Ref.TH.2498-CERN

THE PRICE OF NATURAL PARITY CONSERVATION
IN THE NEUTRAL CURRENT SECTOR

P. Binetruy
LAPP -- Annecy
and
M.K. Gaillard ^{*)} and Z. Kunszt ⁺⁾
CERN -- Geneva

E R R A T U M

Equation (4.3) should be corrected by adding the term

$$\frac{\alpha}{\pi} Q^{p,n} \left(\frac{2M_L^2}{M_{ZA}^2} \ln \left(\frac{M_R}{M_L} \right) + \frac{4}{9} \right) .$$

In the corrected expression, the leading log term of C_1 agrees with that obtained by Marciano and Sanda ²⁸⁾, whom we wish to thank for helpful correspondence.

Ref.TH.2498-CERN

16 June 1978

THE PRICE OF NATURAL PARITY CONSERVATION

IN THE NEUTRAL CURRENT SECTOR

P. Binetruy
LAPP -- Annecy

and

M.K. Gaillard *) and Z. Kunszt +)
CERN -- Geneva

A B S T R A C T

We study the requirement that the neutral current sector in $SU(2)_L \otimes SU(2)_R \otimes U(1)$ gauge models be naturally parity conserving in the tree approximation, and therefore calculable. A class of models which reproduces automatically the Weinberg-Salam neutrino neutral current phenomenology cannot be made natural in this sense and at the same time compatible with charged current phenomenology. At best one has small but uncalculable violations of both Weinberg-Salam neutrino phenomenology and parity conservation in neutral currents, as well as a proliferation of fermion and scalar fields and possible difficulties with neutral strangeness changing transitions. A second class of models has calculable parity violation in neutral currents and allows a more restricted fermion and scalar sector, but the Weinberg-Salam neutrino phenomenology does not emerge naturally. For the latter class we evaluate the parity violation induced by radiative corrections, relevant to atomic physics experiments. While technically calculable, it is not practically so, because of their dependence on unknown parameters of the Higgs sector. We also discuss radiative corrections to Cabibbo universality in these models.

*) Laboratoire de Physique Théorique et Hautes Energies, Orsay (Laboratoire associé au C.N.R.S.).

+) Present address : DESY, Hamburg.

1. Introduction

The motivations for extending the Weinberg-Salam $SU(2)_L \otimes U(1)$ gauge group¹⁾ for weak and electromagnetic interactions to a left-right symmetric group²⁻¹⁵⁾ $SU(2)_L \otimes SU(2)_R \otimes U(1)$ are twofold. One of these is purely aesthetic; many theorists believe that the observed left-right asymmetry of weak interactions should arise as a result of spontaneous symmetry break down rather than as a property of the primary interactions. The other is based on the non-observation^{16,17)} of expected parity violating effects¹⁸⁾ in atomic transitions. If the symmetry of a parity conserving $SU(2)_L \otimes SU(2)_R \otimes U(1)$ model were broken in such a way that the neutral current sector remained parity conserving, this would give a more satisfactory explanation of a null atomic physics result than, for example, the ad hoc addition¹⁹⁾ of an $(e, N)_R$ doublet to the $SU(2) \otimes U(1)$ model.

On the other hand, the charged current couplings observed so far are of the V-A type, including apparently those of the (τ, ν_τ) doublet²⁰⁾ and of the charmed quark: flat y distributions observed²¹⁾ for both ν - and $\bar{\nu}$ -induced dilepton events favor the couplings $(cs)_L$ and $(cd)_L$. These results impose the requirement that either "light" fermions have appreciable right handed couplings only to very heavy fermions, or that the right coupled charged W be much heavier than the left coupled one so that its effective Fermi coupling is suppressed. There are thus two distinct phenomenologically acceptable patterns of spontaneous parity violation:

Parity violation in the fermion sector

$$m_{W_L} = m_{W_R} \quad , \quad U_L \neq U_R \quad ; \quad (1.1)$$

parity violation in the charged boson sector:

$$m_{W_L} \neq m_{W_R} \quad , \quad U_L = U_R \quad . \quad (1.2)$$

Here $m_{W_{L,R}}$ are the charged vector boson masses and $U_{L,R}$ are the left- and right-coupling Cabibbo matrices.

A second phenomenological fact is the success^{22,23)} of the Weinberg-Salam model in describing ν -induced neutral current interactions. One would like this phenomenology to emerge naturally, and it has been remarked by many authors^{6-8,24)} that if one starts with an $SU(2)_L \otimes SU(2)_R \otimes U(1)$ symmetric, parity conserving Lagrangian with only two complex Higgs doublets transforming

according to the $(1/2, 0)$ and $(0, 1/2)$ representations of $SU(2)_L \otimes SU(2)_R$, the spontaneously broken symmetry solution automatically conserves parity to lowest order in the neutral sector and reproduces the Weinberg-Salam neutrino-induced neutral current phenomenology with $m_{W_L} = m_{W_R}$. This model is unsatisfactory as it stands because, as discussed below we must require four-component fermion fields to transform according to $(1/2, 0) + (0, 1/2)$ and they will not acquire masses unless we introduce Higgs multiplets transforming like $(1/2, 1/2)$. In addition, if Υ (9.4)²⁵⁾ is a $(\bar{b}b)$ bound state, the absence²⁶⁾ of a high y anomaly which disfavors a $(ub)_R$ coupling, as well as the evidence on V-A coupling for τ and c , makes the alternative (1.2) more attractive as it does not require a proliferation of very heavy quarks and leptons. Separation of the W_L and W_R masses can be achieved by the introduction of $SU(2)$ triplet Higgs representations, $(1, 0)$ and $(0, 1)$, which do not contribute to neutral boson masses. The correct phenomenology is maintained by giving the $SU(2)_R$ triplet a very large vacuum expectation value, and the $SU(2)_L$ triplet a very small one. However, with these complexifications of the Higgs potential the desired phenomenological features of the simple $(1/2, 0) + (0, 1/2)$ Higgs sector no longer emerge naturally.

There is a certain amount of existing literature on parity violating radiative corrections²⁷⁻²⁹⁾ to Z-couplings which are parity conserving in lowest order, and on the conditions^{10,30)} under which these corrections are finite. However, we do not believe that the phenomenological implications of imposing natural - in the technical sense³¹⁾ - parity conservation on the neutral boson sector have been fully pursued, and it is our purpose to present a systematic study of these implications. For the sake of this paper we assume the following phenomenological "facts" to be true, although the experimental situation is not clear-cut^{17,23)}.

a) There is no enhanced parity violation in atomic physics to lowest order in the effective neutral current Fermi coupling constant. We attribute this to parity conservation in the neutral boson sector and demand that it be true up to finite and calculable corrections.

b) Up to present energies, charged current couplings are predominantly V-A.

c) The phenomenology of neutrino induced neutral currents is to a good approximation identical to that of the Weinberg-Salam model.

We then ask under what conditions the above phenomena emerge naturally, i.e. independently of the parameters of the theory. The answer is that there is no acceptable model in which all can be made natural. There are two alternative⁸⁻¹⁰⁾ "minimal" models, only one of which allows calculable parity violation in neutral currents.

i) The Higgs sector consists of two complex doublets, $\chi_L(1/2, 0)$ and $\chi_R(0, 1/2)$ and at least two complex quartets $\Phi_i(1/2, 1/2)$. Condition c) can be achieved by imposing one "unnatural" condition:

$$|\langle \chi_{L,R} \rangle| \gg |\langle \Phi_i \rangle|. \quad (1.3)$$

Then we get $m_L \simeq m_R$, and condition b) is achieved by adjusting the Cabibbo angle as discussed above. This probably implies the existence of 12 leptons and 8 quarks, as well as at least 18 physical Higgs fields. It is then not possible to maintain condition a), but (1.3) assures that parity mixing in neutral currents will be small although uncalculable. On the other hand, the requirement (1.3) implies at least one strong scalar self-coupling, $\lambda \gg 1$, to be consistent with constraints on flavor changing neutral currents; this presumably renders meaningless the zeroth order mass relations.

ii) The Higgs sector contains at least one complex quartet $\Phi(1/2, 1/2)$ which gives fermion masses, one complex quartet $\varrho(1/2, 1/2)$ containing a doubly charged field, and two triplets $\delta_L(1,0)$ and $\delta_R(0,1)$, giving at least 16 physical Higgs particles (including doubly charged ones). Condition a) is now strictly natural. Condition c) can be achieved at the price of two "unnatural" conditions:

$$|\langle \varrho \rangle| \simeq |\langle \Phi \rangle| \quad (1.4)$$

and

$$4 |\alpha \epsilon| \simeq |\langle \delta_L \rangle| [2|\langle \varrho \rangle|^2 + |\langle \delta_R \rangle|^2]^{1/2} \quad (1.5)$$

where α and ϵ are defined by

$$\langle \Phi \rangle = \begin{pmatrix} \epsilon & 0 \\ 0 & \alpha \end{pmatrix}. \quad (1.6)$$

The mixing between left- and right-coupled charged W's is in general arbitrary, which requires an adjustment of the Cabibbo matrices as in i) to satisfy condition b). The proliferation of fermions which this implies can be avoided by simply imposing an arbitrarily large W_R mass. Then b) and c) require

$$|\langle \delta_R \rangle| \gg |\langle g \rangle| \approx |\langle \Phi \rangle| \gg |\langle \delta_L \rangle|, \quad (1.7)$$

a total of three "unnatural" conditions.

It is of course possible that some of the "unnatural" conditions which we find necessary to impose could become natural in the context of a larger group, for example as discussed by De Rujula, Georgi and Glashow¹²⁾. It may also be that demanding natural parity conservation in the $SU(2)_L \otimes SU(2)_R \otimes U(1)$ neutral current sector is too stringent a condition, and one should treat the strength of the parity violation as a parameter to be adjusted to experiment. However in this case one has lost - at least until a higher symmetry structure should emerge - both the predictive power of the simple Weinberg-Salam model and a natural explanation of the null atomic physics results if they should be confirmed by future experiments. Our purpose here is simply to underline the difficulties encountered if one attempts to construct an $SU(2)_L \otimes SU(2)_R \otimes U(1)$ model which can account for the phenomenological features a), b) and c) in a natural way. In essence, the problem is that in order to keep natural parity conservation in the neutral current sector without encountering phenomenological difficulties, we are forced to introduce so many independent vacuum expectation values that the apparent success of the Weinberg-Salam model becomes an accident.

The outline of our paper is as follows. In sect. 2 we discuss the constraints imposed on the fermion sector by the requirements that parity violation in the neutral current sector and absence of left-right mixing in the charged current sector be calculable, without reference to a specific Higgs potential. We show that left-right mixing in the charged current sector cannot be avoided within the context of $SU(2)_L \otimes SU(2)_R \otimes U(1)$ if we require phenomenologically acceptable fermion masses and coupling matrices. We also show that natural parity conservation in the neutral current sector requires that for every left-coupled lepton there must be a right-coupled lepton of the same mass. This means in particular that there are as many coupled massless right handed neutrinos as there are left handed ones. However cosmological arguments^{32,33)} limit the number of light ($m_\nu \leq 1$ MeV) neutrino helicity states to 3 or 4. Further cosmological arguments^{33,34)} exclude stable ($\tau > 3 - 6 \times 10^{17}$ sec) neutrinos in the range

$50 \text{ eV} < m_\nu < 2 \text{ GeV}$. These arguments would then imply that natural parity conservation can be maintained only if $m_\nu > 1 \text{ MeV}$ and there is a non zero $\nu_e - \nu_\tau$ mixing allowing the decay

$$\nu_\tau \rightarrow e^+ e^- \nu_e. \quad (1.8)$$

In section 3 we discuss in detail the structure of the Higgs sector and the arguments leading to the "minimal" parity conserving models i) and ii) above. The constraints found in sect. 2 will be seen to correspond to constraints on the Higgs-fermion Yukawa couplings required to keep the desired symmetry breaking patterns natural. In section 4 we discuss the strength of parity violation in atomic physics in models in which it is calculable. We emphasize the contribution of physical Higgs scalars, which turn out to be important. This result shows that parity violation in atomic physics, even when calculable in a technical sense, is not calculable in a practical sense because of the unknown parameters in the Higgs potential. In section 5 we comment on radiative corrections to Cabibbo universality, which differ only by 10 % from those of the standard model, within the present level of accuracy of the data. We summarize our results in section 6; calculational details are given in appendices.

2. The fermion sector

In this section we study one fermion loop corrections to vector boson self energies. The purpose is to determine under what conditions the absence of parity mixing in the neutral sector, and the absence of left-right mixing in the charged sector can be imposed up to calculable and finite corrections.

2.1. Z_A, Z_V mixing.

Consider first the tree approximation. We start with fermion-vector boson couplings which are invariant under $SU(2)_L \otimes SU(2)_R \otimes U(1)$ and parity ($g_L = g_R = g$):

$$\mathcal{L} = g \sum_f (\bar{f} \vec{W}_L \vec{I}_L f + \bar{f} \vec{W}_R \vec{I}_R f) + g' \sum_f \bar{f} \not{Y} f, \quad (2.1)$$

$$\text{where} \quad Y = Q - I_3^L - I_3^R, \quad (2.2)$$

$$\text{and} \quad \left. \begin{aligned} \not{Y} &= S_\mu \not{\gamma}^\mu, \text{ etc.} \\ L, R &= \frac{1}{2}(1 \mp \gamma_5) \end{aligned} \right\} \quad (2.3)$$

Spontaneous symmetry breaking induces a mass matrix for the neutral boson system which is diagonalized by a unitary transformation:

$$\begin{pmatrix} W_L^3 \\ W_R^3 \\ S \end{pmatrix} = U \begin{pmatrix} A \\ Z_V \\ Z_A \end{pmatrix} \quad (2.4)$$

where A is the electromagnetic field and we want U to be such that Z_V and Z_A couple to pure vector and axial vector currents, respectively. If $I_{L,R}^3$ and Y are understood as operators acting on the space of physical (mass-diagonal) 4-component spinor fields, this requirement implies the relations

$$\left. \begin{aligned} g U_{1i} I_3^L &= g U_{2i} I_3^R, \quad i=1,2 \\ g U_{13} I_3^L &= -g U_{23} I_3^R \\ g' U_{33} Y &= 0 \end{aligned} \right\} \quad (2.5)$$

From the condition that A couples to Q with strength e we have the additional constraints:

$$\left. \begin{aligned} g' U_{31} &= e \neq 0 \\ (g U_{11} - e) I_3^L &= (g U_{21} - e) I_3^R = 0 \end{aligned} \right\} \quad (2.6)$$

Eq. (2.6) with the first of Eqs. (2.5) gives immediately

$$I_3^L = I_3^R \quad (2.7)$$

and the remaining constraints (2.5) determine uniquely the matrix U giving

$$\left. \begin{aligned} A &= \sin \Theta_w (W_L^3 + W_R^3) + \sqrt{\cos 2\Theta_w} S \\ Z_A &= \frac{1}{\sqrt{2}} (W_L^3 - W_R^3) \\ Z_V &= \sqrt{\frac{\cos 2\Theta_w}{2}} (W_L^3 + W_R^3) - \sqrt{2} \sin \Theta_w S \end{aligned} \right\} \quad (2.8)$$

with

$$\sin \theta_w = e/g, \quad g'/g = \frac{\sin \theta_w}{\sqrt{\cos 2\theta_w}} \quad (2.9)$$

The almost trivial constraint (2.7) immediately rules out models ¹⁵⁾ in which one helicity component of a physical fermion field transforms as a doublet and the other as a singlet, unless there are degenerate fermion fields. However, one might argue that one need only preserve pure vector and axial structures for the couplings to the u, d, e system which is relevant for atomic transitions. It is easy to see that the off diagonal Z_ν, Z_n mass matrix element becomes infinitely renormalized and hence uncalculable if we do not impose (2.7) for all fields.

Suppose there is at least one massive fermion f with $I_3^L \neq 0, Q \neq 0$ and pure axial and vector couplings to Z_n and Z_ν , respectively. Then taking the matrix elements of the operator equations (2.5) and (2.6) between f -states, we again obtain the result (2.8) in lowest order. Then the infinite part of the fermion loop contribution to Z_ν, Z_n mixing, Fig. 1, is proportional to

$$\sum_f m_f^2 (I_3^L - I_3^R)_f [\cos 2\theta_w (I_3^R + I_3^L)_f - 4 \sin^2 \theta_w Y_f] \quad (2.10)$$

As long as the fermion masses are arbitrary parameters of the theory, each term in the sum (2.10) must vanish, as is the case for fermions satisfying (2.7). However it is amusing to note that if $\sin^2 \theta_w = 0.3$, quark doublets with I_3^L or $I_3^R = 0$ and $Q = (2/3, -1/3)$, $Y = 1/6$, give a finite contribution to the diagrams of Fig. 1. Lepton doublets with I_3^L or $I_3^R = 0$, $Q = (0, -1)$, $Y = -1/2$, will always give an infinite contribution.

As noted in the introduction, this result implies that e^- , μ^- , and τ^- -neutrinos must all have right-handed counterparts, which is compatible with cosmological constraints ³¹⁻³³⁾ only if ν_τ has a mass ≥ 1 MeV and is unstable. In addition, unless $m_{W_R} \gg m_{W_L}$ with small W_L, W_R mixing, we must have at least 12 leptons with the doublet structure:

$$\left\{ \begin{array}{cccccc} \left(\begin{array}{c} \nu_e \\ e^- \end{array} \right)_L & \left(\begin{array}{c} \nu_\mu \\ \mu^- \end{array} \right)_L & \left(\begin{array}{c} \nu_\tau \\ \tau^- \end{array} \right)_L & \left(\begin{array}{c} E^0 \\ E^- \end{array} \right)_L & \left(\begin{array}{c} M^0 \\ M^- \end{array} \right)_L & \left(\begin{array}{c} T^0 \\ T^- \end{array} \right)_L \\ \left(\begin{array}{c} E^0 \\ e^- \end{array} \right)_R & \left(\begin{array}{c} M^0 \\ \mu^- \end{array} \right)_R & \left(\begin{array}{c} T^0 \\ \tau^- \end{array} \right)_R & \left(\begin{array}{c} \nu_e \\ E^- \end{array} \right)_R & \left(\begin{array}{c} \nu_\mu \\ M^- \end{array} \right)_R & \left(\begin{array}{c} \nu_\tau \\ T^- \end{array} \right)_R \end{array} \right\} \quad (2.11)$$

and the appropriate phenomenological constraints on the Cabibbo mixing and the E, M, T masses.

In the remainder of this paper we assume $I_3^L = I_3^R = \pm 1/2$ for all fermions.

