^{(A)}(\xi) = 2\int d^2\xi \Gamma(\xi)
\]
\[= 2\int d^2\xi \left[1 - \frac{1}{2}\bar{\sigma}_c(\xi)\right]^A\]

as follows
\[
\overline{\nu} = \frac{1}{\sigma_{r}^{(A)}} \int d^2\xi \sum_{\nu=1}^{A} \nu p(\nu, \xi)
\]
\[= \frac{1}{\sigma_{r}^{(A)}} \int d^2\xi \sum_{\nu=1}^{A} \nu p(\nu, \xi)
\]
is the probability of \(\nu\) hits and \(A - \nu\) misses at the impact parameter \(b\). One computes (A15) using the generating function
\[
\phi(x) = \sum_{\nu=1}^{A} \nu (1 - \bar{\sigma}_c(\xi))^A - \nu \bar{\sigma}_c(\xi)^A x^\nu
\]
\[= \left[1 - \bar{\sigma}_c(\xi) + \bar{\sigma}_c(\xi)^A\right]^A
\]
Since
\[
\frac{\partial \phi}{\partial x} \bigg|_{x=1} = \sum_{\nu=1}^{A} \nu p(\nu, \xi) = A \bar{\sigma}_c(\xi)
\]
we get from (A15), (A18), (A2), (A4) and (A5)
\[\overline{\nu} = \frac{A \sigma_r}{\sigma_r^{(A)}}.
\]
When both colliding objects are treated as composite (e.g. two nuclei with \(A\) and \(B\) nucleons) one can use (A19) to compute, in each object, the average number of hit constituents. The average numbers of hit nucleons in the nuclei \(A\) and \(B\) are, respectively,
\[\overline{W}_A(8) = \frac{A \sigma_r^{(A)}}{\sigma_r^{(A)}}, \quad \overline{W}_B(8) = \frac{B \sigma_r^{(A)}}{\sigma_r^{(A)}}.
\]
where \(\sigma_r^{(A)}\) is the nucleon-nucleus \(B\) reaction cross section and \(\sigma_r^{(AB)}\) is the nucleus \(A\) - nucleus \(B\) reaction cross section. Eqs (A20) are also being used to describe production processes through interactions of constituent quarks (see Section 3). Note that in all nuclear interactions the average (A19) and (A20) can be computed from directly measured quantities.
Fig. 1 Relative multiplicities calculated from nuclear cascading compared with data.

Fig. 2 Relative multiplicities for protons and pions of various energies interacting with various nuclei. The straight line is the relation $(2.6).$ Relations between $A$ and $\bar{\nu}$ are: for protons $A = 3.16 \bar{\nu}^{1.32}$ and for pions $A = 3.35 \bar{\nu}^{1.41}.$

Fig. 3 Pseudorapidity distribution of multiplicities from proton-emulsion interactions at 400 GeV and in proton-proton interactions. The solid line is the leading particle cascading calculated in.

Fig. 4 Attenuation of forward produced particles in $\pi^-$ and $p$ emulsion interactions.
Fig. 5 (a) Rapidity distributions of multiplicities for various nuclei in 200 GeV proton-nucleus collisions.
(b) Rapidity distributions of multiplicities for various energies of incident protons. Also theoretical curves of multiple-chain parton model are shown.

Fig. 6 Attenuation of forward produced particles from ~300 GeV neutron-nuclei interactions given through exponent $\alpha$ in invariant cross sections fitted to the $\sim A^{\alpha}$ dependence. Horizontal solid line is the prediction of the constituent quark model of production.

Fig. 7 Demonstration that large-$p_T$ invariant cross sections fit $\sim A^{\alpha}$ dependence: (a) $\pi^{\pm}$ at $p_T = 1.5$ GeV/c, (b) $\pi^{-}$ at $p_T = 3.5$ GeV/c, (c) $\pi^{+}$ at $p_T = 5.3$ GeV/c, (d) $\pi^{\pm}$ at $p_T = 5.3$ GeV/c.

Fig. 8 The exponents $\alpha$ for various produced particles as functions of $p_T$. 

- 16 -
Fig. 9 Antishadowing ($\alpha > l$) for production of massive dihadron states with net charge zero, $p_\perp = |p_\perp^+ + p_\perp^-|$, $m = p_\perp^+ + p_\perp^-$, with $p_\perp^+ (-)$ being the magnitude of the positive (negative) member of a pair.

Fig. 10 Attenuation of forward produced hadrons in 20.5 GeV electron-nucleus interactions. For the definition of $\alpha'$ see the text.

Fig. 11 Rapidity distribution of multiplicities in 150 GeV $\mu$-emulsion interactions. Also the same distributions for proton-emulsion and pion-emulsion at equivalent energies are shown.

Fig. 12 Total multiplicities in five $\omega$-bins for 150 $\mu$-emulsion and $\mu$-proton interactions.

Fig. 13 Space-time development of multiparticle production in multiperipheral model.
Fig. 14 Comparison of the relative multiplicities in the central region of pseudorapidities $^2$ with the results of the constituent quark model of production $^6$.

Fig. 15 Total cross sections for wide range of energies of neutrons interacting with nuclei compiled in ref. $^{43}$. Dashed curves are calculated from the probabilistic expression for inelastic cross sections and optical theorem (see Appendix). Solid lines include corrections from inelastic shadowing $^{43}$. 
Literature


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25. Data on hadron-hydrogen data were made available by dr L. Voyvodic, private communication.

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Foreword

Due to circumstances beyond my control, the write-up of my lectures was not ready well after the deadline for submittal was passed. The lecture notes will appear as a CERN-TH preprint. What follows is a brief outline of the topics I discussed in a set of five one-hour talks.

1. INTRODUCTION TO QCD

I started by reviewing QED as a gauge theory of (abelian) electric charge. I emphasized how the gauge principle and the renormalizability constraint fix the form of the interactions and "predict" the existence of the photon (See the first attached transparency).

I moved my way towards the concept of color by emphasizing the analogy between electrodynamics and the closely bound states of leptons and the allegedly chromodynamic bound states of quarks. As important as the analogies are the differences: in particular the fact that quarks bind in gleeqs (sets of three). I faced the problem of the spin-statistics puzzle for the baryon wave function and let color, at this point a sort of hidden variable, solve it for me.

For no unique a priori reason other than maximum simplicity, quarks are chosen as color triplets. Having color add in a non-abelian fashion (like angular momentum) allows one to construct mesons and baryons that are color singlets, much as electrodynamical bound states like to be electrically neutral.

Next came the main step: the gauging of the color charge. The audience and myself contemplated the birth of eight massless color gluons, with specific and universal couplings to each other and to quarks (See the second attached transparency). I pronounced some words on asymptotic freedom, infrared

*) This is a short summary of lectures given at the 1978 CERN School of Physics.
slavery and the sense in which quarks have been seen and gluons have been smelled (by theorists). I re-emphasized the color and flavor symmetries of the QED and QCD Lagrangian.

I finished with a Michelin guide to QCD predictions (See third attached transparency), many of which were to be discussed in subsequent lectures.

2. ASYMPTOTIC FREEDOM

QCD is an asymptotically free field theory, somewhat unfortunate adjectives stemming from the well established perversity of their inventor: The theory is not ever a free field theory and it makes predictions that are not asymptotic. The way this works, similarly perverse, originates from the fact that intelligently defined coupling constants are not constants. This lecture was too technical for an outline to be informative. An intermediate set of conclusions is reproduced in the fourth attached transparency.

3. CHROMODYNAMICS VERSUS HADRON SPECTROSCOPY

I reconsidered the symmetries of meson and baryon wave functions as color singlet composites of quarks. The main emphasis was on "hyper-fine" mass splittings between particles with the same quark content and radial quantum numbers but with different spins ($\rho$ and $\pi$, $N^*$ and $N$) or spin wave functions ($\Sigma$ and $\Lambda$). I reviewed hyperfine splittings in electrodynamics and a naive, unbelievably successful study of hadron mass splittings in a chromodynamic one-gluon exchange model. Why the pion, the proton and the $\Lambda$ are respectively lighter than the $\rho$, the $N^*$ and the $\Sigma$, are questions that QCD hopefully made clear to the audience. I revisited the entertaining history of the successes and failures of the predictions of charmed particle masses.

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Transparency 3

Transparency 4
4. INCLUSIVE DEEP INELASTIC LEPTON SCATTERING

I reviewed the subject of quasi-scaling structure functions $F(x, Q^2)$ in the context of QCD, with emphasis on the factorization of their infrared (long-distance) and ultraviolet (short-distance) properties. Very succinctly the Bjorken (or Nachtmann) $x$-dependence is infrared and presently incalculable while the $Q^2$-evolution is short-distance and calculable in an asymptotically free theory.

Somewhat unconventionally I indulged in a non-relativistic model of the positronium, pion and proton structure functions, to emphasize and try to understand the connection between the $x$-dependence of structure functions and the bound state wave functions.

More along the traditional path I briefly reviewed the $\pi$-momentum frame and parton model mnemonotchnics for structure functions, to end up with a short-circuited discussion of the QCD predictions for the $Q^2$ evolution of structure functions.

5. JETOLOGY

I began with a brief review of the experimental information on jets in $e^+e^-$ annihilation. Then I attempted to start from QCD and work towards understanding jets and predicting the future of jet physics. Much time and effort was devoted to the taming of the infrared problems that occur in perturbation theory and the skirting of the non-perturbative aspects of QCD that we do not understand. The jet-physics commandments appear in the fifth attached transparency. I talked about jets in $e^+e^-$ annihilation, both in the "continuum" and on prominent "onium" resonances. Few words were devoted to jets in other processes.

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**PROCEDURE FOR TESTING PERTURBATIVE QCD (A)**

**JET PHYSICS** (e$^+$e$^-$ OR ELSEWHERE)

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1. Thou shalt compute observables in pert. theory in terms of QCD quanta (quarks, gluons).

2. Thou shalt smooth the result over whatever width is bigger (for any obs).

   - A fixed $<p_T>$ "nonperturb" width $\Delta_{NP}$ (erases nonpert sensitivity).
   - An infrared perturbative width $\Delta_{IRP}$ (tames infrared catastrophes).

   QCD quanta $\rightarrow$ jets with same momentum.

3. Thou shalt interpret the results at tests of QCD.

   - Thou shalt relieve thine taxpayer by building nothing sexier than hadron calorins.
6. QUARK AND GLUON LIBERATION

This lecture was heretical: It concerned the possibility that quarks and gluons may be produced as real unconfined particles. The main idea (See the last attached transparency), is that the gluon mass term in the QCD Lagrangian may not exactly vanish and that Archimedes' principle may apply to quarks and gluons: They are light inside hadrons and heavy when extracted. Time limitations stopped me from actually giving this lecture.
Gauge Fields

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Abstract

In these notes we provide some background on the theory of gauge fields, a subject of increasing popularity among particle physicists (and others). We do not present detailed motivations and applications which are covered in the other lectures of this school. In particular we omit the application to weak interactions by referring to the introduction given by J. Iliopoulos a year ago. The aim will be rather to stress those aspects which suggest that gauge fields may play some role in a future theory of strong interactions. Only a few references have been quoted. We apologize to all of those who have contributed and are not cited here.

1. Local Invariance

We are all familiar with the concept of gauge invariance in electrodynamics. The simplest example is provided by a charged particle moving in an electromagnetic field described by a scalar (V) and vector (A) potential. The Schrödinger equation reads

$$\frac{1}{2m} \left( \frac{\mathbf{p}^2}{\hbar^2} - eA \right) \psi = \left( i\hbar \frac{\partial}{\partial t} - eV \right) \psi$$

(1)

An arbitrary change in the phase of $\psi$

$$\psi(\mathbf{x},t) = \exp \left[ \frac{ie}{\hbar} \chi(\mathbf{x},t) \right] \psi'(\mathbf{x},t)$$

(2)

may be compensated by a corresponding change in the potentials

$$A(\mathbf{x},t) = A'(\mathbf{x},t) + \chi'$$

$$V(\mathbf{x},t) = V'(\mathbf{x},t) - \frac{\partial}{\partial t} \chi$$

(3)

without modifying the measurable electromagnetic fields

$$\mathbf{B} = \text{curl} \mathbf{A} = \mathbf{B}'$$

$$\mathbf{E} = -\nabla V - \frac{\partial \mathbf{A}}{\partial t} = \mathbf{E}'$$

(4)

nor the physical content of the Schrödinger equation which takes the same form as (1) in the prime variables.

This aesthetical invariance principle has important consequences, among which are the following:

1) it severely restricts the types of possible couplings,

2) it reduces the number of degrees of freedom in the system,

3) it implies, in the quantum context some non trivial interplay between topology and dynamics, a point which was not sufficiently emphasized until recently,

4) at the level of quantum field theory it plays a crucial rôle when discussing renormalizability.

Point 1) is a very familiar one. For each charged system interacting electromagnetically, it implies the replacement of ordinary derivatives by 'covariant' ones (minimal coupling). Henceforth, we use relativistic notations, with $c=1$, and Minkowskian metric $g^{\mu\nu} = g_{\mu\nu}$, $\delta^\mu_{\nu} = \delta_\nu^\mu$, $0 \leq \mu, \nu \leq 3$, $\eta_{00} = -\eta_{KK} = 1$

ordinary derivative $\partial_\mu = \frac{\partial}{\partial x^\mu}$

4-potential $A_\mu = \{A_0,\mathbf{A}\}$

covariant derivative $D_\mu = \partial_\mu + ieA_\mu$

field strength $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

Sum over repeated indices is understood.

For instance, it is the minimal prescription which gives the electromagnetic coupling of a Dirac field ($\gamma^\mu \equiv \text{Dirac matrices}$)

$$\left\{ \gamma^\mu \left( i\sigma_\mu - eA_\mu \right) - m \right\} \psi = 0$$

(6)

This entails the famous non trivial prediction of a gyromagnetic ratio equal to 2 (forgetting radiative corrections) for a Dirac electron, but could of course be modified by extra ad-hoc terms of the type $\sigma^{\mu\nu} F_{\mu\nu}$ themselves gauge invariant.

With regard to point 3) we observe that we may derive equations of motion for the electromagnetic field from a stationary action principle. This requires the construction of a gauge invariant Lagrangian density involving the minimal number of derivatives. The simplest candidate is
\[ \mathcal{L}_{\text{e.m.}} = -\frac{1}{4} \, F_{\mu \nu} \, F^{\mu \nu} \]  

(7)

from which the Euler-Lagrange equations follow if we write \[ \mathcal{L}_{\text{tot}} = \mathcal{L}_{\text{e.m.}} + \Delta \mathcal{L} \]

\[ \frac{1}{2} \frac{\partial}{\partial \lambda} \left( \frac{2F_{\mu \nu}}{\partial(\sigma A^\mu)} \right) \, F^{\mu \nu} + \frac{3}{3\lambda} \, \Delta \mathcal{L} = 0 \]

i.e.

\[ \frac{3 F_{\mu \nu}}{\mu} = -\frac{3}{3\lambda} \, \Delta \mathcal{L} = j^\nu \]  

(8)

defining the conserved current \( j^\nu \). These are the well known Maxwell equations which we consider here in terms of the potentials. Unfortunately they are not sufficient to prescribe their time evolution, since the equations are not modified by a gauge transformation (3). The description in terms of \( A^\mu \), although it provides a local characterization of the dynamics introduces spurious unphysical degrees of freedom. Eq. (8) is compatible with an infinite set of potentials parametrized by an arbitrary function of \( \chi \) and \( t \). We must make a choice of gauge to select a representative solution. For instance we can impose the Lorentz condition

\[ A^\mu_\mu = 0 \]  

(9)

in which case under suitable boundary conditions, Eq. (8) which reduces to \( \nabla A^\nu = j^\nu (\nabla = \text{d'Alambertian} = \partial^2_\chi - \delta^2_{tt}) \) admits a unique solution

\[ A^\nu = A^\nu_{\text{asympt.}} + \sigma^{-1} j^\nu \]  

(10)

Stated differently if we write explicitly the equation as

\[ \left( \sigma_{\mu \nu} - \sigma_{\mu} \sigma_{\nu} \right) A^\nu = j^\nu \]  

(11)

the operator on the left hand side does not admit a unique inverse formally. In a quantized theory, this inverse is to become the photon propagator. We must therefore cope in a way or another with this difficulty to construct a meaningful photon field.

An example of point \( \psi \) is the Bohm-Aharonov effect. One studies electron diffraction in a Young type experiment in the presence of a coil where a current induces a magnetic field \( \vec{B} \). 

![Diagram](image)

Eventhough no field acts on the electron path one predicts, and observes, a shift in the diffraction pattern through a phase

\[ \theta = \frac{e}{\hbar} \int_{S} \vec{A} \cdot \vec{d}x = \frac{e}{\hbar} \int_{S} \text{curl} \vec{A} \cdot \vec{d}x = \frac{e \Phi}{\hbar} \]  

(12)

where \( \Phi \) is the magnetic flux through the coil. In the empty space surrounding it \( \int_{C} \vec{A} \cdot \vec{d}x \) is an invariant (with respect of continuous deformations of the countour \( C \)) because \( \text{curl} \vec{A} = \vec{0} \). Since the surrounding space has a hole the integral need not vanish. Such interplay between topology and dynamics is to become a major theme in the non-abelian counterpart of the theory.

Finally point \( \psi \) will be elaborated as we go along and is presumably well known to an audience of particle physicists.

It occurred to Yang and Mills\(^2\) that the above construction could be generalized to other invariance groups than the commutative group of phases.

What happens if the dynamics is invariant under changes of local reference frames describing internal degrees of freedom ? To be specific we use the colored quark model as an example ; the ideas are of course general. Assume therefore that the quark states carry a three-color index. We define correspondingly three fields \( \psi_a(\chi) \), \( a = 1, 2, 3 \) which can be linearly transformed according to

\[ \psi_a \rightarrow \psi'_a = U_{ab} \psi_b \]  

(13)

where \( U \) stands for a 3 by 3 unitary matrix with unit determinant, i.e. an element of \( \text{SU}(3) \). Note that we leave aside global phase transformations. An infinitesimal transformation will read

\[ \psi' = \psi + \delta \psi \quad U_{ab} = \delta_{ab} + i \lambda^c_{ab} \delta \xi^c \]

\[ \delta \psi = i \lambda^c \delta \xi^c \psi \]  

(14)

The 8 hermitian traceless matrices \( \lambda^a \) (\( \lambda^a \) by 3 Gell-Mann matrices) are representatives of \( \text{SU}(3) \) generators. This group is non commutative, i.e. in general \( U U' \neq U' U \); one also says non-Abelian. This will be the main source of differences when we try to generalize our previous ideas on local invariance. We insist that the model be invariant when the group element \( U \) appearing in (13) depends on the space-time point \( \chi \). Consequently values of the quark field at distinct points are not directly comparable. In particular the notion of field derivative must be modified. It is clear that if \( \psi' = U \psi \)
\[ \partial_\mu \psi' = U \partial_\mu \psi + (\partial_\mu U) \psi \]  
(15)

where the additional term can be compensated if we allow for a mean of comparing frames at nearby points. This is provided by the analog of the gauge potential which describes an infinitesimal frame transformation as we move from \( x \) to \( x + dx \). It will be convenient to use the following conventions. In a given representation we absorb the factor \( i \) occurring in front of the generators to define the analog of \( i\gamma^\alpha \) as \( \gamma^\alpha \). We denote by \( \gamma \) the corresponding abstract element in the Lie algebra of the group without reference to a specific representation. An \( x \)-dependent infinitesimal transformation corresponding to the path \( x \), \( x + dx \) is

\[ U(x + dx, x; A) = I + t^\alpha A_{\mu, \nu}(x) \, dx^\mu \]  
(16)

This allows to define the covariant derivative \( D_\mu \psi \) through

\[ dx^\mu D_\mu \psi = \psi(x + dx) - U(x + dx, x; A) \psi(x) = dx^\mu \left( \partial_\mu - \gamma^\alpha A_{\mu, \nu}(x) \right) \psi(x) \]  
(17)

We have now the rationale to write

\[ D_\mu = \partial_\mu - A_{\mu}(x) \]  
(18)

where \( A_{\mu} \) stands for the Lie algebra element \( t^\alpha A_{\alpha, \mu}(x) \) or any of its matrix representative according to the context.

A gauge potential (or connexion) is therefore a collection of eight vector fields in the case of \( SU(3) \) in general of as many vector fields as there are generators. To obtain the properties of \( A_{\mu} \) under local transformations, we require of course that in contradistinction to (15)

\[ D^\mu \psi' = U(x) D_\mu \psi \]  
(19)

This is equivalent to say that

\[ U(x + dx, x; A') = U(x + dx, x; A) U^{-1}(x) \]

i.e.

\[ A'_{\mu}(x) = U(x) A_{\mu}(x) U^{-1}(x) + \left[ \partial_\mu U(x) \right] U^{-1}(x) \]  
(20)

For an infinitesimal transformation

\[ U(x) = I + \delta \xi(x) \]

\[ A'_{\mu} = A_{\mu} + \delta A_{\mu} \]

\[ \delta A_{\mu} = \left[ \partial_\mu \delta \xi \right] \]  
(21)

If this is compared with the electromagnetic case (3) we observe the double character of a non-Abelian gauge potential. On the one hand the quantity \( \partial_\mu \delta \xi \) is the analog of the added term in an Abelian gauge transformation. The extra term \( \delta \xi A_{\mu} \) can be interpreted as if \( A_{\mu} \) corresponds to a "charged" field belonging to the adjoint representation. To obtain the latter, consider the structure constants defined through

\[ [\gamma^\alpha, \gamma^\beta] = C^{\alpha\beta} \gamma^{\gamma} \]  
(22)

and consider the matrices

\[ (\gamma^\alpha)_{\beta\gamma} = C^{\alpha\gamma} \beta \]  
(23)

Then

\[ [\delta \xi, A_{\mu}] = [\gamma^\alpha, \gamma^\beta] \delta \xi \gamma^{\alpha} A_{\beta, \mu} = C^{\alpha\beta} \gamma^{\gamma} A_{\beta, \mu}^{\gamma} \]

Note also that (21) can also be written \( \delta A_{\mu} = (D_\mu \delta \xi) \) with \( D_\mu \) interpreted in the adjoint representation.

Geometrically we have introduced the gauge potential to compare the internal frames at nearby points. From a frame at \( x \) we derive a frame at \( x + dx \) using (16). This cannot be done in a coherent fashion throughout space-time unless some integrability conditions are satisfied. The generalized curvature tensor (or field intensity) \( F_{\mu\nu} \) will express this non-integrability. Indeed the frame coherence could be obtained by integrating the differential equations

\[ D_\mu \psi = 0 \]

which would require the operators \( D_\mu \) and \( D_\nu \) to commute. We define therefore the antisymmetric field strength as

\[ F_{\mu\nu} = -[D_\mu, D_\nu] = \partial_\mu A_{\nu} - \partial_\nu A_{\mu} - [A_{\mu}, A_{\nu}] \]  
(24)

\[ F_{\alpha, \mu\nu} = \partial_\mu A_{\alpha, \nu} - \partial_\alpha A_{\mu, \nu} - C^{\alpha\beta} \gamma_{\alpha} A_{\beta, \mu} A_{\beta, \nu} \]

In a local transformation, this field transforms according to the adjoint representation

\[ \delta F_{\mu\nu} = T^\alpha \delta \xi_{\alpha} F_{\mu\nu} \]  
(25)

with \( T^\alpha \) given by (23). This is again a new feature as compared to the Abelian case, where the field strength tensor was invariant, here it is covariant. In matrix notation the Jacobi identity

\[ \left[ B_{\mu}, [D_\nu, D_\rho] \right] + \text{cycl. perm.} = 0 \]  
(26)

also called Bianchi identity in this context, expresses the equivalent of the homogeneous set of Maxwell's equations
Given the fields $F_{\mu\nu}(x)$ it is an interesting, and not yet fully clarified, question to figure out whether the equations

$$[D_{\nu}(A),F_{\mu\nu}] + \text{cycl. perm} = 0$$

require $F$ to be expressed in terms of $A$ as in (21). Counter-examples are known.

The non commutative character of the internal group has induced typical non-linearities characteristic of the Yang Mills case. A familiar concept such as the one of plane wave has no direct physical content, due to the loss of the superposition principle, except in a perturbative treatment of non-linearities. In many respects the parallel treatment of QED and QCD is deceptive as we have come to learn in recent years.

Consider for instance the configurations of zero curvature at a given time. Let us restrict ourselves to situations where $A_{\mu}^0$ vanishes. In the absence of source this temporal gauge condition can be achieved through a gauge transformation in a regular way. According to the geometrical interpretation, this means that we can solve for the equations $\mathcal{D}^\mu_\nu = 0$ throughout space. Starting from a frame gives by three linearly independent $\psi$'s (for SU(3)) at a point we obtain a frame throughout space, related by a unitary transformation $U(x)$ and it is easy to see that

$$\mathbf{A}(x) = \mathcal{U}(U(x)) \ U^{-1}(x)$$

(27)

i.e. we have a pure gauge. This will eventually be identified with a (classical) vacuum configuration very much as in electrodynamics. The question arises whether or not $U(x)$ can be continuously transformed to the unit transformation with $\mathbf{A}$ going to zero throughout space. We expect to pay a price (in energy) if we encounter singularities in this process.

A mean to eliminate the physical role of the points at infinity, is to require $\mathbf{A}$ to vanish as $[\mathbf{A}] = \infty$, hence $U = \text{const}$. This constant can be transformed to the unit element through a global transformation. In this way the physical space $\mathbb{R}^3$ may be identified with a three dimensional sphere $S_3$ which is mapped into the group through $x \rightarrow U(x)$. Equivalence classes of such maps under continuous deformations are called homotopy classes. The crucial point is that in general for an arbitrary compact Lie group such classes do not reduce to the identity. An example which may be (almost) visualized is afforded by the group SU(2). We write a representative matrix

$$U = u_0 + i \mathbf{u} \cdot \mathbf{\sigma}$$

(28)

$\mathbf{\sigma}$, Pauli matrices $u_0^2 + u^2 = 1$

This representation implies that a point in SU(2) may also be thought as a point on a sphere $S_2$. We are left with classifying maps of $S_3$ (the physical space suitably parametrized) onto itself (the group space). Such classes are characterized in this case by the number of times the sphere is mapped onto itself. It can be computed by integrating the Jacobian of the function throughout space and by dividing by the "area" of the sphere $2\pi^2$. The result is an integer

$$n = \int \frac{d^3x}{2\pi^2} \det A_{\alpha k}(x)$$

(29)

In the determinant both $k$ and $\alpha$ run from 1 to 3.

As an example the stereographic mapping

$$u_0 = \frac{1-x^2}{1+x^2}, \ u = \frac{2x}{1+x^2}$$

gives

$$A_{\alpha k} = \frac{4}{(1+x^2)^2} \left[ \frac{1-x^2}{2} \delta_{\alpha k} + x^2 \right] \det A_{\alpha k}(x)$$

(32)

$$\det A_{\alpha k}(x) = \frac{8}{(1+x^2)^3}$$

(33)

$$\int \frac{d^3x}{2\pi^2} \det A_{\alpha k}(x) = \frac{4\pi}{2\pi^2} \int^\infty_0 \frac{x^2 \ dx}{(1+x^2)^3} = 1$$

(34)

and it was obvious that space was mapped once onto the sphere. A representative of the general class $n$ would be obtained by taking the $n$-th power

$$u_0^n(x) = \left( \frac{1-x^2 + 2i0^+\cdot x}{1+x^2} \right)^n$$

(35)

Vacuum configurations fall therefore in inequivalent classes characterized by a topological index $n$. We shall have more to say on this situation as we proceed to the quantum theory.

2. Equations of Motion

It is tempting to generalize the Maxwell Lagrangian (1.7) to the non-Abelian case. Here comes a point that we omitted to discuss up to now. It has to do with the fact that we have not yet introduced a coupling constant. Indeed we wrote $3 \mu^i A^\mu_i$ for
the covariant derivative in QED and $\partial_\mu - A_\mu$ in QCD. It is simply a matter of rescaling the potential $A \to gA$ so as to give a similar appearance to both, and this is what we understand from now on (similarly $F(A) \to \frac{1}{8} F(gA)$). We therefore follow Yang and Mills by defining
\[ \omega_{YM} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \tag{1} \]
The sum over $\alpha$ implies that we have picked a suitable orthonormalized basis in the Lie algebra.

For a simple Lie group there exists only one non-degenerate invariant quadratic form on the Lie algebra. A semi-simple Lie algebra is a direct sum of simple components. Any compact Lie group has a Lie algebra equal to a direct sum of a semi-simple one plus Abelian factors. An example of the last circumstance is the $SU(2) \times U(1)$ group of the Weinberg-Salam model.

To construct this so-called Killing quadratic form, one considers the Lie algebra as a representation vector space and defines the action of any element $X$ on an arbitrary $Y$ through
\[ \text{adj}(X) Y = [X, Y] \tag{2} \]
By virtue of the Jacobi identity $\text{adj}[X_1, X_2] = [\text{adj} X_1, X_2] + [X_1, \text{adj} X_2]$ so that we have indeed a representation. This is the one to which the gauge potential belongs. The Killing form is then nothing but
\[ (X, Y) = C \text{Tr}(\text{adj} X, \text{adj} Y) = C x_\alpha y_\beta \text{Tr} adj_t^\alpha adj_t^\beta \tag{3} \]
if $X$ (respectively $Y$) is written $X = x_\alpha t^\alpha (Y = y_\alpha t^\alpha)$ and $C$ is a constant. The physicists’ choice is such that
\[ (t^\alpha, t^\beta) = -\frac{1}{2} \delta_{\alpha\beta} \tag{4} \]
For $SU(2)$, $[t^\alpha, t^\beta] = -\epsilon_{\alpha\beta\gamma} t^\gamma$
\[ \text{adj} t_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}, \quad \text{adj} t_2 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \quad \text{adj} t_3 = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{5} \]
We recognize the spin 1/2 representation with
\[ \text{Tr} adj_t^\alpha adj_t^\beta = -2\delta_{\alpha\beta} \]
so that $C = \frac{1}{2}$ in this case. In fact if we compare traces in two irreducible representations, they are proportional, the factor being the ratio of Casimir operators times the ratio of dimensions. For $SU(2)$ the normalization (4) corresponds to the trace in the spin 1/2 representation and the ratio $\xi$ is the ratio of values of $(2j+1)$ and $j+1$ with $j = \frac{1}{2}$ and $j = \frac{1}{2}$. The negative sign is due to the fact that we use antihermitian generators.

An alternative notation is therefore
\[ \omega_{YM} = \frac{1}{4} (F_{\mu\nu}, F^{\mu\nu}) \tag{6} \]
We shall sometimes be sloppy and omit the brackets in the scalar products.

It is now a simple matter to derive the classical field equations. In the absence of matter fields they read
\[ [\epsilon^\mu_{\nu}, F_{\mu\nu}] = 0 \tag{7} \]
or equivalently
\[ \varepsilon^\mu_{\alpha\mu\nu} - \frac{1}{2} \epsilon_{\alpha\beta\gamma} \alpha A^\beta \alpha A^\gamma F_{Y,\mu\nu} = 0 \tag{7'} \]
The crowding of indices illustrates the virtue of compact notations, especially if we express $F$ itself in terms of $A$.

These equations can be put in parallel with the Bianchi identities (1.26) and exhibit once more, through their non-linearities, the fact that $A$ acts as a source on itself.

If we define in analogy with electrodynamics the electric (E) and magnetic (B) components of the field, it is easy to see that they contribute additively to the energy density
\[ \varepsilon^i_{\alpha} = F_{i\alpha}, \quad \pi^i_{\alpha} = -\frac{1}{2} c_{ijk} F^{jk}_{\alpha} \tag{8} \]
As anticipated the classical minimal energy configurations are such that $F$ vanishes.

To generalize the model we may introduce matter using a minimally coupled Dirac quark field. This amounts to add a piece
\[ \mathcal{L}_D = -\bar{\psi} (im_D - m) \psi \tag{9} \]
to the Lagrangian. The covariant derivative is taken here in the quark representation and $M$ is the mass matrix. This piece will generate a current $ig\bar{\psi}^\alpha \gamma_\mu \psi$ on the r.h.s. of the field equation (recall that it is hermitian). Similarly Bose fields would lead to
\[ \mathcal{L}_B = (\partial_\mu \phi)^* (\partial^\mu \phi) - V(\phi^*, \phi) \tag{10} \]
where $V$ includes mass terms and non derivative self-
couplings.

Let us concentrate on the gauge field dynamics. As in QED gauge freedom implies that the Yang Mills equations are not sufficient to determine the time evolution of the potential. This is also reflected in the fact that the conjugate momentum to $A^0$ vanishes, which leads to trouble as we attempt to quantize. There are at least two ways to cure this disease, which turn to be ultimately equivalent. In the first approach one uses a canonical procedure. The time components $A^0$ are given the meaning of Lagrange multipliers of constraints on the dynamical variables. Alternatively, one introduces additional terms in the Lagrangian breaking local gauge invariance which is restored at a final stage by a suitable average over the gauge group. The first way is perhaps the most illuminating but it requires some sophisticated analysis which we cannot describe here.

At any rate we have to discuss gauge fixing conditions. A physically appealing procedure in the classical context uses the radiation gauge: $A^0$ is obtained in terms of the source through Gauss's law and the vector potential is required to satisfy $\nabla \cdot A = 0$. Equivalently $A^0$ is traded for $\vec{E}$, so that the fundamental variables $\vec{E}$ and $\vec{A}$ satisfy $\nabla \cdot \vec{E} + \epsilon_\mu \epsilon_\alpha A^\alpha = 0$ (Gauss's law in the absence of matter sources) and $\nabla \cdot \vec{A} = 0$.

As with any gauge condition, this leads immediately to the question: does it determine the potential unambiguously? Gribov has recently pointed out that this may not be the case in the non-Abelian case and that one has therefore to be careful.

As an example consider a vacuum configuration, corresponding to $\vec{A}$ a pure gauge

$$\vec{A} = \vec{\nabla}(U(\vec{x})) \ U^{-1}(\vec{x})$$

(11)

Does the condition $\nabla \cdot \vec{A} = 0$ restrict $U$ to be constant and hence $\vec{A}$ to vanish (an obvious solution). To be definite we have to insist on boundary conditions at infinity where we require $\vec{A}$ to vanish. To see what happens, consider a spherically symmetric SU(2) case with $U(\vec{x}) = \exp i \theta(\vec{x}) \ 0 \ 0$, $\theta(0) = 0$, $\theta(\infty) = \pi$. Then

$$A_{\mu} = \frac{\sin 2\theta}{2\ |\vec{x}|} \left[ \delta_{\mu k} - \frac{(1 - \cos 2\theta)}{\sin 2\theta} \right] \frac{\partial_y}{\partial x_k} + t g \theta \ \epsilon_{\alpha \beta \gamma} \frac{\partial y}{\partial x_k} \frac{\partial y}{\partial x_k}$$

(12)

The radiation gauge requires

$$0 = \frac{\partial_y}{\partial x^2} \left( \frac{d^2}{d|\vec{x}|^2} - \frac{2}{|\vec{x}|} \frac{d}{d|\vec{x}|} - \frac{\sin 2\theta}{|\vec{x}|^2} \right)$$

(13)

In terms of $\tau = \ln |\vec{x}|$ this is a damped pendulum equation

$$\frac{d^2 \theta}{d\tau^2} + \frac{d\theta}{d\tau} = \sin 2\theta = - \frac{d}{d\theta} \left( \frac{\cos 2\theta - 1}{2} \right)$$

(14)

with a potential $V = \frac{\cos 2\theta - 1}{2}$

and

$$\frac{d}{d\tau} \left( \frac{1}{2} \left( \frac{d\theta}{d\tau} \right)^2 + V \right) = - \frac{d}{d\theta} \left( \frac{d\theta}{d\tau} \right)^2 < 0$$

(15)

The desired solution has $\theta = \theta = 0$ at $\tau = -\infty$ for $|\vec{x}| = 0$ and $\theta = \pi$ at $\tau = +\infty$ for $|\vec{x}| = \infty$. This yields a matrix $U(\vec{x})$ behaving as $\exp \left( \frac{i}{\sqrt{2}} \ 0 \ \vec{\sigma} \cdot \vec{x} \right)$ at infinity and $\vec{A}$ decreasing slowly as $i \vec{\sigma} \vec{E} / |\vec{x}|$ for large $|\vec{x}|$. If we insist that $|\vec{x}| / \vec{A}$ goes to zero for $|\vec{x}| = \infty$, a rather mild assumption, it seems that the radiation condition fixes unambiguously (spherically symmetric) vacuum configurations to $\vec{A} = 0$ in contrast to our previous discussion in the temporal gauge. A closer look reveals that there is no contradiction.

