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QUARK MASSES, ISOSPIN BREAKING AND THE VECTOR PIECE OF $\pi \rightarrow e\nu\gamma$

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ABSTRACT

We discuss a direct way to detect isospin breaking generated by the current quark mass matrix. With PCAC the vector form factor of $\pi \rightarrow e\nu\gamma$ can be described by a pure current process. In the chiral symmetry limit we prove, to all orders in QCD, that $\xi = m_d/m_u$ zero or infinity induces a 50% isospin breaking in the physical amplitude. Detailed effects, as function of ξ , are calculated from the quark triangle diagram. Experiments to look for this possible departure from isospin invariance are feasible.

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The standard picture of hadron physics based on quarks and gluons (QCD)¹⁾ implies that the symmetries of strong interactions are determined by the quark mass matrix. In particular, as long as the up and down quark masses are much smaller^{2,3)} than the typical mass scale parameter $\mu \approx 700$ MeV which governs scaling violations, one expects that chiral $SU(2) \times SU(2)$ be a very good symmetry. This symmetry is realized in the Nambu-Goldstone mode: the pions are the almost massless Goldstone bosons, which provides an explanation⁴⁾ for the successes of soft pion techniques.

One striking feature of the predicted current quark mass ratio^{2,3,5)} is the large isospin breaking in the free Lagrangian: $m_d - m_u$ is of the order of m_u, m_d themselves. However, for hadrons, isospin is a good symmetry because $m_d - m_u$ is still very small when compared to μ ; due to this most hadronic processes are insensitive to the small current quark masses. Isospin violations will therefore only be observed in processes where current, and not constituent, quark masses play a role and which furthermore depend strongly on their actual values. Weinberg³⁾ advocates that one such process is zero-energy πN scattering, where isospin breaking should show up in the corresponding scattering lengths.

In this paper we suggest a particularly clean candidate where to observe large isospin violations: the vector form factor of the structure dependent term in the radiative charged pion decay: $\pi \rightarrow e \nu \gamma$. This observable satisfies a number of important requirements which, we believe, makes it unique. (i) By PCAC one can describe it in terms of currents only. The explicit absence of hadrons then allows one to believe that confinement, and therefore constituent quarks, should not be relevant. (ii) Its value is exactly known for $m_u = m_d$, for it is then given, to all orders in strong interactions, by the Adler-Bell-Jackiw anomaly. This sets the scale of the isospin breaking corrections due to different up and down quark masses. (iii) At zero order in the gluon coupling constant, α_c , the result depends only on $\xi = \frac{m_d}{m_u}$ and is infra-red stable inasmuch as it is finite in both limits $\xi \rightarrow 0$ and $\xi \rightarrow \infty$. It satisfies, in this sense, a criterion à la Sterman and Weinberg⁶⁾, so that a perturbative calculation in α_c should make some sense. (iv) To first order in α_c the result is infra-red stable; in fact, it is finite in the chiral symmetry limit $m_u, m_d \rightarrow 0$, ξ fixed. These points justify our main assumption: the chiral symmetry limit is meaningful for the exact calculation. This implies that the exact result depends (in the chiral limit) only on ξ (and not on $\frac{m_u}{\mu}, \frac{m_d}{\mu}$), which is a renormalization group invariant ratio in QCD and has only small electromagnetic and weak corrections at low energies^{5,7)}. A key result from our analysis then is that,

not only for $\xi = 1$, but also for $\xi \rightarrow 0$ and $\xi \rightarrow \infty$ the gluon corrections cancel exactly, so that a zero order calculation gives the full result. Even more important, in these limits, where our result is exact, we find an impressive 50% isospin breaking. This shows that isospin violations are real, in the process under consideration, as they cannot be washed out by strong interaction renormalization.

PCAC. - We consider the usual QCD Lagrangian and work to lowest order in weak and electromagnetic interactions. PCAC is imposed as a definition of the interpolating pion field. This leads, in the soft pion approximation, to an effective γ_5 coupling between pions and quarks $i g_{jk} \bar{q}_j \gamma_5 q_k$ with ($q_u \equiv u$, $q_d \equiv d$)

$$\frac{g_{uu}}{m_u} = \frac{g_{dd}}{m_d} = \frac{2g_{ud}}{m_u + m_d} = -\frac{\sqrt{2}}{f_\pi m_\pi}. \quad (1)$$

It is easy to check that the old Adler-Bell-Jackiw calculation^{8,9)} of the $\pi^0 \rightarrow 2\gamma$ amplitude is not affected by different quark masses. If $v(\mu)$ is the index of the photon with momentum $k(k')$, the decay amplitude is given by

$$\begin{aligned} T_0^{\nu\mu} &= \epsilon^{\mu\nu\alpha\beta} k_\alpha k'_\beta \frac{\alpha}{3\pi} \left(4 \frac{g_{uu}}{m_u} - \frac{g_{dd}}{m_d} \right) \\ &= -\frac{\sqrt{2}\alpha}{\pi f_\pi m_\pi} \epsilon^{\nu\mu\alpha\beta} k_\alpha k'_\beta, \end{aligned} \quad (2)$$

the last equality by virtue of Eq. (1). This result, although obtained to zero order in α_c , is in fact exact to all orders in the strong interactions. This is due to the well-known Adler-Sutherland theorem⁹⁾ which connects it to the axial current anomaly, itself independent of masses or mass ratios, and unrenormalized by strong interactions. Eq. (2) gives the correct magnitude and sign¹⁰⁾ for the $\pi^0 \rightarrow 2\gamma$ decay amplitude.