2.2. W_L, W_R mixing.

A model of type (1.2), which can avoid the introduction of many new fermion states is phenomenologically viable only if mixing of the left- and right-coupled charged vector bosons is small. Therefore it is appropriate to ask under what conditions this mixing can be made zero up to finite and calculable radiative corrections. In this section we impose this as a strict condition and pursue its consequences. The infinite part of the one fermion loop contribution to W_L, W_R mixing, Fig. 2, is proportional to

$$\text{Tr}(m_a U_L m_c U_R^\dagger) \quad (2.12)$$

where $m_{a(c)}$ is the mass matrix for fermions with $I_3^L = I_3^R = +(-) 1/2$, and $U_{L,R}$ are the coupling matrices:

$$\mathcal{L}_{c.c.} = \sum g \bar{f}_a (U_L W_L^\dagger L + U_R W_R^\dagger R) f_c + \text{h.c.} \quad (2.13)$$

If all neutrinos are massless there is no leptonic contribution to (2.11), but one sees immediately that unless $m_a \equiv 0$ or $m_c \equiv 0$ for quarks, solution (2.1) is ruled out. Taking $U_L = U_R = U$, the quantity (2.11) becomes

$$\sum_{i,j} m_i^a m_j^c |U_{ij}|^2 > 0, \quad (2.14)$$

i.e., is positive definite. Therefore, barring an arbitrary adjustment of the renormalized $W_{L,R}$ mixing parameter to zero, we are forced in any case to take $U_L \neq U_R^\dagger$. We could still avoid a proliferation of heavy quarks, however, if we could choose U_R and U_L such that (2.12) vanishes. Aside from some complicated cancellation involving masses and mixing angles, we must require

$$U_{ij}^L = 0 \quad \text{if} \quad U_{ij}^R \neq 0. \quad (2.15)$$

With a non vanishing Cabibbo angle we are then led to eight quarks anyway as we must have

$$U_{ud}^R = U_{us}^R = U_{cs}^R = U_{cd}^R = 0,$$

giving as the minimal doublet structure

$$\left. \begin{array}{cccc} \begin{pmatrix} u \\ d_c \end{pmatrix}_L & \begin{pmatrix} c \\ s_c \end{pmatrix}_L & \begin{pmatrix} t \\ b_c \end{pmatrix}_L & \begin{pmatrix} x \\ y_c \end{pmatrix}_L \\ \begin{pmatrix} u \\ b'_c \end{pmatrix}_R & \begin{pmatrix} c \\ q'_c \end{pmatrix}_R & \begin{pmatrix} t \\ d'_c \end{pmatrix}_R & \begin{pmatrix} x \\ s'_c \end{pmatrix}_R \end{array} \right\} \quad (2.16)$$

where d_c , d'_c , etc. denote Cabibbo mixed states with mixing between d and s and between b and y only, which would have to be achieved through some additional symmetry.

An alternative approach, which could accommodate a six quark structure, has been discussed by De Rujula, Georgi and Glashow¹²⁾. Their scheme involves an extension of the gauge group with an additional zeroth order symmetry so that all Cabibbo angles vanish in lowest order and (2.15) is automatically satisfied, with for example a zeroth order doublet structure:

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \quad \begin{pmatrix} c \\ s \end{pmatrix}_L \quad \begin{pmatrix} t \\ b \end{pmatrix}_L \quad \begin{pmatrix} u \\ b \end{pmatrix}_R \quad \begin{pmatrix} c \\ d \end{pmatrix}_R \quad \begin{pmatrix} t \\ s \end{pmatrix}_R. \quad (2.17)$$

Since non-zero Cabibbo angles now arise from finite and calculable radiative corrections, the infinite part of Fig. 2 vanishes in order g^2 . If the symmetry giving the assignment (2.17) as a natural zeroth order relation also gives zero W_L - W_R mixing as a zeroth order relation, the fermion loop contribution will be finite in all orders.

From a phenomenological point of view the coupling (2.17) with small Cabibbo mixing is permissible if W_L and W_R are unmixed and if $m_{W_L} \gg m_{W_R}$. However, if $m_{W_R} \approx m_{W_L}$, an appreciable $(cd)_R$ coupling is excluded by data on charmed particle decays²²⁾, and would also encounter difficulties³⁵⁾ with strange particle decay phenomenology. Then the radiative corrections must

be such as to essentially "flip" ¹²⁾ s_R and d_R . This is acceptable for neutrino-hadron interactions since the incident neutrinos decouple from W_R , but would become unacceptable if there were clear evidence for a predominantly V-A coupling in charmed particle decay, requiring again an eight quark model. An alternative would be to assume that the masses of either all $Q = 2/3$ or all $Q = -1/3$ quarks arise from radiative corrections. Similar considerations apply to the leptonic sector if cosmology forbids us to force masslessness on more than two 4-component neutrinos.

3. The scalar boson sector

In this section we examine in detail the minimal Higgs structure required to give the symmetry breaking patterns discussed in Section 1. We start by discussing the $\Phi(1/2, 1/2)$ multiplet (s) necessary for generating fermion masses. The results of section 2.1 lead us to impose $I_3^L = I_3^R$ for physical 4-component fermion states. Then fermion masses can be generated only by couplings to neutral Higgs scalars with $I_3^L = -I_3^R$, $Y = 0$. As these scalar fields decouple from $(W_L^3 + W_R^3)/\sqrt{2}$ and S , the symmetry is broken only down to a vector $U(1) \otimes U(1)$. There are two minimal schemes which have been proposed ⁸⁻¹⁰⁾ for breaking the extra unwanted $U(1)$ symmetry. One involves the introduction of the $SU(2)$ doublets $\chi_L(1/2, 0)$ and $\chi_R(0, 1/2)$. As mentioned in the introduction and discussed more fully below, if the χ -fields give the dominant contribution to the vector boson ($Z_A, Z_V, W_L^\pm$) masses, the V -neutral current phenomenology of the Weinberg-Salam model emerges naturally but other phenomenological requirements make it impossible to keep natural the condition $\langle \chi_L \rangle = \langle \chi_R \rangle$ which is necessary to avoid Z_V, Z_A mixing. In the alternative scheme which avoids the χ -fields, the absence of Z_V, Z_A mixing in lowest order is natural, but the neutral current phenomenology must be adjusted by hand.

3.1 Generation of fermion masses

One obvious requisite for natural zeroth order parity conservation in the neutral current sector is the equality of the $SU(2)$ gauge coupling constants: $g_L = g_R$, before symmetry breaking. This is a natural condition if all terms of dimension four ³⁶⁾, and in particular the Higgs-fermion Yukawa couplings conserve "parity". "Parity" in this context means some discrete operation which leaves invariant the gauge couplings

$$\mathcal{L}_g = \sum_{\Phi} |D_\mu \Phi|^2 + \sum_f \bar{f} \not{D} f, \quad D_\mu = \partial_\mu - i g T^a W_\mu \quad (3.1)$$

and under which

$$\vec{W}_L \longleftrightarrow \vec{W}_R$$

Such a definition of parity can always be found if there is a single Yukawa coupling for quarks and for leptons:

$$\mathcal{L}_Y = \bar{q}_R \Phi G q_L + \text{h.c.} + \text{leptonic term}, \quad (3.2)$$

where the field

$$\Phi\left(\frac{1}{2}, \frac{1}{2}\right) = \begin{pmatrix} \varphi^+ & \varphi^0 \\ \varphi^- & \varphi^+ \end{pmatrix} \quad (3.3)$$

is a matrix in $SU(2)_L \otimes SU(2)_R$ space and G is a matrix in flavor space which commutes with the gauge group operators. We can always express G in terms of a real diagonal matrix M and two unitary matrices

$$G = U^\dagger M V. \quad (3.4)$$

Then the interactions (3.1) and (3.2) are invariant under the reflection:

$$\left. \begin{aligned} \vec{W}_L &\longleftrightarrow \vec{W}_R, \quad \Phi \longleftrightarrow \Phi^\dagger \\ q_L &\rightarrow U^\dagger V q_R, \quad q_R \rightarrow V^\dagger U q_L \end{aligned} \right\} \quad (3.5)$$

(and similarly for the leptonic part). However, the interaction (3.2) is insufficient. If

$$\langle \Phi \rangle = \begin{pmatrix} \epsilon & 0 \\ 0 & a \end{pmatrix} \quad (3.6)$$

then the non-diagonal charge $+ 2/3$ (a) and $- 1/3$ (c) quark mass matrices arising from (3.2) are proportional:

$$G_a = \epsilon G, \quad G_c = a G$$

giving $m_u/m_d = m_c/m_s = m_t/m_b = \dots$, and vanishing Cabibbo angles.

If there is only one Higgs quartet which couples to fermions, arbitrary mass matrices are obtained by adding to (3.2) the coupling

$$\mathcal{L}'_Y = \bar{q}_R \tilde{\Phi} \tilde{G} q_L + \text{h.c.} + \text{leptonic term}, \quad (3.7)$$

where

$$\tilde{\Phi} = \tau_2 \Phi^* \tau_2 = \begin{pmatrix} \bar{\theta}^0, & -\theta^+ \\ -\varphi^-, & \bar{\varphi}^0 \end{pmatrix} \quad (3.8)$$

has the same transformation properties as Φ under $SU(2)_L \otimes SU(2)_R \otimes U(1)$. Now parity conservation is no longer trivial. The general set of "parity" operations which leave (3.1) invariant are

$$\left. \begin{aligned} \vec{W}_L &\longleftrightarrow \vec{W}_R \\ \Phi &\rightarrow \eta \Phi^\dagger \quad \text{or} \quad \Phi \rightarrow \eta \tilde{\Phi}^\dagger \\ q_L &\rightarrow S q_R, \quad q_R \rightarrow S' q_L \end{aligned} \right\} \quad (3.9)$$

where S and S' are any unitary matrices in flavor space which commute with the gauge group operators and $|\eta|^2 = 1$. The quark mass matrices are now

$$\left. \begin{aligned} G_a &= \epsilon G + \alpha^* \tilde{G} \\ G_c &= a G + \epsilon^* \tilde{G} \end{aligned} \right\} \quad (3.10)$$

As shown in Appendix B, the requirement that (3.2) and (3.7) be simultaneously invariant under a transformation of the type (3.9) is satisfied if and only if, for some quark basis,

$$G_a G_a^\dagger = G_c^\dagger G_c \quad (3.11)$$

if parity is defined by $\Phi \leftrightarrow \tilde{\Phi}^\dagger$, and

$$G_{a,c} G_{a,c}^\dagger - G_{a,c}^\dagger G_{a,c} = 2i \operatorname{Im}(\epsilon a) (G \tilde{G}^\dagger - \tilde{G} G^\dagger) \quad (3.12)$$

for parity defined by $\Phi \leftrightarrow \Phi^\dagger$. The condition (3.11) is phenomenologically unacceptable since it implies mass degeneracy between quarks of charge $+2/3$ and $-1/3$ ^{37,10}). If the vacuum expectation values are real the left-hand side of Eq. (3.12) vanishes and we get:

$$G_{a,c} G_{a,c}^\dagger = G_{a,c}^\dagger G_{a,c} \quad (3.13)$$

giving $U_L = U_R$ up to a diagonal phase transformation ¹⁰). In Appendix B we show that even with imaginary vacuum expectation values the constraint (3.12) is sufficient to exclude the phenomenologically desired pattern of Cabibbo mixing. In addition as we shall see in section 3.3, the absence of zeroth order Z_A, Z_V mixing holds in the χ_L, χ_R model only if $m_R = m_L$ and either $\operatorname{Im}(\epsilon a) = 0$ or the Higgs potential has only real couplings. The first condition cannot be imposed, and the second will be natural if we impose "CP" invariance $\Phi \rightarrow \Phi^*$ $\chi_{L,R} \rightarrow \chi_{L,R}^*$ which in turn implies real Yukawa coupling matrices. Coupled with invariance under parity, Eqs. (3.9) we obtain the conditions (see Appendix B)

$$G_{a,c} G_{a,c}^\dagger = G_{a,c}^\dagger G_{a,c} \quad (3.14)$$

which is also phenomenologically unacceptable as it implies ^{37,10}) $U_L = U_R^*$. However it turns out that in the one Φ model an infinite parity violating counter term for the Higgs potential appear only in a very high order if at all (see Appendix B), if the only source of CP violation is in the Yukawa coupling. But since this model cannot reproduce the desired Cabibbo matrices we are led to

consider more complicated Higgs sector.

If we introduce more Φ fields which transform differently under parity we are a priori less constrained by the requirement that the Cabibbo matrices are different. For instance we can have under parity

$$\Phi_1 \rightarrow \Phi_1^\dagger, \quad \Phi_2 \rightarrow -\Phi_2^\dagger, \quad q_L \leftrightarrow q_R \quad (3.15)$$

Then Φ_1 couples to fermions via Hermitean matrices and Φ_2 via anti-Hermitean matrices and the mass matrices

$$G_a = \sum_i (\epsilon_i G_i + a_i^* \tilde{G}_i), \quad G_c = \sum_i (a_i G_i + \epsilon_i^* \tilde{G}_i)$$

are in general neither Hermitean nor symmetric. However, from the form of the Higgs potential discussed in section 3.3 and Appendix A, we find that natural zeroth order Z_V, Z_A non-mixing requires either condition on the phases of the vacuum expectation values:

$$\text{Im}(a_1 \epsilon_1) = \text{Im}(a_2 \epsilon_2) = \text{Re}(a_1 \epsilon_2) = \text{Re}(a_2 \epsilon_1) = 0 \quad (3.16)$$

or conditions on the phases of quartic couplings which can be made natural only by imposing CP invariance with $\Phi_1 \rightarrow \Phi_1^*$, $\Phi_2 \rightarrow \Phi_2^*$. The former condition gives (3.13) while the second gives (3.14).

Whether or not the Yukawa couplings conserve "parity", the lowest order Higgs exchange contributions to parity violation in the helicity conserving vector boson-fermion couplings (Fig. 3) are finite since each diagram only involves one coupling matrix and we can always find a quark basis in which (3.12) is satisfied for a single matrix ($G = \tilde{G} = 0$). Infinite parity violating effects should arise from higher order graphs involving the interplay of two coupling matrices which cannot be made to satisfy (3.12) simultaneously. In our evaluation of finite parity violating corrections we neglect those arising from Higgs exchange as they are proportional to fermion masses. (Helicity flip amplitudes will of course have arbitrary parity violation if no restrictions are placed on the Yukawa couplings. We also ignore these, as they vanish in the zero fermion mass limit.)

3.2 Breaking the vector U(1) symmetry: phenomenological constraints 8-14),24)

In addition to one or more Φ fields which can generate fermion masses, we must introduce other Higgs fields with

$$I_3^L + I_3^R \neq 0 \quad (3.17)$$

in order to break $SU(2)_L \otimes SU(2)_R \otimes U(1)$ down to U(1) of QED. The conditions for avoiding mixing of the axial vector with the two vectors in the neutral boson sector are

$$m_{W_{3L}}^2 = m_{W_{3R}}^2 : \sum_i (I_{3L}^2 - I_{3R}^2) |V_i|^2 = 0 \quad (3.18a)$$

and

$$m_{Z_{A,S}}^2 = 0 : \sum_i (I_{3L}^i - I_{3R}^i) Y |V_i|^2 = 0 \quad (3.18b)$$

where the sums are over all the scalar fields in the theory and the V_i are their vacuum expectation values (vevs). In terms of the fields A , Z_A and Z_V , defined in section 2.1, Eq. (2.8), the conditions for approximately reproducing the ν_L and $\bar{\nu}_R$ induced neutral current phenomenology are

$$m_{Z_R}^2 \simeq m_{Z_V}^2 \cos 2\theta_w \simeq m_{W_L}^2, \quad (3.19)$$

which requires

$$|\sum_i I_{3L}^i I_{3R}^i |V_i|^2| \ll \sum_i (I_{3L}^i)^2 |V_i|^2 \quad (3.20a)$$

and

$$\sum_i [(I_{3L}^i)^2 - 3(I_{3L}^i)^2] |V_i|^2 \simeq 0 \quad (3.20b)$$

Finally, the condition for avoiding $W_L - W_R$ mixing is:

$$\sum (v_i^* \{I_L^+, I_R^-\} v_i) \approx 0 \quad (3.21)$$

The problem is that it is difficult to find a simple model which satisfies simultaneously (3.18) and (3.20) while maintaining dominance of left handed couplings in charged current phenomenology.

As previously stated, the full symmetry breaking can be achieved either by introducing the complex SU(2) doublets

$$\chi_L (1/2, 0) = \begin{pmatrix} \chi_L^+ \\ \chi_L^0 \end{pmatrix}, \quad \chi_R (0, 1/2) = \begin{pmatrix} \chi_R^+ \\ \chi_R^0 \end{pmatrix} \quad (3.22)$$

with vevs:

$$\langle \chi_L \rangle = \begin{pmatrix} 0 \\ a \end{pmatrix}, \quad \langle \chi_R \rangle = \begin{pmatrix} 0 \\ b \end{pmatrix} \quad (3.23)$$

or by adding to the Φ multiplet of (3.3), with vev of Eq. (3.6), the hypercharge 1 complex quartet

$$\varphi (1/2, 1/2) = \begin{pmatrix} \varphi^+ & \varphi^{++} \\ \varphi^0 & \varphi^+ \end{pmatrix} \quad (3.24)$$

with vev:

$$\langle \varphi \rangle = \begin{pmatrix} 0 & 0 \\ b & 0 \end{pmatrix} \quad (3.25)$$

All of these contribute equally to both charged vector boson masses:

$m_{W_L^\pm} = m_{W_R^\pm}$. If we wish to separate them, $m_{W_L} \neq m_{W_R}$, we may introduce self-conjugate SU(2) triplets which do not contribute to neutral boson masses:

$$\delta_L = \begin{pmatrix} \delta_L^+ \\ \delta_L^0 \\ \delta_L^- \end{pmatrix}, \quad \delta_R = \begin{pmatrix} \delta_R^+ \\ \delta_R^0 \\ \delta_R^- \end{pmatrix} \quad (3.26)$$

with vevs

$$\langle \delta_L \rangle = \begin{pmatrix} 0 \\ d \\ 0 \end{pmatrix}, \quad \langle \delta_R \rangle = \begin{pmatrix} 0 \\ c \\ 0 \end{pmatrix}. \quad (3.27)$$

We see that the χ -fields satisfy (3.20) and (3.21)⁶⁾ automatically, but (3.18) is satisfied only if $\langle \chi_L \rangle = \langle \chi_R \rangle$. The Φ , ρ and δ fields satisfy (3.18) automatically, but not (3.20). Specifically, (3.20a) requires

$$|b|^2 \simeq |a|^2 + |\epsilon|^2 \quad (3.28)$$

and (3.20b) requires

$$|d|^2 = 0 \quad (3.29)$$

In addition the Φ vev generally violates (3.21) which requires

$$\text{Re}(\epsilon^* a) = 0 \quad (3.30)$$

Specifically, in the A, Z_V, Z_A basis, the neutral boson mass matrix is

$$M_{\text{neut}}^2 = \frac{g^2}{2} \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{2|b|^2 + \frac{1}{2}(|a|^2 + |\rho|^2)}{\cos 2\theta_w} & \frac{|a|^2 - |\rho|^2}{2\sqrt{\cos 2\theta_w}} \\ 0 & \frac{|a|^2 - |\rho|^2}{2\sqrt{\cos 2\theta_w}} & 2(|a|^2 + |\epsilon|^2) + \frac{1}{2}(|a|^2 + |\rho|^2) \end{pmatrix} \quad (3.31)$$

and in the W_L, W_R basis the charged boson mass matrix is:

$$M_{ch}^2 = \frac{g^2}{2} \begin{pmatrix} |a|^2 + |e|^2 + |b|^2 + |d|^2 & -2\text{Re}(a^*e) \\ -2\text{Re}(a^*e) & |a|^2 + |e|^2 + |b|^2 + |d|^2 \end{pmatrix} \quad (3.32)$$

The most attractive solution would be one with the χ -fields only. In this case a parity conserving Higgs potential gives $\alpha = \beta$ as a natural condition and the vector boson mass pattern is the desired one. However a parity conserving Yukawa coupling could be achieved at the price of all massive 4-component fermions appearing in degenerate pairs, $(1/2, 0) \oplus (0, 0)$ and $(0, 0) \oplus (0, 1/2)$, with equal $SU(2)_L$ and $SU(2)_R$ Cabibbo matrices. This is clearly unacceptable phenomenologically, and if we want at least some massive fermions transforming like $(1/2, 0) \oplus (0, 1/2)$ we must introduce the Φ field. Then as shown in sections 3.1 and 3.3, it is not possible to allow for different Cabibbo matrices, $U_L \neq U_R, U_R^*$ without inducing parity mixing in the neutral current sector.