For simplicity we restrict ourselves to a spherically symmetric case and consider an evolution in the temporal gauge from a vacuum configuration $\vec{A}_f = 0$ (at time $\tau_f$) to $\vec{A}_f = \vec{\nabla}(U_f) \ U_f^{-1}$ (at $\tau_f$) corresponding again to a vacuum configuration. At time $\tau_f$ we write $U_f = \exp i \theta(\vec{x}) \vec{\sigma} \cdot \vec{x}$, $\theta(0) = 0$, $\theta(\infty) = \pi$. The topological winding number is easily computed to be $n$. At intermediate time we do not require $F_{\mu \nu}$ to vanish except at spatial infinity where we also insist that $|\vec{x}| / \vec{A}$ vanishes. We attempt to find a time dependent gauge transformation to $\vec{A}_f$ in the radiation gauge, i.e. such that $\vec{\nabla} \cdot \vec{A}' = 0$ and similar conditions at infinity. From the above we conclude that $\vec{A}' = \vec{A}_f = 0$ and in general

$$\vec{A}' = \vec{\nabla}(U(\vec{x}, t)) \ U_f^{-1}(U(\vec{x}, t))$$

with

$$U(\vec{x}, t) = \exp i \theta(\vec{x}, t) \vec{\sigma} \cdot \vec{x}$$

$\theta(0, t) = 0$ and $\theta(\infty, t)$ is an integer multiple of $\pi$ to insure the vanishing of $|\vec{x}| / |\vec{A}'|$ at infinity. It follows that $\theta(\vec{x}, t) = 0$, $\theta(\vec{x}, \tau_f) = -\theta(\vec{x})$, then at infinity $\theta(\infty, t)$ varies from 0 to $\pi$. Being constrained to be a multiple of $\pi$ it can not do so continuously and we must have a discontinuous behavior of the gauge transformation $U(\vec{x}, t)$.
As an alternative requirement, we may impose the covariant Lorentz condition \( \partial \mu A^\mu = 0 \) with similar questions of uniqueness. Fortunately enough when we study quantization, we only look at small deviations around a given configuration, at least when we develop a perturbation theory and associated Feynman diagrams. Unless we ask global questions we may then safely ignore this gauge ambiguities.

### 3. Quantization and Path Integrals

A very useful presentation of quantum theories uses path integrals introduced by R.P. Feynman\(^5\). Their many advantages justify the investment of learning how to deal with these objects. First of all they give a transparent connection between classical and quantum mechanics. They will suggest how to deal with specific difficulties such as gauge fixing conditions in our context. They also provide a fruitful analogy with statistical mechanics and allow therefore a number of speculations on phase transitions and confinement. For the skeptical they can also be thought as a simple algorithm to generate the familiar terms in the relativistic perturbation expansion.

To introduce the subject, let us compute for a one dimensional quantum system with Hamiltonian \( H \), the probability amplitude that a particle at point \( x_i \) at time \( t_i \), it is observed at \( x_f \) at time \( t_f = t_i + t \)

\[
\langle x_f, t_f | x_i, t_i \rangle = \langle x_f | e^{-iHt/N} | x_i \rangle
\]

We set here \( \hbar = 1 \), otherwise we would write \( e^{-iHt/N} \).

In terms of normalized eigenstates \( \psi_n(x) \), \( H\psi_n = E_n\psi_n \), the transition amplitude reads

\[
\langle x_f, t_f | x_i, t_i \rangle = \sum_n e^{-iE_nt/N} \langle x_f | \psi_n \rangle \langle \psi_n | x_i \rangle
\]  

Note that if we take the time purely imaginary \( t = -i\tau \), the only surviving contribution in the limit \( \tau \to \infty \), is the ground state one \( e^{-E_0\tau} \psi_0(x_f) \psi_0^*(x_i) \).

An alternative mean of computation uses the superposition principle. We slice the time interval \( t \) in small parts \( t/N (N \text{ large}) \) and introduce intermediate states

\[
\langle x_f, t_f | x_i, t_i \rangle = \int \cdots dx_N <x_f | e^{-iHt/N} | x_N> \times <x_{N-1} | e^{-iHt/N} | x_N-1> \cdots <x_1 | e^{-iHt/N} | x_0>
\]

If \( t/N \) is small enough \( <x|e^{-iHt/N}|y> \) will only be sizeable when \( x \) is close to \( y \). For simplicity assume that \( H \) is the sum of a kinetic, \( T(P) \), and potential, \( V(X) \), part. Neglecting corrections of order \( (t/N)^2 \) we find

\[
\langle x|e^{-iT}\rangle \simeq \langle x|e^{-iT_P}t/N\rangle e^{-iV(x)t/N} = \int \frac{dp}{2\pi} \exp \left\{ i(p(x-y) - iT_P) \frac{t}{N} - iV(x) \frac{t}{N} \right\} = \int \frac{dp}{2\pi} \exp \left\{ i(p(x-y) - ih(p, \frac{x+y}{2}) \frac{t}{N} \right\}
\]

where \( h(p, x) \) is the classical Hamiltonian and we have symmetrized the configuration argument. Consequently

\[
\langle x_f, t_f | x_i, t_i \rangle = \int \frac{dx}{N} \frac{dp}{\sqrt{2\pi}} \exp \left\{ p(x_{k+1} - x_k) - ih(p, \frac{x_k + x_{k+1}}{2}) \frac{t}{N} \right\}
\]

In the limit of large \( N \) we obtain

\[
\langle x_f, t_f | x_i, t_i \rangle = \int \frac{dx}{N} \frac{dp}{\sqrt{2\pi}} \exp \left\{ i\frac{E}{N} \int_{t_i}^{t_f} dp \Psi(x(t')) - h(p, x) \right\}
\]

i.e., we sum over classical trajectories in phase space, submitted to boundary conditions \( x(c_i) = x_i \), \( x(c_f) = x_f \), the classical action

\[
S_{cf} = \int_{t_i}^{t_f} dp \left[ \frac{p^2}{2} - V(x) \right]
\]

When the momentum appears quadratically in the Hamiltonian, as here, then the Gaussian integral over \( p \) can be performed with the result that the path integral involves only configuration variables

\[
\langle x_f, t_f | x_i, t_i \rangle = \int Dx(t') \exp \left\{ \frac{E}{N} \int_{t_i}^{t_f} dx(t') - V(x(t')) \right\}
\]

This formalism illustrates the relationship between quantum and classical physics if we restore \( \hbar \), i.e., \( S_{cf} = S_{cf}/\hbar \). When \( \hbar \) is small with respect to the relevant quantities of dimension of an action in the problem, we can look for stationary phase contributions to the path integral. They satisfy the condition

\[
\delta S_{cf} = 0
\]

which is the classical stationary action principle.

This can now be generalized to systems with an arbitrary number of degrees (even infinitely
many as in field theory). It can also be modified with respect to boundary conditions. A convenient way to allow for arbitrary boundary conditions is to include a source term for the dynamical variables of interest. The response to suitable variations of the source will provide the answer to various physical problems.

Let us illustrate this by jumping from a one-dimensional system to a self-interacting scalar quantum field in 3+1 Minkowskian space time. The corresponding world of scalar interacting bosons is described by the field \( \varphi(x, t) \) and its conjugate momentum \( \pi(x, t) = \dot{\varphi}(x, t) \). This stands in fact for a collection of dynamical variables indexed at each time by the three space coordinates \( x \). The classical action is the integral of a Lagrangian given in this case by

\[
\mathcal{L} = \frac{1}{2} \left( \dot{\varphi}^2 - (\nabla \varphi)^2 \right) - V(\varphi) \tag{11}
\]

The conjugate momentum to \( \varphi \) is indeed \( \frac{\delta \mathcal{L}}{\delta \varphi} = \pi = \dot{\varphi} \).

The Hamiltonian density

\[
\mathcal{H} = \pi \dot{\varphi} - \mathcal{L} = \frac{\pi^2}{2} + \frac{1}{2} (\nabla \varphi)^2 + V(\varphi) \tag{12}
\]

will be bounded from below if \( V(\varphi) \) is a bounded function of \( \varphi \) and it will be zero at the ground state. The generating vacuum-to-vacuum transition amplitude in the presence of a source \( j \) is written in analogy with (6) as

\[
Z(j) = \int \mathcal{D}[\varphi, \pi] \exp \left[ \int dx \left( \pi \dot{\varphi} - \mathcal{H} \right) + j \varphi \right] \tag{13}
\]

Integration over \( \pi \) leads to

\[
Z(j) = \int \mathcal{D}[\varphi] \exp \left[ \int d^4x \left( \mathcal{L}(\varphi, \varphi) + j \varphi \right) \right] \tag{13'}
\]

The Green functions of the field theory are obtained by considering small variations of the source around \( j = 0 \) at separate time points. They are normalized by dividing out the amplitude for vacuum persistence in the absence of source. For instance the two point Green function, i.e. the full propagator of the field \( \varphi \) is expressed as

\[
\mathcal{G}(x_1, x_2) = \frac{1}{Z(0)} \int \mathcal{D}[\varphi] \exp \{ iS(\varphi) \} \varphi(x_1) \varphi(x_2) e^{iS(\varphi)} \tag{14}
\]

It is unfortunately a hopeless task to compute such an infinite dimensional integral directly, so that one has to resort to some approximation. The most favoured one corresponds to the case of small coupling - with its undeniable successes in QED. Let us see how this is treated in the path integral context. For this purpose let us use a potential including a quadratic (i.e. mass) term and a quartic part:

\[
V(\varphi) = \frac{m^2}{2} \varphi^2 + \frac{g}{4!} \varphi^4 = \frac{1}{g} V(\varphi g) \tag{15}
\]

We purposely write the \( \varphi^4 \) coefficient as \( -g^2 \), a reminder of the classical stability condition. Rescaling the dummy field variable through \( \varphi \rightarrow g^2 \varphi \) we readily discover that up to an uninteresting factor which drops out in the ratio \( Z(j)/Z(0) \), the crucial factor in the integrand can be written

\[
\exp \left[ \frac{iS(\varphi)}{g^2} \right]
\]

exhibiting the dependence on the coupling constant on the same footing as \( H \). The small \( g \) limit is therefore related to the classical limit and can be treated in an analogous way through a stationary phase method. An expansion in powers of \( g^2 \) will also be an expansion in powers of \( H \) (it will also be related to simple topological properties of the corresponding Feynman diagrams in terms of number of loops).

We also learn from this analysis that non-trivial field configurations corresponding to a stationary action with a finite value \( S \) may lead to non-analytic contributions (in \( g^2 \)) to the Green functions of the form \( \exp i\frac{S(\varphi)}{g^2} \).

With this in mind we return to the original self-interaction before rescaling, i.e. \( g^2 \varphi^4/4! \). For definiteness suppose that we continue to consider the particle propagator. Up to a normalizing factor \( Z(0) \), it can be considered as a power series in \( g^2 \) of the form

\[
g(x_1, x_2) = \frac{1}{Z(0)} \int \mathcal{D}[\varphi] \exp \left[ \frac{iS(\varphi)}{g^2} \right] \varphi(x_1) \varphi(x_2)
\]

with \( S_0 \) the free action

\[
S_0 = \int d^4x \frac{1}{2} \left[ \partial \mu \varphi \partial^\mu \varphi - m^2 \varphi^2 \right]
\]

It is then obvious that to obtain the coefficients in the expansion it is sufficient to know the Gaussian integrals of the type

\[
\frac{1}{Z(0)} \int \mathcal{D}[\varphi] \exp \left[ \frac{iS_0(\varphi)}{g^2} \right] \varphi(x_1) \ldots \varphi(x_n)
\]

where we have kept an even number of fields on

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account of the symmetry $\varphi \rightarrow -\varphi$ of the integrand. Integrals such as (18) are easy to evaluate using a rule referred to as Wick's theorem. The result is

$$\int_{\text{all distinct partitions}} \langle \varphi(1) \varphi(2) \rangle \langle \varphi(3) \varphi(4) \rangle \ldots$$  \hspace{1cm} (19)

where the elementary contraction is equal to the free propagator

$$\langle \varphi(1) \varphi(2) \rangle = \frac{1}{2(0)} \int [\mathcal{D}[\varphi] \exp \frac{i S}{\hbar}[\varphi] \varphi(1) \varphi(2)]$$

$$= \frac{1}{\pi^2} \int d^{4}p \frac{-i \rho(x_{1}-x_{2})}{p^2 - \nu^{2} + i\epsilon}$$  \hspace{1cm} (20)

The free propagator in momentum space $\frac{1}{p^2 - \nu^{2} + i\epsilon}$ is the inverse of the kernel $i[\rho + \nu^{2}]$ occurring in the exponent of the path integral. Feynman's prescription is a consequence of taking carefully the boundary conditions into account.

With Eqs. (16) to (20) we have now the necessary ingredients to derive the perturbative contributions to a given process in terms of elementary Feynman graphs and associated rules.

As an example the lowest order diagrams for the two point function in the $\varphi^4$ theory are

\[ \begin{array}{cccccc}
\text{---} & \text{---} & \text{---} & \text{---} & \text{---} & \text{---} \\
\end{array} \]

Lines carry momentum with a factor $\frac{1}{p^2 - \nu^{2} + i\epsilon}$. Vertices conserve energy-momentum (as a consequence of the space-time translational invariance) and carry a factor $\frac{1}{\nu^2}$. Integration over loop momenta is to be performed, and various factors of $\eta$'s, $i$'s and pertaining to symmetries, complete the rules. We postpone a more precise description in the case of gauge fields.

Everyone has heard or experienced the fact that beyond Born terms involving no loop integrals, unfortunate divergences blur this beautiful picture. This is due to the insufficient decrease at infinity of the particle propagator in momentum space. This leads to a full pleeged theory of how to cure these diseases, called renormalization. Renormalizable models depend on a finite number of parameters. Luckily the Yang Mills interactions belong to this category as we have learnt in the early seventies.\(^6\)

Before investigating these matters in more detail we would like to comment on a different aspect having to do with boundary conditions on path integrals. In view of the classical vacuum degeneracy of the gauge field, it seems appropriate to spend some time on this point.

To exhibit the main features of the argument we will discuss first a one-dimensional $\varphi^4$ system with negative mass term. In other words we deal with a quantum mechanical problem with a potential

$$V(\varphi) = -\varphi^2(1 - g^2 \varphi^2)$$  \hspace{1cm} (21)

with two generate minima at $\pm \varphi_c = \pm \frac{1}{\sqrt{2g}}$.

Classically the point $\pm 0$ is unstable. By a shift $\varphi \rightarrow \varphi + \varphi_c$ we can develop a perturbation theory around one of the minima. This would first take into account the harmonic vibrations around the minima with frequency $\omega_c^2(\varphi_c)$ and then anharmonic corrections in a perturbative manner.

Something is however obviously wrong in this picture due to quantum tunnelling through the potential barrier. Indeed quantum mechanics teaches us that the ground state is unique, corresponding to an even wave function with two probability peaks around $\pm \varphi_c$. While in the semi-classical picture, we have two orthogonal ground states centered at $\pm \varphi_c$. To exhibit a signal of this mixing in the semi-classical approach we might examine the transition amplitude over a large time, between the $\pm \varphi_c$ "vacua"

$$\langle \varphi_c(t) \varphi_c(t) \rangle \sim \int [\mathcal{D}[\varphi] \exp \frac{i S[\varphi]}{\hbar}$$

$$S[\varphi] = \int_{i}^{f} dt [\varphi(t)^2 - V(\varphi(t))]$$  \hspace{1cm} (22)

Here we use a rescaling on $\varphi$ so that $V$ appears as $-\varphi^2(1 - \varphi^2)$ and we insist on boundary conditions such that $\varphi(t)$ starts from $-\varphi_c$ and ends at $+\varphi_c$. For small $\varphi$ we attempt to use a stationary phase method but seem to fail miserably since the potential barrier prevents a classical real trajectory to fulfill our condition. This objection would disappear would we
be free to wander in the complex $t$ and $\phi$ planes. It is suggested to compute instead a transition amplitude for imaginary "time" $\tau$ as

$$<\phi_c | e^{-\mathcal{H}_T} | \phi_c>$$

For this Euclidean amplitude, closely connected to statistical mechanics, we can obviously develop a path integral formalism with the result that

$$<\phi_c | e^{-\mathcal{H}_T} | \phi_c> = \int_{\phi_0 \rightarrow \phi_c} \mathcal{D}[\phi] \exp \left( -\frac{1}{2} \mathcal{S}^E(\phi) \right)$$

The Euclidean action is given as kinetic plus potential energy

$$\mathcal{S}^E(\phi) = \int \, dt \left[ \frac{\dot{\phi}^2}{2} + V(\phi) \right]$$

The reason for the relative change in sign is the use of an imaginary time so that the square of the velocity changes sign with respect to the Minkowskian case. It follows that the equations of motion take the form

$$\ddot{\phi} = V'(\phi)$$

$$V'(\phi) = \frac{d}{d\phi} \left( -\phi^2 (1 - \phi^2) \right) = -2\phi + 4\phi^3$$

For large times we may analyse the behavior of the amplitude by retaining only the contributions of the even wave function $|+\rangle$ (with energy $E_+\rangle$) and odd one $|-\rangle$ (energy $E_-\rangle$ corresponding to the almost degenerate ground states. We have $E_+ \sim V(\pm \phi_c)\rangle$, $E_- = E_+ + \Delta E$ so that taking the symmetry of the wave functions into account

$$<\phi_c | e^{-\mathcal{H}_T} | \phi_c> \propto e^{-E_+^T} \tau^\Delta E + \ldots$$

To obtain an estimate of $\Delta E$ it is sufficient to divide out by $<\phi_c | e^{-\mathcal{H}} | \phi_c>$. Looking at the path integral we can estimate it using the steepest descent method which replaces the stationary phase trick.

This means that we look for solutions of the classical Euclidean equations of motion (25) with finite action.

In the present case the trajectory of lowest action is the kink solution which interpolates between $-\phi_c$ and $+\phi_c$ in an (almost) infinite time interval ($\tau = \pm$)

$$\phi_{kink}(\tau') = \frac{1}{\sqrt{2}} \tanh(\tau' - \tau_0)$$

Solutions of higher action play a negligible role to leading order. The Euclidean action of this kink solution is

$$\int_{-\infty}^{\infty} dt' \left[ \frac{\dot{\phi}_c^2}{2} + V(\phi_c) - V(\pm \phi_c) \right] = \frac{1}{2} \int_{-\infty}^{\infty} dt' \frac{\dot{\tau}^2}{\cosh^2 \tau} = \frac{2}{3}$$

Consequently

$$\tau \Delta E |<\phi_c | \pm \rangle|^{-2} e^{-E_+^T} \tau e^{-\frac{2}{3}} \exp -\frac{2}{3} e^{\Delta \tau}$$

The factor $\tau$ in front originates from the integral over the collective mode corresponding to the arbitrary origin $\tau_0$. A more careful handling replaces $\tau \Delta E$ on the left hand side by $(1 - e^{\Delta E})$ which would be reconstructed on the right hand side by summing over multikink solutions.

In summary a classical vacuum degeneracy may arise for topological reasons (here we have a classical disconnected set of vacua $\phi_c$). Euclidean solutions interpolate between these vacua and allow symmetry restoration in the quantum case through barrier tunneling.

This applies almost word for word in the four dimensional Yang-Mills case. Again the vacuum is classically degenerate in inequivalent classes of gauge functions characterized by an integer $n$. The classical energy looks like a periodic function with minima for integer $n$. We therefore ask the question: does there exist Euclidean solutions which interpolate between these vacua with topological numbers $n$ and $m$. The positive answer has recently been provided by Polyakov and 't Hooft and these solutions have been called instantons. 7)

Let us simply describe them in the case $n \geq n + 1$. The Euclidean case allows for a vast simplification of the Yang Mills equations due to the following circumstances. If we compare these equations (2.7) with the Bianchi identities (1.26) (all written in Euclidean space, i.e. with a unit metric) we see that self- (or antiself-) dual field is automatically a solution. By this we mean that

$$\tilde{F}_{\mu\nu} = \pm F_{\mu\nu}$$

where

$$\tilde{F}_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} F^{\rho\sigma}$$

$\epsilon_{\mu\nu\rho\sigma}$ totally antisymmetric tensor. This is clear if

One-dimensional kink
we write the Y.M. equations $[D_{\mu}, \mathcal{F}_{\mu\nu}] = 0$ and the Bianchi identities $[D_{\mu}, \mathcal{F}_{\mu\nu}] = 0$. The reason that we can find solutions to Eqs. (30) in the Euclidean domain is that going from $\mathcal{F}$ to $\mathcal{F}$ simply interchanges $\mathbb{F}$ and $\mathbb{F}$ without relative signs, so that this operation has a square equal to the identity (the corresponding one in Minkowski space has a square equal to minus the identity).

This is a considerable simplification in view of the fact that the condition (30) is a first order equation for the potential. It is convenient to use here the normalization of the potentials such that the Euclidean action reads $\int \frac{1}{4g^2} d^4x \mathfrak{F}_{\mu\nu,\alpha} \mathfrak{F}^{\mu\nu,\alpha}$. Up to a sign this is then also equal to $\int \frac{1}{4g^2} d^4x \mathfrak{F}_{\mu\nu,\alpha} \mathfrak{F}^{\mu\nu,\alpha}$. The latter quantity turns out to be a topological invariant of the type considered previously in Eq. (1.30) except that we work now in four-dimensional space.

For definiteness let us consider the $SU(2)$ case and require that $F$ decreases faster at infinity that $|x|^{-2}$. This means that $|x| \to \infty$ in $\mathbb{R}^4$ along a direction labelled by the unit vector $\xi \in \mathbb{S}_3$, $A_\mu$ tends to a pure gauge: $A_\mu \sim \lambda_\mu(U(\xi)) U^{-1}(\xi)$. The matrix $U(\xi)$ describes a continuous map from $\mathbb{S}_3$ onto $SU(2) \sim \mathbb{S}_3$. The integral of the Jacobian of this mapping has to be an integer giving the winding number (or Pontryagin index). It is related to the previous integral through

$$\int \frac{1}{4g} d^4x \mathfrak{F}_{\mu\nu,\alpha} \mathfrak{F}^{\mu\nu,\alpha} = \frac{4\pi^2 n}{8}$$

A little analysis shows that this integer is the difference in integers characterizing the vacuum configurations at $x_0 = \tau = \pm \infty$.

The solution for Pontryagin index equal to one can be written

$$A_\mu(x) = f(|x|) \left[ \bar{\lambda}_\mu(U(\xi)) \right] U^{-1}(\xi)$$

$$U(\xi) = (\mathbb{S}_3 + i \mathbb{S}_3)$$

Substituting in (30) we find

$$|x| f' = 2f(1-f) f = \frac{x^2}{x^2 + \lambda^2}$$

and

$$A_\mu = \frac{x^2}{x^2 + \lambda^2} \left[ \bar{\lambda}_\mu(U(\xi)) \right] U^{-1}(\xi)$$

Since $U(\xi)$ maps the sphere at infinity once on $SU(2)$, $n = 1$; one may also check Eq. (31) directly.

More general solutions depending on $8n-3$ parameters are now under construction in the general case.

All this means that the quantum vacuum will be a linear superposition characterized by an angle $\theta$ with

$$|\beta\rangle = \sum_{n=0}^{\infty} e^{i\theta n} |\alpha\rangle$$

with the possibility of quantum tunnelling indicated by the non vanishing of the transition amplitude $\langle \alpha | e^{-i\beta} |\alpha\rangle$. The implications of this phenomenon are not yet fully elucidated.

### 4. Perturbation Theory

We return to the subject of proper quantization of gauge fields around a classical configuration. The latter will be taken to be the simplest one $A_\mu = 0$ around which we study fluctuations. This allows a consistent treatment of perturbation theory irrespective of the existence of other saddle points in the path integral.

However the integral $\int \mathcal{D}(A)e^{iS.Y.M. (A)}$ is not defined due to the gauge invariance of the action. As a consequence, we know for instance that the quadratic part in $A$ is degenerate, so that the gluon propagator is not well defined. To cure this defect we would like to introduce a gauge fixing term of the form

$$\int_{x_0}^{x_1} \delta \left( \mathcal{F}'(A;x) \right)$$

where $\mathcal{F}'(A;x) = 0$ is an arbitrary local condition which suppresses the gauge arbitrariness, at least in a perturbative sense around the prescribed configuration (here $A_\mu = 0$). This could be for instance the Lorentz condition

$$\mathcal{F}'_\alpha(A;x) = \delta_{\mu}^{\alpha} A^\mu(x) = 0$$

(1)

to which we stick as an example because of its covariance property. However this is not enough: something must compensate for this explicit gauge breaking.

Under an infinitesimal gauge transformation, we have, cf. Eq. (1.21),

$$\delta \mathcal{F}'_\alpha = \left( \delta \mathcal{F}'_\alpha \right) = \delta_{\beta}^{\alpha} \delta \mathcal{F}'_\beta = \delta_{\beta}^{\alpha} \delta \mathcal{F}'_\beta$$

so that

$$\delta \mathcal{F}'_\alpha = \mathcal{M}_{\alpha} \delta \mathcal{F}'_\beta$$

(3)

In the case (1)

$$\mathcal{M}_{\alpha} = \left( \delta \mathcal{F}'_\alpha + \delta \mathcal{F}'_\beta \right) \delta_{\beta}^{\alpha}$$

(4)
where \( \zeta_{a b y} \) are the totally antisymmetric structure constants of the compact Lie group under consideration (SU(2), SU(3), ...)

It is possible to perform a group average over the various condition \( \mathcal{F}_a^\prime(U(x)_{x,x}) = 0 \), where \( U(x)_{x,x} \) is given by equation (1.20) is the transformed of \( x \) under a finite, x-dependent group element \( U(x) \), with the result that

\[
\left[ \mathcal{D}[U(x)] \right] \frac{\det \mathcal{M}(x)}{\prod_{x,a} \delta^{\mathcal{F}_a^\prime(U(x)_{x,x})}} = \frac{1}{\det \mathcal{M}}
\]

(Hint: use the rule \( \int dz \delta(f(z)) = \frac{1}{|f'(z_0)|} \) with \( z_0 \) such that \( f(z_0) = 0 \), and extend it to several variables. Then (5) can be integrated for an \( x \) which fulfills \( \mathcal{F}_a^\prime = 0 \) and the result can be recognized to be gauge independent. Therefore instead of introducing \( \prod_{x,a} \delta \mathcal{F}_a^\prime(x) \) in the path integral we rather insert the group average

\[
\left[ \mathcal{D}[U(x)] \right] \frac{\det \mathcal{M}(x)}{\prod_{x,a} \delta^{\mathcal{F}_a^\prime(U(x)_{x,x})}}
\]

Using the invariance of the integration measure and of the Yang Mills Lagrangian we obtain after addition of a source term

\[
\left[ \mathcal{D}[A] \right] \frac{\det \mathcal{M}(x)}{\prod_{x,a} \delta^{\mathcal{F}_a^\prime(x)}} \exp i \int d^4 x \mathcal{L}_{Y.M.} - J.A.
\]

(6)

Note that the integral over the group has dropped out. We may even replace \( \mathcal{F}_a^\prime \) by \( \mathcal{F}_a - W_A \) (this does not modify \( \det \mathcal{M} \) and average over \( C \) with a Gaussian measure. This gives the more tractable form of the generating functional

\[
Z(J) = \left[ \mathcal{D}[A] \right] \det \mathcal{M}(x) \exp i \int d^4 x \mathcal{L}_{Y.M.} - \lambda \mathcal{F}_a^2 + J.A.
\]

(7)

obtained by Faddeev and Popov who added the following idea.

The only piece of this integral which is not in the form of the exponential of the operator \( \mathcal{M} \). For an ordinary integral (and at least for finite matrices) we have

\[
\prod_k dz_k d\bar{z}_k \exp(-\bar{z}_k \mathcal{M}_{kk} z_k) \sim [\det \mathcal{M}]^{-1}
\]

(8)

It is possible to generalize this definition (and correspondingly the path integrals) to include "classical" anticommuting variables. This is desired in general to include fermionic fields in the theory. The corresponding Gaussian integral then reads

\[
\prod_k d\eta_k d\bar{\eta}_k \exp(-\bar{\eta}_k \mathcal{M}_{kk} \eta_k) \sim \det \mathcal{M}
\]

(9)

with \( \eta_k, \bar{\eta}_k \) anticommuting.

That this is so can be traced to the fact fermionic loops in a Feynman diagram introduce a minus sign. If we consider the propagation of "a charged" particle in an external field \( A \) it will symbolically be represented by the series

\[
\ldots \cdot \frac{1}{1} \cdot \frac{1}{2} \cdot \frac{1}{3} \cdot \ldots
\]

The probability amplitude that the external field does not create particles will have an expression

\[
<0|0>_A = \exp -\mathcal{W}(A)
\]

with \( \mathcal{W}(A) \) given by the one loop diagrams

\[
\frac{1}{1} \cdot \frac{1}{2} \cdot \frac{1}{3} \cdot \ldots
\]

The factor \( \frac{1}{n} \) reflects the cyclic invariance of the diagram. Simplifying down to caricature setting the free propagator equal to one and omitting the internal integration, this series is

\[
-\mathcal{W}(A) = \sum_{1 \leq n} \frac{A^n}{n} = -\log(1-A) \quad \text{and} \quad \exp -\mathcal{W}(A) = \frac{1}{1-A}
\]

i.e. \( 1-A \) is the analog of \( \mathcal{M} \). If however one includes an extra minus sign for each fermionic loop one finds instead \( \exp -\mathcal{W}(A) = 1-(1-A) \). In both cases we recognize for \( <0|0>_A \) the integrals

\[
\int dz d\bar{z} e^{-\mathcal{W}(A)z} = \frac{1}{1-A}
\]

or

\[
\int d\eta d\bar{\eta} e^{-\mathcal{W}(A)\eta} = (1-A)
\]

characteristic of bosonic or fermionic propagation in the field \( A \).

As a result, with these Faddeev-Popov ghost fields \( \eta, \bar{\eta} \), we may write the gauge field path integral

\[
Z(J) = \left[ \mathcal{D}[A, \eta, \bar{\eta}] \right] \exp i \int d^4 x \mathcal{L}_{	ext{eff}} + J.A.
\]

\[
\mathcal{L}_{\text{eff}} = \mathcal{L}_{Y.M.}(A) - \frac{\lambda}{2} \eta^2 - \bar{\eta} \mathcal{M} \eta
\]

(10)

To obtain the Feynman rules we have simply to separate in \( \mathcal{L}_{\text{eff}} \) the quadratic (and now invertible) part from the remaining and read off propagators and vertices. Writing \( \mathcal{L}_{\text{eff}} \) explicitly, we obtain
Observe that in this case, the ghost is massless.

The vertices are of three kinds

\begin{equation}
\begin{align*}
g \frac{C_{\alpha \beta \gamma}}{(2\pi)^4} & \delta^4(p+q+r) \\
& \left[ g_{\mu \nu} (p-q)_{\rho} + g_{\nu \rho} (q-r)_{\mu} + g_{\rho \mu} (r-p)_{\nu} \right]
\end{align*}
\end{equation}

triple gluon vertex

\begin{equation}
\begin{align*}
-i g^2 (2\pi)^4 \delta^4(p+q+r+s) \\
& \left[ C_{\epsilon \alpha \beta \gamma} \varepsilon_{\mu \rho} [g_{\mu \rho} g_{\nu \sigma} - g_{\mu \sigma} g_{\rho \nu}] \\
& + C_{\epsilon \alpha \beta \gamma} \varepsilon_{\rho \sigma} [g_{\mu \sigma} g_{\nu \rho} - g_{\mu \rho} g_{\nu \sigma}] \\
& + C_{\epsilon \alpha \beta \gamma} \varepsilon_{\mu \sigma} [g_{\nu \rho} g_{\sigma \rho} - g_{\nu \rho} g_{\sigma \rho}] \right]
\end{align*}
\end{equation}

four gluon vertex

\begin{equation}
\begin{align*}
-g \frac{C_{\alpha \beta \gamma}}{(2\pi)^4} & \delta^4(p+k-q) \\
& \left[ g_{\mu \nu} (p-k)_{\rho} + g_{\nu \rho} (k-q)_{\mu} + g_{\rho \mu} (q-p)_{\nu} \right]
\end{align*}
\end{equation}

gluon-ghost vertex

In the last vertex the ghosts play asymmetric roles. When matter fields are coupled in a minimal way, we add to \( \mathcal{L}_\text{eff} \) a fermionic contribution \( \mathcal{L}_F \) or a bosonic one \( \mathcal{L}_B \) or both. Let the infinitesimal generators be represented by the (antihermitian) matrices \( \gamma^A_{ab} \)
so that

\[ D_{\mu} = \gamma_{\mu}^{\nu} + g A_{\mu}^{\nu} T^{\nu} \]

then

\[ \mathcal{L}_F = -i \bar{\psi} \gamma^a \gamma^b \psi - m \bar{\psi} \psi \]

\[ \mathcal{L}_B = (D_{\mu} \phi)^* D_{\mu} \phi - m^2 \phi^* \phi - P(\phi^* \phi) \]

The additional rules are

\[ \begin{array}{c}
\text{Fermion} \\
\begin{array}{c}
\frac{i \delta_{ab}}{p^2 - m^2 + i\epsilon} \\
\end{array}
\end{array} \]

\[ \begin{array}{c}
\text{Boson} \\
\begin{array}{c}
\frac{i \delta_{ab}}{p^2 - m^2 + i\epsilon} \\
\end{array}
\end{array} \]

one gluon vertex

\[ g T_{\mu \nu} (2\pi)^2 \delta^4(p-p'-k) \]

\[ g T_{ab} (p_{\mu} + p'_{\mu}) (2\pi)^2 \delta^4(p-p'-k) \]

two gluon vertex

\[ -ig^2 \delta^4(p^2 - k^2 - k'^2) \]

\[ (14) \]

Extra boson self couplings arising from \( P(\phi^* \phi) \) have not been explicitly indicated.

These formulae are somehow complex and it requires some training in diagramatics to extract from them some useful information.

Not only did we choose arbitrarily the gauge fixing function \( \gamma' \) but once this choice was made, we remain with an arbitrary parameter \( \lambda \). Physically meaningful results must be \( \lambda \) independent. If we are confident enough we may however choose a particular value. If \( \lambda = 1 \) (Feynman gauge) the gluon propagator simplifies to \( \frac{i \delta_{ab}}{k^2 + i\epsilon} \).

The value \( \lambda = \frac{1}{2} \) yields the Landau propagator \( -i \delta_{ab} \frac{g_{\mu \nu} - k_{\mu} k_{\nu}/k^2}{k^2 + i\epsilon} \) at infinity. It may be safer to keep \( \lambda \) arbitrary in order to check calculations and in any case we observe that the gluon as well as the other propagators behave like \( \kappa^{-2} \) at infinity.