Radiative charged pion decay amplitude. - We will study the vector part of the structure contribution to this amplitude in terms of the three current Green's function

$$T_{\mu\nu\lambda} \equiv \int d^4x d^4y e^{iq \cdot x} e^{-ip \cdot y} \langle 0 | T V_\mu^+(x) J_\nu(0) A_\lambda(y) | 0 \rangle, \quad (3)$$

where the indices μ, ν, λ and the momenta q, k, p refer to the charged

vector, electromagnetic and axial current, respectively. For different up and down quark masses the divergence of the charged vector current is no longer vanishing. This means that Sutherland's result⁸⁾, the vanishing of $p^\beta T_{\mu\nu\beta}$ in the soft pion limit, does not hold anymore. Instead, what happens, is that the axial divergence is determined by the vector divergence in this limit. One finds

$$p^\beta T_{\mu\nu\beta} = q^\alpha T_{\alpha\mu\nu}. \quad (4)$$

The rhs of this equation is going to be responsible for the isospin breaking effects in the physical π decay amplitude. This process is given by the Green's function

$$T_{\mu\nu} = \int d^4x d^4y e^{iq \cdot x} e^{-ip \cdot y} \langle 0 | T V_\mu^\dagger(x) J_\nu(0) J_\pi(y) | 0 \rangle, \quad (5)$$

where $J_\pi(y) = (\not{\partial} + m_\pi^2) \phi_\pi(y)$. Because of PCAC and the soft pion approximation the lhs of Eq. (4) is related to Eq. (5) through the anomalous Ward identity

$$p^\beta T_{\mu\nu\beta} = i f_\pi m_\pi T_{\mu\nu} + S_{\mu\nu} \quad (6)$$

where $S_{\mu\nu}$ is the charged axial anomaly. On invariance and dimensional grounds one can write

$$T^{\mu\nu} = - \frac{e}{4\pi^2 f_\pi m_\pi} \epsilon^{\mu\nu\alpha\beta} k_\alpha q_\beta f(\xi), \quad (7)$$

where $f(\xi)$ is a function of only $\xi = m_d/m_u$. The use of PCAC and the equations of motion of QCD allows one to write the rhs of Eq. (4) and Eq. (5) in terms of the bilinears $(m_d - m_u) \bar{u} d$ and $(m_d + m_u) \bar{u} \gamma_5 d$ respectively. Eq. (6) can then be rewritten in the following simple form

$$f(\xi) \equiv (1 + \xi) F(\xi) = 1 - (1 - \xi) G(\xi), \quad (8)$$

where the first 1 of the rhs corresponds to the anomaly. This expression shows explicitly the zeros of $f(\xi)$. Eq. 8 shows that for $\xi = 1$ the ampli-

tude is precisely given by the anomaly, $f(1) = 1$, and is related to the $\pi^0 \rightarrow 2\gamma$ amplitude by an isospin rotation. The function $G(\xi)$ is the one associated with the $\bar{u}d$ vertex and the electromagnetic and axial currents. It is possible to connect it with $F(\xi)$ by displacing the γ_5 from the axial current to the $\bar{u}d$ vertex. It is not difficult to convince oneself that to zero order in α_c (Fig. 1) this just corresponds to changing the sign of the mass in the quark propagator which connects the two vertices. In fact, one can say more. For strong interactions mediated by vector gluons, as in QCD, this result holds to all orders in α_c . The reason is simply that each gluon-quark vertex introduces an extra propagator, so that the net effect of the γ_5 displacement is again a change of sign of the quark mass. This then gives the exact relation

$$G(\xi) = F(-\xi) \quad (9)$$

which, once plugged into Eq. (8) predicts, to all orders in QCD,*)

$$f(\xi=0) = 1/2, \quad f(\xi=\infty) = 1/2, \quad (10)$$

the second condition resulting from reversing the roles of up and down quarks.

Our exact result Eq. (10) shows that isospin breaking in the quark mass matrix actually implies isospin breaking in the physical amplitude. The maximal breaking in the quark masses produces an isospin breaking factor $\frac{1}{2}$ in the amplitude, and this statement is exact in QCD.