However, the phenomenological constraints (3.20) and (3.17) require us to impose

$$|\langle \chi \rangle| \gg |\Phi|. \quad (3.33)$$

Then only a small violation of parity for neutral currents would arise if

$$||\alpha|^2 - |\beta|^2| \sim |e|^2 + |a|^2 \ll |\alpha| \quad (3.34)$$

and we could consider that we have an acceptable theory, (with however, a proliferation of heavy fermions), with (3.18), (3.20) and (3.21) all approximately satisfied at the price of one "unnatural" condition, Eq. (3.33).

However in this model there is (at least) one physical Higgs boson with flavor changing neutral currents ^{24,37}). In the simplest one- Φ model:

$$\mathcal{L}_H = \frac{\sqrt{|e|^2 + |a|^2}}{|e|^2 - |a|^2} \left(\bar{q}_L^a U_L M_c U_R^\dagger q_R^a + \bar{q}_R^c U_R^\dagger M_a U_L q_L^c \right) H + h.c. \quad (3.35)$$

$$H = \frac{\varepsilon \bar{\Theta}^0 - a^* \Phi}{(|e|^2 + |a|^2)^{1/2}}$$

Phenomenology requires that the amplitude for $\Delta S = 2$ processes (Fig. 4) be highly suppressed:

$$\frac{1}{m_\Phi^2} \left[(U_R^\dagger M_\rho U_L)_{sd} \right]^2 \frac{(|e|^2 + |a|^2)}{(|e|^2 - |a|^2)^2} \leq G_F^2 (1 \text{ GeV})^2 \sin^2 \theta_c$$

The matrix $U_R^\dagger M_\rho U_L$ cannot be made diagonal for $U_L \neq U_R$. In fact we require significant right-handed s- and d-couplings only to very heavy quarks:

$$(U_R^\dagger)_{si} (U_L)_{id} \ll \sin^2 \theta_c$$

$$(U_R^\dagger)_{di} \sim 1, \quad m_i \gg 1 \text{ GeV}$$

So it is reasonable to guess

$$(U_R^\dagger)_{di} m_i^2 (U_L)_{is} \sim \sin^2 \theta \times 1 \text{ GeV}^2$$

giving

$$m_\Phi^2 \geq G_F^{-2} \frac{(|e|^2 + |a|^2)}{(|e|^2 - |a|^2)^2} > \frac{G_F^{-2}}{|e|^2 + |a|^2} \quad (3.36)$$

Because of the condition (3.33), only the χ -fields contribute significantly to the boson masses; we find

$$\sqrt{2} G_F^{-1} = \frac{8 m_W^2}{g^2} = 4 |\alpha|^2 \quad (3.37)$$

On the other hand, to the extent that the Φ - and χ -fields are decoupled in the Higgs potential, one expects to find:

$$m_\Phi^2 \sim (|\epsilon|^2 + |\alpha|^2) \lambda_\Phi \quad (3.38)$$

when λ_Φ is the strength of the quartic Φ self-coupling. Combining (3.38) with (3.36) would give

$$\lambda_\Phi > |\alpha|^4 / (|\epsilon|^2 + |\alpha|^2)^2 \gg 1 \quad (3.39)$$

This conclusion is modified by the presence of Φ - χ couplings. As discussed in Appendix A, barring strong cancellations among dimensional parameters in the Higgs potential, Eq. (3.38) becomes replaced by

$$m_\Phi^2 \sim (|\alpha|^2 + |\beta|^2) \lambda_{\Phi-\chi} \quad (3.40)$$

where $\lambda_{\Phi-\chi}$ is also a quartic coupling strength. Eq. (3.40) still gives an unacceptable condition

$$\lambda_{\Phi-\chi} > \frac{|\alpha|^2}{|\epsilon|^2 + |\alpha|^2} \geq 10 \quad (3.41)$$

If at least one coupling constant must be large, perturbation theory will be inapplicable in the Higgs sector, and the zeroth order relations are likely to be meaningless. Adding additional Φ -fields to the theory is unlikely to change this conclusion. The problem of "natural" flavor conservation in neutral current transitions is common to all $SU(2) \otimes SU(2) \otimes U_1$ models³⁷⁾, which generally require $\lambda \gg 1$, but in this model it becomes acute.

3.3 The Higgs scalar potential: Naturality of the phenomenological constraints.

We now examine the Higgs potentials, which are given explicitly in Appendix A, to see whether any of the requirements (3.19) - (3.30) can be made natural in the technical sense.

Consider first a Higgs sector containing the complex doublets χ_L and χ_R . The triplets δ_L and δ_R may be introduced to avoid a proliferation of new fermions, but as discussed by previous authors^{10,30)}, if all couplings are real we obtain the minimum condition:

$$2\lambda_\chi (|\alpha|^2 - |\beta|^2) + \lambda_{\chi-\delta} (|c|^2 - |d|^2) = 0 \quad (3.42)$$

where the λ 's are quartic coupling constants. The phenomenological requirements, including calculable parity mixing in the neutral scalar sector are:

$$|c|^2 \gg |\alpha|^2 = |\beta|^2 \gg |d|^2 \quad (3.43)$$

which is impossible unless $\lambda_{\chi-\delta} = 0$. As follows from Symanzik's theorem³⁶⁾ and as shown by explicit calculation³⁰⁾, this condition is unstable against infinite renormalization via the diagrams of Fig. 5. If we allow for complex couplings, the expression (3.42) is modified to include terms involving the Φ vevs. Since we require $|\Phi|^2 \ll |\alpha|^2$, the correction to (3.42) will be negligible unless $\lambda_{\chi-\Phi} \gg \lambda_{\chi-\delta}$ and we still will not obtain a solution $|\alpha| = |\beta|$ without imposing "unnatural" (renormalization unstable) conditions on the coupling constants.

We are thus led to drop the triplet fields and impose different left and right Cabibbo matrices consistent with charged current phenomenology, as discussed in 3.1 and Appendix B.

The most general parity conserving potential with one Φ -quartet and χ_L, χ_R is given in Appendix A, Eq. (A.2). Here parity is defined by

$$\Phi \rightarrow \Phi^\dagger, \quad \chi_L \leftrightarrow \chi_R \quad (3.44)$$

Eq. (3.42) is now replaced by

$$\lambda_{\phi\chi} \text{Im}(\epsilon a) = [|\alpha|^2 - |\beta|^2] \left[\lambda_{\chi\chi} + \text{Re} \left(\frac{g_1 a + g_2 \epsilon^*}{\alpha \beta^*} \right) \right] \quad (3.45)$$

where g_1, g_2 denote the three-point couplings $g_{\chi\chi\phi}$ and we see that the condition for calculable parity mixing, $|\alpha| = |\beta|$, is that either $\text{Im}(\epsilon a) = 0$ leading to the unacceptable condition (3.13), or that the quartic couplings be real. The latter condition can be achieved by imposing CP invariance $\Phi \rightarrow \Phi^*$, $\chi_{L,R} \rightarrow \chi_{L,R}^*$, but will be natural only if CP invariance is also imposed on the Yukawa couplings, leading to the unacceptable condition (3.14). If we introduce two Φ -fields with opposite parity transformations

$$P\Phi_1 = \Phi_1^\dagger, \quad P\Phi_2 = -\Phi_2^\dagger$$

Eq. (3.45) is replaced by

$$\begin{aligned} & \lambda_{1\chi\phi} \text{Im}(\epsilon_1 a_1) + \lambda_{2\chi\phi} \text{Im}(\epsilon_2 a_2) + \lambda_{3\chi\phi} \text{Re}(\epsilon_1 \epsilon_2^*) + \lambda_{4\chi\phi} \text{Re}(a_1 a_2^*) + \\ & + \lambda_{5\chi\phi} \text{Re}(\epsilon_1 a_2) + \lambda_{6\chi\phi} \text{Re}(\epsilon_2 a_1) = \frac{1}{2} [|\alpha|^2 - |\beta|^2] [\lambda_{\chi\chi} + \text{other terms...}] \end{aligned} \quad (3.46)$$

and we see that the conditions for calculable parity mixing, $|\alpha| = |\beta|$ are those discussed in section (3.1), again giving one of the unacceptable conditions (3.13) or (3.14). In Appendix B it is shown explicitly that for any set of N fields $\Phi_i, N \geq 1$ with $P\Phi_i = \pm \Phi_i^\dagger$, the effective potential induced by the diagram of Fig. 6 will not have $|\alpha| = |\beta|$ at the minimum unless either there is a complicated (unnatural) interplay among coupling parameters, or the Yukawa coupling matrices satisfy constraints leading to (3.13) or to (3.14).

In summary, natural parity conservation for neutral currents in the χ -Higgs model appears to be incompatible with acceptable charged current phenomenology; either $m_{W_R} \gg m_{W_L}$ or $U_L \neq U_R, U_R^*$. Small but uncalculable parity violation, along with the desired γ -interaction phenomenology, may be maintained at the price of one unnatural condition and a proliferation of fermion fields (probably at least 12 leptons and 8 quarks). However, as discussed in section 3.2, the χ -scheme may not be tenable at all because the required

smallness of the $\langle \Phi \rangle$ vev together with the large mass required to avoid strangeness changing neutral transitions may require at least one large quartic scalar coupling constant. While we cannot rule out a strong cancellation among large dimensional parameters in the potential of Eq. A.2, this appears to us to be a very unattractive feature of the model.

We now turn to a Higgs sector with two quartets $\Phi(1/2, 1/2)$ and $\varphi(1/2, 1/2)$ and the triplets δ_L, δ_R . In this case the condition that there be no parity mixing in the neutral sector is strictly natural. We then ask whether any of the other phenomenological conditions discussed in section 3.2 may be made natural.

It is obvious that this is not the case for the condition (3.28) which corresponds to the requirement that the (V-A) and isospin structure of the effective Fermi coupling for ν -induced neutral current interactions is the same as in the Weinberg-Salam model. Condition (3.28) corresponds to a reflection symmetry

$$\Phi \leftrightarrow \varphi$$

which does not commute with the hypercharge gauge transformation, and therefore could only be imposed if $SU(2)_L \otimes SU(2)_R \otimes U(1)$ were embedded in a larger group.

The condition (3.29) corresponds to the requirement that the effective Fermi couplings for charged current V-A interactions and ν -induced neutral current interactions have the same relative strength as in the Weinberg-Salam model. This condition is also clearly unnatural since any extra symmetry requiring $\langle \delta_L \rangle = 0$ but not $\langle \delta_R \rangle = 0$ would not commute with parity. On the other hand the condition (3.30) which insures the absence of $W_L - W_R$ mixing could be achieved by imposing an extra $U(1)$ symmetry which remains unbroken so that no extra Goldstone boson appears. If we require invariance under a local gauge transformation

$$\Phi \rightarrow e^{i\alpha} \Phi \quad (3.47)$$

with all other fields invariant, this is equivalent to introducing a new hypercharge: $Y' = 1$ for Φ , $Y' = 0$ for all other fields. Then if, say

$$Y' \pm (I_3^L - I_3^R) \quad (3.48)$$

remains unbroken we will have $\epsilon = 0$ or $\alpha = 0$. Then the Cabibbo matrices will

not be arbitrary, but this is acceptable if $m_{W_L} \gg m_{W_R}$.

The realization of this symmetry in the Higgs potential and the resulting conditions on the vacuum expectation values are discussed explicitly in Appendix A. However, the symmetry is necessarily broken by the Yukawa couplings. In order to make the Yukawa couplings invariant we must impose a corresponding phase transformation on quark fields. However since Φ and $\tilde{\Phi}$ have opposite transformation properties we must have either $G = 0$ or $\tilde{G} = 0$ [see Eqs. (3.2) and (3.7)], i.e. all $Q = 2/3$ or all $Q = -1/3$ quarks massless. One way around this would be to introduce two Φ fields with vevs

$$\langle \Phi_1 \rangle = \begin{pmatrix} \epsilon & 0 \end{pmatrix}, \quad \langle \Phi_2 \rangle = \begin{pmatrix} 0 & a \end{pmatrix} \quad (3.49)$$

This would allow arbitrary mass matrices: $G_a = \epsilon G_1, G_c = a G_2$ and no $W_L - W_R$ mixing in zeroth order. However the condition (3.49) does not correspond to a conserved quantum number of the type (3.48) and therefore cannot be made natural. This can also be seen by directly examining the Higgs potential. Since Φ_1 and Φ_2 must have the same transformation property under (3.47) in order to keep the Yukawa couplings invariant they will mix. The (infinite) mixing induced by the Yukawa couplings, Fig. 7, is proportional to (for the quadratic term)

$$\text{Tr } G_a G_c^\dagger \quad (3.50)$$

Using the properties

$$\begin{aligned} G_a &= U_a M_a V_a^\dagger, & G_c &= U_c M_c V_c^\dagger \\ U_L &= U_a^\dagger U_c, & U_R &= V_a^\dagger V_c \end{aligned} \quad (3.51)$$

we see that (3.50) is identical to (2.12), the (infinite) fermion loop contribution to $W_L - W_R$ mixing. Thus we need a symmetry which requires (3.50) to vanish. As mentioned in section 2.2, such a symmetry was introduced by De Rujula et al.¹²⁾ in an attempt to make the Cabibbo angles calculable, and this can be achieved only by extending the group. An alternative, since phenomenologically this latter symmetry must be broken, is to break it softly^{12,36)} via a mass term in the Higgs potential. In this case $W_L - W_R$ mixing can be made arbitrarily small in zeroth order, with finite radiative corrections, but proportional to an arbitrary parameter.

Since $W_L - W_R$ mixing cannot be made zero naturally within the context of $SU(2)_L \otimes SU(2)_R \otimes U(1)$, we are forced to consider the full mass matrix (3.31). What is relevant for phenomenology is the inverse mass matrix:

$$\bar{m} = (m_{ch})^{-1} = \frac{g^2}{2} \begin{pmatrix} 2|b|^2 + |c|^2 & 2\text{Re}(a^* \epsilon) \\ 2\text{Re}(a^* \epsilon) & 2|b|^2 + |d|^2 \end{pmatrix} \frac{1}{\text{Det} m_{ch}} \quad (3.52)$$

where we have set $|e|^2 + |a|^2 \approx |b|^2$. Phenomenology dictates that

$$\bar{m}_{W_L W_L} \approx \frac{2}{|g|^2} \times \frac{1}{2|b|^2}$$

giving the condition

$$(2|b|^2 + |c|^2)(2|b|^2 + |d|^2) - 4(\text{Re } a^* \epsilon)^2 = (2|b|^2 + |c|^2) 2|b|^2 \quad (3.53)$$

which is satisfied if

$$4(\text{Re } a^* \epsilon)^2 \approx |d|^2 (2|b|^2 + |c|^2) \quad (3.54)$$

This is the only condition we need impose if $U_L \neq U_R$. However if we wish to avoid the proliferation of fermions implied by the latter we must also impose the smallness of left-right transitions, $\bar{m}_{LR} \ll \bar{m}_{LL}$, or

$$2|b|^2 + |c|^2 \gg |2\text{Re}(a^* \epsilon)| \quad (3.55)$$

and of right-right transitions

$$|c|^2 \gg |d|^2 \quad (3.56)$$

The condition (3.54) is very artificial. However, conditions (3.53), (3.55) and (3.56) will all be approximately satisfied if

$$|c|^2 \gg |b|^2 \gg |d|^2 \quad (3.57)$$

On the other hand we cannot make $|c|^2$ arbitrarily large, since then the radiative corrections to parity violation in atomic physics, discussed in the following section, become arbitrarily large (as $\ln c$). As discussed by Beg et al.¹¹⁾ the present phenomenological constraints are such as to allow a solution which satisfies all requirements, including a parity violation in atomic physics within the limits found by the experiments of ref. 16. The price is however the imposition of three unnatural conditions, the two implied by (3.58) and the condition (3.28). Essentially all observed weak interaction phenomenology appears as accidental, both the apparent success of the Weinberg-Salam model, and the fact that all charged current interactions observed to date are consistent with a V-A coupling.

4. Contributions of physical Higgs scalars to parity violations in atoms

In a consistent left-right symmetric gauge model with parity conserving neutral currents, parity violation in atoms has to be given by calculable radiative corrections. In fact calculability is assured by demanding that parity conserving neutral currents be generated by a "natural" Higgs mechanism. The one-loop radiative corrections to electron-quark interactions (see Fig. 8 and 9), however are not influenced by problems of renormalization in the Higgs sector. So if one assumes that there exist some natural Higgs mechanism, these corrections are calculable.

In fact, the leading logarithmic corrections proportional to $\ln(M_L/M_R)$ have been estimated in Ref. (27). The box diagrams (Fig. 8.b.) and the charge radii contributions are calculated in Ref. (28).

There arises, however, a practical difficulty. The physical Higgs particles also contribute to parity violation in atoms (see Fig. 9). These contributions depend on unknown mass and mixing angle parameters of the Higgs sector. Consequently, one can calculate the magnitude of parity violation in atoms only to the extent that the Higgs particle contributions are negligible.

To understand better the importance of the Higgs contributions we have calculated the radiative corrections given by the diagrams in Fig. 9a and Fig. 9b.

The contributions of the graphs in Fig. 9a have a divergent term, which is cancelled by a similar term given by the diagrams in Fig. 9b. The finite part can be conveniently expressed by means of the self-mass corrections to the neutral boson mass matrix. (The details of this calculation are given in Appendix C.) We find, however, that the results are more transparent when presented in terms of weak charges.