5. Renormalization

Rather than working out renormalization to all orders in a very abstract language, we will content ourselves to exhibit its features on a lowest order calculation based on the above Feynman rules. This will be sufficient to understand the crucial mechanisms and deduce some important information.

For the time being, let us concentrate on the
pure gauge field. We shall indicate later on the necessary extension to include matter fields.

Consider the gluon propagator. It may be obtained by summing the geometric series

\[ \frac{1}{k^2} + \frac{1}{k^2} + \frac{1}{k^2} + \frac{1}{k^2} + \cdots = \frac{1}{k^2 - \sigma} \]

where the dark blob indicates the contribution of one particle irreducible (1PI) diagrams, i.e., those that cannot be separated in two disjoint parts by cutting through one line only. Simplifying the free propagator as \(1/k^2\) and calling the blob \(0\) the series is

\[ \frac{1}{k^2} + \frac{1}{k^2} + \frac{1}{k^2} + \frac{1}{k^2} + \cdots = \frac{1}{k^2 - \sigma} \]

To lowest order \(\left( g^2 \right)\) the blob (or proper two point Green function) is given in terms of three diagrams

which include a "tadpole" one (the second) and all three are divergent since propagators behave as \(\sim 1/k^2\) for large \(k\) and we have at most two propagators for a \(d/k\) integral. The matter is complicated by the structure of vertices and numerators of propagators. Anyhow we seem to have quadratic and logarithmic ultraviolet divergences.

In order to deal with meaningful expressions we have to regularize the theory, i.e., to introduce some ultraviolet cut-off. This might however spoil the very tight algebraic structure of the theory. 't Hooft has suggested to cope with this difficulty by using a dimensional regularization, i.e., to replace integrals in four dimensional space-time by integrals in a \(d+\epsilon\) dimensional space. For \(d\) small enough the integrals will converge and we can extract the infinities close to \(d=4\) by a Laurent expansion in inverse powers of \(\epsilon\).

For reasons which are soon to be clear we shall concentrate on the structure of these infinities. Neglecting vanishingly small contributions as \(\epsilon \to 0\) we find after some (tedious) calculation

\[ \Gamma^{(2) \mu \nu}_A = \frac{g^2}{16\pi^2} C \delta_{A^3} \left[ k^2 \mu \nu - k^2 \mu \nu \right] \left[ \frac{5}{3} + \frac{1 - \lambda}{2} \right] \]

for the two point function \(\Gamma^{(2)}\). The constant \(C\) is defined in the adjoint representation through

\[ \text{tr} \Gamma^{AB}_{\mu \nu} = -C^A B \]

if \(A\) stands for the representation of the generator (\(C\) is equal to \(N\) for \(S U(N)\)). The interesting feature of (1) is its transversity property \(\Gamma_{\mu \nu}^{(2) \mu \nu} = 0\), in spite of the apparent breaking of gauge invariance in the Lagrangian.

How can we deal with the infinite contribution arising in \(\Gamma^{(2)}\). The answer has been provided long ago by the method of renormalization. It amounts to add a piece (a counterterm) in the Lagrangian therefore adding a contribution to the two point function

\[ \Delta \Gamma_{\mu \nu}^{(2)} = (Z_2 - 1) \left\{ \frac{1}{\lambda} \left[ g_{\mu \nu} \delta^{A^3}_A - \lambda g_{\mu \nu} A^3_A \right] \right\} \left[ \lambda \Gamma^{(2) \mu \nu}_{\lambda} - \lambda \Gamma^{(2) \mu \nu}_{\lambda} \right] \]

with

\[ Z_2 = 1 + \frac{\epsilon}{16\pi^2} \left[ \frac{5}{3} + \frac{1 - \lambda}{2} \right] \]

Strictly speaking, when working in dimension \(d = 4 - \epsilon\) the coupling constant originally dimensionless, acquires a dimension and should be replaced by \(g^2 \times (\text{mass})^\epsilon\). More correctly written, the above factor \(\frac{2}{\epsilon}\) should therefore be read

\[ \frac{2}{\epsilon} (A^2 - \mu^2) = \mu^2 \frac{2}{\epsilon} \left[ \left( \frac{A}{\mu} \right)^\epsilon - 1 \right] \sim \ln \frac{A^2}{\mu^2} \]

\[ \frac{2}{\epsilon} \leftrightarrow \ln \frac{A^2}{\mu^2} \]  \hspace{1cm} (4)

Using the same philosophy we can proceed to the computation of the other (proper) Green functions to lowest order. We select those which yield divergent contributions hence contribute to the construction of the corresponding counterterms.

From the three gluon vertex with diagrams

we obtain \(\Gamma^{(3)}\) and the counterterm

\[ \Delta \Gamma_{\mu \nu \lambda} = (Z_1 - 1) \frac{g^2}{2} \left[ g_{\mu \lambda} A^3 - \lambda g_{\mu \lambda} A^3_A \right] \left[ \lambda \Gamma^{(3) \mu \nu \lambda} - \lambda \Gamma^{(3) \mu \nu \lambda} \right] \]

\[ Z_1 = 1 + \frac{\epsilon}{16\pi^2} \left[ \frac{2}{3} + \frac{3}{4} \right] \left( \frac{1 - \lambda}{2} \right) \]

Similarly from the four gluon function \(\Gamma^{(4)}\)

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we derive
\[ \delta \mathcal{L}^4 = (Z_4^{-1}) \left\{ -\frac{2}{4} \left[ A_\mu \delta A_\mu \right] \left[ \delta^\mu A^\nu \delta A^\nu \right] \right\} \tag{7} \]
\[ Z_4 = 1 + \frac{g_2^2}{16\pi^2} C \left[ -\frac{1}{3} + (1-\lambda^{-1}) \left( \frac{2}{\epsilon} \right) \right] \]
Green functions with more external gluon lines are convergent to this order. But this is not the end of the story, since we have also Green functions with external ghost lines. For the two- and three-point functions, we find respectively
\[ \delta \mathcal{L}_{\eta \eta} = (Z_3^{-1}) (-\eta_0 \delta \eta_0) \tag{8} \]
\[ Z_3 = 1 + \frac{g_2^2}{16\pi^2} C \left[ \frac{1}{2} + 1\frac{1-\lambda^{-1}}{4} \left( \frac{2}{\epsilon} \right) \right] \]
\[ \delta \mathcal{L}_{\eta \eta \eta} = (Z_3^{-1}) c_{\delta \eta \eta} A^{\mu \nu} \delta A_{\mu} \delta A_{\nu} \tag{9} \]
\[ Z_1 = 1 - \frac{g_2^2}{16\pi^2} C \frac{\lambda^{-1}}{2} \left( \frac{2}{\epsilon} \right) \]
When matter fields—say fermions—are present, two things occur. New counterterms are needed to compensate the divergences appearing in those Green functions involving fermion external lines. We omit them here. But their presence also modify the gluon functions. To lowest order this adds new contributions to the various \( Z \) constants which turn out to be equal to
\[ \delta Z_3 = \delta Z_1 = \delta Z_4 = - T F \frac{g_2^2}{16\pi^2} \frac{4}{3} \frac{2}{\epsilon} \tag{10} \]
where \( T F \) is given by
\[ \text{tr} T F \delta A^\alpha = - T F \delta A^\beta \tag{11} \]
and \( T F \) stands for the matrices of the generators in the fermion representation (if these belong to the fundamental representation of the group then \( T F = \frac{1}{2} \) according to our conventions for \( SU(N) \)).

We pause now to examine these results. Interestingly enough the added counterterms needed to define a finite theory to order one loop have the same structure as the original monomials of the Lagrangian. If we use the notation \( A \) with \( A^2 = -\frac{1}{2} \left[ A A \right] \) as in the first sections, the complete Lagrangian to this order is
\[ \mathcal{L} + \delta \mathcal{L} = \frac{1}{2} Z_3 (g_2 A_\mu A_\mu - g_2 A_\mu A_\mu) \left( \delta A^\mu A^\nu - \delta A^\mu A^\nu \right) + \frac{1}{2} (\delta A^2) \]
\[ - g_2 Z_3 (g_2 A_\mu A_\mu - g_2 A_\mu A_\mu) \left( \delta A^\mu A^\nu \right) + \frac{g_2^2}{2} Z_4 [\delta A_\mu A_\nu] [\delta A^\mu A^\nu] \]
\[ + Z_3 \mu A_\mu \delta A^\mu + g Z_3 c_{\delta \eta \eta} \mu _\eta \delta A^\mu \eta \gamma \tag{12} \]
It appears that no mass counterterms of the type \( \frac{\delta m^2}{2} A_\mu A^\mu \) has been introduced. This is analogous to a similar phenomenon in QED; as a consequence of gauge invariance, the gluon remains "massless". Similarly no counterterm in \( (\delta A^2) \) is necessary.

Will we say then that the theory is renormalizable (to this order)? The answer will be yes, provided that the relative coefficients (involving infinities unfortunately) are such that some remnant of the gauge invariance has been preserved. More precisely define bare fields and couplings through
\[ A_0 = Z_3^{1/2} A \quad \eta_0 = Z_3^{1/2} \eta \quad \bar{\eta}_0 = Z_3^{1/2} \bar{\eta} \]
\[ g_0 = Z_3^{-3/2} g \quad \lambda_0 = \lambda Z_3^{-1} \tag{13} \]
As a consequence of the rescaling of the field, we have to make a corresponding change in \( \lambda \), eventhough as remarked above there was no counterterm in \( (\delta A^2) \).

Then \( \mathcal{L} + \delta \mathcal{L} \) may be regarded as the initial Lagrangian \( L(A_0, \eta_0, \bar{\eta}_0; g_0, \lambda_0) \) written in terms of bare quantities provided that the following identities hold
\[ \frac{Z_4}{Z_3} = \frac{Z_1}{Z_3} = \frac{Z_4}{Z_3} \tag{14} \]
An additional ratio would appear had we written the fermionic counterterms and (14) expresses the universality of the gluon coupling. These identities which generalize the Ward identities of QED are indeed satisfied by our former expressions. This implies the renormalizability of the non-Abelian theory to the one-loop order.

J.C. Taylor and A. Slavnov were able to write the general form of these identities not only on the (infinite) bare Lagrangian, but also their consequences on the various Green functions. 'T Hooft, Veltman, B. Lee and Zinn-Justin and others have then derived suitable subtraction procedures to all orders fulfilling these identities and thereby proving renormalizability to all orders. The subject then gets very technical.

The essential content of the identities (14) is the universality of coupling constant renormalization. The ratio of bare to renormalized coupling constant is given by
\[ \frac{g_2}{\bar{g}} = 1 - \frac{2}{16\pi^2} \left[ \frac{11}{6} C - \frac{2}{3} T_f \right] \ln \frac{\Lambda^2}{\mu^2} \]  
\hspace{10cm} (15)

In spite of the dependence on the cut-off \( \Lambda \) and the normalization scale \( \mu \), this ratio does not depend on the arbitrary gauge parameter \( \lambda \). We therefore suspect that this relation contains some interesting physics. Indeed we may ask: what is the effect on \( g \) of varying the scale \( \mu \) keeping \( g_0 \) fixed. That is we want to compute \( \mu \frac{\partial}{\partial \mu} g \|_{\Lambda, g_0} \). From (15) we find

\[ \mu \frac{\partial}{\partial \mu} g = \bar{g} (g) = -\frac{3}{16\pi^2} \left[ \frac{11}{3} C - \frac{4}{3} T_f \right] + \ldots \]  
\hspace{10cm} (16)

the \( \ldots \) are of course of order \( g^5 \). This defines a function \( \bar{g} (g) \) which depends only on the renormalized coupling constant. For \( g \) small enough the leading term has been obtained and if there are few enough fermions (if \( T_f < \frac{11}{4} C \)) then \( g \beta (g) \leq 0 \) in the vicinity of zero.

The meaning of \( \beta (g) \) is clear from the above. It tells us how \( g \) varies as we change our scale of investigation. This was emphasized first by Gell-Mann and Low, then reexamined by Callan and Symanzik. The interesting thing about non-Abelian gauge fields, is that they are unique among renormalizable theories to exhibit the behavior that as the momentum scale grows \( g^2 \) decreases down to zero as \( \mu \rightarrow \infty \). This is the celebrated asymptotic freedom, which is the most dramatic argument to select such a model for an explanation of deep inelastic phenomena. The subject will be fully discussed by the other lecturers. Hence we stop here except to say that asymptotic freedom does not mean free field theory. It rather implies that the approach to the trivial scaling limit (\( g=0 \)) is partly under control and that deviations may be computed perturbatively using the small running coupling constant \( g(\mu) \).

6. Towards Confinement

We have just seen that for large momenta the gauge theory tends in a predictable way towards a free field theory. An immediate corollary is that for small momenta (or large distances) the coupling gets stronger and stronger and the perturbation theory becomes useless. This is not entirely true as we may attempt resummation procedures. Eventhough the problem is around since a couple of years no totally satisfactory and convincing picture has been given of what really happens at large distances. Are the quarks really confined? Can one seriously attempt numerical calculations? ...

Rather than describing the various approaches that have been developed I will limit myself to describe the Wilson proposal of a lattice approximation. This is not so much because it might turn out to be the most useful but because it enables us to benefit from a lot of intuition and methods which were developed in statistical mechanics. They might eventually make their way in particle physics. Again this will only be a brief survey to quote some of these ideas.

Let us first allow ourselves the possibility of transforming the path integral to the Euclidean region where space and time are on the same footing. Since small distances correspond to small coupling we may attempt to disconnect the properties at large from those at small distances by introducing an ultraviolet cut-off \( \Lambda \) that respects gauge invariance (as dimensional regularization did). Such a cut-off is provided by replacing the continuum space-time by a discrete lattice of points (we take it hypercubical to be specific) with lattice interval \( \varepsilon \sim \frac{1}{\Lambda} \). The price to pay is that we loose the continuous translation and rotation invariances. If we look however at large distances, in those circumstances where fluctuation of wave length much larger than a occur it can be expected that the discrete lattice will be irrelevant and continuous geometrical invariance restored.

We are left with the problem of expressing a locally invariant theory on a lattice. Following Wilson\(^8\) we attach to each link joining two neighboring points of the lattice, from \( x \to y = x + a \mu \) (\( \mu \) unit vector in the \( \mu \) direction) a group element

\[ U_{xy} = U^{-1}_{yx} \]  
\hspace{10cm} This may be thought as

\[ e^{A_{\mu} (x-y)} = 1 + A_{\mu} (x) + \ldots \]  
\hspace{10cm} in terms of the variables \( A_{\mu} \) of the continuous theory. Note that instead of varying in a linear manifold like \( A_{\mu} \) these \( U \)-variables take their value in the compact group space and transform as

\[ U_{xy} = U_{x} U_{y} U^{-1}_{xy} \].
The ordered product of the U's pertaining to the four links bordering an elementary square of the lattice (a plaquette) is called a plaquette variable
\[ U_p = U_1 U_2 U_3 U_4 \]

We now take the trace of these \( U_p \) variables in a given representation of the group - this is called a character (we assume that it is real, for instance for SU(2) where a group element is characterized by its angle of rotation \( \theta \), \( \chi_j(U) = \text{tr}_j U = \frac{\sin(2j+1) \theta}{\sin \theta/2} \)). Otherwise we add the trace in the complex conjugate representation. We are now in a position to define the discrete Euclidean gauge invariant action up to an additive constant by summing \( \chi(U_p) \) over all plaquettes of the lattice. The generating functional reads
\[
Z = \int \prod \text{d}U_{\text{link}} \exp - \frac{S}{T} \\
S = \sum_p \chi(U_p) 
\]

From our previous discussion \( T \) is proportional to the square of the (bare) coupling constant \( g^2 \). The analogy with problems in statistical mechanics is striking: \( Z \) looks like the partition function, \( T = g^2 \) like the temperature (recall the Boltzman-Gibbs factor) and the Euclidean action like the classical energy of a configuration described by the continuous link variables (or string bits) \( U_{xy} \). The main difference is formal from the mathematical point of view; statistical mechanics at equilibrium deals with three-dimensional systems - whereas here we have a four-dimensional one.

It can be shown that if we expand \( S \) in powers of the "small" lattice constant \( a \), i.e. if we consider those configurations where \( U_{xy} \) varies smoothly - the \( S \) lattice is up to an additive constant proportional to the Euclidean Yang-Mills action.

In this formalism we get as a bonus that gauge invariance is strictly enforced (except by possible external sources that were omitted in (1)) and in particular no Faddeev-Popov trick is necessary a priori unless we want to study the formal continuous limit. This is due to the compactness of the range of integration of group variables.

Things are even better if we use a theorem in statistical mechanics which states that the limit from a finite to an infinite space-time limit is well defined. We can therefore approximate the problem in a finite box with finitely many points and a well defined integral. It would of course be unrealistic to attack the problem with purely numerical means without physical intuition.

Statistical mechanics suggests the possibility of a regime of large wave-length fluctuations in the vicinity of critical (or phase transition) points.

A number of arguments point towards the following result: the most likely candidate for a transition point is \( T = g^2 = 0 \). This would mean \( (I) \) that this point is a non-analyticity point, hence perturbation theory a poor guide (it diverges as we know) to understand the large distance behavior - a mild surprise, \( (\epsilon) \) that some qualitative features of the regime with small \( g^2 \) are already present at large \( g^2 \) (large temperature) since no transition occurs between infinite \( g^2 \) and small \( g^2 \).

Now at high temperature, most systems simplify drastically and tend towards an analog of the perfect gas of non-interacting subsystems. It should be understood that in the statistical interpretation it is the kinetic part of the Lagrangian which is interpreted as an "interaction energy". Consequently there is no contradiction in saying that in the large \( T \) regime "interactions" become negligible. This simply means that the kinetic terms are relatively small and propagation is inhibited. We have in a nutshell the origin of confinement.

To state this more precisely, we can introduce Wilson's confinement criterion. Let us imbue an inert quark pair in the gluon soup. Eventhough we neglect their translational degrees of freedom, we do not so for their internal color quantum numbers, which evolve through multiplication by a string of \( U_{xy} \) joining the initial and final observation points.

For a closed loop \( C \) in space-time we find the ordered product \( \prod_{(C)} U_{xy} \). Summing over quarks internal degrees of freedom yields the trace in the fermionic representation \( \chi_T(\prod_{(C)} U_{xy}) \). We have to average this quantity over the gluon "soup", with the measure (1) i.e. we define

\[
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\]
\[ \mathcal{W} = \langle \chi_f(\mathbb{B} U_{xy}) \rangle \]
\[ = Z^{-1} \prod_{\text{links}} dU \exp - \frac{S}{T} \chi_f(\mathbb{B} U_{xy}) \]  
(2)

When $Z$ gets large we expect a behavior of the type $\mathcal{W} \sim \exp -t E(L)$ where $E(L)$ is the energy required to separate the quarks a distance $L$ apart. If $-\log W$ grows when $t(L)$ goes to infinity, i.e. if $\log W$ decreases faster than the perimeter of $\mathcal{C}$, this is the signal that $E(L)$ grows with $L$ and a possibility of confinement. (If $E(L) \sim \text{cst}$ this would mean that virtual gluons only contribute a finite renormalization to the quark masses, as long as $\mathcal{C}$ is finite).

It is possible to show that for large $\beta$ one has
\[ \mathcal{W}(\mathcal{C}) \sim \exp -\text{cst} A \]
(3)

at least in certain cases, where $A$ is the minimal area enclosed by $\mathcal{C}$. To illustrate this use an Abelian group. Then
\[ \mathbb{B} U_{xy} = \prod_{\text{A}} U_{xy} \]
(4)

For large $T$ we can neglect the correlations between plaquette variables. Therefore
\[ \mathcal{W}(\mathcal{C}) \rightarrow \langle \chi_f(U) \rangle^A \]
(5)
in agreement with (3) since $\langle \chi_f(U) \rangle < 1$. As a result Abelian gauge theories on a lattice always confine for large $T$ ! This is of course not the picture of QED, but there we may assume that the large $T$ and small $T$ regimes are separated at finite $T_c$ by a transition

On one side we have the usual perturbative unconfined phase, on the other a confined phase. Of course the step of Eq.(4) is not valid for a non-Abelian gauge group. Nevertheless we can develop a high $T$ perturbation theory. To leading order the result (5) will remain qualitatively correct provided some selection rule on representations is satisfied. For SU(2) for instance if quarks belong to the fundamental representation and $\chi$ in the action is taken in this representation. Then confinement holds but this is not true for particles belonging to the representation with $j = 1$.

These results are only meaningful as was already stressed if we do not encounter a transition until

$T \to 0$ in the non-Abelian case. This is more or less what is presently believed. But calculations based on the large temperature expansion have still a long way to go to become realistic.

Nevertheless if (5) remains qualitatively true, then we expect the quark-antiquark interaction to grow linearly at large distances, a prediction which seems in fair agreement with the present understanding of high mass $\bar{q}q$ states.

Alternative attempts are now under way to study qualitatively the confinement mechanism, which we cannot describe here for lack of time.

From this brief presentation one can only say that the subject of gauge fields is very lively and might still offer new surprises.

I take this opportunity to thank my colleagues and friends J.B. Zuber and J.M. Drouffe for their help in the study of gauge fields.

   G. 't Hooft and M.T. Veltman, Nucl. Phys. B50, 318 (1972) ;
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Some reviews are the following:
Recent generalizations are actively studied by Atiyah and collaborators.
HIGH ENERGY NEUTRINO REACTIONS
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Abstract:
Recent experimental results on neutrino interactions are discussed in the framework of the quark parton model. The topics discussed include charged currents, neutral currents, dileptons, trileptons and tetraleptons. These lecture notes are not meant to be a complete review article.

1. Introduction

1.1 Neutrinos
The existence of the neutrino has been postulated by W. Pauli\(^1\) in 1930 on grounds of the experimentally observed electron energy spectrum in nuclear beta decays. The fact that the average electron energy is less than the total disintegration energy and the apparent violation of angular momentum conservation in a transition between two states of nuclear spin 0 with emission of a spin \(\frac{1}{2}\) electron could be explained by the simultaneous emission of a neutral massless spin \(\frac{1}{2}\) particle with very small interaction cross-section. Apart from indirect evidence for such a neutrino, the direct observation of antineutrinos from nuclear beta decay was achieved in an experiment using reactor neutrinos by Cowan and Reines\(^2\), who observed the reaction \(\bar{\nu} + p \rightarrow e^+ + n\). The first observation of a different kind of neutrino, the \(\nu\)-neutrino, is due to a BNL-Columbia experiment\(^3\) at the AGS, where neutrinos from the decay \(\pi^+ \rightarrow \mu^+ + \nu_\mu\) were interacting in a spark chamber detector producing a \(\nu^-\): \(\nu^- + N \rightarrow \mu^- + X\). Since 29 events of this type were observed, and there were less than 6 events having an \(e^-\) in the final state in the same sample, it was concluded that neutrinos carry a quantum number, "lepton number", which is separately conserved for \(\nu\)-like and \(e\)-like leptons.

1.2 Lepton number
The lepton numbers given to the \(\nu\)- and \(e\)-like particles are

<table>
<thead>
<tr>
<th>(L_e)</th>
<th>(L_\nu)</th>
</tr>
</thead>
<tbody>
<tr>
<td>+1 (e^-)</td>
<td>+1 (\nu^-)</td>
</tr>
<tr>
<td>+1 (\nu_e)</td>
<td>+1 (\nu_\mu)</td>
</tr>
<tr>
<td>-1 (e^+)</td>
<td>-1 (\nu^+)</td>
</tr>
<tr>
<td>-1 (\nu_e)</td>
<td>-1 (\nu_\mu)</td>
</tr>
</tbody>
</table>

and, to our present knowledge, these lepton numbers are conserved in all interactions:

\[ L_e \mid = \mid L_i \mid = \text{const} \quad \text{and} \quad L_\nu \mid = \mid L_k \mid = \text{const}. \]

Present experimental tests on this conservation law include the upper limit\(^4\) on the decay \(\nu^+ \rightarrow e^+\gamma\),

\[ \Gamma (\nu^+ \rightarrow e^+\gamma) < 3.6 \times 10^{-9}, \]

and the observed\(^5\) lifetime of double beta decay of \(^{130}_{\text{Te}} \rightarrow ^{130}_{\text{Xe}} + 2e^- + 2\bar{\nu}\). The minute amount of \(^{130}_{\text{Xe}}\) found in an old Tellurium-ore allows the determination of a lifetime of \(10^{21.34 \pm 0.12}a\), while calculations assuming lepton conservation lead to \(t_1 = 10^{22+2}a\), and lepton number nonconservation would lead to a life time of \(t_\nu = 10^{15\pm2a}\) for the neutrinoless decay \(^{130}_{\text{Te}} \rightarrow ^{130}_{\text{Xe}} + 2e^-\).

1.3 Lagrangian
The Fermi-type Lagrangian for weak interactions is of current-current type, we write for purely leptonic interactions

\[ \mathcal{L}_L = \frac{G}{\sqrt{2}} J_a^+ \cdot j_a^- + \text{h.c.} \]

for semileptonic interactions

\[ \mathcal{L}_{SL} = \frac{G}{\sqrt{2}} J_a^+ \cdot j_a^- + \text{h.c.} \]

for non leptonic interactions

\[ \mathcal{L}_{NL} = \frac{G}{\sqrt{2}} J_a^+ \cdot J_a^- + \text{h.c.} \]
where \( j_a \) is the leptonic current
\[
j_a^- = \bar{\nu}_e^- (1 - \gamma_5) \gamma_a \nu_e + \bar{\nu}_\mu^- (1 - \gamma_5) \gamma_a \nu_\mu
\]
\[
j_a^+ = \bar{\nu}_e^+ (1 - \gamma_5) \gamma_a \nu_e + \bar{\nu}_\mu^+ (1 - \gamma_5) \gamma_a \nu_\mu
\]
and \( j_a \) is the hadronic current of two fermions. The study of this hadronic current in weak decays of hyperons has shown that they can be successfully described by the Cabibbo current
\[
J_a = J_a^{\Delta S=0} \cos \theta_c + J_a^{\Delta S=1} \sin \theta_c
\]
where the Cabibbo angle \( \theta_c \) describes the observed suppression of decays with change of strangeness, \( \Delta S=1 \). Present experiments \(^6\) on hyperon lepton decays determine \( \sin \theta_c = 0.230 \pm 0.003 \).

1.4 Neutral currents

This picture of weak interactions had to be enlarged when, in 1973, the Gargamelle Collaboration \(^7\) discovered the existence of neutral currents by observing the reactions
\[
\bar{\nu}_\mu + e^- \rightarrow \bar{\nu}_\mu + e^-
\]
\[
\nu_\mu + N \rightarrow \nu_\mu + \text{hadrons}
\]
We have therefore to add to the leptonic charged current the current
\[
j_c^{\Delta S=0} = (\bar{\nu}_e (1 - \gamma_5) \gamma_a \nu_e) + \text{terms for } e \text{ and } \mu
\]
and this current would require the existence of a massive neutral boson \( Z^0 \) together with the charged boson \( W^\pm \). The idea that weak and electromagnetic interactions can then be unified via the interference of \( Z^0 \) and \( \gamma \), is one of the enormous theoretical achievements of the last years.\(^8\) It also offers the possibility of renormalizing the weak interactions.

One problem was left, however. The neutral long-lived \( K \) meson decays mainly via three-body leptonic channels. A decay via a neutral current interaction \( K_L \rightarrow \mu^- \mu^- \) was searched for, but the experimental branching ratio\(^9\)
\[
\frac{\Gamma(K_L \rightarrow \mu^- \mu^-)}{\Gamma(K_L \rightarrow \text{all})} = (12^{+8}_{-4}) \times 10^{-9}
\]
is extremely small and can be explained by a second order weak-electromagnetic process.

It is therefore clear that neutral currents with change of strangeness \( \Delta S = 1 \) are suppressed at least by a factor \( 10^8 \). However there is a process which leads to a much larger decay rate for \( K_L \rightarrow \mu^- \mu^- \):

It was the idea of Glashow, Iliopoulos and Maiani\(^10\) that in order to cancel this diagram, a fourth quark can be instrumental if this quark is coupled to \( s \) and \( d \) quarks in a way exactly orthogonal to the normal \( u \)-quark. The corresponding diagram

\[\text{cancels the diagram above to first order. The remaining difference of order}
\]
\[
\frac{m_c^2}{m_w^2}
\]
is canceled by a process mediated by the \( Z^0 \) boson. This GIM mechanism seemed so convincing that the existence of the fourth quark, the charmed quark \( c \), was a natural postulate.

1.5 Charm

The discovery of hidden charm at ENL\(^11\) and SPEAR\(^12\) in the form of the \( J/\psi \) particle interpreted as \( c\bar{c} \) state was followed by the observation\(^13\) of the \( D^0 = c\bar{u} \) meson. The hadronic current therefore has to be extended in the way prescribed by GIM. We have two quarks of charge \( +\frac{2}{3} \), \( u \) and \( c \), and two of charge \( -\frac{1}{3} \), \( d \) and \( s \), where \( s \) carries strangeness \( \Delta S = -1 \) and \( c \) carries charm \( \Delta C = +1 \), and the charged hadronic current contains four terms:
\[
\begin{align*}
J_a^- &= \cos \theta_c \ (\bar{c} (1 - \gamma_5) \gamma_a u) \\
&+ \sin \theta_c \ (\bar{s} (1 - \gamma_5) \gamma_a u) \\
&- \sin \theta_c \ (\bar{d} (1 - \gamma_5) \gamma_a c) \\
&+ \cos \theta_c \ (\bar{s} (1 - \gamma_5) \gamma_a c)
\end{align*}
\]
where the quark symbols stand for the spinors of the corresponding particles. The two large terms are the ones with \( \Delta S = \Delta C = 0 \) or \( \Delta S = \Delta C = 1 \) and the small ones have \( \Delta S = 1 \)
and $\Delta C = 0$ or $\Delta S = 0$ and $\Delta C = 1$.

1.6 Neutrino-induced reactions

Considering the structure of the hadronic current, neutrinos and antineutrinos can be used in order to induce transitions between different quarks. This tool may serve in two ways: either we can study the quark structure of the nucleon, its dynamics and possible production of new quarks, or we concentrate on the space-time structure of this new and basically unknown interaction described by neutral currents. These lectures therefore are subdivided in the following way:

2. Neutrino beams
3. Neutrino detectors
4. Quark structure of nucleons:
   Charged-current inclusive reactions
5. Space-time structure of neutral currents:
   - $\nu_e e^-$ scattering
   - $\nu_N$ inclusive reactions
6. New particle production
   - dilepton events
   - trimuon events
   - tetraevent events

In preparing these lectures I used the ones by Segal14, Steinberger15 and Dydak16.

2. Neutrino beams

2.1 Principles

In order to produce high-energy neutrinos, protons are accelerated in a synchrotron to the maximum energy, which is 30 GeV for the CERN PS and Brookhaven AGS and 400 GeV for the CERN SPS and the Fermilab synchrotron. The protons are then extracted from the machine and impinge on a target, usually of Be or Cu, where they produce charged $\pi$ and $K$ mesons: $p + N \to \pi^\pm + X$, $K^\pm + X'$. These mesons are long-lived, e.g. a $\pi^-$ meson of 200 GeV has a mean decay length of 14.8 km.

They are allowed to decay in a long region behind the target: $\pi^\pm \to \nu^\pm + \nu(\bar{\nu})$ and $K^\pm \to \nu^\pm + \nu(\bar{\nu})$. A very large mass of material behind this decay region serves as a shield for all hadrons and for the muons produced together with the neutrinos.

Since these muons loose only $\gtrsim 1.2$ GeV per metre of iron shield, their energy determines the length necessary for the absorber.

One example of a neutrino beam is shown in Fig.1, which displays the general layout at the CERN SPS West Area.

![Diagram](image_url)

Figure 1

2.2 Neutrino energy spectrum

The overwhelming majority of the $\pi$ decays (99.98%) and most of the $K$ decays (63.6%) occur through the two-body decays into ($\nu\nu$). In this case the laboratory energy spectrum of the neutrinos for a fixed meson energy is particularly simple. In the forward direction we have

$$E_\nu = p_{\text{Lab}} = \gamma (p_{\text{cm}} \cos \theta^* + \epsilon E_{\text{cm}})$$

$$= \gamma p_{\text{cm}} (\cos \theta^* + 1)$$

where $\gamma$ is the Lorentz factor for the decaying meson $\gamma_{\pi,K} = E_{\pi,K} / m_{\pi,K}$, $p_{\text{cm}}$ is the neutrino momentum in the cm frame, and $\theta^*$ the cm decay angle. A flat distribution in $\cos \theta^*$ leads then to a flat energy spectrum

$$0 \leq E_\nu/E_{\pi,K} \leq 2 p_{\text{cm}} / m_{\pi,K} = 0.43 \text{ for } \pi + \nu\nu$$

$$= 0.95 \text{ for } K + \nu\nu$$

These box spectra have to be folded with the meson laboratory energy spectra in order to obtain the real neutrino spectrum.

There are mainly two ways of treating the mesons produced in the proton collisions:
a) the maximum number of mesons of one charge are bent parallel to the beam line and send their neutrinos into the detector
(wide band beam); b) a system of magnetic bending and focusing elements is used in order to select mesons in a narrow momentum interval; only these mesons are allowed to enter the decay region. These two principles are sketched in Fig.2.

![Figure 2](image)

**2.3 Wide band beams**

The focusing of charged mesons is achieved by a magnetic horn, a current sheet of rotational symmetry (Fig.3), producing a toroidal magnetic field.

![Figure 3](image)

**Particles of one charge are focused towards the beam axis, the others defocused. In order to achieve transverse momentum kicks of the order of 0.5 GeV/c, horns of 6.5 m length with pulsed currents of several 100 kA are used. Focusing is improved by a second horn placed downstream of the first one, called "reflector".**

The neutrino energy spectra are dominated by the strong fall-off of the parent meson energy spectra: For a 400 GeV/c proton beam, the average neutrino energy is around 30 GeV. Since typical $\tau^-/\tau^+$ production ratios are 1/3 and $K^-/K^+$ ratios about 1/10, the antineutrino intensities are lower by these factors at the low ($\nu$) and high ($\bar{\nu}$) energy parts of the spectrum, respectively (Fig.4).

![Figure 4](image)

**The background from wrong-charge mesons is therefore tolerable in neutrino beams (at the percent level), while it becomes severe in antineutrino beams, in particular at the high energy end, where the $\bar{\nu}/\nu$ ratio comes near unity.**

**2.4 Narrow-band beams**

The advantages of such a layout selecting mesons of a fixed momentum are threefold: i) the neutrino spectra are the flat box spectra discussed in 2.2 ii) it is possible to determine the neutrino energy event by event iii) monitoring of the neutrino flux becomes easier than in a wide-band beam.

**2.4.1 Kinematics**

Suppose that all parent mesons have the same momentum and travel exactly parallel to the beam axis. Then, if $\theta$ is the cm decay angle
of the neutrino and $P_{\text{CM}}$ its cm momentum, the transverse momentum of the neutrino relative to the beam direction is

$$P_T = P_{\text{CM}} \sin \theta^\circ.$$ 

On the other hand, this transverse momentum can be obtained from the longitudinal laboratory momentum (2.2) and the laboratory angle $\theta = R/L$ in Fig.2:

$$P_T = \gamma P_{\text{CM}} \cos \theta + 1.$$ 

From these two relations we obtain

$$\cos^2 \theta^\circ = \frac{(1 - \gamma^2 \theta^2)}{(1 + \gamma^2 \theta^2)}$$

and the neutrino energy

$$E_\nu = \frac{2 \gamma P_{\text{CM}}}{(1 + \gamma^2 R^2/L^2)}$$

For $\pi \rightarrow \mu \nu$ decay, $P_{\text{CM}} = 29.9$ MeV, and for $K \rightarrow \mu \nu$ $P_{\text{CM}} = 235.5$ MeV. The relation between $E_\nu$ and $R$ is therefore a Lorentz-shaped curve.