Detailed zero order results. - Although there are some claims that $m_u \ll m_d$ and even¹¹⁾ $m_u = 0$, in which case Eq. (10) gives an exact prediction for isospin breaking, it may very well^{3,5,12)} be that $\xi \sim 2$. We cannot exploit our Eq. (8) in this case. However, since a zero order calculation (no gluonic corrections) reproduces the exact results at $\xi = 0, 1, \infty$, we feel justified to use it as a reasonable interpolation in between. From the diagrams (a) and (b) of Fig. 1 one easily calculates the corresponding functions $f_u(\xi)$ and $f_d(\xi)$, which satisfy $f_d(\xi) = f_u(\xi^{-1})$. (This identity can be shown to be

*) The assumption that a perturbative calculation gives the correct answer is, of course, essential to get this. If they existed, for the process at hand, non-perturbative effects could modify Eq. (10).

valid in general.) The physical amplitude is then obtained from the combination

$$f(\xi) = \sum_{\text{colours}} \{ Q_u^c f_u(\xi) + Q_d^c f_u(\xi^{-1}) \}, \quad (11)$$

where c refers to the colour multiplets and Q_u^c and Q_d^c are the quark charges. In any coloured quark model the result is

$$f(\xi) = \frac{1}{2(\xi^2-1)} \{ \xi^2 - 6\xi - 1 + 2\xi \frac{2\xi^2+1}{\xi^2-1} \log \xi^2 \}. \quad (12)$$

The functions $f(\xi)$ as given in Eq. (12) and $f_u(\xi)$ are shown in Fig. 2. One realizes that the particular structure of $f(\xi)$ comes from the combination imposed by the quark charges, the function $f_u(\xi)$ itself being much smoother. Taking Eq. (12) as an indication of the size of the isospin breaking one gets results ranging from 1.18 to 1.32 when $\xi = 1.5$ to $\xi = 2.6$ ^{3,5,12}). The cross-over through the value 1 for the function $f(\xi)$ takes place ^{*)} at $\xi \approx 20$. We would like to emphasize again that, except for $\xi = 0, 1, \infty$ the result given in Eq. (12) may be modified by gluonic corrections. A detailed study of these is in progress ¹³⁾.

Experimental aspects. - The structure radiation of the positive pion electronic radiative decay has been measured several times ¹⁴⁾. The aim was a determination of the axial form factor, assuming the vector form factor as determined by CVC and isospin symmetry from the neutral pion lifetime. We propose that an independent determination of the charged vector form factor should be undertaken. A separation of the vector and axial vector pieces is possible when the energy spectra of electrons and photons (or the opening angle distribution) is measured. For the positive pion the spectra of the structure dependent contribution have the following form

$$\frac{d^2 \Gamma_+}{dx dy} = \Gamma_{r^0} \frac{G^2 \cos^2 \theta_c}{8\pi^2 e^2} m_\pi^4 \{ (1+y)^2 (1-x)(1-x-y)^2 + (1-y)^2 (1-x)(1-y)^2 \} |f(\xi)|^2, \quad (13)$$

^{*)} Taking this as given, an analysis of the radiative charged kaon decay similar to the one presented here would lead to some -30% effect due to SU(3) breaking in the vector amplitude of the $K \rightarrow e \nu \gamma$ decay.

with $x = 2 E_\gamma / m_\pi$ and $y = 2 E_e / m_\pi$. The parameter γ in Eq. (13) gives the relative contribution of the axial vector form factor versus the vector one. Theoretical calculations¹⁵⁾ of it do not agree with the experimental value, as determined conventionally. This situation could change if $f(\xi) \neq 1$. One should make a two parameter fit to the spectrum and rate which would then give the values of $f(\xi)$ and γ .

Of great help in separating the two form factors would be a comparison of results for both π^+ and π^- . Our theoretical result for $f(\xi)$ in terms of the isospin breaking parameter $\xi = m_d/m_u$ is the same for both charged decays. The spectrum corresponding to $\pi^- \rightarrow e^- \bar{\nu}_e \gamma$ is obtained from Eq. (13) by changing γ into $-\gamma$. This would require measurements of the decay in flight for these radiative processes which, though difficult, appear feasible with pion beams of present day intensities. A complete separation of the vector and axial vector form factors still presents the $V \nleftrightarrow A$ ambiguity. Its resolution would require measurement of an interference with another process. One possibility is offered by the interference with the bremsstrahlung contribution in the same decay. This interference term is quite small for the electronic radiative decay, but some prospects to isolate this interference exist for the muonic radiative decay in the large photon energy region.

The arguments put forward in this paper show that, to say the least, it should be rewarding to revisit radiative charged pion decays asking new theoretical questions and using present experimental capabilities. They offer a possible scenario where isospin breaking effects in the quark mass matrix should manifest themselves as a large departure of the vector amplitude from its value as determined by conventional CVC arguments.

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FIGURE CAPTIONS

Figure 1 : Quark triangle diagrams for the radiative pion decay.

Figure 2 : Isospin breaking in the physical amplitude as given by $f_u(\xi)$ (broken line) and $f(\xi)$ (continuous line).

(a)

(b)

FIG.1

FIG.2