If the effective Hamiltonian is defined as

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} [(\bar{e} \gamma_\mu \gamma_5 e)(\bar{q} C_{1q} \gamma_\mu q) + (\bar{e} \gamma_\mu e)(\bar{q} C_{2q} \gamma_\mu \gamma_5 q)] \quad (4.1)$$

where $e(x)$ and $q(x)$ denote the electron and quark fields, respectively, the weak charge measured in parity violating atomic transitions is

$$Q_W = 2 [C_{1p} + (A-Z)C_{1n}] \quad (4.2)$$

where $C_{1p} = 2C_{1u} + C_{1d}$ and $C_{1n} = 2C_{1d} + C_{1u}$. The prediction of the W-S model is $Q_W = -123^{+1}$.

If we sum up all the contributions (Figs. (8a), (9a), (9b)), except that of the box diagrams (Fig. 8b) and neglect terms of $O(M_c^2/M_W^2)$ we obtain

⁺) We recall that at 90 % confidence level, the measured values ^{16,17)} imply the bounds $-42 < Q_W(B_i) < 27$ (Washington), $-15 < Q_W(B_i) < 58$ (Oxford), $-260 < Q_W(B_i) < -65$ (Novosibirsk), assuming the validity of atomic physics calculations which are ambiguous up to a factor of ~ 2 .

$$\begin{aligned}
 C_{1p,n} = & \frac{\alpha}{4\pi \sin^2 \theta_w} [I_3^{p,n} - 2 \sin^2 \theta_w Q^{p,n}] + \\
 & * [-7/4 + \frac{1}{4} \frac{m_\phi^2}{M_L^2} + \frac{3}{2} \frac{m_\phi^2}{M_L^2} \frac{\ln(m_\phi^2/M_L^2)}{m_\phi^2/M_L^2 - 1} - \frac{1}{4} \frac{\lambda_+ m_\phi^2 + \lambda_- m_\chi^2}{M_R^2} - \\
 & - \frac{3}{2} \lambda_\psi \frac{m_\phi^2}{M_R^2} \frac{\ln(m_\phi^2/M_R^2)}{m_\phi^2/M_R^2 - 1} - \frac{3}{2} \lambda_\chi \frac{m_\chi^2}{M_R^2} \frac{\ln(m_\chi^2/M_R^2)}{m_\chi^2/M_R^2 - 1} + \quad (4.3) \\
 & + \frac{\lambda_+ \lambda_-}{M_L^2} (m_\phi^2 + m_\chi^2 - \frac{2m_\phi^2 m_\chi^2}{m_\phi^2 - m_\chi^2} \ln(m_\phi^2/m_\chi^2))] + \\
 & + \frac{\alpha}{\pi} Q^{p,n} \frac{M_L^2}{M_{Z_n}^2} [5 \ln \frac{M_R}{M_L} + \frac{1}{12} \frac{M_{Z_n}^2}{M_L^2} \ln \frac{m_\phi^2}{M_L^2} + \frac{1}{6} \ln \frac{m_\phi^2}{M_L^2} - \\
 & - \frac{1}{12} \frac{M_{Z_n}^2}{M_R^2} \ln \frac{m_\phi^2}{M_L^2} - \frac{1}{6} \alpha_\psi \ln \frac{m_\phi^2}{M_L^2} - \frac{1}{6} \alpha_\chi \ln \frac{m_\chi^2}{M_L^2}]
 \end{aligned}$$

where θ_w denotes the corresponding "Weinberg angle", M_{Z_n}, M_L, M_R denote the vector meson masses, $m_\phi, m_\chi, m_\psi, m_\delta$ are the masses of the physical Higgs boson of unit charge; the parameters $\lambda_{+,x}, \alpha_{\psi,\chi}$ are factors of $O(1)$ and depend on the vector boson masses and mixing angles (see Appendix C)⁺). Complications given by the diagonalization of the mass matrix of the physical Higgs bosons are neglected. The values of the charges are $I_3^p = 1/2$, $Q^p = 1$ for protons and $I_3^n = -1/2$, $Q^n = 0$ for neutrons. For the contributions of the box diagrams we obtain²⁸⁾

$$C_{1p}^{box} = \frac{\alpha}{4\pi \sin^2 \theta_w} \cdot \frac{7}{2}, \quad C_{1n}^{box} = \frac{\alpha}{4\pi \sin^2 \theta_w} \quad (4.4)$$

We would like to make few comments:

⁺) If $M_L = M_{Z_n} = M_{Z_n} \sqrt{\cos 2\theta_w}$, up to terms of $O(M_L^2/M_R^2)$ we have $\lambda_+ = 1$, $\lambda_- = -1$ and $\alpha_\psi = \alpha_\chi = 1/2$.

i) A recent analysis finds the phenomenological condition ¹⁰⁾ $M_R/M_L > 2.7$. If M_R is not asymptotically large, then other terms than the leading logarithms $\sim \ln M_R/M_L$ could also become important ⁺). In particular, in Eq. (4.3) there is a term, independent of the masses (7/4). It gives similar and as important contributions as the corrections induced by the box diagrams (Eq. 4.4). So it is not consistent to keep the contributions of the box diagrams and neglect the corrections given by this term ²⁸⁾.

ii) As it is seen from Eq. (4.3), the Higgs contributions are very model dependent. They can be large or small depending on the actual values of the masses. To make them negligible it is necessary to demand some inequalities to hold between the vector boson and Higgs boson masses. For the particular Higgs multiplets we have used one must require

$$M_L \gg m_\phi, m_\psi, m_\chi, m_\sigma; \quad M_R \gg M_L \quad (4.5)$$

On the other hand if some of the charged Higgs particles are very heavy, the Higgs contribution may dominate. Let us illustrate this with two numerical examples. If we assume $\ln M_R/M_L \sim 1$ and that the inequalities (4.5) hold we get ($\sin^2 \theta_W = 1/4$)

$$Q_W(B_i) \approx +4.4$$

If we assume, however, that $m_\phi \approx 20 M_L$, $\ln M_R/M_L \approx 1$ and $m_\sigma, m_\psi, m_\chi \ll M_L$ we obtain

$$Q_W(B_i) \approx -\frac{2}{\pi}(A-Z) \left[\frac{m_\phi^2}{4M_L^2} + 6 \ln m_\phi/M_L \right] \approx -35$$

iii) It is illuminating to consider the limit $M_R \rightarrow M_L$. The expression (4.3) then simplifies and we obtain

⁺) The coefficient of the leading log is different from those obtained in Refs. 27) and 28).

$$C_{1p,n} = \frac{\alpha}{4\pi \sin^2 \theta_W} \left[\frac{1}{4} \frac{m_\phi^2 - m_\psi^2}{M_L^2} + \frac{3}{2} \frac{m_\phi^2}{M_L^2} \frac{\ln(m_\phi^2/M_L^2)}{m_\phi^2/M_L^2 - 1} - \frac{3}{2} \frac{m_\psi^2}{M_L^2} \frac{\ln(m_\psi^2/M_L^2)}{m_\psi^2/M_L^2 - 1} \right] \\ + [I_3^{P,n} - 2\sin^2 \theta_W Q^{P,n}] + \frac{\alpha}{\pi} \frac{1}{12} \ln \frac{m_\phi^2}{m_\psi^2} Q^{P,n} \quad (4.6)$$

The Higgs contribution (4.6) vanishes if $m_\phi = m_\psi$. From the study of the Higgs potential, however, (see Appendix C) we can infer that to require the condition $m_\phi^2 = m_\psi^2$ is not natural: there is no symmetry which would forbid different masses for m_ϕ and m_ψ .

iv) If $M_L = M_R$, one expects the Higgs contributions (4.6) to be the dominating radiative corrections ⁺⁾ . The leading contributions induced by fermions ($\theta_L^{\text{Cabibbo}} \neq \theta_R^{\text{Cabibbo}}$) are the charge radii effects of order $\frac{1}{\pi} \ln M_L/m_p$. But large logarithms are obtained only for the "incoherent part" of the potential proportional to $\bar{\psi} \gamma_\mu e \tilde{q} \gamma_\mu \tilde{q} \psi$ so the fermion contributions are negligible for heavy atoms (see Fig. 10).

v) The qualitative conclusions of i-iv) do not depend on the specific properties of the Higgs sector.

Summarizing, in left-right symmetric $SU(2)_L \otimes SU(2)_R \otimes U(1)$ models parity violation in the neutral sector is induced as radiative corrections given by splittings in the W -boson, fermion and Higgs boson masses. The corrections induced by fermions are negligible for parity violating transitions in heavy atoms. Parity violation in atoms is induced dominantly by corrections proportional to $\ln(M_R/M_L)$ if and only if inequalities like (4.5) hold and $M_R \gg M_L$. For large but finite M_R , the Higgs contributions and the mass independent terms may become important. If $M_L = M_R$ the dominant corrections are the Higgs scalar contributions. Without specifying the parameters in the Higgs sector, the radiative corrections cannot be calculated quantitatively.

⁺⁾ unless accidentally $m_\psi^2 = m_\phi^2$

5. Cabibbo universality

In the previous sections we discussed the difficulties which arise in the Higgs sector if we demand calculable parity violating radiative corrections to neutral current couplings. In certain combinations of physical amplitudes, however, the Higgs contributions can cancel. So the confrontation of these corrections with experimental data could test the model, independently of its Higgs sector.

A well-known example is given by radiative corrections to Cabibbo universality of the β -decays of neutrons and muons³⁸⁾. The experimental accuracy is sensitive to radiative corrections and are in agreement with that given by the Weinberg-Salam model³⁹⁻⁴⁰⁾.

In this section we calculate radiative corrections to Cabibbo universality in the $SU(2)_L \otimes SU(2)_R \otimes U(1)$ model. The corrections remain in agreement with the experimental data and differ only slightly ($\sim 10\%$) from those of the standard model.

As Sirlin pointed out Cabibbo universality receives contributions only from the box diagrams³⁸⁾ given in Fig. 11. The contributions of these graphs do not cancel in the ratio of the rates of muon and neutron β -decay. This is understood as arising from the different ordering of γ -matrices in the Feynman amplitudes or alternatively (and more generally) by the difference between the average charge of quark and lepton multiplets. Let us consider the difference between the $SU(2)_L \otimes SU(2)_R \otimes U(1)$ and the standard model only from this point of view.

The neutral current couplings in the standard model and the L-R model with fermions in left-right symmetric doublets are given as

$$\mathcal{L}_{int}^{WS} = e \bar{\Psi} \gamma^\mu Q \Psi + g \bar{\Psi} \gamma^\mu Q_Z \Psi \quad (5.1a)$$

and

$$\mathcal{L}_{int}^{L-R} = e \bar{\Psi} \gamma^\mu Q \Psi + g \bar{\Psi} (\cancel{Z}_V Q_{Z_V} + \cancel{Z}_A Q_{Z_A}) \Psi \quad (5.1b)$$

respectively, where Q is the charge operator and the "Z-charges" are defined as

$$Q_Z = [I_3(1 - \gamma_5) - 2 \sin^2 \theta_w Q] \frac{1}{2 \cos \theta_w} \quad (5.2a)$$

$$Q_{Z_V} = [I_3 - 2 \sin^2 \theta_w Q] \frac{1}{2 \cos 2\theta_w} \quad (5.2b)$$

$$Q_{Z_A} = -\frac{1}{12} I_3 \gamma_5 \quad (5.2c)$$

I_3 denotes the third component of the weak isospin defined for doublets. The contributions of the diagrams in Fig. 1a are of course the same in both models. The Z_A contribution vanishes and the Z_V contribution can be given as

$$T_{Z_V}^{(\beta)} = -T_{0Z_V}^{(\beta)} \frac{3d}{2\pi} \bar{Q}_e \bar{Q}_h 2 \frac{\sin^2 \theta_w}{\cos 2\theta_w} \frac{M_W^2/M_{Z_V}^2}{1 - M_W^2/M_{Z_V}^2} \ln \frac{M_{Z_V}^2}{M_L^2} \quad (5.3a)$$

$$T_{Z_V}^{(\beta)} = -T_{0Z_V}^{(\beta)} \frac{3d}{2\pi} (\bar{Q}_e)^2 2 \frac{\sin^2 \theta_w}{\cos 2\theta_w} \frac{M_W^2/M_{Z_V}^2}{1 - M_W^2/M_{Z_V}^2} \ln \frac{M_{Z_V}^2}{M_L^2} \quad (5.3b)$$

where the index 0 refers to Born amplitudes, $\bar{Q}_e$ and $\bar{Q}_h$ are the average charge of the quarks and leptons. If we replace M_{Z_V} with M_Z , $\cos 2\theta_w$ with $2\cos^2 \theta_w$ we obtain the formula of the Weinberg-Salam model. This simple result can be easily understood. The essential point is that the contributions of different Z bosons are proportional to the average of the corresponding "Z-charge" defined in Eq. (5.2). Since the average of I_3 in a given doublet is zero we have

$$\bar{Q}_{Z_A} = 0, \quad \bar{Q}_{Z_V} = -2 \sin^2 \theta_w \bar{Q} \frac{1}{2 \cos 2\theta_w}, \quad \bar{Q}_Z = -2 \sin^2 \theta_w \bar{Q} \frac{1}{2 \cos \theta_w} \quad (5.4)$$

We notice that if

$$M_Z^2 \cos^2 \theta_w = M_W^2, \quad M_{Z_V}^2 \cos 2\theta_w = M_W^2 \quad (5.5a)$$

as favored by the data, then

$$2 \frac{\sin^2 \theta_w}{\cos 2\theta_w} \cdot \frac{M_{W_L}^2 / M_{Z_V}^2}{1 - M_{W_L}^2 / M_{Z_V}^2} \approx 1 \quad , \quad \tan^2 \theta_w \frac{M_{W}^2 / M_Z^2}{1 - M_W^2 / M_Z^2} \approx 1 \quad (5.5b)$$

If we use the quantity $\Delta_V - \Delta_\mu$, as it is defined in Ref. 41), as a measure of deviation from Cabibbo universality we obtain

$$(\Delta_V - \Delta_\mu)_{LR} \approx \frac{1}{2\pi} \left[3 \ln \frac{M_{W_L}}{m_p} + 2C + 6 \bar{Q}_h \ln \frac{M_{W_L}}{m_A} + \right. \\ \left. + 6(\bar{Q}_h - \bar{Q}_e) \left(2 \frac{\sin^2 \theta_w}{\cos 2\theta_w} \frac{M_{W_L}^2 / M_{Z_V}^2}{1 - M_{W_L}^2 / M_{Z_V}^2} \ln \frac{M_{Z_V}}{M_{W_L}} \right) \right] \quad (5.6)$$

where m_p denotes the proton mass; the first three terms come from the contributions of diagrams given in Fig. 11a and have been calculated by Sirlin³⁸⁾. The second and third term give respectively the subasymptotic and asymptotic contributions controlled by the matrix element of the time ordered product of the electromagnetic and axial currents. m_A is a hadronic mass that describes the onset of the asymptotic behaviour and C is the non-asymptotic contribution. Since C is very model dependent and controlled by strong interactions, only rough estimates of C have been obtained. These estimates give values 10 to 20 times smaller than the other corrections. If we use the relations (5.5), $m_A \approx m_p$ and $\bar{Q}_h = 1/6$, $\bar{Q}_e = -1/2$ we obtain

$$(\Delta_V - \Delta_\mu)_{LR} \approx \frac{1}{\pi} \left[\ln \frac{M_{Z_V}^2}{m_p^2} + C \right] = (\Delta_V - \Delta_\mu)_{WS} + \frac{1}{\pi} \ln \frac{M_{Z_V}^2}{M_Z^2} \\ \approx (\Delta_V - \Delta_\mu)_{WS} + 2.2 \times 10^{-3}$$

Therefore

$$(\Delta_V - \Delta_\mu)_{LR} \approx 2.4 \times 10^{-2} + 10 \% \text{ theoretical uncertainty.}$$

The experimental number is

$$(\Delta_\nu - \Delta_\mu)_{\text{exp}} = (2.2 \pm 0.27) \times 10^{-2}$$

Consequently the corrections to Cabibbo universality are still consistent with the experimental data also in the L-R model, and its numerical value differs from that of the WS model only in 10 %. We remind that there is a 10 % theoretical uncertainty in the calculation of the contribution of the diagrams in Fig. 11a, given by the term C, as well.

6. Conclusions

We have examined in this paper the left-right symmetric $SU(2)_L \otimes SU(2)_R \otimes U(1)$ gauge models with naturally parity conserving neutral current. Motivated by the null result of the Bismuth experiment ¹⁶⁾ and by the aesthetical beauty of having spontaneous parity violation in weak interactions, these models have been extensively analyzed in the literature ⁶⁻¹⁵⁾. Here we presented a systematic study of the phenomenological implications of imposing natural parity conservation on the neutral boson sector. Although some of the difficulties have been realized by various authors ^{8,9,10)}, we believe our study is more complete, our results are independent of assumptions on the reality of coupling constants.

We have shown that if we impose natural parity conservation in the neutral current sector it is necessary to have left-right symmetric fermion multiplets. Furthermore we have pointed out that the models which have one or more complex Higgs quartets $\Phi: (\frac{1}{2}, \frac{1}{2}, 0)$, two Higgs doublets $\chi_L (\frac{1}{2}, 0, -\frac{1}{2}), \chi_R (0, \frac{1}{2}, -\frac{1}{2})$ with or without Higgs triplets $\delta_L (1, 0, 0), \delta_R (0, 1, 0)$ are phenomenologically unacceptable. So we do not agree with previous results ¹⁰⁾ that the $2-\Phi$ version of these models is tenable. In these models either parity conservation is unnatural or we have large V+A currents at low energies. Even if we allow small but uncalculable parity violation, along with the desired ν -phenomenology, the χ, Φ scheme may be unacceptable. Natural flavour conservation in the neutral current transition in all $SU(2)_L \otimes SU(2)_R \otimes U(1)$ models require that the Higgs scalar self-coupling is large ³⁷⁾. In the χ, Φ scheme however this difficulty becomes acute.

If the $W_{L,R}^\pm$ -boson masses are split, it is possible to have naturally parity conserving neutral currents with two different complex quartets $\Phi (\frac{1}{2}, \frac{1}{2}, 0)$

and $\phi(\frac{1}{2}, \frac{1}{2}, -1)$ and two real triplets δ_1, δ_2 . The observed weak interaction phenomenology in the ν -sector, however, appears accidental: it requires at least three "unnatural" constraints on the parameters of the theory. In this model we have calculated atomic parity violation induced by radiative corrections. We have found the embarrassing result that the contributions of the physical Higgs scalars may be large. Consequently the induced parity violation cannot be calculated if we do not know numerical values of the charged Higgs boson masses ⁺⁾ .

In attempting to find arguments favouring or disfavouring the $SU(2)_L \otimes SU(2)_R \otimes U(1)$ model independent of the Higgs sector, we have calculated radiative corrections to Cabibbo universality and we found agreement with the present data.