### 2.4.2 Measurement of neutrino energy

In reality, the decay point and therefore $L$ are known only to within the length of the decay tunnel, which leads to a smearing of the exact relation above. The two bands predicted in the $(E_\nu, R)$ plane for a $200$ GeV narrow band beam are shown in Fig.5 together with experimental data from the CERN-Dortmund-Heidelberg-Saclay (CDHS) experiment$^{52}$, where $E_\text{TOT} = E_\mu + E_h$ is the total visible energy in a charged-current event $\nu_\mu + N \rightarrow \mu^- +$ hadrons. We see from this figure that, e.g., for $K$-neutrinos of $150$ GeV, the measured vertex position $R$ determines $E_\nu$ to $10\%$.

### 2.4.3. Neutrino energy spectra

It is evident from Fig.5 that neutrinos from $\pi$ decay are concentrated in the center of the apparatus, while $K$-decay neutrinos cover a large area, and a $3.6$ m diameter detector can only catch the high energy part of their flat energy spectrum. If one takes radial slices, $200$ mm large, from $0$ to $1600$ mm, the resulting neutrino energy spectra are shown in Fig.6. The sum of these events for a detector of $3.6$ m diameter results (schematically) in the spectra of Fig.7. A real event spectrum of narrow-band beam data from the CDHS experiment (Fig.8) displays these two components of neutrino flux multiplied with the linearly rising total cross-section.
2.5 Flux monitoring

2.5.1 Wide-band beam
Here it is possible to measure the fluxes of \( \pi \) and \( K \) mesons produced at 0° and calculate the resulting neutrino fluxes with Monte Carlo methods. A more direct way utilizes the fact that with every neutrino from two-body decays, a muon is born as well. The flux of these muons is measured in the CERN beam in gaps inside the iron shielding at depths of 30, 50, 70 and 94 meters from the entry face of the shielding. Solid state counters are used for this ionization measurements, they are calibrated against a moveable counter in the gap, and this in turn is calibrated relative to the numbers of muon tracks in an emulsion stack exposed to the same beam flux. From the measured number of muons, the pion and kaon spectra are obtained, and those in turn are used for the calculation of the neutrino fluxes.

2.5.2 Narrow-band beam
The monitoring devices used at CERN include
i) a beam current transformer measuring the total hadron flux at the entry of the decay region
ii) a differential gas Cerenkov counter for a determination of the beam composition entering the decay region
iii) ionization chambers at the downstream end of the decay region and inside the iron shielding
iv) the muon counters mentioned above, situated inside the shielding.

A radial distribution of muons at 4 different depths inside the shielding is shown in Fig.9, where one can see the effect of multiple scattering for depths larger than 50 m. The distribution at 30 m depth allows a determination of the \( \pi \) meson beam divergence, and the integral over the distribution gives a measurement of the total number of muons from \( \pi \) decay neutrinos.
The CITF group has measured the \( \pi/K \) ratios by single particle counting in a Cerenkov counter. In order to do this, they had to reduce the intensity of the proton beam by a factor \( 10^3 - 10^4 \).

Typical flux uncertainties are about 6% for \( \pi \) decay neutrinos and 12% for \( K \) decay neutrinos.

3. Detectors

General requirements for a neutrino detector are:

i) the target must have a large mass because of the smallness of the cross-sections, of order \( 10^{-38} \text{ E}_{\nu} \text{ cm}^2 \text{ GeV}^{-1} \) for \( \nu N \) collisions

ii) the target itself should be a sensitive volume because in general hadronic showers created in the reaction have to be detected

iii) lepton identification is very important in order to measure the outgoing lepton coupled to the incident neutrino through the leptonic current.

These requirements can be fulfilled by bubble chambers and electronic detectors.

3.1 Bubble Chambers

Some characteristic figures for bubble chambers used for neutrino experiments are collected in Table I. Apart from size, the most important parameter is the liquid used in the chamber. Physical properties of these are given in Table II.

<table>
<thead>
<tr>
<th>Name</th>
<th>Lab.</th>
<th>Dim [m]</th>
<th>Vol m(^3)</th>
<th>fid vol.</th>
<th>Magn. field kG</th>
<th>liquids</th>
</tr>
</thead>
<tbody>
<tr>
<td>BEBC</td>
<td>CERN</td>
<td>3.7x2</td>
<td>30</td>
<td>(18)</td>
<td>35</td>
<td>Ne, H2, D2</td>
</tr>
<tr>
<td>GGM</td>
<td>CERN</td>
<td>6.9x4.8</td>
<td>12</td>
<td>(3)</td>
<td>20</td>
<td>Freon, propane</td>
</tr>
<tr>
<td>15'</td>
<td>FNAL</td>
<td>3.8</td>
<td>30</td>
<td>(18)</td>
<td>30</td>
<td>H2, Ne/H2</td>
</tr>
<tr>
<td>12'</td>
<td>ANL</td>
<td>3.8x1.9</td>
<td>20</td>
<td>(11)</td>
<td>18</td>
<td>H2, D2</td>
</tr>
</tbody>
</table>

Figure 9

However, the \( K/\pi \) ratio has to be inferred from a measurement using the Cerenkov counter. Since the intensity of charged particles traversing this counter is around \( 10^8 \) per machine burst, the counter integrates all light from one burst. An experimental curve displaying the amount of light as a function of gas pressure is shown in Fig.10, where the peaks from protons, kaons and pions are clearly visible.

Figure 10
Table II

<table>
<thead>
<tr>
<th>Liquid</th>
<th>Density</th>
<th>Rad.1.X₀ [cm]</th>
<th>Nuclear coll.1. [cm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>H₂</td>
<td>0.063</td>
<td>970</td>
<td>680</td>
</tr>
<tr>
<td>D₂</td>
<td>0.14</td>
<td>890</td>
<td>320</td>
</tr>
<tr>
<td>Ne(213)₀/H₂</td>
<td>0.27</td>
<td>115</td>
<td>100</td>
</tr>
<tr>
<td>C₃H₈ prop</td>
<td>0.41</td>
<td>111</td>
<td>130</td>
</tr>
<tr>
<td>Ne</td>
<td>1.2</td>
<td>24</td>
<td>54</td>
</tr>
<tr>
<td>CF₃Br</td>
<td>1.5</td>
<td>11</td>
<td>50</td>
</tr>
<tr>
<td>freon</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3.1.1 Cryogenic chambers

The advantages of H₂/D₂ fillings are: reactions on free nucleons make nuclear corrections unnecessary; the long radiation length results in small multiple scattering and good momentum measurement, and Λ and K₀ decays can be identified; disadvantages include the small mass of order 1 ton, the small γ conversion probability and the poor electron and muon identification. Cryogenic chambers are therefore usually operated with a H₂/Ne mixture, and an External Muon Identifier (EMI) is added consisting of about 1 m of iron and proportional chambers detecting the particles penetrating this absorber. Fig. 11 shows the BEBC assembly at the CERN SPS.

3.1.2 Heavy liquid chambers

Propane and/or freon fillings due to their short radiation length offer the opportunity for good electron identification, complete γ conversion and a larger mass (10 t) at the price of complications because of nuclear corrections and loss of precision due to multiple scattering. One example is the new Gargamelle setup at the CERN SPS (Fig. 12), where an electronic calorimeter behind the chamber measures the part of the hadron shower leaking out of the chamber and an EMI is used for muon identification as in the case of BEBC.

3.2 Electronic detectors

General purpose detectors for inclusive processes like νX → ν-X or νX consist of a de-

Figure 11

Figure 12

vice measuring the total hadronic energy of the system X and a muon identifier detecting the presence or absence of a penetrating muon complemented by a magnetic measurement of the muon momentum, if there is one.
3.2.1 Calorimetry

The hadron energy is usually measured by a sampling device, called calorimeter. The principle of this technique has been described frequently\textsuperscript{17}. Plates of target material (iron, marble, asphalt) are sandwiched with planes of ionization detectors (scintillator, proportional tubes). The average ionization measured in the sampling scintillators is then proportional to the total energy dissipated by a hadronic shower, but there is a finite width of the measured pulseheight distribution due to statistical fluctuations of the fraction of the energy sampled in the detectors (Fig. 13).

![Figure 13](image)

The energy resolution for fixed incident hadron energy is given mainly by three contributions: \( \Delta E/E = f_1 f_2^2 \), and these three are:

- \( f_1 \): fluctuation in the number of particles traversing the detectors; since the number of traversals \( N \) is inversely proportional to the thickness \( d \) of the target plates, this fluctuation \( f_1 = 1/\sqrt{N} = \sqrt{d} \).

- \( f_2 \): fluctuation in the energy not seen by the detectors because of nuclear excitation and decay via soft photons, escaping neutrinos, and production of heavy particles which give less light per energy loss than fast light particles;

- \( f_3 \): fluctuation in the energy carried off by \( \gamma \)'s; from \( \pi^0 \) decay; since these will give electromagnetic showers, there will be no missing energy for this part of the shower, and a fluctuation in the amount of \( \gamma \)'s will add a fluctuation to the pulseheight seen.

The response and energy resolution of hadron calorimeters is usually calibrated using hadron beams of fixed energy, and as an example, Fig. 14 gives the resolution and mean pulseheight (in nep = number of equivalent particles) for an effective 5 cm iron sampling calorimeter for hadron energies between 15 and 140 GeV. Typical values for the hadron energy resolution are\textsuperscript{18}:

\[
\frac{\sigma_E}{E} = 0.8 \frac{1}{\sqrt{E \text{(GeV)}}}
\]

while for electrons \( f_2 = f_3 = 0 \) which leads to \( \sigma_E/E < 1/2 \) \( (\sigma_E/E)_{\text{had}} \).

![Figure 14](image)
The amount of missing energy in a hadron shower can be estimated from the measured ratio of pulseheight P generated by an electron or hadron (pion) shower, which comes out to be [8]

\[ P(e)/P(\pi) = \begin{cases} 
0.87 \pm 0.02 & \text{at 30 GeV} \\
0.94 \pm 0.02 & \text{at 140 GeV} 
\end{cases} \]

Another characteristic difference between electron and hadron showers can be obtained from Fig. 15. Here the measured number of shower particles is plotted versus the depth inside the calorimeter, measured in cm of iron. Due to the radiation length being shorter than the nuclear interaction length, the electron shower is much shorter than the hadronic one. For pion-induced showers the shower length (defined as the length of a box inside which 95% of the energy are contained) varies from 60 cm Fe at 15 GeV to 80 cm Fe at 140 GeV.

![Figure 15](image)

Similarly, a lateral size with 95% energy containment can be defined, which, at a depth of 90 cm Fe shrinks slowly with energy, from 30 cm Fe at 50 GeV to 20 cm Fe at 140 GeV energy.

3.2.2 Measurement of hadron shower direction. The different longitudinal developments of the hadronic and electronic part of a hadron shower in iron, as illustrated by Fig. 15, also induce lateral fluctuations of the shower which make a measurement of shower direction too inaccurate to be relevant. For such a measurement another target material with low Z is needed, where radiation length and nuclear interaction length become approximately equal. Such a detector has been built by the CHARM Collaboration [19] using Carrara marble as target material. For a 22 GeV proton initiated shower, an angular resolution of 38 mrad (rms) is obtained (Fig. 16). Using two sets of detectors between their 8 cm marble plates, they obtain a resolution for the projected angle of

\[ \sigma(\theta_H) = 41 \frac{600}{E_H/\text{GeV}} \text{ mrad} \]

![Figure 16](image)

3.2.3 Measurement of muon momentum

In principle, the best momentum measurement can be done with air core magnets because there is only negligible multiple scattering. On the other hand, such magnets would be very uneconomical for neutrino experiments because
the iron would be used for a flux return yoke only, and the power consumption would be enormous. Experiments therefore use iron core magnets with toroidal field. The momentum resolution is then limited by the multiple scattering of the muons in the iron. The multiple scattering deflection angle in a magnet of length L and radiation length \( X_0 \) is \( \theta_{\text{MS}} = 0.0144 \sqrt{L/X_0} / p \) with \( p \) in GeV/c, and the corresponding angle from magnetic deflection in a saturated iron core magnet is \( \theta_M = 0.5 L/p \) with \( L \) in m and \( p \) in GeV/c. For iron, \( X_0 = 1.8 \) cm, such that the resolution
\[
\Delta p / p \sim \frac{0.2}{\sqrt{L}}
\]
with \( L \) in m.

For an iron core magnet of 4 m length, this gives \( \Delta p / p \sim 1\% \).

3.2.4 Configurations.

The two main parts of an electronic neutrino detector can be arranged in two ways: either the target calorimeter section is physically separated from the magnet section placed downstream. This solution was adopted by most experimenters, e.g. the HPW\(^\text{a}\) and CITF\(^\text{b}\) Collaborations at FNAL and the CHARM Collaboration at CERN.

The other possibility, used by the CDHs\(^\text{c}\) group at CERN, is the integration of target and magnet sections by using a magnetized iron calorimeter. While the magnetic field has no influence on the properties of the calorimeter, this detector offers magnetic analysis for muons in a very large part of the angular range of these muons.

### Table III

<table>
<thead>
<tr>
<th>Detector</th>
<th>HPWF</th>
<th>CITF</th>
<th>CDHS</th>
<th>CHARM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Target Scintillator</td>
<td>Fe</td>
<td>Fe</td>
<td>Marble</td>
<td></td>
</tr>
<tr>
<td>Material</td>
<td>tillator</td>
<td>T.Dimensions</td>
<td>3x3m(^2)</td>
<td>1.5x1.5m(^2)</td>
</tr>
<tr>
<td></td>
<td>9m long</td>
<td>20m long</td>
<td>22.4m</td>
<td>15.6m</td>
</tr>
<tr>
<td>T.Mass</td>
<td>70t</td>
<td>120t</td>
<td>1240t</td>
<td>180t</td>
</tr>
<tr>
<td>Fiducial mass</td>
<td>20t</td>
<td>50t</td>
<td>800t</td>
<td>100t</td>
</tr>
<tr>
<td>Sampling thickness</td>
<td>10cm</td>
<td>5/15cm</td>
<td>6cm</td>
<td></td>
</tr>
<tr>
<td>Separated functions</td>
<td>yes</td>
<td>yes</td>
<td>no</td>
<td>yes</td>
</tr>
<tr>
<td>Integrated functions</td>
<td>no</td>
<td>yes</td>
<td>no</td>
<td></td>
</tr>
<tr>
<td>Typ. angular acceptance</td>
<td>175mr</td>
<td>75mr</td>
<td>400mr</td>
<td>230mr</td>
</tr>
<tr>
<td>Density</td>
<td>1</td>
<td>3.5</td>
<td>5.3</td>
<td>1.1</td>
</tr>
<tr>
<td>Hadron shower direction</td>
<td>no</td>
<td>no</td>
<td>no</td>
<td>yes</td>
</tr>
<tr>
<td>Electron identification</td>
<td>no</td>
<td>no</td>
<td>yes? (indirect)</td>
<td></td>
</tr>
</tbody>
</table>

3.2.5 Examples of electronic detectors

Table III gives a few properties of four detectors used in the last years or coming into operation. Figs. 17 shows side and front views of the detector of the CERN-Dortmund-Heidelberg-Saclay group. In this detector the functions of neutrino target, hadron calorimeter, muon identifier and muon magnetic spectrometer are integrated. It consists of 19 toroidal modules of magnetized

---

**Figure 17**

- 53 -
iron plates interspaced with 19 triple plane drift chambers. The diameter of the toroids, 3.75 m is matched to the 90°c.m. decay angle of neutrinos from $\tau \to \mu \nu$ decay such that nearly all $\tau$ decay neutrinos hit the detector, while about half of the neutrinos from $\kappa$ decay miss the apparatus. The thickness of these magnets is 75 cm, composed of 15 plates of 5 cm thickness for the first seven modules and of 15 cm plates for the other twelve. In each gap between two plates is inserted a plane of 8 mm plastic scintillators viewed by two phototubes at each end. The sum of right and left pulseheights is used for calorimetry, while their ratio serves for determining the shower position along the counter. Pulseheight calibration and measurement of light attenuation in the counters is done using cosmic muons continuously between machine bursts.

The drift chambers are hexagonal and consist of three independent gaps with wires in the vertical direction and at ± 60° relative to the vertical. The wire spacing is 6 cm, the measurement accuracy 1 mm and the efficiency typically 99.5% per gap.

Fig. 18 is a pictorial of the detector of the CERN-Hamburg-Amsterdam-Rome-Moscow (CHARM) group, which comes into operation now. Its special features are the ability of measuring the hadron shower direction due to the choice of marble (low Z) as a target material and the possibility of distinguishing electron showers from hadron showers. The target consists of 78 slabs of 8 cm marble, each followed by ionization detectors, 3 cm plastic scintillators and/or proportional drift tubes. Iron toroids behind this target serve for identification of muons and for measurement of their momentum. This detector is well suited for a measurement of the $x$-distribution in neutral current reactions and possibly for a study of $\nu_\mu e^-$ scattering.

4. Inclusive $\nu$-hadron charged-current reactions

4.1 Kinematics

The usual kinematic variables defined for the process

$$\nu + p + \mu^- + X \rightarrow (\text{lepton l})$$

where $\nu + p + \mu^- + X$ are the following:

$$(l = \text{lepton}, h = \text{hadronic system})$$

$$q = k - k', \quad Q^2 = -q^2 = -(k - k')^2$$

$$W = \text{inv. mass of hadronic System}$$

$$= \sqrt{(p^l)^2 + (k+p-k)^2}$$

$$= \sqrt{(k-k')^2 + p^2 + 2(k-k') \cdot p}$$

where $Q^2 = M^2 + 2v$

The kinematic region in the $(Q^2, \nu)$ plane is bounded by the condition

$$0 \leq W^2 = -Q^2 + M^2 + 2\nu$$

and therefore

$$Q^2 \leq 2\nu + M^2$$

This boundary is indicated in Fig. 20 where the diagonal corresponds to $W^2=0$, i.e. to elastic scattering.

Instead of $Q^2$ and $\nu$ we can use the dimensionless Bjorken scaling variables:

$$x = \frac{Q^2}{2\nu}$$

$$y = \frac{\nu}{k \cdot p} = \frac{E_h - M}{E_\nu} - \frac{E_h}{E_\nu}$$

4.2 Structure functions

For charged currents assuming current-current interaction, we obtain the inclusive cross-section in terms of three structure functions

$$W_i(Q^2, \nu);$$

Figure 18
4.3 Bjorken scaling

A simple dimensional argument can be made if the neutrino scatters from a point like object: The dimension of the Fermi coupling constant $G$ is

$$(G) = \{\text{erg cm}^3\} = \{\frac{1}{\text{MeV}^2}\} \text{ for } c = 1$$

$$(G^2) = \{\frac{\text{cm}^2}{\text{MeV}^2}\}$$

The cross-section has to be proportional to $G^2 \sigma (\nu -$ point $) = G^2 \cdot F$

Dimension: $\{\text{cm}^2\} \quad \{\frac{\text{cm}^2}{\text{MeV}^2}\} \quad \{\text{MeV}^2\}$

Since there is no scale available, only invariants can be used for $F$; therefore $F = s = 2E_\nu$ and the cross-section is linearly rising with $E_\nu$, $\sigma = G^2E_\nu f$ where the function $f$ is independent of $E_\nu$.

The precise meaning of this conjecture, called scaling, was given by Bjorken: For fixed $x$ and $Q^2 \to \infty$, $\nu \to \infty$, the structure functions become functions of $x = \frac{Q^2}{2\nu}$ only:

$$W_1(Q^2, \nu) \to F_1(x)$$
$$\frac{\nu}{M^2} W_2(Q^2, \nu) \to F_2(x)$$
$$\frac{\nu}{M^2} W_3(Q^2, \nu) \to F_3(x)$$

Then we obtain

$$\frac{d^2\sigma}{dxdy} = \frac{G^2M E_\nu}{\pi} \{ (1-y) F_2^2 \nu \overline{\nu} (x) + xy^2 F_1 \nu \overline{\nu} (x) \}$$

where the common factor is

$$\frac{G^2M E_\nu}{\pi} = \sigma_0 = 1.52 \times 10^{-38} \text{ cm}^2 E_\nu$$

Consequences of Bjorken scaling are:

1) $\sigma = E_\nu$
2) $\frac{d^2\sigma}{dxdy} = F_\nu \cdot \text{ distributions independent of } E_\nu$

Integrating over $y$, we obtain:

$$\frac{d\sigma}{dx} |_{\nu} = \sigma_0 \{ \frac{1}{2} F_2^2 \nu \overline{\nu} (x) + \frac{1}{3} x F_1 \nu \overline{\nu} (x)$$

$$\pm \frac{1}{3} x F_3 \nu \overline{\nu} (x) \}$$

So far this is valid for neutrino scattering on nucleons, and we have 4 sets of 3 structure functions for $\nu p$, $\nu n$, $\overline{\nu} p$ and $\overline{\nu} n$ scattering.

For neutrino scattering on isoscalar nuclear targets, we assume charge symmetry, then $F_\nu \nu n = F_\nu \nu p$; $F_\nu \overline{\nu} n = F_\nu \overline{\nu} p$, $\nu = 1, 2, 3$

which is an exact relation for $F_1$ and $F_2$.

Figure 19

where $G$ is the Fermi constant and $M$ the nucleon mass. The structure function $W_2$ is analogous to the electric scattering in the case of $l - p$ scattering, while $W_1$ corresponds to magnetic scattering and $W_3$ is a parity-violating term not present in $l - p$ scattering. In terms of $x$ and $y$ we obtain

$$\frac{d^2\sigma}{dxdy} = \frac{G^2M E_\nu}{\pi} \{ (1-y) \frac{M}{2E_\nu} W_1 \nu \overline{\nu} (x) + \nu \overline{\nu} (x) \}$$

For $E_\nu > M$, the term $\frac{M}{2E_\nu}$ can be neglected. It appears that the common factor in front of the bracket is proportional to $2M E_\nu = s$, the square of the center-of-mass energy.
but only valid for $F_3$ if strange sea quarks in the nucleon are neglected. For a nucleus $N$ with equal proton and neutron numbers, this leads to $F_L^{\nu N} = F_L^{\bar{\nu} N}$.

Then

$$
\frac{d^2 \sigma}{dxdy} = \sigma_0 \left\{ (1-y)F_2(x) + xy^2F_1(x) \right\} \\
\hspace{1cm} \pm \left( y - \frac{1}{2} \right) \times F_3(y)
$$

We see that for $y = 0$, where only $F_2$ contributes, charge symmetry gives an exact prediction:

$$
\frac{d\sigma}{dy} \bigg|_{y=0} = \frac{d\sigma}{dy} \bigg|_{y=0} = \sigma_0 \int_0^1 F_2(x) \, dx
$$

4.4 Quark-parton model

In this model, the nucleon consists of 3 point-like objects (partons) identified with the spin $\frac{1}{2}$ quarks and a sea of $q\bar{q}$ pairs. The scattering of a neutrino from a nucleon can then be calculated as the incoherent sum of scattering cross-sections from the individual partons $j$:

$$
\frac{d^2 \sigma}{dxdy} = \sum_j f_j(x) \frac{d\sigma_j}{dy} \quad j = 1, 2, \ldots \text{ type of parton}
$$

where $x$ is the fraction of the nucleon momentum carried by the interacting quark. This $x$ coincides, in inclusive reactions, with the Bjorken $x$ defined above.

In the quark model the nucleon consists of 3 valence quarks and a $q\bar{q}$ sea of all flavours. The distribution functions of quarks inside a proton are called $u(x), d(x), s(x), c(x), b(x), \bar{u}(x), \bar{d}(x), \bar{s}(x), \bar{c}(x), \bar{b}(x)$. In our energy region $30 < E_\nu < 200$ GeV, the charged and bottom sea, $\bar{c}(x)$ and $\bar{b}(x)$ is probably negligible. If the sea is $SU(3)$ symmetric, then $\bar{u}(x) = \bar{d}(x) = \bar{s}(x)$. Since the nucleon has no strangeness, strange quarks are only present in the sea, and $s(x) = \bar{s}(x)$. For the $u$-quark distribution, on the other hand, two valence quarks are present in the proton ($u\bar{u}$):

$$
\int (u(x) - \bar{u}(x)) \, dx = 2
$$

Similarly for the $d$ quark

$$
\int (d(x) - \bar{d}(x)) \, dx = 1
$$

and for the $s$ quark

$$
\int (s(x) - \bar{s}(x)) \, dx = 0
$$

Since the neutron consists of $(ddu)$, the neutron distributions can be obtained from proton distributions by exchanging $u \leftrightarrow d$.

4.5 $y$ dependence of elementary cross-sections

Consider individual $\nu - q$ quark scattering where the quark has spin $\frac{1}{2}$, is light and left-handed (predominantly) and the neutrino has spin $\frac{1}{2}$, is massless and left-handed. Then in the center-of-mass system the following simple consideration can be made:

1) for neutrino-quark scattering, both partners are left-handed and the total spin adds up to $J = 0$:

2) for neutrino-antiquark scattering, the spins add up to $J = 1$:

The scattering is therefore isotropic in the center-of-mass angle $\theta^$; i.e. there is a flat distribution in $\theta^$.

Since $\cos \theta^$ is related to the Bjorken scaling variable $y$ by the relation $y = \frac{1}{2} (1 - \cos \theta^)$, this leads to a flat distribution in $y$ between 0 and 1.

In particular, scattering by $180^\circ(y=1)$ is possible.

iii) the scattering of antineutrinos from quarks resembles $\nu q$, i.e. a $(1-y)^2$-distribution is obtained, and $\bar{\nu}P$ is equivalent to $\nu q$, i.e. we get a flat distribution.

This consideration together with the strength of the different hadronic currents in the G.I.M. model can be used to calculate the individual neutrino-quark cross-sections.
Some of these are listed in Table IV.

Table IV

<table>
<thead>
<tr>
<th>Neutrino-quark cross-sections</th>
</tr>
</thead>
<tbody>
<tr>
<td>reaction</td>
</tr>
<tr>
<td>v+d + u^- + u</td>
</tr>
<tr>
<td>v+s + u^- + u</td>
</tr>
<tr>
<td>v+s + u^- + c</td>
</tr>
<tr>
<td>v+d + u^- + c</td>
</tr>
<tr>
<td>v+u^- + u^- + d</td>
</tr>
<tr>
<td>v+u^- + u^- + s</td>
</tr>
<tr>
<td>v+u^- + u^- + d</td>
</tr>
<tr>
<td>v+u^- + u^- + s</td>
</tr>
</tbody>
</table>

4.6. y dependence for isoscalar targets

Neglecting terms of order $\sin^2 \theta_C$, we obtain from table IV:

$$\frac{d \sigma^-}{dy} = \sigma_0 \times (u+d+s+\bar{u}+\bar{d})(1-y)^2$$

$$\frac{d \sigma^+}{dy} = \sigma_0 \times (u+d)(1-y)^2 + (\bar{u}+\bar{d}+2\bar{s})$$

The shape of the y-distributions therefore is a sum of a flat component and a component decreasing with y according to $(1-y)^2$. This form we obtain also from the general expression in sect. 4.3 if the Callan-Gross relation holds, i.e. if $F_2(x)=2xF_1(x)$. This Callan-Gross relation can be derived if the partons (quarks) have spin $\frac{1}{2}$ and if they have no primordial transverse momentum inside the nucleon. Then

$$\frac{d \sigma^-}{dy} = \sigma_0 \left\{ (1-y+\frac{y^2}{2}) F_2(x) \times \frac{(1-y)}{2} + (u+d)(1-y)^2 \right\}$$

which leads to

$$\frac{d \sigma^-}{dy} = \sigma_0 \left\{ \frac{(F_2-xF_1)}{2} \times (1-y)^2 + \frac{(F_2+xF_1)}{2} \right\}$$

$$\frac{d \sigma^+}{dy} = \sigma_0 \left\{ \frac{(F_2-xF_1)}{2} \times (1-y)^2 + \frac{(F_2+xF_1)}{2} \right\}$$

By comparing the two forms of $\frac{d \sigma}{dy}$ we obtain for neutrinos $\frac{(F_2-xF_1)}{2} = x(u+d+2s)$

and defining the function $q(x) = x(u(x) + d(x) + s(x))$, we get

$$F_2 = x(u+\bar{u}+d+\bar{d}+s+\bar{s}) = q(x) + \bar{q}(x)$$

$$-xF_3 = x(d+u+2s-\bar{u}-\bar{d}) = q-\bar{q} + x2s$$

Similarly for antineutrinos

$$F_2^\nu = q + \bar{q} = F_2^\nu$$

$$-xF_3^\nu = q - \bar{q} - x2\bar{s}$$

4.7. Questions to be answered by experiment

A. Space-time structure of hadronic charged current

A.1 Total cross-section ratio

Integrating the Cross-sections for isoscalar targets over y and x we obtain

$$\frac{1}{A} \int \sigma^- = \int \sigma_0 \left\{ \frac{(1-y)}{2} + \frac{(u+d+2s)}{2} \right\}$$

$$\frac{1}{A} \int \sigma^+ = \int \sigma_0 \left\{ \frac{(u+d)}{2} + \frac{(u+d+2s)}{2} \right\}$$

Defining the numbers:

$$Q = \int (u+d+s)\, dx$$

$$\bar{Q} = \int (u+d+s)\, dx$$

$$S = \int (u+d)\, dx$$

the cross-section ratio becomes:

$$R = \frac{\sigma^+}{\sigma^-} = \frac{1}{3} (Q-S) + \frac{3}{1} (Q-S)$$

$$= \frac{(Q+S)}{2} - \frac{(Q+S)}{2}$$

In particular, if there would be no sea $(\bar{Q}=S=0)$, then $R = \frac{1}{3}$. A measurement of $R$ is sensitive to the antiquark contents of the nucleon, and this measurement is independent of the one of the y distributions and can be used as a consistency check.

A.2 y distributions

In order to analyze the y distributions, we can assume the Callan-Gross relation and extract the flat and the decreasing parts of the y distribution for neutrino and antineutrino reactions. With the definitions of $Q$, $\bar{Q}$ and $S$ above, we obtain then from antineutrino data the quantity $(\bar{Q}+\bar{S})/(Q+\bar{Q})$, from neutrino data we get $(Q-S)/(Q+\bar{Q})$, and combining the two informations, we obtain $\bar{Q}/(Q+\bar{Q})$ which is the fraction of momentum carried by the sea antiquarks in the nucleon.
A.3. Test of Callan-Gross relation

If this relation is not valid, we get (1-y) terms in the y distributions in the following way

\[
\frac{d\nu}{dy} = \nu(x) + \int (q+x^2)dx - \int (q-x)dx \quad (1-y)^2
\]

Experimentally, one can then fit the two y distributions with these 3 components and obtain the magnitude of the violation of the Callan-Gross relation:

\[
\frac{d\nu}{dy} \quad (1-y)^2 + \int q_u dx (1-y) + \int q_{\bar{s}} dx
\]

A.4. Test of charge symmetry

Such a test can be done at y=0, where only F_2 contributes to the cross-section and therefore

\[
\frac{d\nu}{dy} \bigg|_{y=0} = \frac{d\nu}{dy} \bigg|_{y=0}
\]

This test can be done only if absolute cross-sections are measured.

B. Quark structure of nucleon

B.1. The fraction of momentum carried by all quarks and antiquarks can be measured by using the relation

\[
\int (q+\bar{q}) dx = \frac{1}{N} F_2(x) dx = \left( \int \frac{d\sigma}{dy} \bigg|_{y=0} \right) dx
\]

If this number is smaller than unity, then something else has to exist inside the nucleon.

B.2. Assuming Callan-Gross, we can determine the structure functions F_2(x, Q^2) and xF_3(x, Q^2); integrating over y, we obtain:

\[
\int \frac{d\sigma}{dy} \bigg|_{y=0} = \sigma_o \left[ (u+d+2s+u\bar{d}+2\bar{s}) + \frac{4}{3} (u+d+u\bar{d}) \right]
\]

\[
\int \frac{d\sigma}{dy} \bigg|_{y=0} = \sigma_o \left[ \frac{2}{3} (F_2(x) + \frac{2}{3} \sigma (s+\bar{s})) \right]
\]

The structure function of the sea quarks can be obtained from the cross-sections in this way:

\[
\frac{1}{A} \int \frac{d\nu}{dx} = \sigma_o x [ (u+d+3 \bar{u}+3 \bar{d}) - \bar{d} + \bar{d} \frac{1}{3} ]
\]

C. Scaling violations

In asymptotically free field theories (e.g. QCD), the strong coupling constant \( \alpha_s \) becomes smaller with \( Q^2 \rightarrow \infty \)

\[
\alpha_s(Q^2) \rightarrow \frac{12}{\pi} \frac{\alpha_s(Q^2)}{(33-2N_f) \ln (Q^2/\Lambda^2)}
\]

where \( N_f \) is the number of flavours, and asymptotic freedom occurs for \( N_f \leq 16 \).

This logarithmic decrease of \( \alpha_s(Q^2) \) causes logarithmic scale violations in the structure functions. In particular, the moments of these structure functions vary in a way calculable by QCD. These moments are defined as

\[
M_l (N, Q^2) = \int \frac{d\nu}{dx} x^l \nu^2(x, Q^2) dx
\]

The \( Q^2 \) dependence of these moments in QCD is given by

\[
M_l (N, Q^2) = C_l^N (\ln \frac{Q^2}{\Lambda^2}) - d_N
\]

where the \( C_l^N \) are computable constants, \( \Lambda \) is one common free parameter of the theory, and \( d_N \) are the anomalous dimensions of the renormalization group

\[
d_N \sim \frac{33-2N_f}{4} \left[ 1 - \frac{4}{N(N+1)} + \frac{N}{4} \right]
\]

Qualitatively, in QCD quarks lose momentum by gluon bremsstrahlung, and therefore the structure functions \( F_2(x, Q^2) \) and \( F_3(x, Q^2) \) shrink in x with increasing \( Q^2 \). Also with increasing \( Q^2 \), the production of \( \bar{q}q \) pairs via gluons becomes stronger, such that the fraction of sea quarks rises with \( Q^2 \).

These effects should be observable in the \( Q^2 \) range from 2 - 200 GeV\(^2\) accessible to neutrino experiments. It has to be kept in mind that asymptotic freedom requires \( Q^2 \) to be much larger than the square of any relevant mass involved.
4.8 Experimental results

A.1 Total cross-sections

Already the early measurements using the Gargamelle heavy liquid bubble chamber exposed to low energy (2-10 GeV) wide band neutrino and antineutrino beams from the CERN PS showed the linear rise of both $\nu$ and $\bar{\nu}$ total cross-sections with neutrino energy (Fig.20) and therefore supported the conjecture of scaling and of a point-like substructure in the nucleon. The data are from $1.4 \times 10^6$ pictures taken with a freon filled chamber, and above $E_\nu=2$ GeV there are 2490 neutrino and 1700 antineutrino events.

Results for neutrino energies above 20 GeV are available now from three high-energy experiments done by the BEBC, CITFR, and CDHS groups.

Some information concerning these experiments is collected in table V.

<table>
<thead>
<tr>
<th>Expt.</th>
<th>Beam</th>
<th>Events</th>
<th>Flux determination</th>
</tr>
</thead>
<tbody>
<tr>
<td>BEBC</td>
<td>NBB</td>
<td>1120</td>
<td>$\mu$ flux, $\hat{\nu}$ flux, $\hat{\nu}$ flux</td>
</tr>
<tr>
<td>CITFR</td>
<td>NBB</td>
<td>18000</td>
<td>charged particle flux, $\hat{\nu}$ flux, $\hat{\nu}$ flux</td>
</tr>
<tr>
<td>CDHS</td>
<td>NBB</td>
<td>23000</td>
<td>$BCT$, $\hat{\nu}$ flux, $\hat{\nu}$ flux</td>
</tr>
</tbody>
</table>

The main problem of this measurement is the determination of the neutrino flux. The methods used by the groups differ, but the main points are

i) a measurement of the charged particle flux entering the decay pipe per incident proton, for neutrino and antineutrino beam settings; this is done by the beam-current-transformer (BCT) during the run for the CDHS experiment, while the CITFR group did a special survey experiment at low proton intensity.

ii) a measurement of the beam composition, i.e. the $K^-/\pi^-$ and $p/K^+/\pi^+$ ratios done with threshold Čerenkov counters.

An alternative to method i) used by the BEBC group consists in determining the flux of decay muons inside the iron shielding at depths larger than 30 m.

Results of these experiments on the total cross-sections per nucleon for neutrinos and antineutrinos are shown in fig. 21. The measured slopes $\sigma/E$ are given in table VI.

The high-energy data are consistent with each other and with no energy variation of the slopes. If one takes into account the low-energy Gargamelle point, there is an indication for a decrease of the neutrino slope with neutrino energy,
while the antineutrino slope is still compatible with being constant.

Figure 21

This phenomenon can also be seen by considering the cross-section ratio \( R = \frac{\sigma^+}{\sigma^-} \). Fig. 22 is a semilogarithmic plot of \( R \) for two of the experiments.

According to the values quoted in Table VI, this ratio seems to be higher in the energy domain 30 - 200 GeV than at 2 - 10 GeV.

Table VI.
Slopes and ratios of total cross-sections:

\[
\sigma_i \times 10^{38} \text{cm}^2 \text{ GeV}^{-1}
\]

<table>
<thead>
<tr>
<th>Energy (GeV)</th>
<th>GGM</th>
<th>CITFR</th>
<th>BEBC</th>
<th>CDHS</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 - 10</td>
<td>0.72 ± 0.05</td>
<td>0.61 ± 0.02</td>
<td>0.61 ± 0.04</td>
<td>0.62 ± 0.04</td>
</tr>
<tr>
<td>20 - 100</td>
<td>0.62 ± 0.03</td>
<td>0.55 ± 0.04</td>
<td>0.63 ± 0.05</td>
<td>( \sigma^- )</td>
</tr>
<tr>
<td>100 - 200</td>
<td>0.29 ± 0.02</td>
<td>0.28 ± 0.01</td>
<td>0.25 ± 0.02</td>
<td>0.30 ± 0.02</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.34 ± 0.03</td>
<td>0.32 ± 0.04</td>
<td>0.31 ± 0.04</td>
</tr>
<tr>
<td>2 - 10</td>
<td>0.40 ± 0.02</td>
<td>0.48 ± 0.05</td>
<td>0.48 ± 0.025</td>
<td>0.49 ± 0.05</td>
</tr>
<tr>
<td>20 - 100</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>100 - 200</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

According to the relation given in 4.7.A.1, this corresponds to an increase of the quark sea component with \( E_v \) (and therefore with \( Q^2 \)), in line with the expectation from QCD.

A.2 \( y \) distributions

A large deviation from scaling behaviour (high \( y \) anomaly) had been found earlier by the HPWF group 28). This was not confirmed by recent experiments from CDHS 29), CITFR 30), and BEBC 31). We only discuss here the recent results. Important features for a measurement of \( y \) distributions are

- uniform muon acceptance up to the
largest scattering angles (large $y$) and a precise hadron energy calibration. A useful check can be done in a NB beam, where the neutrino energy reconstructed from the vertex position has to agree with the sum of $E_\mu + E_{\text{had}}$.

As an example, fig.23 shows the $y$ distribution from the CDHS group$^{27}$ in the energy range 30 - 200 GeV. The $\nu$ distribution is consistent with being the sum of a flat distribution with a small $(1 - y)^2$ admixture from scattering off antiquarks, and the $\bar{\nu}$ distribution is predominantly $(1 - y)^2$ with a small constant admixture. The shape of these distributions does not change very much with neutrino energy, as can be seen by comparing the distribution for the energy range 30 - 90 GeV in fig.24 with that for the energy range 90 - 200 GeV in fig.25. This fact can also be seen by displaying the first moments of the distributions, the average $\langle y \rangle$, as a function of neutrino energy, which are compatible with no energy dependence, but allow for a mild increase with energy as required by QCD. (Fig.26)
If we analyze the $y$ distributions shown in the simple Quark-Parton-Model, i.e. assuming the Callan-Gross relation, we obtain from the flat and $(1 - y)^2$ parts the antiquark contents (4.7.A.2).

The results (preliminary) of the CDHS group are:

From $\bar{v}$
\[ \frac{\bar{Q} + \bar{S}}{Q + Q} = 0.16 \pm 0.02 \]

From $v$
\[ \frac{\overline{Q} - \bar{S}}{\overline{Q} + Q} = 0.08 \pm 0.04 \]

From $\bar{v}$
\[ \frac{\bar{Q}}{\overline{Q} + Q} = 0.12 \pm 0.025 \]

and $v$
\[ \frac{Q - \bar{Q}}{Q + Q} = 0.76 \pm 0.05 \]

The last result means that valence quarks carry 76% of the total quark momentum at these energies, and sea quarks account for the rest. The fraction of sea quark momentum in $\bar{v}$ reactions, $(\overline{Q} + \bar{S})/(\bar{Q} + Q)$, can also be obtained as a function of neutrino energy. This is shown in fig. 27, where it appears that the data are compatible with the kind of increase of the sea demanded by QCD.

Figure 27

A.3 Test of Callan-Gross relation

An analysis of the $y$ distributions including $(1 - y)$ terms can in principle give the magnitude of deviations from this relation. Fig. 28 shows the quantity $A = 2 \times F_1/F_2$ as a function of $\ln Q^2$ from BEBC and GGM data.

![Figure 28](image)

The mean value of $A$ extracted is

\[ \langle A \rangle = 0.96 \pm 0.20 \ (0.09) \]

where the figure in brackets is the statistical error. A similar result from the CTF group was reported:

\[ \langle A \rangle = 0.83 \pm 0.09 \]

A preliminary analysis of the CDHS group (neglecting for the moment the radiative corrections) gives
However, radiative corrections will affect this result because they predominantly occur at low $y$.

For the moment the experiments are not conclusive as to whether $A$ is exactly equal to 1 as assumed by the Callan-Gross rule or $A$ differs from unity. QCD would predict $A < 1$ for finite $Q^2$, but $A \rightarrow 1$ for $Q^2 \rightarrow \infty$.

A.4 Test of charge symmetry

The equality of differential $\nu$ and $\bar{\nu}$ cross-sections at $y = 0$ (4.7.A.4) has been tested in several experiments and has been established at the $5 - 10\%$ level. E.g. the CITFTR group obtains this equality within $5\%$, and the CDHS group\(^\text{27}\) quotes

$$\frac{d\sigma^{\nu}}{dy}\bigg|_{y=0} = (1.05 \pm 0.07) \frac{d\sigma^{\bar{\nu}}}{dy}\bigg|_{y=0}$$

B.1 Fraction of quark momentum

The absolute differential cross-section at $y = 0$ can be used for a determination of the fractional momentum carried by the quark constituents of the nucleon (4.7.B.1). As an example, the CDHS group obtains

$$\int_0^1 (q + \bar{q}) \, dx = \int_0^1 F_2(x) \, dx = 0.45 \pm 0.03$$

in agreement with the GGM result at lower energies ($0.51 \pm 0.06$). This means that half of the nucleon momentum is carried by objects different from quarks, which can be identified with the 8 gluons in QCD mediating the strong color force.

B.2 Structure functions

Using normalized $\nu$ and $\bar{\nu}$ cross-sections, we can integrate over $y$ and extract the structure functions $F_2(x)$ from $\sigma^{\nu+\sigma^{\bar{\nu}}}$, $x F_3$ from $\sigma^{\nu-\sigma^{\bar{\nu}}}$ and $\bar{q}(x)$ from $3 \sigma^{\nu-\sigma^{\bar{\nu}}}$ (see 4.7.B.2).

From the CDHS data between 30 and 200 GeV one obtains then the x-distributions shown in fig.29, normalized arbitrarily to

$$\int_0^1 F_2(x) \, dx = 1.$$  

These shapes can be fitted with powers of $(1 - x)$, and the result is\(^\text{27}\)

$$\bar{q}(x) \propto (1 - x)^{6.7 \pm 0.5}$$

$$q(x) - \bar{q}(x) \propto \sqrt{x} \, (1 - x)^{3.5 \pm 0.5}$$

Similar shapes are obtained by the BEBC\(^{31}\) group, viz.

$$\bar{q}(x) \propto (1 - x)^{4.9^{+2.4}_{-1.7}}$$

$$q(x) \propto (1 - x)^3$$

for $x > 0.3$

![Figure 29](image-url)

It appears therefore that the $q\bar{q}$ sea is compressed very much at low $x$, while valence quarks have higher fractional momentum. This can also be seen by considering $\bar{\nu}$ and $\nu$ data at $y > 0.8$ (Fig.30). For the $\bar{\nu}$ data mainly the sea quarks contribute, and the fall-off is very steep, like $\propto (1 - x)^7$, while for the $\nu$ data valence quarks dominate, and the distribution is much wider ($\langle x \rangle = 0.22$ compared to $\langle x \rangle = 0.12$ for $\bar{\nu}$ data). The sea seen in these $\bar{\nu}$ reactions is

$$\bar{u}(x) + \bar{d}(x) + 2 \bar{s}(x),$$

and it therefore remarkable that the $\bar{s}(x)$ sea seen in single charm production (dimuon events

$$\bar{u} + \bar{s} + g + \bar{c} \rightarrow \mu^- \bar{\nu}$$

has approximately the same shape $(1 - x)^7$ in the CDHS data.
C. Scaling violations

If we now want to see whether the structure functions obtained in this way vary with $Q^2$, or, as a reflection of this, with $E_\nu$, we have to repeat the analysis in different $E_\nu$ bins.

Before doing this, we can look at an average quantity as a test of scaling, the ratio $\langle Q^2 \rangle / E = 2M \langle x y \rangle$, which is independent of energy if scaling holds. Fig. 31 shows data on this quantity from several experiments. Taking the low energy GGM points together with the data at high energy ($E > 30$ GeV), one obtains a definite decrease of $\langle Q^2 \rangle / E$ which is compatible with the scaling deviations expected from QCD. From the high energy data alone it is not possible to draw such a conclusion.

This fact was utilized by the BEBC group\textsuperscript{31}) in a very thoughtful way. They used BEBC reon data from the NBB exposure in the interval 20 - 200 GeV and GGM data from a low energy WBB run (2 - 10 GeV). The data sample contains 1100 $\nu$ and 250 $\bar{\nu}$ events from BEBC, and 3000 $\nu$ and 2000 $\bar{\nu}$ events from GGM. The GGM data reach down to a $Q^2$ of 0.1 GeV$^2$, while the highest $Q^2$ values of BEBC events are 100 GeV$^2$.

After integrating over $y$, the data are binned in $(x, Q^2)$ intervals, and the structure functions $F_2(x, Q^2)$ (from the sum of normalized $\nu$ and $\bar{\nu}$ data) and $xF_3(x, Q^2)$ (from the difference $\sigma^\nu - \sigma^{\bar{\nu}}$) are obtained. The results for $F_2(x, Q^2)$ are shown in fig.32 together with data from inelastic electron-deuteron scattering from SLAC multiplied with $\sqrt{Q^2}$, where $Q^2$ is the sum of the squares of the charges of $u$ and $d$ quarks. The agreement of these three sets of data is quite remarkable, and, taken together, they clearly indicate the presence of scaling violations: $F_2$ is increasing with $Q^2$ at low $x$ ($x < 0.1$), and decreasing with $Q^2$ at large $x$ ($x > 0.4$). This is qualitatively the behaviour expected from QCD.

Using the GGM data for large $x$ and the BEBC data for small $x$, the BEBC authors then integrate over $x$, and the resulting $\int F_2(x, Q^2)dx$ is shown in fig.33 as a function of $\ln Q^2$. Quasi-elastic events are included in this integral, such that the main part of the decrease of the integral is due to these elastic events. The value of $\int F_2 dx$ at the highest $Q^2 \approx 15$ GeV$^2$ is $\approx 0.45$, very near to the asymptotic value for 4 flavours in QCD, which is 0.43.

A similar analysis has been done for $xF_3$, and the result can be seen in fig. 34. Again the quasi-elastic contribution has to be subtracted. The integral $\int xF_3 dx$ falls with $Q^2$, and at the highest $Q^2 \approx 15$ GeV$^2$ it is still far from the anticipated asymptotic value zero.

The observed scale violations correspond to a shrinking $F_2(x)$ with increasing $Q^2$. Since the average $Q^2$ is connected to the neutrino energy $E_\nu$, this corresponds to a shrinking of $x$ distributions with increasing $E_\nu$. This can be seen from fig.35, where parametrisations of data on $F_2(x)$ for different energies are shown. The shrinking is small, but observable even in the high energy region between 30 and 200 GeV, as can be seen from fig.36 with preliminary CDF data.

The most instructive plot again is the structure function $F_2(x, Q^2)$ as a function of $Q^2$ for different $x$ bins (Fig.37).
Scale breaking is evident, and its size is consistent with the one observed in ep and μp scattering.

A quantitative comparison of these scaling violations with QCD can be performed by extracting the moments of the structure functions defined in 4.7.C. If one takes the logarithm of two different moments of the same structure function, e.g. $x F_3$, then for the asymptotic values of QCD,

$$\ln M_3(N', Q^2) = \text{const} + \frac{dN'}{dN} \ln M_3(N, Q^2)$$

such that in a double-logarithmic graph we expect a straight line with a predicted slope $dN'/dN$. Fig. 38 shows
results from the ABCLOS - Collaboration using BEBC and GGM data in the way described above. The integration of the moments of the structure function for fixed $Q^2$ uses BEBC and GGM data in different kinematical regions. The results are in complete agreement with a functional dependence as given above, and the
slopes obtained are:

<table>
<thead>
<tr>
<th>Experiment</th>
<th>QCD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d_6/d_4$</td>
<td>$1.29 \pm 0.06$</td>
</tr>
<tr>
<td>$d_5/d_3$</td>
<td>$1.50 \pm 0.08$</td>
</tr>
<tr>
<td>$d_7/d_3$</td>
<td>$1.84 \pm 0.020$</td>
</tr>
</tbody>
</table>

This probably constitutes the first quantitative test of QCD independent of the free parameter $\Lambda$.

A determination of this parameter has also been undertaken by the ABCLOS - group using the $Q^2$-dependence of moments of $xF_3$. If one takes a moment $M_3(N, Q^2)$ to the power $-1/d_N$, then

$$M_3(N, Q^2)^{-1/d_N} = \text{const (ln } Q^2 - \text{ln } \Lambda^2)$$

and such a plot for even and odd moments of $xF_3$ (Fig.39) indeed shows such behaviour for $Q^2 > 1$ GeV$^2$. It is not clear whether the extrapolation of such a straight line in order to obtain the intercept $Q^2 = \Lambda^2$ is permitted. If this procedure is correct, then $\Lambda$ comes out around 0.7 GeV.

We can summarize this chapter by stating

i) charged-current reactions are remarkably well described by the simple quark parton model

ii) in the energy domain up to 200 GeV the scattering is mainly due to left-handed quarks; valence quarks dominate, but the magnitude, shape and composition of the $q\bar{q}$ - sea are measured

iii) scaling violations exist, and quantitative comparisons with QCD can be made

5. Neutral current reactions

5.1 $\nu_e - e$ scattering

This is probably the cleanest example of a neutral current interaction, and the first one to be discovered, in spite of its enormously small cross-section.

If we assume the neutral lepton current to be composed of $V$ and $A$ currents, then this current is given by

$$j_a^{NC} = \bar{\nu}_e (1-\gamma_5) \gamma_a \nu_e + \bar{\mu} (1-\gamma_5) \gamma_a \nu_\mu$$

$$+ C_V (\bar{e} \gamma_a e + \bar{\mu} \gamma_a \mu)$$

$$+ C_A (\bar{\mu} \gamma_5 \gamma_a e + \bar{\mu} \gamma_5 \gamma_a \mu)$$

and the neutral current Lagrangian is

$$\mathcal{L}_{NC} = \frac{G}{2} (j_a^{NC} \cdot j_a^{NC})$$

The kinematics of the $\nu e$ process is analogous to the neutrino-quark scattering (sect.4.5).
In terms of the c.m. scattering angle $\theta^*$, we have

$$E_e = \frac{E_\nu}{2} \left(1 - \cos \theta^*\right)$$
$$\theta^* = \sqrt{\frac{2m_e E_\nu}{E_\nu} \sin \theta^*}$$
$$\gamma = \frac{E_e}{E_\nu} = \frac{(1-\cos \theta^*)}{2}$$

The characteristic feature of the reaction is the very small laboratory angle of the electron relative to the neutrino direction ($\theta^*_e < 5^\circ$ for $E_\nu = 10$ GeV and $\cos \theta^* > 0.85$). The cross-section for this scattering process between two point-like objects has to be proportional to the center-of-mass energy squared, $s = 2m_e E_\nu$.

As a function of the strength of $V$ and $A$ coupling of the electron current, the cross-section comes out\(^{22}\)

$$\frac{d\sigma^V}{dy} = \frac{G^2 m_e E_\nu^2}{8\pi} \left[ (C_V - C_A)^2 + (C_V + C_A)^2(1-\gamma)^2 \right]$$
$$\frac{d\sigma^A}{dy} = \frac{G^2 m_e E_\nu^2}{8\pi} \left[ (C_V - C_A)^2(1-\gamma)^2 + (C_V + C_A)^2 \right]$$

This is in line with our earlier observation that a $V - A$ current between two left-handed fermions results in a flat $y$ distribution. One can designate the left-handed and right-handed couplings by

$$g_L = C_V - C_A; \quad g_R = C_V + C_A$$

In the Weinberg-Salam model,\(^{81}\) the neutral current is a sum of the neutral component $j_\alpha^N$ of the normal $V - A$ current and the electromagnetic current $j_\alpha^{\text{em}}$

$$j_\alpha^N = j_\alpha^3 - 2 \sin^2 \theta_W j_\alpha^{\text{em}}$$

where the electromagnetic current is a pure vector

$$j_\alpha^{\text{em}} = e \gamma_\alpha + \bar{\nu} \gamma_\alpha \nu$$

This model leads to $C_V = 1/2 - 2 \sin^2 \theta_W$ and $C_A = -1/2$. The only free parameter here is the Weinberg mixing angle $\theta_W$, which has to be determined from experiment. The expected $\nu_e$ cross-sections as a function of this angle are shown in fig.40.

For $\sin^2 \theta_W = 0$, we have pure $V - A$ coupling, and $\sigma(\nu_e) = 3$ because of the different $y$ distributions, while for $\sin^2 \theta_W = 0.25$, $\sigma(\nu_e) = 0.7$.

![Figure 40](image)

Experiments on these processes have been done by the GGM\(^{33}\) and the Aachen-Padova\(^{34}\) groups at the CERN PS and by the GGM\(^{35}\) and BNL-Columbia\(^{36}\) groups at high energy. The neutrino energy spectra at the PS are shown in fig.41, and the AC-PD detector in fig.42. It consists of a spark chamber array of 1 cm Al plates, 2 x 2 m\(^2\) large, with a total mass of 30 tons and a fiducial mass of 19 tons. Electromagnetic showers are identified by their longitudinal development sampled every 1/9 radiation length, and the energy measurement is done by spark counting.

Results on the cross-sections are given in table VII.

<table>
<thead>
<tr>
<th>Table VII</th>
</tr>
</thead>
<tbody>
<tr>
<td>Evt.</td>
</tr>
<tr>
<td>$\nu$</td>
</tr>
<tr>
<td>$\nu$</td>
</tr>
<tr>
<td>$\bar{\nu}$</td>
</tr>
<tr>
<td>$\bar{\nu}$</td>
</tr>
<tr>
<td>$\bar{\nu}$</td>
</tr>
<tr>
<td>$\bar{\nu}$</td>
</tr>
</tbody>
</table>
The low-energy experiments are in fair agreement with the Weinberg-Salam model, they prefer a value $\sin^2 \theta_W = 0.38 \pm 0.10$, still in agreement with the value 0.25 found in inclusive neutral current reactions; they also agree with $V + A$ or pure $V$ or $A$ (Fig.43). However, a serious surprise comes from the GGM2 result, which is incompatible with the Weinberg-Salam model if $\sin^2 \theta_W = 0.25$,
or gives \( \sin^2 \theta_W > 0.74 \) with 90% confidence if this parameter is left free. There is no explanation at the moment for this discrepancy, which becomes evident in fig. 44, where the values of \( C_V^u \) and \( C_A^u \) allowed by different experiments (including \( \bar{\nu}_e e^- \) scattering data) are given by shaded areas, and the area allowed by the GM2 experiment does not overlap with the one given by all other experiments.

Very recently, another measurement of the \( \nu_u \bar{e}^- \) cross-section at high energies has given a result in agreement with the Weinberg-Salam model. It comes from a BNL-Columbia group \(^{36} \) working with a heavy (64 at %) neon-hydrogen mixture in the FNAL 15'-chamber. From 11 events, a cross-section of \( (1.8 \pm 0.8) \times 10^{-42} \text{cm}^2/\text{GeV} \) \( E_{\nu} \) and a Weinberg angle around \( \sin^2 \theta_W = 0.2 \) is obtained. The exposure has 4 times more sensitivity than the GM2 experiment, and the results are mutually inconsistent.

5.2 Inclusive inelastic neutral current scattering on isoscalar targets

5.2.1 Phenomenology

If the neutral hadronic current is assumed to be composed of V and A parts and diagonal in the quarks, we obtain in the G.M.10) schema

\[
J_{a \mu}^\nu = \gamma_\mu (C_V^u + C_A^u) u (x) \\
+ \bar{c} (x) y_\nu (C_V^u + C_A^u) c (x) \\
- \bar{d} (x) y_\nu (C_V^d + C_A^d) d (x) \\
- \bar{s} (x) y_\nu (C_V^s + C_A^s) s (x)
\]

Neglecting \( \Delta S = 1 \) transitions, we get the neutrino quark cross-sections from neutral currents (Table VIII) which are shightly more complicated than the ones from charged currents (Table IV) because of the presence of right-handed and left-handed couplings.

### Table VIII

<table>
<thead>
<tr>
<th>Reaction</th>
<th>( d\sigma/dy ) in units of ( G_2 M_\nu \times \cos^2 \theta_W / 2\pi )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \nu_\mu + u^- \rightarrow \nu_\mu + u )</td>
<td>( (C_V^{u} - C_A^{u})^2 + (C_V^{u} + C_A^{u})^2 (1 - y)^2 )</td>
</tr>
<tr>
<td>( \nu_\mu + d^- \rightarrow \nu_\mu + d )</td>
<td>( (C_V^{d} - C_A^{d})^2 + (C_V^{d} + C_A^{d})^2 (1 - y)^2 )</td>
</tr>
<tr>
<td>( \bar{\nu}<em>\mu + u^- \rightarrow \bar{\nu}</em>\mu + u )</td>
<td>( (C_V^{u} + C_A^{u})^2 + (C_V^{u} - C_A^{u})^2 (1 - y)^2 )</td>
</tr>
<tr>
<td>( \bar{\nu}<em>\mu + d^- \rightarrow \bar{\nu}</em>\mu + d )</td>
<td>( (C_V^{d} + C_A^{d})^2 + (C_V^{d} - C_A^{d})^2 (1 - y)^2 )</td>
</tr>
</tbody>
</table>

For the Weinberg-Salam-Model, the coupling constants are

\[
C_V^u = \frac{1}{2} - \frac{4}{3} \sin^2 \theta_W \quad C_A^u = -\frac{1}{2} \\
C_V^d = -\frac{1}{2} + \frac{2}{3} \sin^2 \theta_W \quad C_A^d = \frac{1}{2}
\]

The inclusive cross-sections can then be calculated, and in analogy to the CC case, one obtains \( y \) distributions consisting of a flat and a \((1 - y)^2\) component. If we assume \( \theta^c = 0 \) and neglect the contributions from the strange sea, then the quark distribution is \( q(x) = (u(x) + d(x)) \) and we obtain

\[
d^2d_\nu \frac{d\sigma}{dy} = \sigma_0 (y_L + y_R (1 - y)^2) \\
d^2d_\bar{\nu} \frac{d\sigma}{dy} = \sigma_0 (y_L (1 - y)^2 + y_R)
\]

where the "left-handed" component for \( \nu \) comes from scattering off quarks with a left-handed coupling or scattering off anti-quarks with a right-handed coupling:

\[
g_L = q(x) g^+ \bar{q}(x) g^+ \\
g_R = q(x) g^+ \bar{q}(x) g^-
\]

and

\[
4 g^- = (C_V^u - C_A^u)^2 + (C_V^d - C_A^d)^2 \\
4 g^+ = (C_V^u + C_A^u)^2 + (C_V^d + C_A^d)^2
\]

With the Weinberg-Salam values for the couplings, one obtains

\[
g^- = \frac{1}{2} - \sin^2 \theta_W + \frac{5}{9} \sin^4 \theta_W \\
g^+ = \frac{5}{9} \sin^4 \theta_W
\]

Integrating over \( y \) we have

\[
d_\nu \frac{d\sigma}{dy} = \sigma_0 (g^- + g^+/3) \\
= \sigma_0 \frac{1}{2} - \sin^2 \theta_W + \frac{20}{27} \sin^4 \theta_W
\]

and the ratio

\[
R_\nu = \frac{d_\nu \frac{d\sigma}{dy}}{d_\nu \frac{d\sigma}{dy}} = \frac{1}{2} - \sin^2 \theta_W + \frac{20}{27} \sin^4 \theta_W
\]

is 0.30 for \( \sin^2 \theta_W = 0.25 \).

Similarly for antineutrinos

\[
R_\bar{\nu} = \frac{d_{\bar{\nu}} \frac{d\sigma}{dy}}{d_{\bar{\nu}} \frac{d\sigma}{dy}} = \frac{1}{2} - \sin^2 \theta_W + \frac{20}{9} \sin^4 \theta_W
\]

is 0.39 for \( \sin^2 \theta_W = 0.25 \).
5.2.2 **Measurement of the NC/CC cross-section ratio on isoscalar targets**

After the discovery of inclusive hadronic neutral currents, the most important question was a precise measurement of the Weinberg angle $\theta_W$. Important for such a measurement is a clean neutral current event sample, which can be achieved by a detector with large muon acceptance for suppression of CC events, and a large event sample. A narrow band beam gives kinematical constraints which are very helpful in the analysis.

The main problem of the analysis is the separation of charged current (CC) and neutral current (NC) events. As an example, the CITF group used a criterion based on the total length of an event in their steel detector, from the vertex up to the last signal of the event, called "penetration". Such a penetration curve is given in fig.45, for neutrino and antineutrino events. Events penetrating more than 20 collision lengths of iron are CC events, but one can see an excess of events at short penetration length due to NC events. The experimental task now consists in extrapolating the CC background below the peak at short penetrations. This corresponds to an extrapolation in the $y$ distribution of CC events from large $y$ to $y = 1$. We know from the studies of CC interactions that these $y$ distributions are nearly flat for $\nu$ interactions and also approximately flat in the high $y$ domain for $\bar{\nu}$ interactions because of scattering from the $gq$ sea.

This extrapolation therefore poses serious systematic problems if it is based on an experimental CC event sample with penetrations of 3-5 m iron (20 to 35 interaction lengths).

The amount of CC background to be subtracted clearly depends on the muon acceptance of the apparatus, and the penetration curve of the CDHS experiment, fig. 46, shows a signal-to-background ratio of 10 : 1 rather than 2 : 1 in the CITF Neutrino events.

![Figure 45](image1)

**Figure 45**

![Figure 46](image2)

**Figure 46**

Another source of background in these narrow band beam experiments comes from wide-band beam background, i.e., from neutrinos originating from $\pi/K$-decays (or from new prompt sources of neutrinos).
upstream of the momentum-defining collimator in the beam. This background was determined in these experiments by taking data with the collimator closed and subtracting them on a statistical basis.

A summary of the most recent experimental results on the cross-section ratios
\[ R_\nu = \frac{\sigma_\nu}{\sigma'} \text{ (NC)}/\frac{\sigma''}{\sigma'(CC)} \]

\[ R^\nu_\nu = \frac{\sigma^\nu_\nu}{\sigma'} \text{ (NC)}/\frac{\sigma''}{\sigma'(CC)} \]

and the values of \( \sin^2 \theta_W \) derived from them is given in table IX. Fig. 47 shows \( R_\nu \) and \( R_\nu^\nu \) compared to the prediction of the Weinberg-Salam-model. It can be seen that all these recent experiments agree reasonably well with each other and with the Weinberg-Salam model. The values of \( \sin^2 \theta_W \) are displayed separately in fig.48. The most precise result is that of the CDHS group

\[ \sin^2 \theta_W = 0.24 \pm 0.02 \]

which is pointing to the magic value 0.25 where \( \sigma(\nu e) = \sigma(\bar{\nu} e) \), and the neutral electron current is pure axial vector.

Table IX. shows in addition that \( R_\nu \) and \( R_\nu^\nu \) seem to be largely energy-independent between GGM and the high-energy experiments.

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Beam</th>
<th>Energy (GeV)</th>
<th>( R_\nu ) and ( R_\nu^\nu )</th>
<th>( E_H ) cut-off</th>
<th>( \sin^2 \theta_W )</th>
</tr>
</thead>
<tbody>
<tr>
<td>GGM (^{40})</td>
<td>WBB</td>
<td>2</td>
<td>( R_\nu = 0.26 \pm 0.04 ) ( R_\nu^\nu = 0.39 \pm 0.06 )</td>
<td>( E_H &gt; 0 )</td>
<td>0.32 \pm 0.05</td>
</tr>
<tr>
<td>HPWF (^{41})</td>
<td>WBB</td>
<td>85 ( \Rightarrow 40 )</td>
<td>( R_\nu = 0.29 \pm 0.04 ) ( R_\nu^\nu = 0.39 \pm 0.10 )</td>
<td>( E_H &gt; 4 \text{ GeV} )</td>
<td>0.23 \pm 0.06</td>
</tr>
<tr>
<td>CIFT (^{42})</td>
<td>NBB</td>
<td>90 ( \Rightarrow 70 )</td>
<td>( R_\nu = 0.28 \pm 0.03 ) ( R_\nu^\nu = 0.35 \pm 0.11 )</td>
<td>( E_H &gt; 12 \text{ GeV} )</td>
<td>0.33 \pm 0.07</td>
</tr>
<tr>
<td>CDHS (^{39})</td>
<td>NBB</td>
<td>100 ( \Rightarrow 90 )</td>
<td>( R_\nu = 0.293 \pm 0.010 ) ( R_\nu^\nu = 0.35 \pm 0.03 )</td>
<td>( E_H &gt; 12 \text{ GeV} )</td>
<td>0.24 \pm 0.02</td>
</tr>
<tr>
<td>BEBC (^{43}) (Preliminary)</td>
<td>NBB</td>
<td>100 ( \Rightarrow 90 )</td>
<td>( R = 0.32 \pm 0.04 ) ( R^\nu = 0.38 \pm 0.07 )</td>
<td>( E_H &gt; 15 \text{ GeV} )</td>
<td>0.21 \pm 0.04 (stat. error only)</td>
</tr>
</tbody>
</table>
5.2.3 Measurement of the hadron energy spectrum in NC reactions

The variable $y = E_h/E_\nu$, which is readily calculable in CC reactions, is not known precisely in NC reactions because of the marginal measurement accuracy on $E_\nu$ even in narrow-band beams. It is therefore preferable to use the experimental quantity $E_h$ directly in order to obtain information on the $y$ distributions.

A study of these $E_h$-distributions has been done by the CDHS group. $E_h$ distributions of NC and CC events are shown in fig. 49. The analysis is done in terms of the ratio NC/CC. This reduces systematic errors from the hadron energy calibration and from resolution effects. The data are divided in bins of radial distance of the vertex from the center line of the beam, which corresponds to a selection of neutrino energy bins. Fig. 50 shows the results from the CDHS experiment. The ratios do not deviate very much from unity, which leads to the conclusion that the $y$-distributions of NC and CC are rather similar.

If the $\nu$ and $\bar{\nu}$ data are treated separately, and if the $y$-distributions are parametrized by one parameter ($a_{NC}$ for $\nu$ and $\tilde{a}_{NC}$ for $\bar{\nu}$) in the form

$$\frac{NC}{CC} (y) = R_0 \frac{1 + a_{NC} (1 - y)^2}{1 + a_{CC} (1 - y)^2} \quad \text{for} \quad \nu$$

$$= \frac{R_0 (1 - y)^2 + \tilde{a}_{NC}}{(1 - y)^2 + a_{CC}} \quad \text{for} \quad \bar{\nu}$$

then, for an assumed value of $a_{CC} = 0.1$, the parameters are

$$a_{NC} - a_{CC} = 0.09 \pm 0.18$$

$$\tilde{a}_{NC} - a_{CC} = 0.10 \pm 0.07$$

and the deviation of the NC distribution from the CC distribution is not significant.
5.2.4 Space-time structure of hadronic neutral currents

More information can be obtained of the $\nu$ and $\bar{\nu}$ data are analyzed together in the framework of the differential cross-sections in terms of left-handed and right-handed couplings (sect. 5.2.1). Taking into account small corrections due to the strange sea and to the nonisoscalarity of the Fe target, the CDHS group\(^{44}\) obtains

$$g_L = 0.300 \pm 0.012$$
$$g_R = 0.050 \pm 0.005$$

if a sea $(\bar{Q} + \bar{\Sigma})/(Q + Q) = 0.1$ is assumed. Taking the value from sect. 4.8 would modify this result. The result is graphically illustrated in fig. 51.

![Diagram](https://via.placeholder.com/150)

**Figure 51**

A similar result has been obtained by the CITF group\(^{42}\):

$$g_L = 0.199 \pm 0.023$$
$$g_R = 0.110 \pm 0.037$$

assuming a larger amount of sea,

$$(\bar{Q} + \bar{\Sigma})/(Q + Q) = 0.17.$$ 

From experiment one can now draw the conclusion that a $V + A$ structure or a pure $V$ or $A$ structure are excluded for the neutral current interaction. In particular, this means that the NC must be parity violating (since $V$ and $A$ are present). The actual structure is composed of a dominating $V - A$ component with a smaller ($V + A$) admixture.

5.2.5 Outlook

The neutral current phenomena discovered in neutrino interactions should show up also in other elementary interactions, e.g. the electron-proton or electron-neutron interactions. They should therefore be present in atomic physics, where their earmark compared to the overwhelming electromagnetic interaction should be their parity violating character. Experiments in atomic physics have been done in heavy atoms, e.g. Bi, trying to observe the interference of parity violating and parity conserving atomic transition amplitudes via an optical rotation of polarized laser light in Bi vapor. Two experiments at Oxford\(^{45}\) and Seattle\(^{46}\) did not find the effect at the level predicted by the Weinberg-Salam model. If

$$R = \text{Im} (E_{\nu}^{PV}/M_{\nu}) \times 10^3$$

is the ratio of the parity violating and the parity-conserving amplitudes, then the upper limits given by these experiments are

$$R_{\text{ex}}/R_{\text{th}} < 0.25$$ for Oxford
$$R_{\text{ex}}/R_{\text{th}} < 0.16$$ for Seattle.

Quite recently, there has been a report from an experiment at Novosibirsk\(^{47}\), which observes definitely parity violation in atomic physics and quotes $R_{\text{ex}}/R_{\text{th}} = 1.3 \pm 0.4$ in agreement with Weinberg-Salam. The disagreement between these experiments has not yet been resolved, but even more recently has been a result from a SLAC experiment on $e - p$ and $e - d$ scattering with longitudinally polarized electrons demonstrating a parity violating interaction of the magnitude predicted by the Weinberg-Salam model.