In summary the $SU(2)_L \times SU(2)_R \times U(1)$ gauge models pay an embarrassingly high price for parity conservation in the neutral sector at low energies (see the Table). Should parity conservation be confirmed in e-N weak interactions, it is likely to have a deeper origin.

Acknowledgements

We have enjoyed many helpful discussions with our colleagues. We take pleasure in thanking Rod Crewther, Alvaro De Rujula, John Ellis, Douglas Ross, Tony Sanda and Gary Steigman. One of us (Z.K.) is grateful to Jacques Prentki for his extended hospitality in the CERN Theoretical Physics Division.

⁺⁾

While this work was being completed we received calculation of the Higgs contribution by Senjanović and Sokorac ²⁹⁾, we agree with their results. They have neglected, however, the mass independent terms.

Appendix A

We have shown in Sections 2 and 3 that the $SU(2)_L \otimes SU(2)_R \otimes U(1)$ models with naturally parity conserving neutral currents have necessarily complex Higgs sector. In this Appendix we present some details of the arguments obtained from the study of the Higgs potential. First we discuss the pattern with one or two complex quartets $\Phi(\frac{1}{2}, \frac{1}{2}, 0)$ and two doublets $\chi_L(\frac{1}{2}, 0, \frac{1}{2}), \chi_R(0, \frac{1}{2}, \frac{1}{2})$. Then we investigate the pattern with two different complex quartets ϕ, ϑ and two real triplets δ_L, δ_R (see Eqs. (3.24)-(3.27)).

Since we discuss only vacuum expectation values and neutral Higgs masses, we need only consider the part of the potential $V(\phi, \chi_L, \chi_R)$ containing neutral Higgs. There are fewer invariant terms than for the full potential. They can be obtained either from terms in the most general $SU(2)_L \otimes SU(2)_R \otimes U(1)$ potential or directly by imposing on $V(\varphi^0, \vartheta^0, \chi_L^0, \chi_R^0)$ hermicity, parity ($\varphi \leftrightarrow \bar{\varphi}, \vartheta \leftrightarrow \bar{\vartheta}, \chi_L \leftrightarrow \chi_R$), conservation of I_3^L, I_3^R and $I_3^L \otimes I_3^R$ reflection ($\varphi^0 \leftrightarrow \bar{\varphi}^0, \chi_{L,R}^0 \leftrightarrow \bar{\chi}_{L,R}^0$) where $\varphi^0, \vartheta^0, \chi_L^0, \chi_R^0$ are the field components defined as

$$\phi = \begin{pmatrix} \varphi^0 & \varphi^+ \\ \vartheta^- & \vartheta^0 \end{pmatrix} \quad \chi_L = \begin{pmatrix} \chi_L^+ \\ \chi_L^0 \end{pmatrix}, \quad \chi_R = \begin{pmatrix} \chi_R^+ \\ \chi_R^0 \end{pmatrix} \quad (A1)$$

So we obtain the formula (dropping the superscript 0)

$$\begin{aligned} V_n = & -\mu^2 \{ |\varphi|^2 + |\vartheta|^2 \} - \nu^2 (\varphi \vartheta + \bar{\varphi} \bar{\vartheta}) - M^2 (|\chi_L|^2 + |\chi_R|^2) \\ & + g_1 [(\bar{\chi}_L \vartheta \chi_R) + (\bar{\chi}_R \bar{\vartheta} \chi_L)] + g_2 [\bar{\chi}_L \bar{\varphi} \chi_R + \bar{\chi}_R \varphi \chi_L] \\ & + \lambda_1 \{ |\varphi|^4 + |\vartheta|^4 \} + \lambda_2 |\varphi|^2 |\vartheta|^2 + \lambda_3 (|\varphi|^2 + |\vartheta|^2) (\varphi \vartheta + \bar{\varphi} \bar{\vartheta}) + \\ & + \lambda_4 (\varphi^2 \vartheta^2 + \bar{\varphi}^2 \bar{\vartheta}^2) + (\lambda_5 |\varphi|^2 + \lambda_6 |\vartheta|^2) (|\chi_L|^2 + |\chi_R|^2) + \\ & + \lambda_7 (|\chi_L|^2 + |\chi_R|^2) (\varphi \vartheta + \bar{\varphi} \bar{\vartheta}) + \\ & + i \tilde{\lambda}_7 (|\chi_L|^2 - |\chi_R|^2) (\varphi \vartheta - \bar{\varphi} \bar{\vartheta}) + \\ & + \lambda_8 (|\chi_L|^4 + |\chi_R|^4) + \\ & + \lambda_9 (|\chi_L|^2 - |\chi_R|^2)^2 \end{aligned} \quad (A2)$$

If we impose CP defined as

$$\varphi \rightarrow \bar{\varphi}, \quad \theta \rightarrow \bar{\theta} \quad \text{and} \quad \chi_{L,R} \rightarrow \bar{\chi}_{L,R}$$

we eliminate the coupling $\tilde{\lambda}_7$.

To have a minimum, the partial derivatives $\frac{\partial V}{\partial \chi_L}, \frac{\partial V}{\partial \chi_R}, \frac{\partial V}{\partial \varphi}, \frac{\partial V}{\partial \theta}$ must vanish. If we calculate the real part of the linear combination

$$\frac{1}{\tilde{\chi}_L} \frac{\partial V}{\partial \chi_L} - \frac{1}{\tilde{\chi}_R} \frac{\partial V}{\partial \chi_R} \quad (A3)$$

we obtain the condition (3.45)

$$4 \tilde{\lambda}_7 \text{Im}(\langle \varphi \rangle \langle \theta \rangle) = [|\langle \chi_L \rangle|^2 - |\langle \chi_R \rangle|^2] [\lambda_9 - 2\lambda_8 + \text{Re}(\frac{g_1 \langle \theta \rangle + g_2 \langle \varphi \rangle^*}{\langle \chi_L \rangle \langle \chi_R \rangle^*})]$$

Consequently to have the condition

$$|\langle \chi_L \rangle|^2 - |\langle \chi_R \rangle|^2 = 0 \quad (A4)$$

naturally, either $\text{Im}(\langle \varphi \rangle \langle \theta \rangle) = 0$ or we must impose CP conservation. In both cases however we have the same Cabibbo angle (up to phases) for the left and right-handed fermions. (See Appendix B). It does not help if we introduce another Higgs quartet

$$\Phi_2 = \begin{pmatrix} \varphi_2^0 & \varphi_2^+ \\ \theta_2^- & \theta_2^0 \end{pmatrix}$$

If we again impose on the Higgs potential hermiticity and parity⁺) defined as

+) If we define φ_i, θ_i to be even under parity we have $\theta_L^{\text{Cabibbo}} = \theta_R^{\text{Cabibbo}}$ independently of the naturalness of the condition (A4). There are alternative definitions of parity, e.g., like $\varphi_1 \leftrightarrow \theta_1, \varphi_2 \leftrightarrow \theta_2$. We have checked that then the condition (A4) cannot be obtained.

$$\begin{aligned} \varphi_1 &\rightarrow \bar{\varphi}_1 & \varphi_2 &\rightarrow -\bar{\varphi}_2 \\ \theta_1 &\rightarrow \bar{\theta}_1 & \theta_2 &\rightarrow -\bar{\theta}_2 \end{aligned} \quad \chi_L \rightarrow \chi_R$$

the neutral part of the Higgs potential, containing fields χ_L, χ_R consists of terms

$$\begin{aligned} V_H = & -M^2(|\chi_L|^2 + |\chi_R|^2) + g_1(\bar{\chi}_L \theta_1 \chi_R + \bar{\chi}_R \bar{\theta}_1 \chi_L) + i g_2(\bar{\chi}_L \theta_2 \chi_R - \bar{\chi}_R \bar{\theta}_2 \chi_L) + \\ & + g'_1(\bar{\chi}_R \varphi_1 \chi_L + \bar{\chi}_L \bar{\varphi}_1 \chi_R) + i g'_2(\bar{\chi}_R \bar{\varphi}_2 \chi_L - \bar{\chi}_L \bar{\varphi}_2 \chi_R) + \\ & + [\lambda_1 |\varphi_1|^2 + \lambda_2 |\varphi_2|^2 + \lambda'_1 |\theta_1|^2 + \lambda'_2 |\theta_2|^2 + \\ & + \mu_1^2 \text{Re}(\varphi_1 \theta_1) + \mu_2^2 \text{Re}(\varphi_2 \theta_2) + \rho_1 \text{Im}(\varphi_1 \bar{\varphi}_2) + \\ & + \rho_2 \text{Im}(\theta_1 \bar{\theta}_2) + \rho_3 \text{Im}(\varphi_1 \theta_2) + \rho_4 \text{Im}(\varphi_2 \theta_1)] [|\chi_L|^2 + |\chi_R|^2] + \\ & + [\tilde{\mu}_1^2 \text{Im}(\varphi_1 \theta_1) + \tilde{\mu}_2^2 \text{Im}(\varphi_2 \theta_2) + \tilde{\rho}_1 \text{Re}(\varphi_1 \bar{\varphi}_2) + \tilde{\rho}_2 \text{Re}(\theta_1 \bar{\theta}_2) + \\ & + \tilde{\rho}_3 \text{Re}(\varphi_1 \theta_2) + \tilde{\rho}_4 \text{Re}(\varphi_2 \theta_1)] [|\chi_L|^2 - |\chi_R|^2] + \\ & + \delta_1 (|\chi_L|^4 + |\chi_R|^4) + \delta_2 |\chi_L|^2 |\chi_R|^2 \end{aligned} \quad (A5)$$

If we impose CP conservation

$$\begin{aligned} \varphi_1 &\rightarrow \bar{\varphi}_1 & \theta_1 &\rightarrow \bar{\theta}_1 \\ \varphi_2 &\rightarrow -\bar{\varphi}_2 & \theta_2 &\rightarrow -\bar{\theta}_2 \end{aligned} \quad \chi_{L,R} \rightarrow \bar{\chi}_{L,R} \quad (A6)$$

the couplings $\tilde{\mu}_i, \tilde{\rho}_i$ must vanish. If we calculate again the linear combination (A3) we obtain the equation

$$\begin{aligned} & \tilde{\mu}_1^2 \text{Im}(\varphi_1 \theta_1) + \tilde{\mu}_2^2 \text{Im}(\varphi_2 \theta_2) + \tilde{\rho}_1 \text{Re}(\varphi_1 \bar{\varphi}_2) + \tilde{\rho}_2 \text{Re}(\theta_1 \bar{\theta}_2) + \\ & + \tilde{\rho}_3 \text{Re}(\varphi_1 \theta_2) + \tilde{\rho}_4 \text{Re}(\varphi_2 \theta_1) = \frac{i}{2} [|\chi_L|^2 - |\chi_R|^2] [2\delta_1 - \delta_2 + \dots] \end{aligned}$$

We again conclude that in order to obtain the condition (A4) naturally we must either impose CP conservation or we must require that the phase of the φ_L, θ_L fields differs from that of the fields φ_R, θ_R by $\pi/2$. Again in both cases we have related Cabibbo angles in the left and the right-handed sector (see Appendix B).

Let us consider now the order of the magnitude of the Higgs masses given by the potential (A2), in order to understand the importance of the phenomenological constraint

$$m_\phi^2 > O(v^4/v_0^2) \quad (A7)$$

given by the suppression of the flavour changing neutral currents (see Eqs. (3.36) and (3.37)). v_0^2 and v^2 denote some characteristic values of orders of magnitude

$$\begin{aligned} v^2 &\approx O(|\chi_L|^2, |\chi_R|^2) \\ v_0^2 &\approx O(|\langle \theta \rangle|^2, |\langle \varphi \rangle|^2), \end{aligned} \quad (A8)$$

respectively, and

$$v^2/v_0^2 \gg 1 \quad (A9)$$

We can see from the explicit form of the potential (A2) that the trace of the mass matrix is

$$\text{Tr}(M^2) = -2M^2 - 2\mu^2 + q_1 v_0^2 \lambda_{\phi\phi}^i + q_1' v^2 \lambda_{\phi\chi}^i + q_1'' v^2 \lambda_{\chi\chi}^i \quad (A10)$$

where the dimensionless coefficients q_1, q_1', q_1'' are of magnitude $O(1)$. We can use the linear condition

$$\text{Re} \left(\frac{1}{\phi} \frac{\partial V}{\partial \varphi} + \frac{1}{\theta} \frac{\partial V}{\partial \theta} + \frac{1}{\chi_L} \frac{\partial V}{\partial \chi_L} + \frac{1}{\chi_R} \frac{\partial V}{\partial \chi_R} \right) = 0$$

to eliminate the mass parameters M^2, μ^2 and we get

$$\text{Tr}(M^2) = a_i v_0^2 \lambda_{\phi\phi}^i + b_i v^2 \lambda_{\phi\chi}^i + c_i v^2 \lambda_{\chi\chi}^i + d_i v^2 g^i / v_0 \quad (\text{A11})$$

If the Higgs masses are dominated by the last term we obtain from the condition (A7) that ⁺⁾

$$|g^i / m| > 1 \quad (\text{A12})$$

This condition is not obviously stronger than $\lambda \geq 1$ as required in any $\text{SU}(2) \otimes \text{SU}(2) \otimes \text{U}(1)$ model³⁷⁾. We may attempt to estimate the order of magnitude of three-point couplings in terms of $\lambda^i v_0$. It is easy to see from Eq. (A2), that a suitable combination

$$x_1 \text{Im} \left(\frac{1}{\phi} \frac{\partial V}{\partial \varphi} + \frac{1}{\theta} \frac{\partial V}{\partial \theta} \right) + x_2 \text{Re} \left(\frac{1}{\phi} \frac{\partial V}{\partial \varphi} - \frac{1}{\theta} \frac{\partial V}{\partial \theta} \right)$$

where the dimensionless coefficients x_1, x_2 are of $\mathcal{O}(1)$, gives a condition of the form

$$\bar{a}_i \lambda_{\phi\phi}^i v_0^2 + \bar{b}_i \lambda_{\chi\phi}^i v^2 + \bar{c}_i g^i v^2 / v_0 = 0$$

where again $\bar{a}_i, \bar{b}_i, \bar{c}_i$ are $\mathcal{O}(1)$. Consequently if no important cancellation occurs between the contribution of the terms $\bar{c}_1 g_1 v^2 / v_0$ and $\bar{c}_2 g_2 v^2 / v_0$ we obtain either

$$g^i \sim \mathcal{O}(\lambda_{\chi\phi} v_0) \quad \text{or} \quad g^i \sim \mathcal{O}(\lambda_{\phi\phi} v_0 \sim v^2 / v_0) \quad (\text{A13})$$

In both cases instead of the inequality (A12) we have the stronger conditions either

$$\lambda_{\phi\phi} > (v^2 / v_0^2)^2 \gg 1 \quad \text{or} \quad \lambda_{\chi\phi} > v^2 / v_0^2 \gg 1 \quad (\text{A14})$$

⁺⁾ If the other terms dominate we have instead the condition that for some of the couplings $|\lambda^i| > v^2 / v_0^2 \gg 1$.

Let us consider now the other pattern (Eqs. 3.24-3.27). The most general $SU(2)_L \otimes SU_R(L) \otimes U(1)$ and parity invariant Higgs potential consists of 28 terms as follows (where $(\Phi^\dagger \Phi) = \text{Tr} (\Phi^\dagger \Phi)$, etc.)

$$\begin{aligned}
 V(\phi, \varphi, \delta_L, \delta_R) = & \mu^2 (\Phi^\dagger \Phi) + M^2 (\varphi^\dagger \varphi) + \frac{1}{2} m^2 (\vec{\delta}_L^2 + \vec{\delta}_R^2) + k^2 [(\tilde{\Phi}^\dagger \Phi) + (\Phi^\dagger \tilde{\Phi})] + \\
 & + \lambda_1 (\Phi^\dagger \Phi)^2 + \lambda_2 (\tilde{\Phi}^\dagger \Phi) (\Phi^\dagger \tilde{\Phi}) + \lambda_3 (\Phi^\dagger \Phi) (\varphi^\dagger \varphi) + \lambda_4 (\Phi^\dagger \varphi) (\varphi^\dagger \Phi) + \\
 & + \lambda_5 (\tilde{\Phi}^\dagger \varphi) (\varphi^\dagger \tilde{\Phi}) + \lambda_6 [\text{Tr} (\varphi^\dagger \varphi \Phi^\dagger \Phi) + \text{Tr} (\varphi \varphi^\dagger \Phi \Phi^\dagger)] + \lambda_7 (\Phi^\dagger \Phi) (\vec{\delta}_L^2 + \vec{\delta}_R^2) + \\
 & + \lambda_8 \delta_L^i \delta_R^i \text{Tr} (\Phi^\dagger \sigma^i \Phi \sigma^i) + \lambda_9 [\delta_L^i \text{Tr} (\Phi^\dagger \sigma^i \Phi) + \delta_R^i \text{Tr} (\Phi^\dagger \sigma^i \Phi)] + \\
 & + F_1 [(\tilde{\Phi}^\dagger \Phi)^2 + (\Phi^\dagger \tilde{\Phi})^2] + F_2 [(\Phi^\dagger \Phi) (\tilde{\Phi}^\dagger \Phi) + (\Phi^\dagger \Phi) (\Phi^\dagger \tilde{\Phi})] + \\
 & + F_3 (\varphi^\dagger \varphi) [(\tilde{\Phi}^\dagger \Phi) + (\Phi^\dagger \tilde{\Phi})] + F_4 [(\tilde{\Phi}^\dagger \varphi) (\varphi^\dagger \Phi) + (\varphi^\dagger \tilde{\Phi}) (\Phi^\dagger \varphi)] + \\
 & + F_5 [(\tilde{\Phi}^\dagger \Phi) + (\Phi^\dagger \tilde{\Phi})] (\vec{\delta}_L^2 + \vec{\delta}_R^2) + i F_6 [(\tilde{\Phi}^\dagger \Phi) - (\Phi^\dagger \tilde{\Phi})] (\vec{\delta}_L^2 - \vec{\delta}_R^2) + \quad (A15) \\
 & + F_7 \delta_L^i \delta_R^i \text{Tr} (\tilde{\Phi}^\dagger \sigma^i \Phi \sigma^i + \Phi^\dagger \sigma^i \tilde{\Phi} \sigma^i) + \\
 & + F_8 [\delta_L^i \text{Tr} (\tilde{\Phi}^\dagger \sigma^i \Phi) + \delta_L^i \text{Tr} (\Phi^\dagger \sigma^i \tilde{\Phi}) + \delta_R^i \text{Tr} (\Phi^\dagger \tilde{\Phi} \sigma^i) + \delta_R^i \text{Tr} (\tilde{\Phi}^\dagger \Phi \sigma^i)] + \\
 & + h_1 (\varphi^\dagger \varphi)^2 + h_2 (\tilde{\varphi}^\dagger \tilde{\varphi}) (\varphi^\dagger \varphi) + h_3 (\varphi^\dagger \varphi) (\vec{\delta}_L^2 + \vec{\delta}_R^2) + \\
 & + h_4 \delta_L^i \delta_R^i \text{Tr} (\varphi^\dagger \sigma^i \varphi \sigma^i) + \\
 & + h_5 [\delta_L^i \text{Tr} (\varphi^\dagger \sigma^i \varphi) + \delta_R^i \text{Tr} (\varphi^\dagger \sigma^i \varphi)] \\
 & + g_1 [(\vec{\delta}_L^2)^2 + (\vec{\delta}_R^2)^2 + 2 \vec{\delta}_L^2 \vec{\delta}_R^2] + g_2 \vec{\delta}_L^2 \vec{\delta}_R^2
 \end{aligned}$$