If the remaining experimental discrepancies in atomic physics and in $\nu e$ scattering can be cleared up, it seems therefore that neutral currents will emerge as predicted by Weinberg and Salam, with an angle around

$$\sin^2 \theta_W = 0.25.$$ 

6. New particle production

New particles can be recognized in neutrino experiments by their decay into leptons. One example was the observation of opposite-sign dimuon events by the HPSF group\(^{48}\) in 1974 before the discovery of the $J/\psi$ - particle.
6.1 Eilepton production

6.1.1 Opposite sign dimuon events

After the first observation of this type of event, further experiments of the HPWF group\textsuperscript{49} and the CTTP\textsuperscript{50} group gave support to the explanation of this process by production and semileptonic decay of a charmed particle.

The largest sample of kinematically reconstructed events induced by neutrinos and antineutrinos has been obtained by the CDHS group\textsuperscript{51}. These data were taken in the narrow-band-beam in a neutrino exposure corresponding to $3 \times 10^{17}$ protons and an antineutrino exposure of $6 \times 10^{17}$ protons, yielding 53 000 charged current events for $\nu$ and 15 000 for $\bar{\nu}$. Fig. 52 shows the spatial reconstruction of a dimuon event. From the reconstructed vector momenta of the two mesons, the neutrino direction and the measured hadron energy $E_h$ we obtain the following kinematical variables:

- $E_{\mu_1}$: the energy of the leading muon, i.e. the one carrying the same lepton number as the incident (anti)neutrino $E_{\mu_2}$: the energy of the non-leading muon

$$x = \frac{(P_\nu - P_{\mu_1})^2}{[2M_p (E_h + E_{\mu_2})]}$$

$\hat{\nu} = \nu - \nu_1$ the direction of the hadron shower

$\phi$ the azimuthal angle between the projection of the momenta of $\nu_1$ and $\mu_2$ onto the plane perpendicular to the neutrino direction

$P_{TSH}$ the transverse momentum of $\mu_2$ relative to the shower axis $\hat{\nu}$

Fig. 53 is a scatter plot of $E_{\mu_1}$ vs. $E_{\mu_2}$ for neutrino and antineutrino events, showing a marked asymmetry between the two muons; the averages (for $\nu$) $\langle E_{\mu_2} \rangle = (13.7 \pm 1)$ GeV and $\langle E_{\mu_1} \rangle = (45 \pm 2)$ GeV differ considerably, in line with a reaction mechanism where the leading $\nu_1$ is produced at the lepton vertex, and the other $\nu_2$ at the hadron vertex. The data are contaminated with a background where $\mu_2$ comes from the decay of $\tau$ and $K$ mesons in the hadron shower of a charged-current event. This is calculated to be $(13 \pm 4)$ of the events.

In order to investigate the nature of the the dimuon signal, we then plot the $x$ distributions for neutrino- and antineutrino induced events (Fig. 54).

![Figure 52](image1)

![Figure 53](image2)

The distribution for antineutrinos is shifted towards low $x$, $<x>_\nu = 0.15 \pm 0.02$ compared to the neutrino distribution $<x>_\bar{\nu} = 0.24 \pm 0.01$, indicating that for $\bar{\nu}$,
the dimuons are produced predominantly off the sea quarks. The $x$ distribution for $\bar{\nu}$ is compatible with a form $(1-x)^7$ which agrees with the $x$ distribution of the sea as obtained from charged current events.

Further clarification of the origin of the second muon comes from the distributions shown in fig.55, where a) is a scatter plot of $E_{\nu_2}$ vs. the azimuthal angle $\phi$ between the two muons, and b) its projection onto the $\phi$ axis. If the origin of the two muons would be the production and decay of a new heavy lepton $L^0$ via $\nu + N \rightarrow L^0 + X$ and $\nu^0 + \mu^+ \mu^- \nu$, then one would expect in general a rather flat distribution in $\phi$, independent of $E_{\nu_2}$. If on the other hand, the second muon is the decay product of a particle produced at the hadron vertex, then one expects an anticorrelation in $\phi$ between leading $\nu_1$ and the $\nu_2$ following the hadron shower direction. The most probable $\phi$-angle is then $180^\circ$, i.e. back-to-back emission of the two muons in the transverse direction. The correlation becomes stronger when $E_{\nu_2}$ increases because then $\nu_2$ and its parent carry a larger fraction of the total hadron momentum. The data of fig.55 favour clearly models with hadronic origin of the second muon, and the curves shown are calculated according to such a model of charm production with a fragmentation function $D(z) = (1-z)^2$.

The same conclusion can be drawn from fig.56 which shows that the transverse momentum of the second muon relative to the shower axis is small even if $E_{\nu_2}$ becomes large. This is not so if one plots the transverse momentum of $\nu_2$ relative to the $\nu_1$ direction.

The rates of dimuon production relative to charged-current reactions are shown in fig.57 together with the experimental detection efficiency calculated according to the charm production model mentioned above. The uncertainty in this model lies in the choice of the quark fragmentation function $D(z)$. Since $D(z)$ is related to the spectrum of $E_{\nu_2}$, we can limit the ranges of possible functions $D(z)$. For those fragmentation functions compatible with the $E_{\nu_2}$ spectrum we find that the corrected cross-section ratios $R_{\nu} = \sigma(2\mu)/\sigma(1\mu)$ are flat within our neutrino energy range for $\nu$ and $\bar{\nu}$.

All of these observations are in agreement with the GIM model, where neutrinos can produce charm via $\nu + d \rightarrow \bar{u} + c$ and $\nu + s \rightarrow \bar{u} + c$ with relative rates of
\[ \sin^2 \theta_c \text{ and } \xi_S \cos^2 \theta_c \] (where \( \xi_S = \frac{\int x s(x)dx}{\int x d(x) dx} \) is the amount of strange sea in the nucleon), while antineutrinos can only produce anticharm from the sea \( \bar{\nu} + \bar{s} \rightarrow \nu^c + \bar{c} \) with a rate of \( \xi_S \cos^2 \theta_c \). From the measured dimuon rates \( R_{\bar{\nu}} \) and \( R_{\nu} \) we therefore deduce the amount of sea, \( \xi_S = 0.035 \pm 0.01 \).

The dimuon production from antineutrinos would increase drastically if a right-handed b-quark would exist, which was suggested \(^{55}\) as an explanation of anomalies in the antineutrino scattering \(^{28}\). In fig. 58 the predicted increase in the double ratio \( D = \frac{R_{\bar{\nu}}}{R_{\nu}} \) for b-quark masses of 5 GeV and 7 GeV are compared to the CDHS data. These results do not support the existence of an (ub)\( _R \) current, or if this b quark would exist with \( m_b = 5 \) GeV, then the branching ratio for muonic b-decay must be less than 8 \% of the one for semileptonic charm decay.

Similar results have been reported recently by the CITFR group \(^{50}\).
6.1.2 $\mu e$ events

Dilepton events, if due to charm, should also show up in the variety, since semileptonic decays of $D$'s into $\mu$ or $e$ should be of same strength. This type of event can at present only be observed in bubble chambers, and has been observed in GGM$^{56}$, the FNAL 15' chamber$^{57-60}$ and BEBC$^{51})$. The properties of these events are consistent with the ones of the (larger) sample of $\mu^+\mu^-$ events in counter experiments, at least concerning the two leptons involved. However, the bubble chambers are able to observe details of the hadronic shower produced in the interaction, and are therefore in a position to check an additional prediction of the G.I.M. picture: because of the dominance of currents with $\Delta S = \Delta C$, we expect in the case of charm production from a $d$-quark ($\Delta C = 1$) predominantly the production of a $s$-quark ($\Delta S = 1$) which has to show up amongst the final state hadrons as a $K^0, K^+$ or hyperon. The calculated number of strange particles per $\mu^-e^+$ event (induced by neutrinos) is 1.5 and the average number of $K^0$'s per $\mu^-e^+$ event is $\langle n_{K^0} \rangle = 0.8$.$^62$.

Experiments are not quite in agreement about a value of $n_{K^0}$. Table X. summarizes the results.

Table X.

<table>
<thead>
<tr>
<th>Chamber</th>
<th>Filling</th>
<th>Rate $\mu^-e^+/\mu^-$</th>
<th>$\langle n_{K^0} \rangle$</th>
</tr>
</thead>
<tbody>
<tr>
<td>FNAL 15' Ref.57,58</td>
<td>Light neon/ $H_2$mix</td>
<td>$(0.77 \pm 0.3)%$</td>
<td>1.84$^{+0.63}_{-0.55}$</td>
</tr>
<tr>
<td>FNAL 15' Ref.59</td>
<td>Heavy neon/ $H_2$mix</td>
<td>$(0.7 \pm 0.15)%$</td>
<td>0.5$\pm0.2$</td>
</tr>
<tr>
<td>BEBC$^{51)}$</td>
<td>Heavy neon/ $H_2$mix</td>
<td>$(0.7 \pm 0.3)%$</td>
<td>1.7$\pm0.7$</td>
</tr>
</tbody>
</table>

The enhancement of $K^0$'s in these $\mu^-e^+$ samples is clear and confirms, at least qualitatively, the GIM current. The quantitative discrepancies, however, are not yet understood.

6.1.3 Charmed D meson production

In one of the bubble chamber experiments,$^{63)}$ it has been possible to see a direct D meson signal in the invariant mass ($K_{e^+\pi^-}$). A small peak at $m = 1.85$ GeV is seen, indicating the production of the D meson in neutrino interactions (Fig.59).

Figure 59

There are 64 events in the D region above a background of 180 events, giving the peak a statistical significance of more than 4 standard deviations. The production of the $D^0$ followed by its decay into $K_{e^+\pi^-}$ is found to be $(0.7 \pm 0.2)$% of all charged current neutrino interactions.

6.2 Trimuon events

6.2.1 Production mechanisms

Trimuons were first seen in the CITF$^{64)}$ and HPWF$^{65)}$ experiments. The CDHS collaboration has obtained 3 events in the narrow-band beam$^{66}$ and 76 events in the wide-band beam$^{67)}$.

Apart from the trivial case of dimuons with an additional muon from a pion or Kaon decaying in the shower, trimuons may originate in several kinds of processes$^{68,69)}$

a) a charged-current interaction with electromagnetic production of a muon pair, either from the outgoing muon or from the incoming or outgoing quark
b) a charged-current interaction with hadronic production of a muon pair, e.g. by the decay of a vector meson

c) a charged-current interaction with charm-anticharm production at the hadron vertex and subsequent decay of both c and c̄ to muons

![Diagram of particle interactions](image)

More complete references to theoretical work can be found in a review of Barger.

6.2.2 CDHS data

We then turn to experimental data and we consider the largest event sample from the CDHS experiment. One example of a trimuon event $\nu + Fe \rightarrow \mu^- \mu^- \mu^- X$ in the CDHS detector exposed to the wide-band beam is shown in fig.6C in three 120° views. From the measured vector momenta of all three muons, the hadron energy $E_h$ and the neutrino direction we can reconstruct the kinematics of the event. In particular the total visible energy is $E = E_\mu^1 + E_\mu^2 + E_\mu^3 + E_h$. A plot of trimuon event numbers vs. $E$ is shown in fig.61 for two different exposures together with the corresponding dimuon- and charged-current rates. The measured ratio $R(3\mu/1\mu)$ ranges between $(1.2 \pm 0.3) \times 10^{-5}$ at 50 GeV and $(5 \pm 2) \times 10^{-5}$ at 120 GeV neutrino energy. The energy dependence of the ratio $R$ corrected for background from $\pi/K$ decay is shown in fig.62. The background in the
trimuon sample due to dimuons with a π or 
K meson of the hadronic cascade decaying 
into (μν) was obtained by measuring the 
π/K decay rate of hadronic showers pro-
duced by pion beams in the same detector. 
One expects 6μ⁻μ⁺μ⁻ events and 6.6 
μ⁻μ⁺μ⁺ from this background in the total 
sample. The 5 observed μ⁻μ⁺μ⁻ events are 
compatible with this number, while for the 
μ⁻μ⁺μ⁺ events this amounts to a background 
of 8%. The calculated rates for electro-
magnetic processes are around (1 to 2)×10⁻⁵ 
including experimental cuts, and the char-
anticharm production is estimated to be 
less than 10⁻⁶. A simple model on hadronic 
μ pair production was made using experi-
mental data on dimuon production in π N 
reactions and assuming a resemblance be-
tween the interaction of the virtual W boson 
and this π - N interaction. The resulting 
trimuon rate is 2×10⁻⁵.

We see that from the rate alone a reliable 
choice between different production mecha-
nisms is difficult to make. We therefore 
turn to kinematical properties of the trio-
uon events. Since there are two μ⁻ and one 
μ⁺ in these events, we have to choose the lead-
ning μ⁻ amongst the two. In analogy to the 
dimuon events, we define here as leading 
uon (μ̃) the one for which the sum of the 
absolute values of the transverse momenta of 
the other negative muon (μ⁻) and the posi-
tive muon (μ⁺) relative to the direction of the virtual 
W-boson, ̃μ = μ⁻ - μ₁ is minimal. Fig. 63 
then illustrates a difference between the 
leading and non-leading μ⁻. Here the 
invariant dimuon masses M⁻ and M⁺ are 
plotted against the trimuon invariant 
mass Mₐ⁻. For M⁻ there are two possible 
combinations; the one indicated by a triangle 
is the one between μ⁺ and μ₁, the one given 
by a dot is the combination of μ⁺ with the 
non-leading μ₂. It appears from the data that 
there is little correlation between the lea-
ding μ₁ and both μ₂⁻ and μ₂⁺, while the in-
variant mass of μ⁺ with μ₂⁻ is bounded below 
1.5 GeV, indicating a possible common ori-
gin of those two muons.

A similar conclusion can be drawn from the 
momentum asymmetries aₘ(μ₀⁻μ₁⁻) and aₘ(μ₀⁺μ₁⁺) 
of pairs (1, k) of muons. While the asym-
metries between the one from dimuons, the combi-
nation (μ₂⁻μ⁺) is symmetrical within the 
experimental error. The invariant masses 
corresponding to these combinations are 
displayed in fig.64. They again show the 
striking difference between the (μ₂⁻μ⁺) 
pairing and the other combinations. 
The low values of m (μ₂⁻μ⁺) are of course 
reproduced well by the electromagnetic 
μ pair production models and by the 
experimental data on hadronic μ pair 
production which are dominated by 
71,72) production. The leptonic cascade models 
can only reproduce the data if the mass of 
the neutral heavy lepton L₀ is around 1.5 GeV,
which means a severe restriction on these models. Also the heavy quark cascade models with a b-quark mass \( m_b = 4.5 \) GeV predict a higher average invariant mass of this pair. We conclude from this that less than 10% of the events are due to a heavy quark cascade, with 90% confidence. A more decisive test can, as in the case of dimuons, be done on the basis of the projections of the muon momenta on the plane perpendicular to the neutrino direction.

If we then compute the angle \( \Delta \varphi_{1,23} \) in this plane between the leading \( \nu_1^- \) and the vector sum of \( \nu_2^- \) and \( \nu_3^- \), we expect quite different distributions for the models in question: the electromagnetic models yield a distribution in \( \Delta \varphi \) which has enhancements at \( 0^\circ \) and \( 180^\circ \) and a minimum around \( 90^\circ \) due to the interference of amplitudes from the radiation of the leading \( \nu_1 \) and the incoming quarks. The relative strengths of forward and backward radiation differ slightly between the models. Lepton cascade models yield a rather flat distribution in \( \Delta \varphi \), while the hadronic production will occur predominantly around \( \Delta \varphi = 180^\circ \). Fig. 65 shows distributions according to several models and the data. Here we require that all muons have a transverse momentum of at least 200 meV/c relative to the neutrino direction in order to improve on the resolution in \( \Delta \varphi \).
60 events survive this cut. The curves are normalized to the same event number. It appears that the large peak of events around 180° is well reproduced by the hadronic \( \mu \) pair production model, while the component of events below 60° could be due to electromagnetic production. If we fit this distribution by a sum of hadronic and electromagnetic production mechanisms, we obtain a rate \( R (3\mu/1\mu) = (0.8 \pm 0.5) \times 10^{-5} \) for trimuons of electromagnetic origin and \( R (3\mu/1\mu) = (2.2 \pm 0.5) \times 10^{-5} \) for trimuons of hadronic origin.

Additional information is obtained from the distribution of events in \( p_{T}^{23} \), the transverse momentum of the pair \((\mu_{2}, \mu_{3})\) relative to the shower axis (Fig. 66). This transverse momentum is, for all but two events, below 2.2 GeV/c, which is difficult to reconcile with a lepton cascade origin of the events, as indicated by the full line (for masses \( M^{-} = 9 \text{ GeV} \) and \( L_{0} = 1.5 \text{ GeV} \)). This leads to an upper limit on heavy lepton production: with 90% CL less than 17% of the trimuons are due to such a heavy lepton origin.

Summing up the CDHS results on trimuons \((\mu^{-}\mu^{-}\mu^{+})\), we find

i) The experimental \( R (3\mu/1\mu) \) rises from \( (1.2 \pm 0.3) \times 10^{-5} \) at 50 GeV to \( (9.0 \pm 2.0) \times 10^{-5} \) at 120 GeV neutrino energy, where the variation is mostly due to the experimental cut of 4.5 GeV on muon energies.

ii) The 76 events can be explained by a sum of two mechanisms: internal pair bremsstrahlung with \( R (3\mu/1\mu) = (0.8 \pm 0.5) \times 10^{-5} \) and hadronic pair production with \( R (3\mu/1\mu) = (2.2 \pm 0.5) \times 10^{-5} \).

iii) Upper limits for other mechanisms are:
- less than 17% of the events (90% CL) are due to a heavy lepton cascade \( (M^{-}/L_{0}) \) - less than 10% of the events (90% CL) are due to a heavy quark cascade with \( m_{b} = 4.5 \text{ GeV} \).

6.2.3 HPWF Data

These data have been taken in different wide-band beams produced by 400 GeV protons. The beam characteristics and event numbers are collected in Table XI.

There are 13 events from FHPWR in total, and the conclusion of this group is that a fair fraction of their events cannot be due to the conventional mechanisms discussed above. This conclusion was based partially on the rate of 3\( \mu \) events of \( 5 \times 10^{-4} \) per neutrino interaction, which is substantially larger than any of the rates expected for those conventional mechanisms. The most recent rates reported from this experiment are lower:

\[ R (3\mu/1\mu) = (2.6 \pm 1.5) \times 10^{-4}, \ E_{\nu} > 100 \text{ GeV} \]

\[ = (0.9 \pm 0.5) \times 10^{-4}, \ E_{\nu} > 30 \text{ GeV} \]

They are compatible with the CDHS rates and do not, therefore, require unconventional explanations. In addition, Fig. 63 contains the effective \( 2\mu \) and \( 3\mu \) masses of a subsample of 8 of the 13 events (with \( E_{\mu} > 4.5 \text{ GeV} \) in analogy to the CDHS cuts), marked by a star. These events fit into the distribution of CDHS events.
Table XI.

<table>
<thead>
<tr>
<th>Beam</th>
<th>Sign selection of mesons</th>
<th>Proton energy</th>
<th>Protons on target</th>
<th>No. of $3\mu$ events</th>
<th>Experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quadrupole tripled beam</td>
<td>$\nu + \frac{2}{3} \bar{\nu}$</td>
<td>$5.8 \times 10^{17}$</td>
<td>6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bare target $\nu$ beam</td>
<td>$\nu (+10% \bar{\nu})$</td>
<td>400 GeV</td>
<td>$3.3 \times 10^{17}$</td>
<td>6</td>
<td>HPWFR</td>
</tr>
<tr>
<td>Bare target $\bar{\nu}$ beam</td>
<td>$\bar{\nu} (+3% \nu)$</td>
<td></td>
<td>$18.8 \times 10^{17}$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Horn focused WBB + plug</td>
<td>$\nu (+3% \bar{\nu})$</td>
<td>350 GeV</td>
<td>$17.0 \times 10^{17}$</td>
<td>63</td>
<td>CDHS</td>
</tr>
<tr>
<td>Horn focused WBB + plug</td>
<td>$\nu (+3% \bar{\nu})$</td>
<td>400 GeV</td>
<td>$2.3 \times 10^{17}$</td>
<td>13</td>
<td></td>
</tr>
</tbody>
</table>

Two super-events (Fig.67), however, have attracted considerable interest because of their peculiar properties (Nr. 119-017991 and Nr. 281-147196). Their transverse momentum components are shown in Fig.68, where the numbers in parentheses are the muon charge and total momentum. These events have the unusual property that nearly all of the visible energy is in the leptons, very little in the hadrons. Since this is so, the transverse momenta of the three muons should balance approximately if one $\mu^+ \mu^-$ pair comes from a hadronic or electromagnetic origin.

![Diagram for Event 119-017991](image1)

**Event 119-017991**

![Diagram for Event 281-147196](image2)

**Event 281-147196**

Figure 68

Figure 67
On the contrary, the events do not show such a balance, and additional particles (neutrinos?) are required to cure this.

No satisfactory explanation of these events has been found yet, and since their visible energy is above 200 GeV, where the neutrino flux from the 350 GeV CERN beam becomes negligible, the CDHS experiment could have seen such events only in their 400 GeV run provided they are produced only above an energy threshold at 200 GeV. Comparison of the fluxes then leads to an expectation of one event of this type in the CDHS 400 GeV run, compatible with the observation of none.

6.3 Tetralepton events

In the wide-band neutrino beam exposure the CDHS group observed a first example of an event with four muons in the final state. The computer reconstruction of the event is shown in fig.69. Since the number of charged-current events in the same exposure is estimated on the basis of a sample to be $3 \times 10^6$ events and the dimuon number $\sim 7 \times 10^3$, this tetra-muon event corresponds to a rate of $1.4 \times 10^{-4}$ relative to dimuons. The measured parameters of the event are contained in table XII.

Table XII

<table>
<thead>
<tr>
<th>Measured quantities of the CDHS tetra-muon</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Total observed energy</td>
</tr>
<tr>
<td>91.4 ± 7.3 GeV</td>
</tr>
<tr>
<td>2. Hadron shower energy</td>
</tr>
<tr>
<td>58 ± 7 GeV</td>
</tr>
<tr>
<td>3. Momenta of the muons (GeV/c)</td>
</tr>
<tr>
<td>$P_x$</td>
</tr>
<tr>
<td>Muon 1 (+)</td>
</tr>
<tr>
<td>Muon 2 (-)</td>
</tr>
<tr>
<td>Muon 3 (+)</td>
</tr>
<tr>
<td>Muon 4 (-)</td>
</tr>
</tbody>
</table>

The largest background contribution comes from trimuons with a $\pi \rightarrow \mu$ or $K \rightarrow \mu$ decay in the hadron shower. For an average $E_\mu = 28$ GeV we expect $(1.2 \pm 0.4) \times 10^{-2}$ background tetra-muon events from this source.

If we now consider possible origins of the event, three conventional processes can be invoked (Fig.70): 1) charm production...
together with electromagnetic $\nu$ pair production from the leading $\nu$; 2) charm production together with $\nu$ pair production in the hadron shower; 3) charm-anticharm production by a charmed quark produced at the weak vertex. For the total contribution of processes 1) - 3) we estimate a rate of 0.2 events. Additional processes involving new particles could include a quark cascade with new flavours or a heavy lepton cascade with charm production.

Also the HPWPR group reported about a tetramuon event\(^{77}\). The momenta of the four muons are $-25$, $+10$, $+5$ and $-5$ GeV/c.

Even more surprisingly, there has been the observation of a $\mu^+e^-e^-e^-$ event in an anti-neutrino exposure of the $15'$ FNAL chamber\(^{81}\). This event (Fig.71) seems to be compatible with charm production and decay into $e^-$ plus a $\rho$ meson from the hadron vertex decaying into $e^+e^-$, though the probability of obtaining such a rare process in this exposure is very small.

![Figure 71](image)

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QUARKS IN HIGH ENERGY INTERACTIONS*

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1. Scope of the Lectures

The great interest of the quark parton model is that it seems to provide a successful way of relating together a variety of apparently very different reactions. In these lectures I review the principal applications of the model, which means that I discuss the following reactions:

- Deep inelastic scattering of electrons, muons and neutrinos
- Production of lepton pairs, J/\( \psi \) and W in hadronic collisions
- Electron-positron annihilation
- Large transverse momentum hadronic processes.

A striking feature of most reactions is supposed to be the presence of jets of hadrons in the final state. This jet physics is now an important area of experimental and theoretical activity, and a large part of these lectures is devoted to it. Another important area of study nowadays is the modifications to the simple parton model expected from the effects of quantum chromodynamics (QCD). I shall discuss the simple parton model first, since to a very good approximation it seems to be sufficient to describe much of the existing data, and then consider some of the expected modifications towards the end of the lectures.

Various of the topics that I discuss are treated in much more depth in other lectures at the 1978 CERN school. I mention particularly the lectures on QCD by De Rujula and the neutrino lectures of Kleinknecht.

Otherwise, more details on deep inelastic scattering may be found in my review article\(^1\) written with Osborn, and for more about large transverse momentum there is a recent article\(^2\) written with Jacob.

2. Deep Inelastic e and \( \mu \) Scattering

I shall assume some familiarity with deep inelastic scattering, and I therefore give only a reminder about this subject.

Consider, then, the process

\[
e\bar{p} \rightarrow eX
\]

which occurs predominantly through the exchange of a single virtual photon: see figure 2.1. We are

![Figure 2.1](image)

Figure 2.1. Electron or muon scattering through the exchange of a virtual photon, which is absorbed by the target hadron and causes it to break up.

interested in those events where the momentum transfer \( q^2 \) carried by the photon is large. Then the wavelength of the photon is small and so the interaction probes the short-distance structure of the nucleon target. Although it is important to explore spin effects\(^3\), I shall confine the discussion to the case where both initial particles are unpolarised.

From the usual Feynman rules, the matrix element corresponding to figure 2.1 is

\[
M = \left[ \bar{u}(A') \gamma^\mu u(A) \right] \frac{\alpha^\mu}{q^2} A^\mu (q \bar{p} \rightarrow X),
\]

where \( A \) is the amplitude corresponding to the bubble in the figure and describes the interaction of the photon with the target. We are interested in the squared modulus of the matrix element, summed over the possible systems \( X \) of final-state hadrons. This has the form

\[
\sum_X |M|^2 = L_{\mu\nu} \frac{1}{q^2} W^{\mu\nu}.
\]

Here, the first factor is the leptonic part, calculated from the terms in square brackets in (2.1), and the second factor comes from the photon propagator.
These are known factors, so that the experimental data give information about \( W^{\mu \nu} \), which corresponds to the interaction of the photon with the target nucleon. The term \( W^{\mu \nu} \) is a sum over the hadronic systems \( X \) of the squared modulus of the amplitude \( A \). This is depicted diagrammatically in figure 2.2, which shows also how the optical theorem relates \( W^{\mu \nu} \) to the imaginary part of the forward virtual Compton-scattering amplitude.

\[
W^{\mu \nu} = \sum_X A^* A^{\mu \nu} = 2 \text{ Im} W_{\mu \nu}.
\]

Figure 2.2. Definition of \( W^{\mu \nu} \) in terms of the amplitude \( A \) of (2.1), together with its relation to the forward virtual Compton amplitude through the optical theorem.

\( W^{\mu \nu} \) is a tensor, and it has the decomposition

\[
W^{\mu \nu} = -(g^{\mu \nu} - \frac{q^\mu q^\nu}{q^2}) W_1 + \left( p^\mu - \frac{p^\nu q^\nu}{q^2} \right) \left( p^\nu - \frac{p^\mu q^\mu}{q^2} \right) W_2.
\]

(2.3)

Here \( W_1 \) and \( W_2 \) are two Lorentz scalar functions; they depend on the pair of Lorentz scalars \( q^2 \) and \( \nu = p \cdot q \). To arrive at the decomposition (2.3), one needs, in addition to Lorentz invariance,

(i) parity conservation: this excludes a possible term \( e^{\mu \nu \rho \sigma} q_\rho p_\sigma W_3 \), which is present in the case of neutrino scattering.

(ii) current conservation: \( q_\mu W^{\mu \nu} = W^{\mu \nu} q_\nu = 0 \).

In (2.3), an average is implied over the two possible spin configurations of the target proton, as is appropriate for experiments in which the proton is unpolarised.

Notice that, because it is a momentum transfer, \( q^2 < 0 \). To get a feeling for the significance of \( \nu = p \cdot q \), notice that in the rest frame of the target it is equal to \( M q^* \), where \( q^* \) is the energy carried by the virtual photon. It is usual to define

\[
\omega = \frac{q^*}{-q^2}.
\]

(2.4)

Then, because baryon conservation requires that \( (p + q)^2 \geq M^2 \), where \( M \) is the nucleon mass, it must be that \( \omega \geq 1 \).

If we write

\[
W_1 = F_1(\omega, q^2),
\]

\[
W_2 = F_2(\omega, q^2),
\]

(2.5)

then it is possible to check that both \( F_1 \) and \( F_2 \) are functions that are dimensionless. Experimentally, it is found that to a good approximation they become functions only of \( \omega \) when \( \nu \) and \( q^2 \) are large. This is Bjorken scaling. It implies that dependence on any fixed dimensional parameter (mass or length) has disappeared.

Figure 2.3 shows data for large \( \omega \); there is very little variation with \( q^2 \). Just how good Bjorken scaling is, and what is the nature of the deviations from it, is an important question in current experimental and theoretical research. Discussion of this question is obscured by the fact that Bjorken scaling is expected to be, at best, an asymptotic property, valid at rather large values of \( \nu \) and \( q^2 \). Much of the data correspond to only moderate values of these variables, and this raises the question whether it is meaningful to plot such data in terms of some modified variable, say

\[
\omega_m = \frac{2\nu + a}{-q^2},
\]

(2.6)

where \( a \) is some constant parameter. At sufficiently large \( \nu \) and \( q^2 \), \( \omega_m \) and \( \omega \) become indistinguishable, but at small values of \( \omega \) existing data seem to display Bjorken scaling much better if \( a \) is chosen.
suitably than if $\omega$ is assumed to be the appropriate variable. This is shown in figure 2.4, which shows

![Diagram](image)

Figure 2.4. Data for $F_1(\omega, q^2)$ for $\omega^{-1}$ between 0.6 and 0.7, where $\omega^{-1}$ is defined by (2.6) with the three choices (i) $a = 0$, (ii) $a = M^2$, (iii) $a = 1.5$ GeV$^2$.

the $q^2$ dependence of data in the range $0.6 < \omega^{-1} < 0.7$ for the three choices $a = 0$, $a = M^2$ and $a = 1.5$ GeV$^2$. With the last choice, there is little dependence on $q^2$, and the question arises whether this is significant and has a simple explanation: in terms of this variable, will the lack of dependence on $q^2$ survive up to very much larger values of $q^2$? I will return to this question in my last lecture.

3. The Parton Model

To the extent that Bjorken scaling is a valid property, so that dependence on any fixed dimensional parameter has disappeared, the conclusion is that whatever structure in the proton is responsible for scattering the photon, it has no size. This leads to the picture where the proton is composed of pointlike (or, perhaps, almost pointlike) partons.

The virtual photon then scatters on one of the partons, figure 3.1.

![Diagram](image)

Figure 3.1. Deep Inelastic Scattering in the Parton Model.

Assume that the momentum-space wave function of the proton is sufficiently compact that, in a frame where the nucleon momentum $P$ is very large, the moment of each parton is almost parallel to it. That is, before it is struck by the virtual photon the parton's moment is approximately $x P^\mu$, where $x$ is some fraction between 0 and 1. Then the energy of the parton is $(x P^\mu + \mu)^{1/2}$, where $\mu$ is the parton mass, and for large $P$ this is approximately $\propto P^\mu$. Hence the whole parton four-momentum is almost equal to $x P$.

Calculating from the Feynman diagram of figure 3.1 gives

$$F_2 = Q^2 \propto S(\propto - \sqrt{\omega}),$$

(3.1)

where $Q^2$ is the parton charge. So scaling is obtained, and also $\omega^{-1}$ is identified as the fractional momentum $x$ of the parton before it is struck. This last result is obtained by setting the square $(x P + \mu)^2$ of the 4-momentum of the parton after it is struck equal to the square of the mass of the parton.

The result (3.1) for $F_2$ applies whatever the spin of the parton, but the corresponding result for $F_1$ varies according to the spin:

$$F_1 = \begin{cases} 0 & \text{spin 0 partons} \\ \pm \omega F_2 & \text{spin 1 partons} \end{cases}$$

(3.2)

The data are in good, though not exact, agreement with the second relation. The prediction (3.2) is again an asymptotic one, valid for very large $\nu$ and $q^2$; the existing data satisfy it well enough to lead us to assume that spin 1/2 partons are overwhelmingly important. The study of how $(F_1 - \frac{1}{2} \omega F_2)$ varies with $q^2$, and the question of whether it goes to zero at very large $q^2$, is important but experimentally difficult; existing data do not agree too well with
each other.

Given that the partons have spin $\frac{1}{2}$, the most natural and economical assumption is that they are fractionally-charged quarks, since these are already believed to be an important component of hadron structure, as a result of spectroscopic considerations. This introduces the immediate problem that the parton-model calculation seems to suppose that the parton that is struck by the photon is knocked out of the proton while, as far as is known, no fractionally-charged particles are actually present in the final state. The precise resolution of this confinement problem is not at all well understood, but it is generally believed to be something like this. The quark that is knocked out afterwards breaks up into a bunch of fragments, at least one of which must carry fractional charge. Likewise, among the remaining fragments of the proton there must be at least one that has fractional charge. See figure 3.2. A theoretical ideas can confirm the model, it is useful to reformulate the model in terms of quantum field theory. After all, nowadays field theory is widely believed to provide the basis of the dynamics of particles in high energy physics.

The naive parton model calculates the cross section from the squared matrix element in the left-hand side of figure 3.3. The optical theorem says

$$\sum \left| \begin{array}{c} \text{out} \\ \text{in} \end{array} \right|^2 = 2 \text{ Im} \begin{array}{c} \mathbf{T} \\ \text{out} \end{array}$$

Figure 3.3. Summing the squared modulus of figure 3.2 over final states, and using the optical theorem, gives the "handbag" diagram.

that when we sum over final states we obtain the "handbag" diagram shown in the figure. In order to justify Bjorken scaling, it is necessary to establish first that the handbag diagram (with possibly some additional internal lines to represent the confining force) does indeed dominate in the large $q^2$, large $y$, limit. Then one must also show that the handbag diagram really does scale. These questions can be examined in particular field-theory models, with the nucleon/parton-interaction amplitude T regarded as a complete sum of strong-interaction Feynman graphs.

It is found that scaling is true in $\phi^3$ field theory. But this theory contains only spin-zero particles; in theories that contain spin-$\frac{1}{2}$ fields, necessary to describe quarks and nucleons, scaling can be badly broken. The closest that one can get to scaling is found in quantum chromodynamics (QCD), where the strong interaction is mediated by the exchange of spin-one gluons, much as the electromagnetic interaction is mediated by the exchange of spin-one photons. In QCD scaling is almost satisfied. For this reason, QCD plays a central role in present-day thinking about strong interactions. But because QCD predicts that scaling is almost good, as a first approximation it is sensible to pretend that scaling really is exact, and then consider the deviations from scaling later.
Valence and Sea Quarks.