This long expression can be simplified if we demand the Higgs potential to be symmetric under an independent phase transformation on the field

$$\Phi \rightarrow e^{i\alpha} \Phi$$

Furthermore we can also demand invariance with respect to the simultaneous change

of signs of the $\vec{\delta}_L, \vec{\delta}_R$ fields. ⁺⁾ In this case the couplings F, λ_5, h_5 are eliminated. Substituting the fields (Eq. 3.6, 3.25, 3.27) into the potential we obtain the formula

$$\begin{aligned} V(a, \epsilon, c, d, b) = & \mu^2 (|a|^2 + |\epsilon|^2) + M^2 |b|^2 + \frac{1}{2} m^2 (c^2 + d^2) \\ & + \lambda_1 (|a|^2 + |\epsilon|^2) + 4\lambda_2 |a|^2 |\epsilon|^2 + (\lambda_3 + \lambda_6) |b|^2 (|a|^2 + |\epsilon|^2) + \\ & + \lambda_7 (|a|^2 + |\epsilon|^2) (c^2 + d^2) + \lambda_8 (|\epsilon|^2 + |a|^2) cd + \quad (A16) \\ & + h_1 |b|^4 + h_3 |b|^2 (c^2 + d^2) + g_1 (c^2 + d^2)^2 + g_2 c^2 d^2 \end{aligned}$$

This potential is independent of the phase of ϵ/a , which indicates that there is no Goldstone boson associated with the $U(1)_\phi$ symmetry, but instead we have a left-over symmetry in the broken potential. From the requirement that the derivatives of V with respect to a, ϵ, b, c, d must vanish we obtain 5 homogeneous equations as follows (for simplicity we take real vevs)

$$\begin{aligned} a [\mu^2 + 2\lambda_1 (a^2 + \epsilon^2) + (\lambda_3 + \lambda_6) b^2 + \lambda_7 (c^2 + d^2) + \lambda_8 cd] + \\ + \epsilon [4\lambda_2 a\epsilon] = 0 \end{aligned} \quad (A17a)$$

$$\begin{aligned} \epsilon [\mu^2 + 2\lambda_1 (a^2 + \epsilon^2) + (\lambda_3 + \lambda_6) b^2 + \lambda_7 (c^2 + d^2) + \lambda_8 cd] + \\ + a [4\lambda_2 a\epsilon] = 0 \end{aligned} \quad (A17b)$$

$$\begin{aligned} c [m^2 + 2\lambda_7 (a^2 + \epsilon^2) + 2h_3 b^2 + 4g_1 (c^2 + d^2)] + \\ + d [\lambda_8 (\epsilon^2 + a^2) + 2g_2 cd] = 0 \end{aligned} \quad (A17c)$$

$$\begin{aligned} d [m^2 + 2\lambda_7 (a^2 + \epsilon^2) + 2h_3 b^2 + 4g_1 (c^2 + d^2)] + \\ + c [\lambda_8 (\epsilon^2 + a^2) + 2g_2 cd] = 0 \end{aligned} \quad (A17d)$$

⁺⁾ One is not allowed to require symmetry with respect to the independent change of signs of δ_L and δ_R since it would lead to unwanted $SO(3)_S$ symmetry and subsequently to charged Goldstone bosons.

$$b[M^2 + (\lambda_3 + \lambda_4)(a^2 + \epsilon^2) + 2h_1 b^2 + h_3(c^2 + d^2)] = 0 \quad (A17e)$$

We can easily see from Eqs. (A17a), (A17b) that one has non-trivial solutions only if either

$$a = \epsilon \quad \text{or} \quad a\epsilon = 0 \quad (A18)$$

In the first case we would have vanishing Cabibbo angles, so we choose $\epsilon = 0$. If we would add to the potential the $U(1)_\phi$ symmetry breaking term $k^2[(\tilde{\Phi}\Phi) + (\Phi^\dagger\tilde{\Phi})]$ then we would have solutions with $\epsilon^2 \propto O(k^2)$.

In this case one can have the most general fermion mass matrix. This feature of the Higgs potential has been noticed in Ref. (14) in the simpler case when $c = d = b = 0$.

It is also easy to see from Eq. (A17a,c,d,e) that if the couplings of the triplet fields δ_1, δ_2 are much weaker than the other couplings [$\lambda_1, \lambda_2, h_1, h_2, g_1, g_2 \sim O(a^2/c^2\lambda_1)$ for instance] then the equations (A17a,c,d,e) have "natural" solutions satisfying the conditions (see Eqs. (3.28), (3.57) with $\epsilon = 0$)

$$c \gg a, b \gg d \approx O(a^2/c) \quad (A19)$$

But we cannot explain why the triplet field couplings are weaker and the equality of the parameters a, b remains a numerical accident. In this sense the successful phenomenology in the ν -sector is not obtained naturally. As discussed in section 3.3 the invariance under $\Phi \rightarrow e^{i\alpha}\Phi$ cannot be respected by realistic Yukawa coupling and in general we must have $a\epsilon \neq 0$. This complicates the argument, but the conclusions are the same.

Appendix B

In this appendix we discuss the connections between P and CP transformations and the Cabibbo matrices. We show that, on this ground, models with χ_L, χ_R and Φ fields are not phenomenologically viable.

B.1 Parity

B.1.1 One Φ field

As shown in the text, the Yukawa coupling to be taken is

$$\mathcal{L}_Y = \bar{q}_L \Phi G q_R + \bar{q}_L \tilde{\Phi} \tilde{G} q_R + h.c.$$

The most general "parity" transformation is then

$$\left\{ \begin{array}{l} q_L \rightarrow S' q_R \\ q_R \rightarrow S'' q_L \end{array} \right. \quad \text{and} \quad \left\{ \begin{array}{l} \Phi \rightarrow \eta \Phi^\dagger \\ \Phi \rightarrow \eta \tilde{\Phi}^\dagger \end{array} \right. \quad \text{or}$$

where S and S' are unitary matrices and $|\eta|^2 = 1$.

By redefining the Φ field and the quark basis, we can simplify this transformation

$$\left\{ \begin{array}{l} q_L \rightarrow q_L \\ q_R \rightarrow S q_R \end{array} \right. \quad \text{and} \quad \left\{ \begin{array}{l} \Phi \rightarrow \tilde{\Phi}^\dagger \\ \Phi \rightarrow \Phi^\dagger \end{array} \right. \quad \begin{array}{l} \text{case (a)} \\ \text{case (b)} \end{array}$$

Remember that the mass matrices are $G_a = \epsilon G + a^* \tilde{G}$

$$G_c = a G + \epsilon^* \tilde{G}$$

Case (a)

"Parity" invariance states that

$$\mathcal{L}_Y = \bar{q}_R \tilde{\Phi}^\dagger G S q_L + \bar{q}_R \Phi^\dagger \tilde{G} S q_L + h.c.$$

that is

$$G S = \tilde{G}^\dagger \quad \text{and} \quad \tilde{G} S = G^\dagger$$

Then

$$G_a S = \epsilon \tilde{G}^\dagger + a^* G^\dagger = G_c^\dagger$$

$$G_c S = a \tilde{G}^\dagger + \epsilon^* G^\dagger = G_a^\dagger$$

Note that $G_a S = S G_a$ and $G_c S = S G_c$. So the S matrices are not entirely arbitrary. Moreover, in most cases, one can show that $S = 1$. We have

$$G_a G_a^\dagger = G_c^\dagger G_c \quad G_a^\dagger G_a = G_c G_c^\dagger \quad (3.11)$$

Since $(G^\dagger G)_{a,c}$ has eigenvalues $M_{a,c}^2$, we get from (3.11) a degeneracy of a quark and c quark masses.

Moreover, if $M_{a(c)}$ is the diagonal mass matrix

$$M_{a(c)} = U_{a(c)}^\dagger G_{a(c)} V_{a(c)}$$

then

$$U_{a(c)} \text{ diagonalises } G_{a(c)} G_{a(c)}^\dagger$$

$$V_{a(c)} \text{ diagonalises } G_{a(c)}^\dagger G_{a(c)}$$

So we get from (3.11), up to phases $U_a = V_c$, $U_c = V_a$. The Cabibbo matrices being

$$U_L = U_a^\dagger U_c$$

$$U_R = V_a^\dagger V_c$$

we have, up to phases $U_L = U_R^\dagger$.

Case (b)

"Parity" invariance gives $\mathcal{L}_Y = \bar{q}_L \Phi^\dagger G S q_L + \bar{q}_R \tilde{\Phi}^\dagger \tilde{G} S q_L + \text{h.c.}$

that is $GS = G^\dagger$ and $\tilde{G}S = \tilde{G}^\dagger$

Then

$$G^\dagger G = G S S^\dagger G^\dagger = G G^\dagger$$

Likewise $\tilde{G}^\dagger \tilde{G} = \tilde{G} \tilde{G}^\dagger$, $G^\dagger \tilde{G} = G \tilde{G}^\dagger$, $\tilde{G}^\dagger G = \tilde{G} G^\dagger$

$$G_a G_a^\dagger = |\epsilon|^2 G G^\dagger + |a|^2 \tilde{G} \tilde{G}^\dagger + \epsilon a G \tilde{G}^\dagger + \epsilon^* a^* \tilde{G} G^\dagger$$

$$G_a^\dagger G_a = |\epsilon|^2 G G^\dagger + |a|^2 \tilde{G} \tilde{G}^\dagger + \epsilon a \tilde{G} G^\dagger + \epsilon^* a^* G \tilde{G}^\dagger$$

And

$$\left. \begin{aligned} G_a G_a^\dagger - G_a^\dagger G_a &= 2i \operatorname{Im}(\epsilon a) (G \tilde{G}^\dagger - \tilde{G} G^\dagger) \\ G_c G_c^\dagger - G_c^\dagger G_c &= 2i \operatorname{Im}(\epsilon a) (G \tilde{G}^\dagger - \tilde{G} G^\dagger) \end{aligned} \right\} \quad (3.12)$$

In the same way

If ϵa is real,

$$G_{a(c)} G_{a(c)}^\dagger = G_{a(c)}^\dagger G_{a(c)}$$

Since

$$\begin{aligned} U_{a(c)} &\text{ diagonalises } G_{a(c)} G_{a(c)}^\dagger \\ V_{a(c)} &\text{ diagonalises } G_{a(c)}^\dagger G_{a(c)} \end{aligned}$$

we get

$$U_{a(c)} = V_{a(c)} \quad \text{and} \quad U_L = U_R$$

up to phases. However, we would like to show that case (b) is entirely ruled out by requiring the phenomenologically desirable Cabibbo mixing in the χ_L, χ_R, ϕ model. Indeed, whenever the matrix element U_{ij}^L between state i and state j is big, we must demand that U_{ij}^R be small, and vice versa.

Let us write, from the above equations,

$$G_a G_a^\dagger = |\lambda|^2 G_a^\dagger G_a + |\mu|^2 G_c^\dagger G_c + \lambda^* \mu G_a^\dagger G_c + \lambda \mu^* G_c^\dagger G_a \quad (B.1)$$

where

$$\lambda = \frac{\epsilon^* \epsilon - a^2}{|\epsilon|^2 - |a|^2} \quad \text{and} \quad \mu = \frac{a\epsilon - a^* \epsilon^*}{|\epsilon|^2 - |a|^2}$$

Notice that $|\lambda|^2 = 1 + |\mu|^2$ and μ is purely imaginary. By writing

$$\begin{aligned} G_a^\dagger G_a &= V_a M_a^2 V_a^\dagger \\ G_e^\dagger G_e &= V_e M_e^2 V_e^\dagger \\ G_a^\dagger G_e &= V_a (M_a U_L M_e U_R^\dagger) V_a^\dagger \end{aligned}$$

we get, by taking the trace of (B.1),

$$|\mu| \left(\sum_i m_i^{a2} + \sum_i m_i^{e2} \right) = 2(\text{Im} \lambda) \text{Tr} (M_a U_L M_e U_R^\dagger) \quad (\text{B.2})$$

But our phenomenological requirement can be stated in the following way

$$\left| \sum_j U_{ij}^{L*} U_{ij}^R \right| \leq \delta \quad \left| \sum_i U_{ij}^{L*} U_{ij}^R \right| \leq \delta \quad (\text{B.3})$$

where δ is some number much smaller than one.

We have from (B.3)

$$\begin{aligned} 2 \left| \text{Tr} (M_a U_L M_e U_R^\dagger) \right| &= 2 \left| \sum_{ij} m_i^a U_{Lij} m_j^e U_{Rij}^* \right| \\ &\leq \left| \sum_{ij} (m_i^{a2} U_{ij}^L U_{ij}^{R*} + m_{ij}^{e2} U_{ij}^L U_{ij}^{R*}) \right| \\ &\leq \delta \left(\sum_i m_i^{a2} + \sum_j m_j^{e2} \right) \end{aligned}$$

and comparing with (B.2)

$$|\mu|/|\lambda| \leq \delta$$

Since $|\lambda|^2 = 1 + |\mu|^2$, we finally obtain that

$$|\lambda| = O(1) \quad \text{and} \quad |\mu| = O(\delta)$$

Comparing with (B.1) we find that

$$G_a G_a^\dagger - G_a^\dagger G_a = U_a M_a^2 U_a^\dagger - V_a M_a^2 V_a^\dagger = o(\delta)$$

this means that $V_a^\dagger U_a$ commutes with M_a^2 , up to order δ and unless M_a^2 has degenerate eigenvalues, $V_a^\dagger U_a$ is diagonal up to order δ :

$$U_a = V_a D + o(\delta)$$

where D is a diagonal matrix.

Since $V_a^\dagger U_a$ is unitary,

$$|D_{ii}| = 1 + o(\delta^2)$$

Similarly,

$$U_c = V_c D' + o(\delta)$$

where D' is a diagonal matrix. Then

$$U_L = U_a^\dagger U_c = D^\dagger V_a^\dagger V_c D' + o(\delta) = D^\dagger U_R D' + o(\delta)$$

And

$$(U_L)_{ij} = (U_R)_{ij} + o(\delta)$$

This is in contradiction with our initial requirements. The model with χ_L, χ_R and one Φ field is then rejected on this basis.

B.1.2 Two Φ fields

Let us call these two fields Φ_1 and Φ_2 .

As seen before, we must also introduce

$$\Phi_3 = \tilde{\Phi}_1 = \tau_2 \Phi_1^* \tau_2 \quad \text{and} \quad \Phi_4 = \tilde{\Phi}_2 = \tau_2 \Phi_2^* \tau_2$$

The Yukawa coupling is then

$$\mathcal{L}_Y = \sum_{i=1}^4 \bar{q}_L \Phi_i G^i q_R + h.c.$$

(B.4)

We denote the "parity" transformation for the Φ fields in the following way

$$\begin{pmatrix} \Phi_1 \\ \Phi_2 \\ \Phi_3 \\ \Phi_4 \end{pmatrix} \rightarrow P \begin{pmatrix} \Phi_1^+ \\ \Phi_2^+ \\ \Phi_3^+ \\ \Phi_4^+ \end{pmatrix}$$

where P is a 4×4 matrix.

Since the Φ_i 's are not independent, P has the structure

$$P = \begin{pmatrix} A & B \\ B^* & A^* \end{pmatrix}$$

where A and B are 2×2 matrices.

Requiring that the Φ kinetic energy terms are invariant under parity gives

$$\begin{aligned} A^\dagger A + (B^\dagger B)^T &= 1 \\ B^\dagger A + A^\dagger B &= 0 \end{aligned} \quad (B.5)$$

Note that this implies that P is unitary. Requiring that L_Y is invariant under parity gives

$$G^{\dagger\dagger} = G^i P_{ij} \quad (B.6)$$

This last equation is possible only if

$$G^i P_{ij} = G^i P_{ji} \quad (B.7)$$

Let us come back to (B.5). Three cases arise:

Case (a): $B = 0$, A is unitary.

By redefining Φ_1 and Φ_2 , one can diagonalize A

$$A = \begin{pmatrix} \eta_1 & 0 \\ 0 & \eta_2 \end{pmatrix} \quad |\eta_1|^2 = |\eta_2|^2 = 1$$

If we require P to be real, then, up to phases

$$A = 1 \quad \text{or} \quad A = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

If the vacuum expectation values $\langle \Phi_i \rangle$'s are real, the first choice gives equal left and right Cabibbo matrices. Indeed, (B.6) gives $G^{\dagger\dagger} = G^i$. The mass matrices have the form

$$G = \sum_i \langle \Phi_i \rangle G^i$$

and

$$G^\dagger = G$$

So $U_L = U_R$.

Case (b): $A = 0$, B is unitary

Then we can choose

$$B = \begin{pmatrix} \eta_1 & 0 \\ 0 & \eta_2 \end{pmatrix}$$

If we require P to be real, then

$$B = 1$$

or

$$B = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

The first choice gives, with real vacuum expectation values

$$U_L = U_R^\dagger$$

Case (c): A and $B \neq 0$.

Equations (B.5) and (B.7) strongly restrict the form of matrix P . An example is

$$A = \frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \quad B = \frac{1}{2} \begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix}$$

B.2 CP violation

CP violation may arise in various ways from complex Yukawa couplings -- that is, complex G_i matrices, as defined in (B.4), from complex vacuum expectation values of the Φ_i fields, or from complex coupling constants in the Higgs potential. We would like to stress here the connection between different sources of CP violation and improve the results of the preceding section by studying the influence of CP violation in the phenomenology of the theory.