From the naive parton model,

\[ F_2(x) = \sum_r Q_r^2 \left[ F^{(\text{r})}(x) + F^{(\bar{\text{r}})}(x) \right], \]  

(3.5)

where the sum is over the various possible flavours \( r = u, d, s, c, \ldots \) of the struck parton and \( Q_r \) denotes the parton charge. From the calculation of figure 3.1, it is found that

\[ F^{(\text{r})}(x) = x f^{(\text{r})}(x), \]  

(3.4)

where \( f^{(r)}(x) \text{ dx} \) is the expectation value of the number of partons of flavour \( r \) having fractional momentum in the interval \( (x, x+\text{dx}) \). The antiquark contributions \( F^{(\bar{r})}(x) \) are obtained similarly. It follows directly that

\[ \int_0^1 \frac{dx}{x} F^{(\text{r})}(x) = N^{(\text{r})}, \]
\[ \int_0^1 \frac{dx}{x} F^{(\bar{r})}(x) = N^{(\bar{r})}, \]

(3.5)

where \( N^{(r)} \) is the expected number of partons of flavour \( r \) in the proton and \( N^{(\bar{r})} \) is the average total fractional momentum that they carry.

Experimental data show that, for the second sum rule in (3.5),

\[ \sum_{r = u, d, s, c} \left( N^{(\text{r})} + N^{(\bar{\text{r}})} \right) \approx \frac{3}{2}. \]  

(3.6)

It is widely believed that the other half of the proton's momentum is carried by the gluons. Because these are supposed to have zero charge, the virtual photon does not couple to them directly. In deep inelastic scattering, one can only hope to detect their presence indirectly, through them generating quark-antiquark pairs which themselves can couple to the photon: see figure 3.4. I shall consider this again in the last lecture, where I discuss also possible more direct ways of establishing the existence of gluons. As for the other sum rule in (3.5), Regge theory predicts that for small \( x \), where \( q^2 \ll M^2 \), \( F^{(\pi)}(x) \) and \( F^{(\bar{\pi})}(x) \) should be dominated by pomeron exchange and so be essentially constant. The data for \( F_2 \) seem to verify this. In consequence, the first integral in (3.5) diverges at \( x = 0 \). This means that the proton contains an infinite number of quarks and antiquarks, nearly all having very small \( x \) so that they contribute little to the total momentum. That is,

\[ \text{Proton} = \text{udd} + \text{"sea" of infinite number of quark-antiquark pairs}. \]  

(3.7)

The \text{udd} are the "valence" quarks that give the proton its quantum numbers, while the sea has neutral quantum numbers. This means that

\[ N^{(u)} - N^{(\bar{u})} = 2, \]
\[ N^{(d)} - N^{(\bar{d})} = 1, \]
\[ N^{(s)} - N^{(\bar{s})} = 0 = N^{(c)} - N^{(\bar{c})}, \]

(3.8)

and the sea contribution cancels between the two terms in each of these four relations.

Now, for a proton target

\[ F_2^{(p)} = \left( \frac{2}{3} \right)^2 \left[ u(x) + \bar{u}(x) \right] \]
\[ + \left( \frac{1}{3} \right)^2 \left[ d(x) + \bar{d}(x) + s(x) + \bar{s}(x) \right] \]
\[ + c, \bar{c} \text{ terms at high energy} \]

(3.9)

where I have written

\[ u(x) = F_p^{(u)}(x) = u(x). \]

Because the strong interactions are charge independent, charge symmetry gives

\[ F_n^{(u)} = F_p^{(d)} = d(x), \quad F_n^{(d)} = F_p^{(u)} = u(x), \]
\[ F_n^{(s)} = F_p^{(c)} = s(x), \]

and similarly for the antiquark contributions. Hence, for a neutron target,

\[ F_2^{(n)} = \left( \frac{1}{3} \right)^2 \left[ d(x) + \bar{d}(x) \right] \]
\[ + \left( \frac{2}{3} \right)^2 \left[ u + \bar{u} + s + \bar{s} \right]. \]

(3.10)

Now assume (and this may be wrong) that the sea is neutral and SU(2) symmetric for each \( x \), so that the sea contributions satisfy

\[ u(x) = \bar{u}(x) = d(x) = \bar{d}(x) = s(x) = \bar{s}(x) = c(x) = \bar{c}(x). \]

(3.11)
(Maybe there is even SU(3) symmetry, so that \( S = S_\chi \), but this is hard to investigate experimentally.) Then

\[
F_2^e + F_2^{en} = \frac{1}{3}(\bar{u} - d)
\]

(3.12)

and only valence quarks contribute to this difference. The number sum rule in (3.5) gives, from this,

\[
\int_0^1 x F_2^e(x) + F_2^{en}(x) = \frac{\alpha_s}{\pi}.
\]

(3.13)

This seems to be satisfied experimentally, though precise neutron-target data are not available, if only because they necessarily come from deuteron targets and the Fermi-motion corrections are very large and poorly understood.

The literature contains many analyses of deep inelastic scattering in the valence-quark/sea-quark picture. Donnachie and I assumed\(^7\) that the valence \( u \) and \( d \) distributions are the same shape as functions of \( x \) and so satisfy \( u(x) = 2d(x) = V(x) \).

Then \( F_2^e + F_2^{en} = \frac{1}{3} V(x) \), which may be extracted from the data, though there are large error bars.

Then \( F_2^e = V + \frac{1}{3} S + \frac{2}{3} S_\chi \), so that the difference between \( V(x) \) and the \( e^+ p \) data determines the sea contribution. Our resulting curves are sketched in figure 3.5. Other authors make slightly different assumptions and interpret the data in slightly different ways, and therefore obtain slightly different results. But they all agree qualitatively. The sea contributions are least well determined; the only rather sure thing is that they are very small for \( x \geq 2/3 \).

There is room for much more work on this, by both experimentalists and theorists. Is it true that \( u(x) \) and \( d(x) \) are the same shape as functions of \( x \), or does the ratio \( d(x)/u(x) \) instead vanish as \( x \to 1 \)? Does \( u(x) \) behave like \((1-x)^3\) as \( x \to 1 \), as many theorists would like?\(^8\) Or is it rather \((1-x)^4\)? If so, why? And how about \( S(x) \): some popular choices for the power of \((1-x)\) here are 5, 7 or even 10.

Then there are the Fermi-motion corrections for deuterium, which I have already mentioned. Lastly, there is the theoretical question of whether the decomposition "valence + sea", ignoring quantum-mechanical interference effects, can make good sense, or at least nearly so.


Deep inelastic neutrino reactions provide a direct test of the validity of the valence-sea parton picture. There are two types of inclusive reaction, the charged-current processes

\[
\nu \bar{p} \rightarrow \mu^- + \text{hadrons} \quad (\text{or } e^- + \text{hadrons})
\]

(4.1)

and the neutral-current processes

\[
\nu \bar{p} \rightarrow \nu + \text{hadrons}
\]

(4.2)

\[
\bar{\nu} \bar{p} \rightarrow \bar{\nu} + \text{hadrons}
\]

Rather more is known about the charged-current reactions than the neutral ones, both theoretically and experimentally, so I shall discuss only these. For more details, and information also on the neutral-current processes, see the lectures by Kleinhecht at this 1978 CERN School.

Just as deep inelastic \( e^- \) or \( \mu^- \) scattering occurs through the exchange of a single virtual photon, neutrino scattering is believed to be mediated by the exchange of a single virtual \( W \) (figure 4.1). According to whether the incoming particle is a neutrino or antineutrino, the \( W \) carries charge + or −; this is why the reactions are called "charged-current".

The neutral-current reactions are though: similarly to be mediated by a neutral particle, the \( Z_0 \).

The diagrams of figure 4.1 give contributions to the cross-sections that correspond to (2.2):

\[
L_{\mu\nu} \frac{g}{(q^2 - M_W^2)^2} W_{\mu\nu}.
\]

(4.3)

Here, \( L_{\mu\nu} \) is again the leptonic factor; it is different from the leptonic factor in (2.2) because the
\[ W^{\mu \nu} = -g^{\mu \nu} W_1 + p^\mu p^\nu W_2 + \frac{i}{2} e \epsilon^{\mu \nu \alpha \beta} q_\alpha p_\beta W_3 + \ldots, \] (4.6)

where \( G \) is the Fermi weak-interaction constant. A similar approximation is valid in the analysis of \( \lambda \)-decay or neutron \( \beta \)-decay, because there the momentum transfer is certainly small, and the value of \( G \) is, of course, well determined from these decays. A remarkable discovery of the high-energy neutrino experiments is that the structure of weak interactions deduced from the very-low-momentum-transfer decay processes gives good predictions for very much larger momentum transfers. In particular, this implies from presently available data that the \( W \) mass must be rather greater than 20 GeV.

The third factor in (4.3) describes the hadronic part of the interaction. Again, because the \( W \) and the \( \gamma \) have different types of coupling to hadrons, this factor is not the same as the corresponding one in (2.2). Also, it is not the same for \( W^* \) as for \( W \).

Suppose that the beam energy \( E \) is not so large that \(-q^2\) can be comparable with \( M_W^2 \). If Bjorken scaling is valid, there should then be no dependence on any fixed dimensional parameter. The simplest consequence of this concerns the total cross section \( \sigma \). To lowest order in the weak interactions, \( \sigma \) is proportional to \( q^2 \). The parameter \( G \) is not dimensionless; to obtain a cross-section it must be multiplied by a factor with the dimensions of squared momentum. Lorentz invariance demands that this factor be a Lorentz scalar, and the only available factor is therefore one proportional to \( p \cdot k \), where \( p \) and \( k \) are respectively the 4-momentum of the target proton and of the neutrino. In the laboratory frame, \( p \cdot k = ME \), so

\[ \sigma \propto G^2 E. \] (4.5)

Figure 4.2 shows data for \( \sigma/E \) for both neutrino and antineutrino beams. The result (4.5) seems to be verified.

Corresponding to (2.3), there is an expansion of \( W^{\mu \nu} \):

\[ \begin{array}{l}
10^{38} \sigma/E \\
(\text{cm}^{-2}/\text{GeV})
\end{array} \]

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{figure4.2.png}
\caption{\( \sigma/E \) in the two processes (4.1) - data of the CDHS collaboration.}
\end{figure}

where the terms that I have not written explicitly give contributions that are proportional to the muon mass, and therefore are small, when \( W^{\mu \nu} \) is multiplied by \( L_{\mu \nu} \). Notice that the structure (4.6) is not identical with (2.3); this is because the weak-interaction current is not conserved, and there is no parity conservation. In the parton model, \( W_1 \), \( W_2 \) and \( W_3 \) are calculated from the diagram of figure 3.1, just as for electron or muon scattering. The only difference is the coupling at the vertex; instead of being determined by the electromagnetic current

\[ J_{EM}^\mu = \frac{2}{3} \bar{u} y^\mu u - \frac{1}{3} \bar{d} y^\mu d - \frac{1}{3} \bar{s} y^\mu s \]

\[ (+ c, \bar{c}, \bar{c}, \ldots \text{ terms}), \]

it corresponds to the charged weak hadronic current

\[ J_{W}^\mu = \bar{u} y^\mu (1-y_8) [\lambda \cos \Theta_c + s \sin \Theta_c] \]

\[ (+ \ldots \), \] (4.7)

where \( \Theta_c \) is the Cabibbo angle, \( \Theta_c \approx 15^\circ \). If the same valence and sea distributions are used in this calculation as are extracted from \( e \) and \( \mu \) scattering, excellent agreement is obtained with data from \( \nu \) and \( \bar{\nu} \) scattering. As an example, figure 4.3 shows the quark and antiquark distributions required to fit the data from BEBC. Remember

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the valence $u$ quarks. Hence a measurement of the ratio $F_{2u}^h/F_{2d}^h$ at large $x$ gives directly the ratio $d(x)/u(x)$, with no problems about deuterium corrections. In this way, it will be possible to settle the question whether this ratio is fixed at 2, or whether it goes to zero as $x \to 1$.

Again, if one wishes to study how a $u$ quark materialises into hadrons after it has been ejected from the proton by the $W$, one should look at $\nu\bar{\nu}$ reactions at large $x$. Then, unlike in any other process, one can be sure that the quark being studied is a $u$ rather than, say, a $d$ or a $\bar{u}$. I shall come back to this later.

5. Production of lepton pairs.

One of the most important reactions being actively studied in a number of experiments is

$$pp \to \mu^+\mu^- \text{ (or } e^+e^-) + \text{ hadrons.} \quad (5.1)$$

To lowest order in the electric charge, this occurs through the production of a virtual photon,

$$pp \to \gamma + \text{hadrons,} \quad (5.2)$$

which subsequently decays into the $\mu^+\mu^-$ or $e^+e^-$. It is important to distinguish between resonance production and continuum production; the two are beautifully illustrated in the famous data of figure 5.1. I first discuss the continuum.

Figure 4.3. Fractional momentum content in the nucleon, as measured at BEBC with $-q^2$ in the range 3 to 100 GeV$^2$. Notice that the antiquark distribution is entirely nonvalence, while the quarks are a mixture of valence and sea. Compare the curves in figure 3.5 (though note that the normalisation of these is defined differently).

that the antiquarks occur only in the sea, and that the quarks are the sum of valence and sea terms, and compare this figure with figure 3.5.

The data of figure 4.3 are on a heavy liquid target, so that they average the contributions from proton and neutron targets. It is of importance to have more data from hydrogen targets. The reason for this is that if the Bjorken variable $x$ is large enough for the contribution from sea quarks to be negligible, then one can be sure that $\nu$ scattering is probing the valence $d$ quark in the proton. (The $W$ is absorbed by the $d$ quark and a $u$ quark is emitted. A $u$ quark cannot absorb the positively charged $W$. See figure 4.4). Similarly, $\bar{\nu}$ scattering probes

Figure 4.4. Dominant contributions to the processes (4.1) at large values of $x$.

Figure 5.1. Continuum and resonance contributions to the reaction (5.1) - data from the Columbia-Fermilab-Stony Brook collaboration.
Let the 4-momentum of the $\gamma$ be $q$, so that $\sqrt{q^2}$ is the invariant mass $M_{4\ell}$ of the dilepton system. Now $q$ is timelike, $q^2 > 0$, while in the $e^+e^-$ and $\mu^+\mu^-$ scattering processes it was spacelike, $q^2 < 0$. Nevertheless, the parton model allows a calculation of the dilepton production in terms of the quark distributions measured in the scattering processes.

In the centre-of-mass frame, define Feynman's variable

$$x_F = \frac{\text{longitudinal momentum } p_y \text{ of the dilepton}}{\text{its maximum possible value}}$$ (5.3)

An alternative to $x_F$ is the rapidity

$$y = \pm \log \frac{E + p_y}{E - p_y},$$ (5.4)

where $E$ is the energy of the dilepton. This is almost equal to the pseudorapidity

$$\eta = -\log \tan \frac{1}{2} \theta,$$ (5.5)

where $\theta$ is the centre-of-mass frame angle at which the dilepton (or, equivalently, the virtual photon) emerges. One may choose to work with any one of the three dimensionless variables $x_F$, $y$, $\eta$, together with another dimensionless variable $\tau = q^2/s$, or $\sqrt{\tau} = M_{4\ell}/s$, where $\sqrt{s}$ is the total centre-of-mass energy.

If there is scaling, so that dependence on fixed dimensional parameters disappears for sufficiently large $M_{4\ell}$ and $s$, then

$$\frac{d^2\sigma}{d\eta dy} \text{ or } \frac{d^2\sigma}{d\tau dy} \text{ or } \frac{d^2\sigma}{d\tau d\eta}$$

$$= \frac{1}{M_{4\ell}^2} x \left\{ \text{a dimensionless function of } \right\} \{x_F \text{ or } y \text{ or } \eta \}.$$ (5.6)

This is tested by the data in figure 5.2. To the extent that the data points lie on a single curve, the scaling is verified.

The Drell-Yan Mechanism.

In the parton model, the virtual photon in (5.2) is produced by the fusion of a quark from one of the initial hadrons with an antiquark from the other. This is the Drell-Yan mechanism, shown in figure 5.3. In this figure, $p_1 \propto x_1 k_1$, and $p_2 \propto x_2 k_2$. - see the discussion

![Figure 5.2. Test of scaling of the Drell-Yan continuum in preliminary data from the Columbia-Fermilab-Stony Brook collaboration. The plot is of $M_{4\ell}^2 d^2\sigma/d\tau dy$ at $y = 0.2$ in proton-nucleus collisions (logarithmic vertical scale).](image)

![Figure 5.3. The Drell-Yan Mechanism. Additional initial and final state interactions are not shown - see text and figure 5.4. at the beginning of §3. Hence $q = x_1 p_1 + x_2 p_2$, from which we obtain, asymptotically, $x_1 + x_2 = \tau = M_{4\ell}^2/s$ and $x_1 - x_2 = x_F$.](image)
For given $\tau$ and $x_F$, these equations may be solved so as to determine $x_1$ and $x_2$. In particular, when

$$\theta = 90^\circ \text{ so that } x_F = 0,$$

we have $x_1 = x_2 = M_{ee}^2 / \sqrt{s}$. That is, production of large dilepton masses probes the parton distributions at large fractional momenta $x_1$ and $x_2$. Other useful formulae are

$$y = \frac{1}{2} \log \frac{x_1}{x_2},$$

$$2E_F / \sqrt{s} = x_1 + x_2.$$

(5.8)

Calculation of the Drell-Yan diagram gives

$$\frac{d^2\sigma}{d^2x_F dy} = \frac{2E_F}{\sqrt{s}} \frac{d^2\sigma}{d^2x_F dx_F},$$

$$= \frac{1}{3} \frac{8\pi \alpha^2}{3\alpha_s^2 \sqrt{s}} F(\sqrt{s}, y \text{ or } x_F),$$

(5.9a)

where, in the same notation as (3.3)

$$F = \sum_{\tau = u, d, ...} Q^2 \left[ F^{(\tau)}(x_1) F^{(\tau)}(x_2) + F^{(\bar{\tau})}(x_1) F^{(\bar{\tau})}(x_2) \right].$$

(5.9b)

The factor $\frac{1}{3}$ in (5.9a) is a consequence of colour: only quarks of the same colour may fuse to form the virtual photon, and there is a 1 in 3 chance of having the same colour.

It must be mentioned that initial and final state strong interactions modify the simple Drell-Yan diagram of figure 5.3. A final-state interaction is shown in figure 5.4, resulting in the production of additional hadrons. In so far as the "ordinary" strong interactions are concerned, there is a theorem$^1$ that in the calculation of the inclusive cross-section (5.9), where one is not interested in the final-state hadrons, there is destructive interference and the effects of the initial and final state interactions exactly cancel. That is, one may forget them in the calculation of the inclusive cross-section (5.9), and pretend that only the simple Drell-Yan diagram of figure 5.4 matters. Nevertheless, they do affect the distribution of hadrons in the final state. The theorem applies only to the ordinary strong interactions; there must also be quark-confining interactions if there are to be no fractionally-charged particles in the final state, and the effect of these is, of course, not understood. The hope is that they also can be ignored in the calculation of the inclusive cross-section.

The Drell-Yan process is of very great importance. One reason is that the antiquark distributions enters multiplicatively in (5.9), so that the cross-section is sensitive to them. Measurements of $d^2\sigma / d^2x_F dy$, give a more accurate determination of the antiquark distributions than is possible from lepton scattering, where the antiquarks enter additively rather than multiplicatively. Figure 5.5 shows the effect of changing the shape of the antiquark distribution. In comparing data with calculations such as those in figure 5.5, remember that most of the data are for heavy nuclear targets. Experimentally, it seems that cross-sections for massive dilepton production are linear in the mass-number $A$ of the target, but for a heavy target any small discrepancy from this $A$-dependence can have substantial numerical effect. In any case, there is the additional need to correct the data for the effects of the Fermi motion of the nucleons in the nuclear target, and for secondary interactions. The ultimate test of the Drell-Yan mechanism will be to use a hydrogen target, adjust the parton distributions to give a good fit to the

![Figure 5.4. A final-state interaction, which modifies the Drell-Yan mechanism.](image)

![Figure 5.5. The effect on the Drell-Yan calculation of changing the sea distribution in the nucleon (from reference 9). The calculations are for $M_{ee}^2 d\sigma / dM_{ee} d\eta$ at $y = 0$.](image)
energy variation of the data at fixed $x_F$, and then see whether the resulting calculated distributions fit the $x_F$-dependence at each energy.

Another important feature of the Drell-Yan mechanism is that, by changing to a different beam, one can use it to measure parton distributions in other types of hadron. This is, of course, not possible through lepton scattering experiments. Notice that if the beam is a $\bar{p}$ or a $\pi$, the antiquark needed for the fusion can be a valence parton, whereas for a proton beam it has to belong to the sea. Hence when the dilepton mass $M_{\ell\ell}$ is large enough for $x_1$ and $x_2$ to be beyond the range of values where the sea distribution is appreciable, one expects that production for $\bar{p}$ or $\pi$ beams will be rather greater than from a proton beam.

This is verified in the data of figure 5.6, which shows also the results of typical Drell-Yan calculations, including target corrections. For the pion beams, the results of two calculations are shown. The solid curves assume parton distributions in the pion to be not very different from those shown in figure 3.5 for the nucleon; see the solid curves in figure 5.7. The dotted curves assume that the parton distributions in the pion are represented by the dashed curves in figure 5.7, as is proposed by Feynman and Field. For reasons that I shall return to later, it is of importance to have more dilepton production data from pion beams, in order to determine which type of parton distribution in the pion is more correct.

Comparing production with $\pi^+$ and $\pi^-$ beams gives another useful test of the Drell-Yan mechanism. Suppose that the target has equal numbers of protons and neutrons, for example carbon, so that it contains equal numbers of $u$ and $d$ quarks. Take $M_{\ell\ell}$ large enough to be reasonably sure that contributions from valence partons in both beam and target dominate, so that with a $\pi^+$ beam its valence $d$ participates in the fusion, and with a $\pi^-$ it is the valence $u$. Then the cross-section ratio at large $M_{\ell\ell}$ should be

$$\frac{\sigma(\pi^+)}{\sigma(\pi^-)} = \left( \frac{\text{charge of } d}{\text{charge of } u} \right)^2 = \frac{1}{4}. \quad (5.10)$$

Data for this ratio are shown in figure 5.8. For a hydrogen target, the ratio should be $1/8$ if the ratio of the quark distributes $u(x)$ and $d(x)$ in the proton is equal to 2 at large $x$. 

Figure 5.7. Valence and sea distributions in the pion, as proposed in references 7 (solid curves) and 10 (dashed curves).

Figure 5.6. Data from the Omega Beam Dump experiment for $d^2\sigma/dM_{\ell\ell} dx_F$ (in nb/nucleus/GeV) plotted against $x_F$ for $M_{\ell\ell}$ in the range 1.9 to 2.7 GeV. The different beams are at 40 GeV/c and the target is copper. Calculations by L. Kenyon using the distributions of references 7 (solid curves) and 10 (dotted curves).
The Drell-Yan mechanism provides an obvious mechanism for the production of the weak-interaction vector bosons in hadron-hadron collisions. The virtual photon in figure 5.3 is replaced by a $W^\pm$ or a $Z^0$, which now couples to the quarks through the weak current instead of the electromagnetic current. The hope is that the vector boson will be detected either its leptonic decay, $W \rightarrow \mu\nu$ or $Z^0 \rightarrow \mu^+\mu^-$, or through its hadronic decays.

In the case of the $W^\pm$, the strength of its coupling to quarks is given by (4.4). Gauge theory does not enter into that relation. However, in order to predict the mass $M_{W}$, and both the mass and the coupling of the $Z^0$, it is necessary to assume a specific gauge theory. The Salam-Weinberg model gives

$$M_W = M_Z \cos \theta_W = \left(\frac{G_F}{\sqrt{2}}\right) \frac{M_Z}{\sin \theta_W} \left(\frac{37.5 \text{ GeV}}{\sin \theta_W}\right).$$

(6.1)

Here $\theta_W$ is the Weinberg angle. Data on neutral-current neutrino reactions find that $\sin \theta_W$ is somewhere between about 0.38 and 0.25, so that $M_W$ is in the region of 60 to 75 GeV, with $M_Z$ a little higher

For $W^\pm$ production, the formula analogous to (5.9) is

$$\frac{d\sigma}{dy} = \frac{\pi G_F}{\sqrt{s}} \left(\frac{M_W}{\sqrt{s}}\right)^2 \left(\frac{y - \alpha_x}{1 - y}\right),$$

(6.2a)

where

$$H = \left[ F^{(\alpha)}(x_\alpha) + F^{(\beta)}(x_\beta) + F^{(\gamma)}(x_\gamma) \right] \cos \theta_c$$

$$+ \left[ F^{(\delta)}(x_\delta) + F^{(\epsilon)}(x_\epsilon) \right] \sin \theta_c,$$

(6.2b)

with $\theta_c$ again the Cabibbo angle. Predictions based on this formula are shown in figure 6.1. $W^\pm$ and $Z^0$ production may be calculated similarly; the results are not very different.

![Graph showing $d\sigma/dy$ vs. $y$ for different values of $M_W/\sqrt{s}$](image)

Figure 6.1. Predictions for $W^\pm$ production at $\sqrt{s} = 200$ and 600 GeV (Chase and Stirling).

The cross-sections are expected to be at the nanobarn level, which should give a reasonable production rate in the storage rings now being planned. But it is important to discuss the backgrounds that may obscure the signal; I return to this later.

7. $J/\psi$ and $\Upsilon$ Production.

Mechanisms of the Drell-Yan type have been proposed also for the production of the $J/\psi$ and the $\Upsilon$ in hadronic collisions. There are basically two different types of mechanism. The first is the fusion of ordinary quarks $q\bar{q}$, so as to produce the $J/\psi$ or $\Upsilon$ with a coupling that is Zweig-violating and therefore very small. At least in the case of the $J/\psi$, the strength of this coupling is calculated from the observed width of the particle, assuming that the hadronic decays occur through a $q\bar{q}$ intermediate state. For $pp$ collisions, where the $q\bar{q}$ involved in the fusion is necessarily non-valence, the production rate calculated from the mechanism is much less than that which is observed. However, for $\pi p$ and $p p$ collisions the $q\bar{q}$ can be a valence parton of the beam particle, so that at fairly low energy where the fractional momentum $x_\perp$ that is required for the $q\bar{q}$ is large the mechanism gives an appreciable contribution. One concludes that at
fairly low energy the cross-section for $J/\psi$ production from $\bar{p}$ and $\bar{\pi}$ beams should be rather larger than from a proton beam, as is seen in the data of figure 7.1.

Figure 7.1. Data from the Omega Beam Dump experiment for $d\sigma/dx_p$ (in nb/nucleus) plotted against $x_p$ for $J/\psi$ production with 40 GeV/c beams on a copper target. Calculations by L. Kenyon using the model of reference 7.

The other mechanism, which must also be present in order to explain the data, involves the fusion of a pair of partons whose coupling to the $J/\psi$ or $\Upsilon$ does not violate Zweig's rule and therefore is larger. However, both partons are non-valence whatever the type of beam, so that the resulting cross-section is only appreciable in magnitude when the energy is so high that the necessary fractional momenta $x_1$ and $x_2$ are fairly small (see (5.7)), and the mechanism contributes equal production rates for $p$ and $\bar{p}$ beams. Different authors make different assumptions as to the nature of the fusing partons: in the case of $J/\psi$ production some take them to be $c\bar{c}$ while others take them to be a pair of gluons. In either case the appropriate shape of the parton distribution within the nucleon or pion is chosen by fitting the energy variation of the data; see figure 7.2. The strength of the coupling of the partons to the $J/\psi$ is determined from the normalisation of one of the data points. The $x_p$ dependence of the data at each energy then fits very well to the calculations, as is illustrated in figure 7.1.

It is not known at present which of the two guesses about the nature of the partons in the second mechanism is the right one. The $c\bar{c}$ choice has the apparent problem that, when a charmed quark is pulled out of each beam particle, non-zero charm remains in the residual beam fragments. However, final-state interactions between the two sets of residual fragments can remove the necessity of there being any charmed particles in the final state, particularly at fairly low energies where phase space considerations predict that production of charmed particles in association with the $J/\psi$ is inhibited because of their relatively high mass. No strong experimental evidence for such associated production has been found. In the case of the $gg$ fusion, the system that is produced directly by the fusion has the wrong C-parity. It has to be assumed that this is put right by the radiation of either a photon or a soft gluon, and evidence for the production of photons in association with the $J/\psi$ is indeed found. But it should be noted that the $c\bar{c}$ mechanism can produce states of positive C-parity also.

8. $e^+e^-$ Annihilation.

QED Processes.

The simplest $e^+e^-$ annihilation reaction is $e^+e^- \rightarrow \mu^+\mu^-$. This is a pure QED process, with just one Feynman graph in lowest order (figure 8.1). This graph is easy to calculate, but even without calculation one might guess that, when the energy $E$ of each colliding beam is much greater than the electron mass $m_e$, the cross-section will not depend on
the masses. Then, from dimensional considerations, 
\[ \sigma \sim \frac{1}{s} \], where \( \sqrt{s} = 2E \) is the invariant centre-of-mass energy. Calculation of the Feynman graph gives

\[ \sigma \sim \frac{4\pi\alpha^4}{3s} \]

\[ \frac{d\sigma}{d\Omega} \propto 1 + \cos^2\theta. \]  

(8.1)

Another interesting process that is purely a QED process is the two-photon-exchange reaction of figure 8.2, \( e^+e^- \rightarrow e^+e^-e^+e^- \). Evidently, the cross-section for this is proportional to \( \alpha^4 \), and therefore it appears to be much smaller than the cross-section (8.1). However, there is a special feature that enhances the contribution from the Feynman graph of figure 8.2. To obtain the cross-section, one integrates over the momenta of the particles in the final state and in this integration the momentum transfers \( q_1^2 \) and \( q_2^2 \) carried by the two photons vary. At high \( E \), \( q_1^2 \) and \( q_2^2 \) can take very small values, so that the photon propagators \( 1/q_1^2 \) and \( 1/q_2^2 \) are very large. The result of the integration is that

\[ \sigma \propto \frac{\alpha^4}{\alpha^2} \left( \log \frac{E}{m_e} \right)^3. \]

(8.2)

Unlike (8.1), this rises with increasing \( E \).

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Hadronic Processes.

The two-photon process leads also to hadron production, figure 8.3. The cross-section is calculated in terms of the total cross-section for \( \gamma\gamma \) scattering, which may be estimated assuming factorisable pomeron exchange. The conclusion is that even at \( E = 15 \text{ GeV} \) the two-photon process contributes more than 95% of the total \( e^+e^- \) cross section. Its cross section rises slowly with increasing energy, unlike the single-photons processes which fall like \( s^{-1} \), and so its relative importance rises very rapidly with increasing \( E \). Notice that, because the dominant contribution arises from small momentum transfers \( q_1^2 \) and \( q_2^2 \), where the photons move in a direction almost parallel to the incoming beams, nearly all the hadrons are produced close to the beam directions. However, the chance of there being hadrons produced with large transverse momentum is not negligible.

Most of the interest in hadron production from \( e^+e^- \) colliding beams lies in the one-photon process. In the parton model, this corresponds to a diagram just like figure 8.1, but with the \( \mu^+\mu^- \) replaced by a quark-antiquark pair. There has to be some final-state interaction if there are to be no fractionally-charged particles in the final state, but it is assumed that there is unit probability of this occurring, so that the cross-section is calculated just from the \( e^+e^- \rightarrow q\bar{q} \) part of the reaction. Because of the similarity of the coupling of the photon to \( \mu^+\mu^- \) and \( q\bar{q} \), the prediction then is that

\[ R = \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} = 3 \sum_{r=u,d,s} Q_r^+ \]

(8.3)

where, as usual, \( Q_r \) is the charge carried by the quark of flavour \( r \). The factor of 3 occurs if each flavour is found in three colours. For \( r = u, d, s \) the prediction (8.3) is \( R = 2 \), while if \( c \) is added \( R \) rises to 10/3. Data from both SPEAR and PLUTO find
that, below the charm threshold, R is just a little greater than 2. Above the charm threshold and the various structures associated with it, PLUTO finds a value for R quite close to 10/3, while SPEAR finds it to be about one unit higher.

Jets.

After the q and the $\overline{q}$ have been produced, each materialises as a jet of hadrons. It is assumed that the quark-confining interaction between these two jets substantially affects only their slowest component particles. Since the $e^+e^-$ annihilation occurs in the centre-of-mass frame, the intermediate virtual photon is at rest, and so the pair of jets emerges back-to-back in any direction. At fairly low beam energies $E$, the axis that defines the jet direction is not immediately evident from inspection of the event, and it has to be found by a sphericity analysis. This analysis amounts to guessing an axis, calculating the sum over all the particles of the square of the momentum component $p_\perp$ transverse to the axis and varying the direction of the axis until this quantity is minimised. Distributions of $p_\perp$ relative to the axis so chosen are shown in figure 8.4. Notice that $\langle p_\perp \rangle$ remains close to about 300 MeV/c even at large energy.

This, then, is the operational definition of a jet, as deduced from experimental data: a jet is a collection of particles whose momentum component perpendicular to their total momentum vector is limited to a few hundred MeV/c. When the total momentum of the jet is 2.5 GeV/c, on average one finds two charged particles and one neutral; at 5 GeV/c these average multiplicities are nearly doubled. Combining these pieces of information, one can sketch scale plan drawings of typical jets, as I have done in figure 8.5. These jets do not look very jet-like,

![Figure 8.5](image)

Figure 8.5. Scale drawings of "typical" jets; the dashed lines represent neutral particles.

which is why at low energies the sphericity analysis is needed to reveal their presence. However, as the energy increases the jet structure begins to become apparent to the naked eye; figure 8.6 shows an exam-

![Figure 8.6](image)

Figure 8.6. A pair of jets from PLUTO. Each jet has $E = 4.68$ GeV; only charged particles are seen.

of a not untypical event at energy $2E = 9.35$ GeV.

When the jet axis has been determined from event to event, one can ask what is its angular dist-

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tribution relative to the initial $e^+e^-$ beam directions when the events are summed. Because the virtual photon has spin 1, the answer must be of the form $1 + \lambda \cos^2 \Theta$, for some $\lambda$. If the jets each have spin $\frac{1}{2}$, then $\lambda = 1$ as in (8.1). For spin 0, $\lambda = -1$. The data give $^{15}\lambda = 0.97 \pm 0.14$, in agreement with the expected value for quarks.

A distribution of some importance is that of $z$, where $z$ is the fractional longitudinal momentum of the component particles in the jet. Data from PLUTO are shown in figure 8.7, which indicates also the

![Figure 8.7. Momentum distribution of charged hadrons at PLUTO. The dashed line indicates corresponding data from SPEAR. The distribution shown here is in $x = 2P / \sqrt{s}$; that in $z$ is similar.](image)

In deep inelastic $e, \mu$ or $\nu$ scattering, the parton that absorbs the virtual $\gamma$ or $W$ is ejected and materialises as a jet: see figure 3.2. This jet is expected to be similar to those found in $e^+e^-$ annihilation. Superficially, such a similarity is indeed found, but more exact experimental comparison is needed.

**Quark Jet Fragmentation.**

There is a widely-held belief that when a quark fragments to form a jet of hadrons, the fastest hadron tends to be such that it contains the quark as a valence quark. For example, a $u$-quark jet commonly contains a fast $\pi^+, \pi^0$ or $K^+$, but more rarely a fast $\pi^-$ or $K^-$. The best way to investigate this belief experimentally is in $\nu$ and $\overline{\nu}$ interactions on a hydrogen target. I explained that then, when $\omega$ is not large, the flavour of the emerging quark is known. The available data support the belief, but more investigation is needed.