B.2.1 Renormalization of Higgs couplings via Yukawa interaction

We consider the following quartic couplings of the Higgs potential:

$$V = (\lambda_{ij}^L |\chi_L|^2 + \lambda_{ij}^R |\chi_R|^2) \text{Tr}(\Phi_i \Phi_j^\dagger) + \eta_{ij}^L \chi_L^\dagger \Phi_i \Phi_j^\dagger \chi_L + \eta_{ij}^R \chi_R^\dagger \Phi_i \Phi_j^\dagger \chi_R$$

Hermiticity of the potential requires

$$\lambda_{ij}^{L,R*} = \lambda_{ji}^{L,R} \quad \eta_{ij}^{L,R*} = \eta_{ji}^{L,R}$$

We compute radiative corrections to these constants that arise via Yukawa couplings, that is diagrams with one fermion loop as shown in Fig. 6. A straightforward calculation gives the following effective couplings

$$\tilde{\lambda}_{ij} = \lambda_{ij} + 2 \lambda_{\mu\mu} \text{Tr}(G_i G_j^\dagger G_e G_\mu^\dagger) I + (2 \lambda_{\mu\mu} + \eta_{\mu\mu}) \text{Tr}(G_j^\dagger G_i G_\mu^\dagger G_e) I$$

(B.8)

$$\tilde{\eta}_{ij} = \eta_{ij} + 2 \eta_{\mu\mu} \text{Tr}(G_i G_j^\dagger G_e G_\mu^\dagger) I$$

where

$$\eta_{ij}, \lambda_{ij} \dots \text{ stand for } \eta_{ij}^L \text{ or } \eta_{ij}^R, \lambda_{ij}^L \text{ or } \lambda_{ij}^R, \dots$$

and I is an infinite integral. One can check that

$$\tilde{\eta}_{ij}^* = \tilde{\eta}_{ji} \quad \text{and} \quad \tilde{\lambda}_{ij}^* = \tilde{\lambda}_{ji}$$

We now apply these results to the two cases discussed in Section B.1 and restrict ourselves to the following choice of Higgs: χ_L, χ_R and Φ_i 's.

B.2.2 One Φ field

We have seen in B.1.1 that case (a) gives $U_L = U_R^\dagger$, which is ruled out. We now stick to case (b).

As shown in Appendix A, in order to get $|\alpha| = |\beta|$ naturally, we must require the quartic couplings to be real. We then must ask that the effective couplings given in (B.8) are real, that is:

$$\tilde{\lambda}_{ij} = \tilde{\lambda}_{ji} \quad \tilde{\eta}_{ij} = \tilde{\eta}_{ji} \quad (B.9)$$

Using the facts that

$$i = 1 \text{ or } 2 \quad \text{and} \quad G_i^\dagger = G_i$$

it is straightforward to check that Eq. (B.9) is always fulfilled. This means that, at least at this level, one can require "CP" conservation in the Higgs potential and "CP" violation in the Yukawa coupling: this violation does not propagate via radiative corrections (at least the lowest order ones). But we have also seen that the model is anyway ruled out by phenomenological Cabibbo rotation requirements.

B.2.3 Two Φ fields

As shown in Appendix A, in order to get naturally the condition $|\alpha| = |\beta|$ the only possibility has the following features

- parity is given by

$$A = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad \text{and} \quad B = 0$$

- all quartic couplings are real except the λ_{ij} (or η_{ij}) such that i belongs to $\{1,3\}$ and j to $\{2,4\}$ or vice versa.

Namely,

$$G_i^\dagger = \epsilon_i G_i \quad \text{where } \epsilon_1 = \epsilon_3 = 1, \quad \epsilon_2 = \epsilon_4 = -1$$

we have

$$\lambda_{ij} = \lambda_{ij}^* \epsilon_i \epsilon_j \quad \text{and} \quad \eta_{ij} = \eta_{ij}^* \epsilon_i \epsilon_j$$

and we require

$$\begin{cases} \tilde{\lambda}_{ij} = \tilde{\lambda}_{ij}^* \epsilon_i \epsilon_j = \tilde{\lambda}_{ji} \epsilon_i \epsilon_j \\ \tilde{\eta}_{ij} = \tilde{\eta}_{ij}^* \epsilon_i \epsilon_j = \tilde{\eta}_{ji} \epsilon_i \epsilon_j \end{cases}$$

Using Eqs. (B.8) we immediately get the condition to be fulfilled

$$\eta_{\text{Re}} \text{Tr } G_i G_j G_e G_R = \eta_{\text{Re}} \text{Tr } G_j G_i G_e G_R$$

The simplest and most natural solutions are:

$$(V_i) \quad G^i = G^{iT}$$

the mass matrices

$$G = \sum_i \langle \Phi_i \rangle G_i$$

verify

$$G = G^T$$

and we get

$$U_L = U_R^*$$

which is ruled out.

Another way to get this result is to use explicitly CP transformation. As shown in Appendix A, in order to get the features described above, we must take as a CP transformation the following one

$$CP \Phi_i = \epsilon_i \Phi_i^* \quad CP \chi_{L,R} = \chi_{L,R}^*$$

and demand the potential to be CP conserving.

Following Symanzik's theorem, we must then also require the Yukawa couplings to be CP conserving. We then obtain

$$\mathcal{L}_Y = \sum_{i=1}^4 \bar{q}_R \Phi_i^\dagger G^{iT} q_L + \text{h.c.}$$

and comparing with (B.4)

$$G^{iT} = \epsilon_i G^{iT}$$

From parity conservation, we had

$$G^i = \epsilon_i G^{iT}$$

So $G^i = G^{iT}$

- $\forall (i, j) \quad G_j G_i = G_i G_j$

This gives for a mass matrix $G = \sum_i \langle \Phi_i \rangle G_i$:

$$G G^\dagger = G^\dagger G$$

and $U_L = U_R$ up to phases. This too is ruled out.

Appendix C

Because of the importance of the parity violating effects given by the real Higgs contributions, in this Appendix we summarize some of the details of the calculation. Particularly, we present the Feynman rules and the parity violating contributions of the real Higgs particles to the neutral vector-boson self-energies.

As for the Feynman rules we recall that the gauge-boson-couplings to unphysical Higgs, Faddeev-Popov ghosts are independent of the choice of the particular Higgs mechanism. These rules can be inferred either from the high-energy behaviour of tree-diagrams or can be derived directly from the Lagrangian with any specific (but simple) choice of Higgs sector. The gauge-boson-couplings to physical Higgs boson couplings depend in general on some unknown mixing angle (given by the Higgs boson mass matrix), but the divergent contribution is independent of such details.

It is convenient to introduce the notations

$$\begin{aligned}\vec{Z}^i &= (A_\mu, Z_\nu, Z_A) \\ \vec{e}_i^L &= \left(\frac{|e|}{g}, \frac{|e|}{\sqrt{2}g}, \frac{1}{\sqrt{2}} \right) \\ \vec{e}_i^R &= \left(\frac{|e|}{g}, \frac{|e|}{\sqrt{2}g}, -\frac{1}{\sqrt{2}} \right)\end{aligned} \quad i=1,2,3 \quad (C1a-d)$$

$$M_1^2=0, \quad M_2^2=M_{Z_\nu}^2, \quad M_3^2=M_{Z_A}^2$$

and to define the tensors

$$N_{\mu\nu, \lambda\sigma} = (2g_{\mu\nu}g_{\lambda\sigma} - g_{\mu\lambda}g_{\nu\sigma} - g_{\mu\sigma}g_{\nu\lambda}) \quad (C2a)$$

$$V_{\beta\alpha\mu} = [g_{\alpha\beta}(\ell'+\ell)_\mu - g_{\alpha\mu}(\ell-q)_\beta - g_{\beta\mu}(\ell'+q)_\alpha] \quad (C2b)$$

then the Feynman rules for the gauge boson couplings of Figs. 12(a)-12(c) are:

$$12(a): \quad -ig \vec{e}_i^{L,R} V_{\rho\mu\mu}(\ell', \ell, q) \quad (C3a)$$

$$12(b): \quad -ig^2 N_{\mu\nu, \lambda\sigma} \quad (C3b)$$

$$12(c): \quad -ig^2 \vec{e}_i^{L,R} \vec{e}_j^{L,R} N_{\mu\nu, \lambda\sigma} \quad (C3c)$$

(no $W_L W_R$ couplings)

The ghost (Faddeev-Popov or unphysical Higgs)-gauge boson couplings in the Feynman-'t Hooft gauge of the diagrams of Figs. (13a)-(13d) are:

$$13(a): \quad \begin{aligned} & \pm ig \ell'_\mu \vec{e}_i^L \\ & \pm ig \ell'_\mu \vec{e}_i^R \end{aligned} \quad (C4)$$

$$13(b): \quad ig M_{W_{L(R)}} g_{\lambda\mu} \vec{e}_i^{L(R)} \left(1 - \frac{M_i^2}{M_{L(R)}^2}\right) \quad (C5a)$$

$$13(c): \quad ig (\ell + \ell')_\mu \vec{e}_i^{L(R)} \left(1 - \frac{1}{2} \frac{M_i^2}{M_{L(R)}^2}\right) \quad (C5b)$$

(no st couplings)

$$13(d): \quad 2ig^2 \vec{e}_i^{L(R)} \vec{e}_j^{L(R)} g_{\mu\nu} \left(1 - \frac{1}{2} \frac{M_i^2}{M_{L(R)}^2} - \frac{1}{2} \frac{M_j^2}{M_{L(R)}^2}\right) \quad (C5c)$$

The vector bosons - fermion couplings (Figs. (14a)-(14d)) can be given as

$$14(a): \quad i |e| Q(f) \gamma_\mu \quad (C6a)$$

$$14(b): \quad i \frac{1}{\sqrt{2} \sin \theta_w \sqrt{\cos 2\theta_w}} [I_3(f) - 2 \sin^2 \theta_w Q(f)] \gamma_\mu \quad (C6b)$$

$$14(c) \quad -i \frac{g}{\sqrt{2}} I_3(f) \gamma_\mu \gamma_5 \quad (C6c)$$

$$14(d): \quad i \frac{g}{\sqrt{2}} \gamma_\mu \frac{1 + \gamma_5}{2} \tau_\pm \quad (C6d)$$

$$\text{where } \tau_\pm = \frac{1}{2}(\tau_1 \pm i\tau_2)$$

$Q(f)$ denotes the electric charge of the fermion in units of e , $I_3(f)$ denotes the value of the third component of the weak isospin of the fermion f (since the fermions are in L-R symmetric doublets $I_3(f)$ can be defined independently of the L,R indices).

The physical Higgs couplings to neutral vector-bosons given by Figs. (15a)-(15d) are the following ⁺)

$$15(a): \quad 2ig^2 g_{\mu\nu} \lambda_{ab}^i \quad (C7a)$$

$$15(b): \quad ig(2k+q)_\mu V_{ab}^i \quad (C7b)$$

$$15(c): \quad ig g_{\mu\nu} X_{L(R),a}^i M_{L(R)} \quad (C7c)$$

$$15(d): \quad ig(2k+q)_\mu S_{L(R),a}^i \quad (C7d)$$

The couplings are not independent; without specifying the Higgs sector we obtain the relations

$$V_{ab}^i V_{ab}^{i*} + 2 S_{L,a}^i S_{L,a}^{i*} + 2 S_{R,a}^i S_{R,a}^{i*} = -e_1^L e_3^L \left[\left(1 - \frac{M_1^2}{2M_L^2}\right) \left(1 - \frac{M_3^2}{2M_L^2}\right) - (M_L \leftrightarrow M_R) \right] \quad (C8a)$$

⁺) Only Higgs particles of unit charge can give parity violating effects at one loop level.

$$X_{L,a}^i X_{L,a}^{\bar{Z}_A} M_L^2 + X_{R,a}^i X_{R,a}^{\bar{Z}_A} M_R^2 = -e_i^L e_3^L + \left[\frac{M_3^2 M_3^2}{M_L^2} - \frac{M_3^2 M_3^2}{M_R^2} \right] \quad (C8b)$$

$$S_{L,a}^i S_{L,a}^{\bar{Z}_A} M_L^2 + S_{R,a}^i S_{R,a}^{\bar{Z}_A} M_R^2 = \frac{1}{4} X_{L,a}^i X_{L,a}^{\bar{Z}_A} M_L^2 + \frac{1}{4} X_{R,a}^i X_{R,a}^{\bar{Z}_A} M_R^2 \quad (C8c)$$

$$\left(\sum_b V_{ab}^i V_{ab}^{\bar{Z}_A} \right) + S_{L,a}^i S_{L,a}^{\bar{Z}_A} + S_{R,a}^i S_{R,a}^{\bar{Z}_A} - \lambda_{aa}^{\bar{Z}_A} = 0 \quad (C8d)$$

(In Eqs. (C8) repeated indices are summed, except in Eq. (C8d)).

The contribution of the diagrams in Fig. 9(a) has a divergent part which is exactly cancelled by the divergent part of the contribution of the diagrams in Fig. 9b. This cancellation is assured by Eqs. (C8).

The finite parity violating contributions can be summarized in terms of $-i\Pi_{\mu\nu}^{\bar{Z}_A A}$ and $-i\Pi_{\mu\nu}^{\bar{Z}_A \bar{Z}_V}$, the self-energy contributions to the vector boson propagators.

For the sake of simplicity we present the contribution of the real Higgs particles for a particular Higgs sector consisting of the fields $\Phi, \varphi, \tilde{\delta}_L, \tilde{\delta}_R$ with $\epsilon=0$, $d=0$ (see Eqs. 3.24 - 3.27), but the structure of the more general Higgs contributions must be very similar. Then we have four real Higgs fields of unit charge which we denote as ϕ, ψ, χ and δ .

Finite contributions of the diagrams of Fig. 9a ⁺⁾ :

$$-i\Pi_{\mu\nu}^{(4)} \bar{Z}_A^A = \left(-\frac{ig^2}{16\pi^2} \right) \tilde{e}_1^L \tilde{e}_3^L (q_\mu q_\nu - g_{\mu\nu} q^2) + \left(3 + \frac{M_{2W}^2}{6M_A^2} \right) \ln \frac{M_A^2}{M_L^2} \quad (C9a)$$

⁺⁾ Terms of form $\sim \ln m^2$ and $\sim g_{\mu\nu}$ which must be cancelled are not given here.

$$\begin{aligned}
 -i \Pi_{\mu\nu}^{(1) Z_A Z_V} &= \left(-\frac{ig^2}{16\pi^2} \right) g_{\mu\nu} \vec{e}_2^L \vec{e}_3^L M_{Z_A}^2 M_{Z_V}^2 \cdot \\
 &\quad + \left[\frac{1}{2} \left(\frac{1}{M_{Z_A}^2} - \frac{1}{M_{Z_V}^2} \right) - 2 \left(\frac{1}{M_{Z_A}^2} + \frac{1}{M_{Z_V}^2} \right) \ln \frac{M_{Z_V}^2}{M_{Z_A}^2} \right]
 \end{aligned} \quad (C9b)$$

Finite terms given by the diagrams of Fig. 9b:

$$\begin{aligned}
 -i \Pi_{\mu\nu}^{(1) Z_A A} &= \left(-\frac{ig^2}{16\pi^2} \right) \vec{e}_1^L \vec{e}_3^L (q_\mu q_\nu - g_{\mu\nu} q^2) \cdot \\
 &\quad + \left[\frac{1}{6} \frac{M_{Z_A}^2}{M_L^2} \ln \frac{m_\phi^2}{M_L^2} + \frac{1}{3} \ln \frac{m_\phi^2}{M_L^2} - \frac{1}{6} \frac{M_{Z_A}^2}{M_{Z^0}^2} \ln \frac{m_\phi^2}{M_{Z^0}^2} - \right. \\
 &\quad \left. - \frac{1}{3} \kappa_\phi \ln \frac{m_\phi^2}{M_L^2} - \frac{1}{3} \kappa_\chi \ln \frac{m_\chi^2}{M_L^2} \right]
 \end{aligned} \quad (C10a)$$

and

$$\begin{aligned}
 -i \Pi_{\mu\nu}^{(2) Z_A Z_V} &= \left(-\frac{ig^2}{16\pi^2} \right) g_{\mu\nu} \vec{e}_2^L \vec{e}_3^L M_{Z_A}^2 M_{Z_V}^2 \cdot \left[\frac{5}{4} \left(\frac{1}{M_{Z^0}^2} - \frac{1}{M_L^2} \right) + \right. \\
 &\quad + \frac{1}{4} \frac{m_\phi^2}{M_L^4} + \frac{3}{2} \frac{m_\phi^2}{M_L^2} \frac{\ln(m_\phi^2/M_L^2)}{m_\phi^2 - M_L^2} - \frac{1}{4} \frac{\lambda_\phi m_\phi^2 + \lambda_\chi m_\chi^2}{M_{Z^0}^2 M_L^2} - \\
 &\quad - \frac{3}{2} \lambda_\phi \frac{m_\phi^2}{M_L^2} \frac{\ln(m_\phi^2/M_L^2)}{m_\phi^2 - M_{Z^0}^2} - \frac{3}{2} \lambda_\chi \frac{m_\chi^2}{M_L^2} \frac{\ln(m_\chi^2/M_L^2)}{m_\chi^2 - M_{Z^0}^2} \\
 &\quad \left. + \frac{1}{4} \lambda_\phi \lambda_\chi \frac{1}{M_L^4} \left(m_\phi^2 + m_\chi^2 - \frac{2m_\phi^2 m_\chi^2}{m_\phi^2 - m_\chi^2} \ln \frac{m_\phi^2}{m_\chi^2} \right) \right]
 \end{aligned} \quad (C10b)$$

where $\kappa_\phi, \kappa_\chi, \lambda_\phi, \lambda_\chi$ are some parameters of $O(1)$ depending on the vector boson masses and on some mixing angles but

$$\kappa_\phi + \kappa_\chi = 1, \quad \lambda_\phi + \lambda_\chi = M_L^2/M_{Z^0}^2$$

In particular if $M_L = M_{Z_A} = M_{Z_V} \sqrt{\cos 2\theta_W}$, we have (see Eqs. C13)

$$\lambda_\phi = 1, \quad \lambda_\chi = -1 + M_L^2/M_{Z^0}^2 \quad (C11a)$$

$$\kappa_\psi = \frac{1}{2} (1 - M_L^2/M_R^2) \quad , \quad \kappa_\chi = \frac{1}{2} (1 + M_L^2/M_R^2) \quad (C11b)$$

It is illuminating to investigate the limit, when $M_L \rightarrow M_R$. In this limit $\lambda_\psi, \kappa_\chi \rightarrow 1$ and $\lambda_\chi, \kappa_\psi \rightarrow 0$, furthermore $m_\chi^2 \rightarrow m_\psi^2$ and therefore no contributions remain from Fig. 9a but there is a residual contribution from Fig. 9b as follows

$$\begin{aligned} -i \Pi_{\mu\nu}^{(2) Z A} = & \left(-\frac{ig^2}{16\pi^2} \right) (q_\mu q_\nu - g_{\mu\nu} q^2) \vec{e}_1^L \vec{e}_3^L + \\ & + \frac{1}{6} \frac{M_{Z\psi}^2}{M_L^2} \ln \frac{m_\psi^2}{m_\chi^2} \end{aligned} \quad (C12a)$$

$$\begin{aligned} -i \Pi_{\mu\nu}^{(2) Z V Z A} = & \left(-\frac{ig^2}{16\pi^2} \right) g_{\mu\nu} \vec{e}_1^L \vec{e}_3^L \frac{M_{Z\psi}^2 M_{Z\chi}^2}{M_L^2} + \left[\frac{1}{4} \frac{m_\psi^2 - m_\chi^2}{M_L^2} + \right. \\ & \left. + \frac{3}{2} \left(\frac{m_\psi^2}{m_\psi^2 - M_L^2} \ln \frac{m_\psi^2}{M_L^2} - \frac{m_\chi^2}{m_\chi^2 - M_L^2} \ln \frac{m_\chi^2}{M_L^2} \right) \right] \end{aligned} \quad (C12b)$$

The contribution vanishes if $m_\psi^2 = m_\chi^2$. However, as can be readily checked by direct computation, there is no reason to assume that $m_\psi^2 = m_\chi^2$. Particularly to obtain $m_\psi^2 = m_\chi^2$ we would have to put some unnatural constraint on the Higgs potential.