The question has been studied theoretically in a cascade model. Consider, for definiteness, the pair of jets produced in $e^+e^-$ annihilation. As the $q$ and $\overline{q}$ separate, a colour field is set up between them. It is assumed that this field is such that the energy contained in it increases as the $q$ and $\overline{q}$ separate, so that new $q\overline{q}$ pairs are created in the field. The fragmenting quark meets an antiquark belonging to a pair formed in this way, and fuses with it to form a meson. The quark that originally belonged to the pair then continues on in more or less the direction of the initial quark, until it in turn meets an antiquark. The process repeats itself until almost all the energy is shed, and the remaining quark is so slow that it readily fuses with a slow antiquark arising elsewhere in the reaction. See figure 8.8. The first meson produced in the cas-

![Figure 8.8. Cascade model for fragmentation of a quark into mesons. Eventually, a slow quark remains, which annihilates with a slow antiquark from elsewhere in the reaction.](image)
fastest meson. Again, the first meson produced may be an unstable one, and its decay products can then be slower than some of the other mesons produced further down the chain. So in this model the retention of quantum numbers by the fastest hadron in the jet is not perfect, though it does occur approximately.

9. Exclusive Processes at Large \( t \).

Deep inelastic processes involving leptons are associated with a \( \gamma \) or \( W \) that has very short wavelength. This therefore probes the short-distance structure of the hadrons and that is why the reaction mechanisms have the simplicity displayed by the parton model. Purely hadronic reactions in most cases are not associated with short-distance effects: when two protons collide, their collision is usually a glancing one which involves only their outer structure. It is not clear whether the parton model has anything simple to say about such collisions, though there is a possibility that it may, as I shall discuss later. Rather rarely, however, two protons will collide head-on, so that then the interaction does directly involve their short-distance structure and it may be hoped to have a simple parton-model description. The way to pick out the events in which there has been a head-on collision is to study those that contain a large momentum transfer.

I consider first the exclusive processes at large \( t \):

\[
A + B \rightarrow C + D.
\]

(9.1)

Although the theoretical understanding of these is not as well-based as in the case of leptonic processes, there is fairly good reason to suppose that, at least to a good approximation, the large-\( t \) differential cross-section should take the form

\[
\frac{d\sigma}{dt} \sim s^{-m} F(\theta).
\]

(9.2)

Here \( \theta \) is the centre-of-mass scattering angle and the parameter \( m \) is fixed. It has been proposed that there is a simple rule\(^8\) that predicts the value of \( m \):

\[
m = n_H + n_q + n_c + n_D - 2,
\]

(9.3)

where \( n_H \) is the number of valence partons in hadron \( H \). This rule is called the dimensional counting rule. For \( pp \) elastic scattering at large \( t \), the rule predicts \( m = 10 \). This is well satisfied by data at PS energies, for \( -t \gtrsim 2.5 \text{ GeV}^2 \), as is shown in figure 9.1. In that figure, the straight lines actually correspond to \( m = 9.7 \). In will be interesting to have good data for other types of beam, to check whether \( m \) changes according to the prediction (9.3).

![Figure 9.1. \( pp \) elastic scattering data at PS energies. The straight lines correspond to (9.2) with \( m = 9.7 \).](image)

Figure 9.1 shows data only at PS energies. If one plots the energy dependence of \( d\sigma/dt \) at fixed \( t \), as in figure 9.2, one sees that over the PS energy range it falls sharply. Data from Fermilab find that the fall-off has become more gentle, while over the IRS energy range there is no detectable energy variation at all. This suggests that a new dynamical mechanism may have become dominant at very high energy.

![Figure 9.2. \( pp \) elastic scattering data at \( -t = 6 \text{ GeV}^2 \) (reference 17).](image)

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A concrete realisation of the dimensional-counting result, \( m = 10 \), is provided by the constituent-interchange diagram of figure 9.3a. Here, the process consists of three quarks, bound together by gluon exchange, and they interact by interchanging a quark between them. It has been known for some time that a different kind of mechanism gives instead \( m = 8 \), so that at any fixed \( \Theta \) it should dominate over the constituent-interchange mechanism when the energy becomes large enough. This mechanism is shown in figure 9.3b: each quark in one proton scatters at the same angle \( \Theta \) on one of the quarks in the other, so that after the scatterings the three components of each proton are again moving together in a new direction and can readily re-combine. If the wide-angle scatterings occur through the exchange of a spin-one gluon, and if \( t \) is large but \( \ll s \), one finds that figure 9.3b gives

\[
\frac{d\sigma}{dt} \sim t^{-8}.
\]

(9.4)

This is independent of \( s \), as are the ISR data. It is compared with the \( t \)-dependence of the data in figure 9.4.

10. Inclusive Processes at Large Transverse Momentum.

Consider the inclusive production of pions at \( 90^\circ \):

\[
\bar{p}p \rightarrow \pi X.
\]

(10.1)

In the typical reaction, where the protons have only a glancing collision, the pion has only small transverse momentum. At small \( p_T \), the spectrum falls sharply with increasing \( p_T \):

\[
E \frac{d\sigma}{dp_T^2} \sim e^{-6p_T},
\]

(10.2)

and it varies rather slowly with energy. However, for \( p_T \gg 1 \text{ GeV}/c \), when presumably the proton-proton collision is more head-on, the spectrum is found to fall much less sharply and at a fixed value of \( p_T \) there is a marked rise with increasing energy. This is seen in the early data of figure 10.1, which shows also the extrapolation of the exponential fit (10.2) to small-\( p_T \) data.

The dynamics of small-\( p_T \) hadronic reactions is complicated, but the change in the character of the spectrum at large \( p_T \) encourages the hope that there is a new and simple dynamics has set in. It is usual to introduce the dimensionless variable

\[
x_T = 2p_T/\sqrt{s},
\]

(10.3)

together with the centre-of-mass angle \( \Theta \) at which the pion is produced. Then if there is scaling, in the sense that there is no dependence on any fixed dimensional parameter, one expects

\[
E \frac{d\sigma}{dp_T^2} \sim p_T^{-n} F(x_T, \Theta)
\]

(10.4)
\( \Theta = 90^\circ \) data for \( p_t^8 \) vs \( d\sigma/d^3p \), when plotted against \( x_t \), lie on a single curve for a wide range of energies. It is widely believed that experiments at large values of \( p_t \) and \( s \) may show that an \( n = 4 \) term is also present. There is a hint that this may be so in recent data at very large \( p_t \) from the ISR, but the situation is far from clear-cut and it will be of great interest to explore it further in the new colliding-beam facilities that are being planned.

Most of the recent experimental work on large-\( p_t \) processes, particularly at the CERN ISR, has been concerned with the study of correlations. It has been found that large-\( p_t \) events contain an interesting jet structure.

In the ordinary, low-\( p_t \) events the final-state particles are found to emerge in two jets, one in the direction of each of the colliding beams. In the large-\( p_t \) events, these longitudinal "beam fragment" jets are still present, but there are in addition two transverse jets. One of these contains the high-\( p_t \) particle used to trigger the detection apparatus. The transverse momentum of the jet on the other side approximately balances that of the trigger-side jet, but it emerges in a different direction from event to event. The characteristic four-jet structure is shown in figure 10.3.

![Figure 10.3. Four-jet structure in high-\( p_t \) events. The two transverse jets have been exaggerated in this figure. The longitudinal jets are apparently similar to those seen in low-\( p_t \) events.](image)

Figure 10.4 shows some of the data that verify the existence of the opposite side jet. These data are obtained with a \( 90^\circ \) trigger pion, having \( p_t \) in the range 2.5 to 4.5 GeV/c. They plot the rapidity difference \( \Delta y \) between pairs of away-side particles, for those events in which there are at least two such particles having \( p_t > 800 \) MeV/c. This restriction is imposed in order to reduce the possibility that the two particles belong not to the away-side transverse jet but instead to one of the two longitudinal jets. This possible confusion always exists and adds to the difficulty of disentangling the final-

![Figure 10.2. \( p_t^8 \) vs \( d\sigma/d^3p \) for \( pp \rightarrow \pi^+X \) at \( 90^\circ \). The upper points are from the British-Scandinavian experiment at the ISR, and the lower ones are from the Chicago-Princeton experiment at Fermilab.](image)
state structure, but it should be reduced when higher trigger $p_T$ values can be achieved. The clear peak seen in the $\Delta y$ distribution at small values of $\Delta y$, both for the case where the two particles have opposite charge and when they have the same charge, is the evidence for the presence of the away-side jet. It is not clear whether the points having large $\Delta y$ correspond to events where one of the two particles still belongs to one of the longitudinal jets, or whether they indicate the presence of some other structure. But at least it is seen that a single away-side transverse jet is a prominent feature of the data.

The trigger-side jet.

Most of the large-$p_T$ experiments so far have triggered on a single high-$p_T$ particle. This means that the trigger selects the special class of high-$p_T$ events where the trigger-side jet gives most of its momentum to one particle, the trigger particle. Normally, a jet does not choose to fragment into hadrons in this way; its momentum is shared more equally among the particles. That is, if one uses a calorimeter trigger and demands a certain $p_T$ in the calorimeter, so that one is triggering on a jet rather than a single particle, most of the events will not contain any one particle that has particularly large $p_T$. Because the single-particle trigger biases towards the selection of rather unusual events, it reduces the magnitude of the cross-section. From a simple study of correlation data from the ISR it was predicted that the cross-section for a calorimeter trigger set at a given value of $p_T$ should be some 2 orders of magnitude greater than for a single particle of the same $p_T$. This appears to have been verified in recent experiments at Fermilab, and so the use of calorimeter triggers is likely in the future to make possible the study of events in which the total $p_T$ is much higher than up to now. The main difficulty will be to design the calorimeter so as to accept the whole of the transverse jet, and only the transverse jet.

With a single-particle trigger, most of the trigger-side jet momentum is usually given to the trigger particle. But occasionally this is not so, and there is a second fast particle in the trigger-side jet. With a trigger $\pi^0$, a fast $\pi^+$ or $\pi^-$ is sometimes found, and plots of the invariant mass of the pair of pions show a clear signal. From this, it is deduced that production of a $\zeta$ with a given $p_T$ is comparable with that of a single pion having the same $p_T$. However, it is far from true that the trigger-side jet structure is fully described by the $\zeta$; one also finds strong correlations between pairs of $\pi^0$, or pairs of pions of the same charge. In fact, in the literature there are two main alternative suggestions as to the identity of the trigger-side jet: (i) it is a fragmenting quark, like an $e^+e^-$ jet, and (ii) it is a quark system, that is either a $\pi$, a $\zeta$, an $\omega$, and higher meson systems, with also perhaps a $\zeta$ continuum.

It is not yet known whether either of these two suggestions is correct. However, there is one problem with the suggestion that the trigger-side jet is a fragmenting quark. This is a measurement of the total momentum in the trigger-side jet. In order to understand this, let us use the following approximate parametrisation of the cross-section for the production of a jet of large transverse momentum (integrated over the angle of production):

$$\frac{d\sigma}{dp_T} = \frac{A}{p_T^{n-1}}$$

(10.5)

It is found that with this parametrisation, $A$ and $n$
are only slowly varying with $P_T$ when the energy is fixed. Assume that the jet fragmentation function scales, as is approximately found in the $e^+e^-$ data of figure 8.7. That is, the probability $F(z)dz$ of finding a hadron whose fractional longitudinal momentum is in the range $z$ to $z + dz$ is a function of $z$ only. Then the cross-section for production of a particle with large transverse momentum $P_T$ is

$$\frac{d\sigma}{dp_T} = \int dP_T \frac{A}{P_T^{n-1}} \int dz \ F(z) \delta(p_T - zP_T)$$

$$= \frac{A}{P_T^{n-1}} \int dz \ z^{-n-2} F(z).$$

(10.6)

This result has two interesting features. First, the powers of $p_T$ and $P_T$ are the same; this is the parent-child relation. Secondly, the data show that $n$ is large, $n \approx 10$, so that the factor $z^{-n-2}$ in the integral means that values of $z$ close to 1 dominate in the integration. This is just the trigger-bias effect: most of the jet momentum is given to the trigger particle. The average total momentum in the jet is calculated by including an extra factor $P_T$ under the integral (10.6):

$$\langle P_T \rangle = \int dP_T P_T \frac{A}{P_T^{n-1}} \int dz \ F(z) \delta(p_T - zP_T) \frac{d\sigma}{dp_T}$$

(10.7)

If the jet is similar to the $e^+e^-$ jet, this can be calculated from the data for $F(z)$ in figure 8.7.

The SPEAR data, where $F(z)$ is constant as $z \to 1$, give an answer close to $1.2P_T$, while the DESY data rather give an answer closer to $1.5P_T$. If, on the other hand, the jet is a q̅q system, any value is possible, because we have no advance knowledge of the relative importance of the different components π, ρ, ω, ...

Figure 10.5 shows data for the total momentum of the charged particles accompanying a trigger particle of transverse momentum $P_T$, approximately within a cone of half-angle 45° surrounding the trigger particle. The straight-line fit is drawn because (10.7) predicts that $\langle P_T \rangle \propto P_T$. Some of the detected momentum corresponds to particles that do not belong to the trigger jet, but rather to the background contributed by the two longitudinal jets. As is indicated in figure 10.5, it is assumed that the contribution from this background varies little with the trigger $P_T$. After correcting for acceptance and for the undetected neutrals, the conclusion then is that $\langle P_T \rangle < 1.1P_T$. Hence the proposal that the jet is similar to an $e^+e^-$ jet is not favoured, even if it is not correct to assume that the background is not constant. However, this needs more study and it would be premature to reach a firm conclusion.

Figure 10.5. Total momentum of charged particles accompanying trigger particle (British-French-Scandinavian).

**Hard-Scattering Models**

Most of the theoretical study of large $p_T$ inclusive processes nowadays is within the framework of hard-scattering models, figure 10.6. In a sense, the statement that a hard-scattering model is appropriate

![Figure 10.6](image)

Figure 10.6. The hard-scattering mechanism. is directly equivalent to the statement that the final state has the four-jet structure shown in figure 10.3. The transverse jets of figure 10.3 are the objects C and D of figure 10.6, and if these are to emerge with large transverse momentum they must have been produced through a wide-angle scattering of some constituent A of one incoming particle on some constituent B of the other. The central question, whose answer we do not yet know, is what are A, B, C and D?

If one makes the simplest assumptions, the calculation of figure 10.6 leads to an inclusive cross-
section having the structure (10.4). I shall discuss later how these assumptions might be modified, so that a different result is obtained. If one uses the dimensional-counting rule (9.2) and (9.3) for the central wide-angle scattering, one obtains

\begin{align}
\text{qq} & \rightarrow \text{qq}, \text{qg} \rightarrow \text{qg}, \text{gg} \rightarrow \text{gg etc} \quad n = 4 \\
\text{q\overline{q}} & \rightarrow \text{M}, \text{qM} \rightarrow \text{qM} \quad n = 8,
\end{align}

(10.8)

where g denotes a gluon and M a q\overline{q} system. The data strongly favour \( n = 8 \), but I shall explain later how perhaps the processes that give \( n = 4 \) with the simple assumptions might be adapted so as to come closer to the data. The interest in this is that these processes are all associated with quantum chromodynamics in a direct way.

### 11. The Longitudinal Jets.

The large \( p_T \) event structure of figure 10.3 contains a pair of longitudinal beam-fragmentation jets, in addition to the transverse jets. A pair of beam-fragmentation jets is a feature also of small-\( p_T \) processes. Because such processes do not involve any hard scattering, it is not clear whether they have any simple parton-model description. However, the literature contains several apparently very successful calculations of beam fragmentation in parton-model pictures, so that even if we are not able to give these calculations any fundamental justification they are worth considering seriously.

Perhaps the most obvious calculation does not work: it gives too few fast hadrons, compared with experimental data. In this approach, the quarks are supposed to be distributed in the beam particle with the longitudinal momentum distributions measured in deep inelastic lepton scattering, and they then fragment into hadrons with the fragmentation functions measured in \( e^+e^- \) annihilation. The literature contains two apparently different modifications of this approach. Each seems rather successful, but they cannot both be correct.

In the first picture, the quark again fragments as in \( e^+e^- \) annihilation, but instead of being given a variable fractional longitudinal momentum within the beam particle it is assumed to take almost all its momentum. The excuse for this assumption is that then the other constituents of the beam particle must be moving rather slowly, and this is said to be necessary in order that there may be an appreciable interaction with the target. In the second picture, the quark is given the longitudinal momentum distribution within the beam particle as measured in lepton scattering, but it does not fragment. Rather, it annihilates with a slow antiquark produced in some other part of the interaction.

### 12. Backgrounds in \( W \) Production.

Figure 12.1 shows data for the production of direct electron production (with contributions from decay of pions subtracted out), compared with production of pions at the same transverse momentum. As is indicated in the figure, these electrons are thought to be produced from a variety of sources, mostly rather uninteresting ones. It is not clear whether there is also a substantial signal from some more interesting origin, such as charmed-particle production. The ratio \( e/\pi \) is seen to be at the \( 10^{-4} \) level, and this is found to be the case also for the ratio \( \mu/\pi \). The direct muon production will provide a background in experiments that aim to detect the \( W \) through its decay \( W \rightarrow \mu \nu \).

The \( W \) is expected to decay through the simple vertex Feynman graph of figure 12.2. The coupling at the vertex is g, given in (4.4), and the fermion lines are \( \mu \nu \) for the leptonic decay (or \( e^\gamma \nu \)), and a quark-antiquark pair in the case of a hadronic decay. Each diagram gives a readily calculable contribution to the width of the \( W \), so the partial width into any channel is known, but the total number of channels available for \( W \) decay is not known. Considering only
and four quark flavours gives

\[
\frac{\Gamma(\mu\nu)}{\Gamma_{\text{total}}} = \frac{1}{8},
\]

(12.1)

but this estimate omits the heavy lepton \( \tau \), the new quark that constitutes the \( T \), and all the other new quark flavours that will surely be found to have mass less than \( \frac{1}{2} M_W \), that is less than 30 to 40 GeV. So the estimate (12.1) could well be too large by a factor of 2 or more.

Figure 2.2. \( W \) decay; the coupling is either to leptons or \( \tau \) quarks.

Using the branching ratio (12.1) and convoluting the \( W \rightarrow \mu\nu \) decay distribution with a \( W \) production calculation such as is shown in figure 6.1, the muon signal at \( 90^\circ \) is as is shown in figure 12.3. There

\[
\frac{d^3 \sigma}{d\not{p}_T \, d\phi \, dx}
\]

Figure 12.3. Production of muons at \( 90^\circ \) via a \( W \), for three possible values of \( M_W \), in \( pp \) collisions at \( \sqrt{s} = 400 \) GeV. The dashed curve represents the estimated background from direct muons (reference 21).

is a peak at \( p_T = \frac{1}{2} M_W \); the shape of the rise to this peak is determined mainly by the longitudinal momentum distribution of the \( W \) predicted from the Drell-Yan production mechanism, while the right-hand side of the peak has a fall-off largely determined by the total width of the \( W \). If the total width is twice as much as is assumed in (12.1), the peak in figure 12.3 is correspondingly wider and \( \sqrt{p} \) the height.

The calculation has included no transverse momentum for the \( W \); there might well be several GeV/c of transverse momentum, which would smudge out the peak. The dotted line in figure 12.3 is an estimate of the background from direct muon production, assuming that \( \mu/\pi \) remains at about \( 10^{-4} \) and making what is in this context the most pessimistic prediction about pion production at these large values of \( p_T \) and \( s \), namely that the value of \( n \) in (10.4) has changed to 4.

Figure 12.3 encourages the hope that the \( W \) may be readily detectable from the muon signal alone, if it is not, things can be improved by using a hadron calorimeter opposite to the muon detector, so as to pick out those events where the large transverse momentum of the muon is balanced on the other side by an undetected neutrino instead of the hadron jet expected in the case of the background muons.

In the case of the hadronic decays of the \( W \), the quarks in figure 12.2 fragment into hadrons and a pair of jets is expected exactly similar to those seen in \( e^+e^- \) annihilation. However, from what we know about jet production in ordinary large \( p_T \) reactions it is expected that the background of jets will swamp the signal from the \( W \), even if \( n = 4 \) in (10.4) has not yet been reached. But perhaps it will be possible to establish the presence of the \( W \) from a peak in the invariant mass distribution of the combined pair of transverse jets.


In the various reactions that I have discussed, scaling seems to be a remarkably good feature of the data. Nevertheless, there are clear deviations from scaling, as is seen in the deep inelastic muon and neutrino scattering data shown in figures 2.3 and 13.1. At small \( x \), \( F_2 \) rises slowly with increasing \( -q^2 \), while at large \( x \) it falls.

Figure 13.1. Measurements of \( F_2(x,q^2) \) in neutrino scattering (CDHS collaboration).
Mass Effects.

A large part of the deviations from scaling is due to parton mass effects. As I shall now explain, unfortunately we do not know how to calculate these mass effects at all exactly, so that we cannot be sure how much of the breaking of scaling has a more interesting dynamical origin.

Consider the simple parton model that eventually is supposed to scale at large enough values of $\nu$ and $-q^2$ (figure 13.2). The question that must be answered is how this scaling is approached, so that one needs to calculate the next-to-leading order contribution, which contributes terms of order $\nu^{-1}$ to $F_2(x)$. It is found that, correct to terms of order $\nu^{-1}$, the contribution from figure 13.2 is found essentially by replacing $x$ by

$$\frac{\xi}{2\nu} = x + \frac{\langle s \rangle - x^2 M^2}{2\nu},$$

(13.1)

where $M$ is the mass of the target hadron and $\langle s \rangle$ is the average value of the squared invariant mass of the quark jet. That is, $F_2$ is constant at fixed $\xi$. Notice that this result is correct only up to terms in $\nu^{-1}$, which may not be good enough for a completely accurate analysis of present data. But there is a more serious problem that it takes account only of the impulse-approximation contribution of figure 13.2.

In the scaling parton model, this contribution is the only one that survives in the ultimate scaling limit, and in leading order it satisfies the current-conservation conditions $q_{\mu} W^{\nu} = W^{\mu} q_{\nu} = 0$. But in non-leading orders it does not, so that in non-leading orders there must be contributions from other, more complicated diagrams. We have no idea of how to calculate these.

If we assume that nevertheless the variable $\xi$ provides at least an approximate description of the mass effects, we explain a large part of the deviations from scaling. Notice that we must consider the contribution from each quark flavour separately, because $\langle s \rangle$ varies with the flavour. We do not understand the effects of the quark-confining forces, but presumably $\langle s \rangle$ describes the mass of the fragmenting quark before the confining force has had a chance to act, and so for $u$ and $d$ quarks it is less than $(350 \text{ MeV})^2$. These quarks dominate $F_2$ for $x$ large, where only the valence contributions matter. Hence for large $x$, $\xi \approx x - x^2 M^2/2\nu$. So if $\xi$ is fixed $\xi$, the large $x$ part of the plot of $F_2$ against $x$ approaches the ultimate scaling curve from the left as $\nu$ or $-q^2$ increases. Taking account of the shape of the curve, this means that, at fixed large $x$, $F_2$ decreases with increasing $-q^2$. As we have seen in figures 2.4 and 13.1, the data do show a decrease of $F_2$ at fixed large $x$. Near $x = 0, \xi \approx x + \langle s \rangle/2\nu$, so that the variation is in the opposite direction, as also is seen in the data. The variation resulting from heavy quarks can be particularly rapid, until values of $\nu \gg$ their squared mass $\langle s \rangle$ are reached.

It is an interesting question how much of the rise seen at small $x$ is due to the contribution from charmed quarks. Notice that a charmed quark may contribute to $F_2$ without any charmed particles appearing in the final state; just as the final-state interaction that must be added to figure 13.1 annihilates fractional charge, so also it may annihilate charm. Indeed, there is a kinematic incentive for it to do so, until $\nu$ becomes very large, because charmed particles are relatively massive.

QCD Effects.

Once the mass effects are subtracted out (if only we knew how to do this), the remaining scale-breaking is interpreted as coming about because the wavelength of the virtual photon decreases as $-q^2$ increases, and so structure in the partons themselves is progressively revealed.

A concrete model in which this occurs is provided by quantum chromodynamics. Rather as in quantum electrodynamics the force is transmitted by the spin-one photon, in QCD it is transmitted by the spin-one gluon. However, while the photon couples to charge but itself carries no charge, the gluon couples to colour and does carry colour. Consequently, in QCD there is a three-gluon vertex which has no analogue in QED, so that there are vital differences between the two theories.

The theory contains a bare coupling between gluons and quarks, figure 13.3a. This is dressed by various insertions; some of the lowest-order ones are shown in figure 13.3b. The various terms must be
large $x$ is to decrease $F_2$ with increasing $-q^2$. Another possible QCD effect is shown in figure 13.4b. Here a gluon constituent of the target hadron couples to a quark-antiquark pair. Because the gluon has zero charge, it cannot absorb the virtual photon directly, but the photon can couple to the $q\bar{q}$ pair. This pair production occurs mainly at small values of $x$, and there it produces a rise in $F_2$ as $-q^2$ increases.

Another QCD effect is that a quark may radiate a gluon before it fragments to form a jet of hadrons. This can occur in deep inelastic scattering, as shown in figure 13.4c. It turns out that the interference between this diagram and that of figure 13.4a also contributes to the breaking of scaling. The radiation of a gluon before the quark fragments can also occur in the $e^+e^-$ annihilation diagram, as in figure 13.5a.

A quark may radiate a gluon, which then fragments to form a jet of hadrons. In deep inelastic $e$ or $\mu$ scattering, this may happen before the quark absorbs the virtual photon, as in figure 13.4a. This means that the fractional momentum probed by the photon is reduced, some of it having already been radiated away by the gluon. So the curve $F_2(x)$ is shifted to the left in the $x$ plot, and the more so the larger the value of $-q^2$. Once again, the effect at fixed $x$ is to decrease $F_2$ with increasing $-q^2$. Another possible QCD effect is shown in figure 13.4b. Here a gluon constituent of the target hadron couples to a quark-antiquark pair. Because the gluon has zero charge, it cannot absorb the virtual photon directly, but the photon can couple to the $q\bar{q}$ pair. This pair production occurs mainly at small values of $x$, and there it produces a rise in $F_2$ as $-q^2$ increases.

Another QCD effect is that a quark may radiate a gluon before it fragments to form a jet of hadrons. This can occur in deep inelastic scattering, as shown in figure 13.4c. It turns out that the interference between this diagram and that of figure 13.4a also contributes to the breaking of scaling. The radiation of a gluon before the quark fragments can also occur in the $e^+e^-$ annihilation diagram, as in figure 13.5a.

$$\alpha_s = \frac{12\pi}{25 \log Q^2/\Lambda^2}.$$ (13.3)

Here $Q^2$ is the squared 4-momentum of whichever leg in the diagram of figure 13.3c is being taken off-shell, and $\Lambda$ is a parameter that cannot be calculated. It would be helpful to have a direct physical interpretation of the mass-scale set by $\Lambda$ (or the length-scale set by $\Lambda^{-1}$), but none seems to have been given yet. It is usually supposed that $\Lambda \approx 500$ MeV. This

Figure 13.3. The quark-gluon vertex: a) the bare vertex, b) some lowest-order corrections to it, c) the complete vertex.

Figure 13.4. Low-order QCD corrections to figure 13.2.
comes from fits to the scale-breaking in deep inelastic $e$ or $\mu$ scattering. However, because of the problem of how correctly to allow for the mass effects, and because, particularly at small $x$, the scale breaking obtained from the diagrams of figure 13.4 is sensitive to the shape of the unknown distribution of gluons in the hadron, this value for $\Lambda$ cannot be accepted as final. Figure 13.6 illustrates the results of one particular calculation of $F_2$. With $\Lambda = 500$ MeV, $\alpha_s = \frac{1}{3}$ in the range $10 \text{ GeV}^2 < Q^2 < 50 \text{ GeV}^2$, so that $\alpha_s/\pi \approx 10\%$ and the QCD effect in (13.2) is actually rather small.

![Graph showing QCD corrections to $F_2(x,Q^2)$](image)

Figure 13.6. The effect of QCD corrections to $F_2(x,Q^2)$. At small $x$ the curves are sensitive to the shape of the unknown distribution of gluons in the proton. (Reference 23).

Notice that, except in deep inelastic scattering where there is an alternative, operator formalism not based directly on individual perturbation-theory diagrams, there is always a problem in how to identify $Q^2$ with the external variables in a reaction. Theoretical manipulations work to leading order in logarithms, and the problem is that to leading order one has, for example $\log Q^2 = \log \frac{1}{2} Q^2 = \log (Q^2 + \text{const.})$. Thus while in $e^+e^-$ annihilation it seems rather natural to identify $Q^2 = s$, the theoretical work cannot show that it should not rather be, say, $Q^2 = \frac{1}{2} s$. Asymptotically it makes no difference but at present values of $s$ it does matter which choice is made.

Drell-Yan in QCD.

The Drell-Yan formula (5.9) was initially derived assuming that the parton distributions scale and so are functions of $x$ only. If the $e$ or $\mu$ scattering experiments find that there is also a variation with $q^2$, one might guess that this variation should be reflected in (5.9). In deep inelastic scattering, $q^2$ is a momentum transfer and so $q^2 < 0$, while in the Drell-Yan process the large variable is $M_{LL}^2 > 0$. Perhaps the obvious guess is to insert into the Drell-Yan formula the deep inelastic scattering data obtained at $q^2 = -M_{LL}^2$. In so far as the QCD effects are concerned, this has been verified to lowest order in $\alpha_s$ and to leading order in logarithms. So again there is the problem whether one should not really have for example $q^2 = -\frac{1}{2} M_{LL}^2$, or some other choice that is equivalent to leading order in logarithm.

Asymptotically it makes no difference, but at present accessible values of $M_{LL}$ the different choices do give large numerical differences. In addition, the mass effects in deep inelastic scattering and the Drell-Yan process are certainly different, but cannot really be calculated in either case. Figure 13.7,

![Graph showing predictions for scale-breaking effects in continuum dilepton production](image)

Figure 13.7. Predictions for scale-breaking effects in continuum dilepton production (reference 24).
which shows predictions for scale-breaking in the Drell-Yan process, should be viewed with these difficulties in mind.

14. The Transverse Momentum of Partons.

The transverse momentum of large-$M_{LL}$ lepton pairs produced in proton-nucleus collisions is measured to be surprisingly large: see figure 14.1. The transverse momentum of the dilepton is equal to that of the virtual photon. In the Drell-Yan mechanism, this is in turn equal to the sum of the transverse momenta of the quark and antiquark that fuse to form the photon. Since the sum here is a vector sum, $Q_T = k_T^1 + k_T^2$, and the relative orientation of $k_T^1$ and $k_T^2$ is presumably random, one has $\langle Q_T^2 \rangle = \langle k_T^1 \rangle^2 + \langle k_T^2 \rangle^2$. Assuming that the average transverse momenta of the quark and the antiquark are equal, each must therefore be equal to $\langle Q_T^2 \rangle / 2 = 850$ MeV/c.

Part of this transverse momentum can originate from the QCD effects shown in figure 14.2. In these diagrams, the large $q_T$ of the dilepton is balanced, at least partly, by a gluon jet or a quark jet recoiling with transverse momentum in the opposite direction. Calculations find that the observed $\langle Q_T^2 \rangle$ is too large for all of it to be balanced by the recoiling quark or gluon jets. That is, the quark or gluon constituents in the hadron wave function must have some "primordial" transverse momentum before the QCD effects occur. It is not known how large this primordial component should be expected to be. It probably varies with the fractional longitudinal momentum $x$ of the constituent. One does not really know how to combine it with the transverse momentum generated by the QCD effects; the calculations shown in figure 14.1 simply add on a constant 500 MeV/c to the QCD-generated contributions.

This is a rather typical problem encountered in trying to compare the predictions of QCD with experiment. Although asymptotic freedom makes it possible to calculate the short-distance effects in a strong interaction by perturbation theory, in most cases there are parts of any reaction that do not depend on short-distance structure. These cannot be calculated by perturbation theory, and their effects can well swamp the ones that can be calculated.

15. QCD and Large $p_T$.

In the hard scattering diagram of figure 10.6, an obvious choice for the central wide-angle scattering is $qq \rightarrow qq$. If it is assumed (i) that this occurs through the exchange of a single gluon with constant coupling, (ii) that the distributions of quarks in the initial hadrons scale, (iii) that the fragmentation functions of the quarks into hadrons scale, and (iv) that the quarks have negligible transverse momentum before they scatter each other, one obtains (10.4) with $n = 4$. If each of the assumptions (i) to (iv) is relaxed, the qq scattering diagram provides a much better fit to the data, which prefer the value $n = 8$.

With present knowledge, relaxing each of the four assumptions introduces a considerable amount of arbitrariness into the calculation. It has been shown that in QCD the assumptions (i), (ii) and (iii) should be modified so that the constant coupling of the gluon is replaced by the "running" coupling (13.3), and the quark distributions and fragmentation functions should be replaced by the nonscaling ones as measured in lepton scattering and $e^+e^-$ annihilation. (In the latter case, the break-
(ing of scaling is predicted but not yet seen.)

There are the usual problems about mass effects and how to identify $Q^2$ with the variables in the diagram. When the modifications to the assumptions (i) and (iii) are made, the contribution from the mechanism no longer takes the simple form (10.4) with a fixed value of $n$. However, it is useful to talk in terms of the resulting effective value of $n$. According to the calculations of Field\textsuperscript{25}, using the running coupling changes $n$ from 4 to 4.9, introducing breaking of scaling into the quark distributions changes $n$ from 4.9 to 5.2, and breaking the scaling of the quark fragmentation changes $n$ from 5.2 to 5.9.

In the framework of the hard-scattering model, the large $p_T$ cross-section is small because the central wide-angle scattering cross-section is small. However, if the constituents A and B in figure 10.6 have transverse momentum before the scattering, and if this is aligned towards the trigger, the central scattering does not have to be through such a wide angle and so the magnitude of the cross-section is enhanced. Field finds that putting in an average transverse momentum of 850 MeV/c for each quark before the scattering changes the effective value of $n$ from 5.7 to 7.1. Further, the calculated magnitude of the cross-section then fits data well for $p_T \gtrsim 6$ GeV/c.

To improve the fit for smaller values of $p_T$, contributions from other choices for the central scattering are included. In QCD, natural choices are $qg \to qg$, $gg \to gg$, $q\bar{q} \to gg$ and vice versa, where $g$ denotes a gluon. Expected sizes for different contributions are shown in figure 15.1. When all the contributions are added together, agreement with the data is surprisingly good, though certainly not perfect. However, I must stress again that there are several arbitrary ingredients in the calculation.

Finally, I mention direct photon production at large $p_T$ (that is, photons that do not come from the decay of a $\pi^0$ or other particle). In QCD, this can be calculated from the diagrams of figure 14.2 by replacing the virtual photon with its attached lepton by a real, high $p_T$ photon. Predictions are shown in figure 15.2, together with a representation of data for $\pi^0$ production. The $Y$ production is expected to be rather large. Similar predictions have been made in some other models; see for example figure 15.3. The "data" in that figure should probably be interpreted as upper limits, because of the great difficulty of the experiment.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure15.1.png}
\caption{Contributions of different hard scatterings to $90^\circ$ pion production at $\sqrt{s} = 53$ GeV (reference 27).}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure15.2.png}
\caption{QCD predictions for direct photon production at $90^\circ$ with $\sqrt{s} = 53$ GeV, and data for $\pi^0$ production (reference 24).}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure15.3.png}
\caption{Direct photon production. Data from CERN 412, prediction from reference 28.}
\end{figure}

The most striking feature of the quark parton model is how well it seems to agree with experiment, even in its simplest version. It is important to try and achieve understanding of quark confinement, and of its effects on the structure of final states, but it seems that these effects may be fairly simple.

Quantum chromodynamics provides the only full field theory of strong interactions available at present, and the immediate future will see much work towards trying to establish its connection with experiment. This is not straightforward, because only certain quantities in the theory may be calculated in perturbation theory, and it is not easy in the data to separate these from those that can not be calculated. However, with a theory that makes predictions for such a wide range of different reactions there is the exciting prospect that, by working closely together, experimentalists and theorists can gradually piece together a complete picture of interactions at high energy and short distance.

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