So we conclude that in the case when $M_L = M_R$, the leading parity isolating effects for the **B_i** experiment are given by the real Higgs particle contribution. (We recall that in this case we must also have heavy fermions, but they give parity violating contributions only to the incoherent part of the electron-quark interactions and so they are negligibly small in heavy atoms.) Some details concerning the statement that if $M_L^2 = M_R^2$ then it does not follow that $m_\psi^2 = m_\chi^2$, are given below.

In this case we can work with the Higgs fields ϕ and ρ only. The most general $SU(2)_L \otimes SU(2)_R \otimes U(1)$ symmetric potential has the form (where the symbol (...) means that the trace must be taken)

$$\begin{aligned}
 V = & \mu^2 (\phi^\dagger \phi) + M^2 (g^\dagger g) + \lambda_1 (\phi^\dagger \phi) + \lambda_2 (\tilde{\phi}^\dagger \phi) (\phi^\dagger \tilde{\phi}) + \\
 & + \lambda_3 (\phi^\dagger \phi) (g^\dagger g) + \lambda_4 (\phi^\dagger g) (g^\dagger \phi) + \lambda_5 (\tilde{\phi}^\dagger g) (g^\dagger \tilde{\phi}) + \\
 & + \lambda_6 [(\phi^\dagger \phi g^\dagger g) + (\phi^\dagger g g^\dagger \phi)] + h_1 (g^\dagger g)^2 + h_2 (\tilde{g}^\dagger g) (g^\dagger \tilde{g}) .
 \end{aligned}$$

This potential has a minimum for the configuration

$$\phi_0 = \begin{pmatrix} 0 & 0 \\ 0 & a \end{pmatrix} , \quad g = \begin{pmatrix} 0 & 0 \\ b & 0 \end{pmatrix}$$

The value of the potential at the minimum is:

$$V(a, b) = \mu^2 a^2 + M^2 b^2 + \lambda_1 a^4 + \lambda_3 a^2 b^2 + \lambda_6 a^2 b^2 + h_1 b^4$$

At the minimum we have the relations

$$\mu^2 + 2\lambda_1 a^2 + (\lambda_3 + \lambda_6) b^2 = 0$$

$$M^2 + 2h_1 b^2 + (\lambda_3 + \lambda_6) a^2 = 0 .$$

If we parametrize the fields as

$$\phi = \begin{pmatrix} \phi_1^0 & \frac{a}{\sqrt{2}} \phi^+ \\ \frac{a}{\sqrt{2}} \psi^- & a + \frac{1}{\sqrt{2}} u \end{pmatrix} , \quad g = \begin{pmatrix} -\frac{b}{\sqrt{2}} \phi^+ & g^{++} \\ b + \frac{1}{\sqrt{2}} v & \frac{b}{\sqrt{2}} \psi^+ \end{pmatrix}$$

We obtain the following quadratic terms

$$(\lambda_4 + \lambda_6) (a^2 + b^2) \psi^+ \psi^- + (\lambda_5 - \lambda_6) (a^2 + b^2) \phi^+ \phi^- .$$

There is therefore no reason to assume that $m_{\vec{\chi}}^2 = m_{\vec{\Phi}}^2$.

There is no Goldstone-boson in the neutral sector - but we have a left-over symmetry: namely the two neutral fields associated with Φ^0 remain degenerate in mass.

For completeness we list here the coupling constants

$$\lambda_{ab}^{AZH}, \lambda_{ab}^{ZVZH}, V_{ab}^{ZAh}, X_{L,a}^i, X_{R,a}^i, \\ S_{L,a}^i, S_{R,a}^i$$

assuming a Goldstone mechanism⁺)

$$\Phi_0 = \begin{pmatrix} 0 & 0 \\ 0 & a \end{pmatrix}, \varphi_0 = \begin{pmatrix} 0 & 0 \\ b & 0 \end{pmatrix}, \vec{\delta}_L^0 = 0, \vec{\delta}_R^0 = (0, 0, c).$$

We define the Higgs fields in the unitary gauge as follows

$$\Phi_u = \begin{pmatrix} \phi_1^0 & \frac{a}{\sqrt{2}} \phi_1^+ \\ \phi_2^- & \frac{1}{\sqrt{2}} u^0 \end{pmatrix}, \varphi_u = \begin{pmatrix} -\frac{a}{b} \phi_1^+ & \varphi^{++} \\ \frac{1}{\sqrt{2}} v^0 & \varphi^+ \end{pmatrix}$$

where

$$\phi_2^+ = \frac{\beta}{a} \sin \omega \psi^+ + \frac{\sigma}{a} \cos \omega \chi^+$$

$$\varphi^+ = \frac{\beta}{b} \cos \omega \psi^+ - \frac{\sigma}{b} \sin \omega \chi^+$$

and

$$\delta_{Ru}^+ = \frac{1}{c\sqrt{2}} (a \phi_2^+ - b \varphi^+), \delta_{Ru}^0 = 0, \vec{\delta}_{uL} = \vec{\delta}_L$$

+) Although the vacuum expectation values in this form do not represent a real minimum of the potential (see Appendix A), it can be used to determine Feynman rules, since the Feynman rules of the Higgs particle - vector boson couplings are independent of the specific form of the Higgs mechanism.

$\alpha, \beta, \gamma, \omega$ are determined by the relations

$$\frac{1}{\beta^2} = \frac{1}{a^2} \sin^2 \omega + \frac{1}{b^2} \cos^2 \omega + \frac{1}{2c^2} (1 - \sin 2\omega)$$

$$\frac{1}{\gamma^2} = \frac{1}{a^2} \cos^2 \omega + \frac{1}{b^2} \sin^2 \omega + \frac{1}{2c^2} (1 + \sin 2\omega)$$

$$\left(\frac{1}{a^2} - \frac{1}{b^2} \right) \sin 2\omega = \frac{1}{c^2} \cos 2\omega$$

$$\frac{1}{a^2} = \frac{1}{b^2} + \frac{1}{c^2}$$

The fields ψ and χ couple only to the right-handed gauge bosons, the fields ϕ and δ_L couple only to the left-handed gauge bosons and the coupling matrices are diagonal for ϕ and δ_L . So the non-vanishing couplings are the following

$$V_{\delta}^R = e/g, \quad \lambda_{\delta}^{AZ_n} = e/(g\sqrt{2}), \quad \lambda_{\delta}^{Z_n Z_n} = e/(2g')$$

$$V_{\phi}^R = e/g, \quad \lambda_{\phi}^{AZ_n} = \frac{e}{g\sqrt{2}} \cdot \frac{\alpha^2}{b^2}, \quad \lambda_{\phi}^{Z_n Z_n} = \frac{e}{2g'} \cdot \frac{\alpha^2}{b^2} (1 - g'^2)$$

$$X_{L,\phi}^{Z_n} = g g' \alpha / M_L, \quad X_{L,\phi}^{Z_n} = -g \alpha / M_L, \quad \mathcal{N}_{L,\phi}^{Z_n} = \frac{1}{2} X_{L,\phi}^{Z_n}, \quad \mathcal{N}_{L,\phi}^{Z_n} = \frac{1}{2} X_{L,\phi}^{Z_n}$$

and

$$V_{\alpha\beta}^R = \frac{e}{g} \delta_{\alpha\beta}$$

$$V_{\alpha\beta}^{Z_n} = \frac{e}{g'\sqrt{2}} \begin{bmatrix} 1 - g'^2 \frac{\beta^2 \cos^2 \omega}{b^2} & \frac{\beta \gamma}{2a^2} \sin 2\omega + g'^2 \frac{a^2}{b^2} \\ \times & 1 - g'^2 \frac{\gamma^2 \sin^2 \omega}{b^2} \end{bmatrix}$$

$$V_{\alpha\beta}^{Z_n} = -\frac{1}{\sqrt{2}} \begin{bmatrix} 1 - \frac{\beta^2 \sin^2 \omega}{a^2} & -\frac{\beta \gamma}{2a^2} \sin 2\omega \\ \times & 1 - \frac{\gamma^2 \cos^2 \omega}{a^2} \end{bmatrix}$$

$$\lambda_{\alpha\beta}^{AZ_n} = \frac{e}{g} V_{\alpha\beta}^{Z_n}$$

$$\lambda_{\alpha\beta}^{Z_n Z_n} = -\frac{e}{2g_1} \begin{bmatrix} 1 - \frac{\beta^2 \sin^2 \omega}{a^2} - g'^2 \frac{\beta^2 \cos^2 \omega}{b^2}, -\frac{\beta\gamma}{2a^2} \sin 2\omega (1 - g'^2 \frac{a^2}{b^2}) \\ \times, 1 - \frac{\gamma^2 \cos^2 \omega}{a^2} - g'^2 \frac{\gamma^2 \sin^2 \omega}{b^2} \end{bmatrix}$$

$$X_{L(R),\alpha}^A = 0 \quad S_{L(R),\alpha}^A = 0$$

$$M_R X_{R,\alpha}^{Z_n} = gg' [\beta \cos \omega, -\gamma \sin \omega]$$

$$M_R X_{R,\alpha}^{Z_n} = g [\beta \sin \omega, \gamma \cos \omega]$$

$$M_R S_{R,\alpha}^{Z_n} = \frac{1}{2} gg' [\beta \cos \omega, -\gamma \sin \omega]$$

$$M_R S_{R,\alpha}^{Z_n} = \frac{1}{2} g [\beta \sin \omega, \gamma \cos \omega]$$

where the indices α, β take the values χ and ψ .

Finally, the parameters $\lambda_\psi, \lambda_\chi, \mathcal{X}_\psi, \mathcal{X}_\chi$ are defined as follows

$$\lambda_\psi = \frac{\beta^2(a^2+b^2)}{2a^2b^2} \sin 2\omega \quad (C13a)$$

$$\lambda_\chi = -\frac{\gamma^2(a^2+b^2)}{2a^2b^2} \sin 2\omega \quad (C13b)$$

$$\mathcal{X}_\psi = 1 - \frac{\beta^2 \sin^2 \omega}{a^2} - \frac{a^2}{b^2} \quad (C13c)$$

$$\mathcal{X}_\chi = 1 - \frac{\gamma^2 \sin^2 \omega}{a^2} \quad (C13d)$$

References

- 1) S. Weinberg, Phys. Rev. Letters 19 (1967) 1264 and
A. Salam, Proc. of the 8th Nobel Symposium, Stockholm, ed. Svartholm
(Almqvist and Wiksells, Stockholm 1968, p. 367).
- 2) S. Weinberg, Phys. Rev. Letters 29 (1972) 388.
- 3) J.C. Pati and A. Salam, Phys. Rev. D10 (1974) 275.
- 4) P.N. Mohapatra and J.C. Pati, Phys. Rev. D11 (1975) 566 and D11 (1975)
2588.
- 5) G. Senjanović and R.N. Mohapatra, Phys. Rev. D12 (1975) 1502.
- 6) H. Fritzsch and P. Minkowski, Nucl. Phys. B103 (1976) 61.
- 7) R.N. Mohapatra and D.P. Sidhu, Phys. Rev. Letters 38 (1977) 667 and Phys.
Rev. D16 (1977) 2843.
- 8) J.C. Pati, S. Rajpoot and A. Salam, Phys. Letters 71B (1977) 387.
- 9) A. De Rújula, H. Georgi and S. Glashow, Ann. Phys. 109 (1977) 242.
- 10) P.N. Mohapatra, F. Paige and D.P. Sidhu, Brookhaven preprint BNL 23/86
(1977).
- 11) M.A. Bég, R. Budny, R.N. Mohapatra and A. Sirlin, Phys. Rev. Letters 38
(1977) 1252.
- 12) A. De Rújula, H. Georgi and S. Glashow, Ann. Phys. 109 (1977) 258.
- 13) E. Ma, Phys. Letters 73B (1978) 49.
- 14) P.N. Mohapatra and D.P. Sidhu, Brookhaven preprint (1977).
- 15) E. Ma, Phys. Letters 68B (1976) 468 ; Nucl. Phys. B12 (1977) 421.
- 16) L.L. Lewis, J. Hollister, D.C. Soreide, E.G. Lindahl and E.N. Fortson,
Phys. Rev. Letters 39 (1977) 795 ; P.E.G. Baird, M.W.S.M. Brimicombe,
R.J. Hunt, G.J. Roberts, P.G.H. Sandars and D.N. Stacey, Phys. Rev.
Letters 39 (1977) 798.
- 17) However a group in Novosibirsk has recently reported a positive result,
compatible in sign and magnitude with the Weinberg - Salam model;
L.M.B. Barkov and M.S. Zolotarev, Pisma JETP 26 (1978) 379, and private
communication.
- 18) M.A. Bouchiat and C. Bouchiat, Phys. Letters 48B (1974) 111.

- 19) A. De Rújula, H. Georgi, S.L. Glashow and H. Quinn, Rev. Mod. Phys. 40 (1974) 391, for recent discussions see A. DeRujula, H. Georgi and S.L. Glashow, Harvard Univ. preprint HUTP - 77/A046; S. Weinberg Proc. of 1977 International Symposium on Lepton and Photon Interactions at High Energies (DESY pub. ed. F. Gutbord) p.619 (1977).
- 20) See talks by M. Perl and A. Diamant-Berger, Proceedings of the 13th Rencontre de Moriond (1978) ed. by J. Tran Thanh Van, to be published.
- 21) M. Holder et.al., Phys. Letters 69B (1977) 377; B.C. Barish et.al., Phys. Rev. Letters 39 (1977) 981; P.C. Bosetti et.al., Phys. Letters 73B (1977) 380.
- 22) For a recent review see the Proceeding of the Hamburg Conf. (DESY publication ed. F. Gutbord) (1977).
- 23) An exception is the recently reported Gargamelle result on elastic scattering: P. Alibran et.al., CERN preprint submitted to Phys. Letters B (1978).
- 24) See also H. Georgi and S. Weinberg, Harvard preprint HUTP - 77/A052 (1977) and G.G. Ross and T. Weiler preprint CALT - 68-620 (1977).
- 25) S.W. Herb et.al., Phys. Rev. Letters 39 (1977) 252.
- 26) M. Holder et.al., Phys. Rev. Letters 39 (1977) 433.
- 27) Q. Shafi and Ch. Wetterich, Phys. Letters 69B (1977) 464.
- 28) W.J. Marciano and A.I. Sanda, Rockefeller Univ. preprint C00-2232B-124(1978) G. Senjanović, City College, N.Y. preprint (1977).
- 29) G. Senjanović and A. Sokorac, to be published in Phys. Letters B (1978).
- 30) H. Mani, J.C. Pati and A. Salam, Trieste preprint IC/77/80 (1977).
- 31) S. Weinberg, Phys. Rev. Letters 29 (1973) 388.
- 32) G. Steigman, D.N. Schramm and J.E. Gunn, Phys. Letters 66B (1977) 202.
- 33) For a review see G. Steigman, Proc. Ben Lee Memorial Int'l Conf. on Parity Non-conservation, Weak Neutral Currents and Gauge Theories (Fermi Lab, 1977) to be published.
- 34) J.E. Gunn, B.W. Lee, I. Leiche, D.N. Schramm, G. Steigman and S. Weinberg, Submitted to Ap. J. (1977).
- 35) E. Golowich and B.R. Holstein, Phys. Rev. Letters 35 (1975) 831.
- 36) K. Symanzik in Fundamental Interactions at High Energies, eds. A. Perlmutter et al. (Gordon and Breach, 1970).
- 37) M.S. Chanowitz, J. Ellis and M.K. Gaillard, Nucl. Phys. B128 (1977) 506.
- 38) A. Sirlin, Nucl. Phys. B71 (1974) 29 and B100 (1975) 291.

- 39) D.H. Wilkinson in "Interaction Studies in Nuclei", Eds. H. Jochin and B. Ziegler, North-Holland Publishing Company, Amsterdam (1975), p.147 and Nature 257, No. 5523 (1975) 189;
D.H. Wilkinson and D.E. Alburger, Phys. Rev. C13 (1976) 2515.
- 40) We thank J. Ellis for calling these data to our attention.
- 41) L.H. Ryder, Phys. Reports C34 (1978) 57, Δ_v and Δ_p are the measure of radiative corrections in the corresponding decay rates.

Figure captions

- Fig. 1 : Fermion loop contribution to Z_V - Z_A mixing.
- Fig. 2 : Fermion loop contribution to W_L - W_R mixing.
- Fig. 3 : Lowest order Higgs exchange contribution to helicity conserving vector boson - fermion couplings, $i = L$ or R .
- Fig. 4 : $\Delta S = 2$ process induced by the exchange of one neutral physical Higgs boson.
- Fig. 5 : W-loop contribution to the χ - δ quartic coupling in the effective Higgs potential.
- Fig. 6 : Effective Higgs couplings induced by Yukawa couplings, at the two loop level.
- Fig. 7 : Mixing between two Φ fields induced by the Yukawa couplings.
- Fig. 8a: Lowest order vertex corrections leading to parity violating neutral current effects.
- Fig. 8b: Box diagrams involving the exchange of charged W_L vector bosons.
- Fig. 9a: Self-energy corrections to Z_A - A and Z_A - Z_V mixing without the physical Higgs contributions.
- Fig. 9b: Self-energy corrections to Z_A - A and Z_A - Z_V mixing involving real Higgs particles.
- Fig. 10 : Charge radii diagrams.
- Fig. 11a: Some basic photon corrections in β - decay and the associated corrections in μ - decay.
- Fig. 11b: Box diagrams involving Z_A, Z_V or Z boson exchanges in β - decay and μ - decay.
- Fig. 12a - 12c: Gauge boson vertices.
- Fig. 13a: Faddeev-Popov ghost-gauge boson coupling.
- Fig. 13b - 13d: Unphysical Higgs - gauge boson couplings.
- Fig. 14a - 14d: Vector boson - fermion vertices.
- Fig. 15a - 15d: Physical Higgs couplings to neutral vector bosons.

Table

Price of natural parity conservation in the neutral current sector

FIG. 1

FIG. 2

FIG. 3

FIG. 4

FIG. 5

FIG. 6

FIG. 7

FIG. 8-a

FIG. 8-b

FIG. 9a

FIG. 9b

FIG. 10

FIG.11a

FIG.11b

(a)

(b)

(c)

FIG.12

(a)

(b)

(c)

(d)

FIG.13

(a)

(b)

(c)

(d)

FIG.14

(a)

(b)

(c)

(d)

FIG.15