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UNORTHODOX BRANCHING RATIOS OF THE ψ''' (4.414) AS RADIAL
EXCITATION OF CHARMONIUM

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ABSTRACT

Considering the vector state at 4.414 GeV as the third radial excitation of a $c\bar{c}-^3S_1$ system, it is shown how the nodal structure of the decay amplitudes leads to unexpected branching fractions for the charmed meson channels. Specifically, it is predicted that this state should (practically) not decay into $D^* \bar{D}^*$ nor into FF. The same momentum dependence (which is closely connected with the composite nature of the mesons) yields also an explanation for the disturbingly small total width of the 4.414 state.

Recent experiments¹⁾ on the production rates of charmed meson pairs in e^+e^- annihilation at around $\sqrt{s} = 4.03$ GeV have yielded the astonishing result that charmed vector meson pairs ($D^*\bar{D}^*$) are produced much more abundantly than one would expect from naïve (spin-) comparison with $D\bar{D}$ and $D\bar{D}^*$ production. Earlier attempts in interpreting these data as a signature for the existence of a (quasi-molecular) $D^*\bar{D}^*$ resonance could not be hardened in a more detailed model calculation, although the total e^+e^- cross-section would admit such a structure somewhere in the energy region around 4.0 GeV. A different explanation for the unorthodox production rates starts from interpreting the bump at 4.03 (4.028 at SLAC, 4.040 at DESY-DASP) as the second radial excitation of ψ , which we will call here ψ''^*). Then, model estimates^{2),3)} of the momentum dependence of the coupling constants $f_{\psi''D\bar{D}}, f_{\psi''D\bar{D}^*}, f_{\psi''D^*\bar{D}^*}$ yield a vanishing of the corresponding production amplitudes at momenta which are very near to the CM momenta of the $D\bar{D}$ and $D\bar{D}^*$ channels. Thus, the latter decay amplitudes taken at the correct momentum values are very small as compared to the $D^*\bar{D}^*$ amplitude.

We have studied these effects quantitatively within a non-relativistic model where the spatial wave functions of the contributing mesons are taken from solving the Schrödinger equation with the "charmonium potential"^{4),5)}

$$V(r) = -\frac{4}{3}\alpha_s \frac{1}{r} + \lambda r + V_0. \quad (1)$$

Since the above-mentioned nodal structure of the decay amplitude is closely connected to the fact that a radially excited meson (ψ'') is involved in the decay, it will be even more interesting to look at the next radial excitation ψ''' . This state has been predicted⁵⁾ by the charmonium modelists at 4.471 GeV and, indeed, a clear resonance signal has been found in e^+e^- annihilation nearby this energy, namely at 4.414 GeV.

Interpreting this bump as the ψ''' , we present here the results of our calculations concerning the various decay modes of this state.

The model which we used for computing three meson vertices is described in detail in Ref. 6). Here, we only mention the main features. We consider the three mesons A,B,C which form the vertex $A \rightarrow B+C$, as non-relativistic quark-antiquark

*) We would strongly plead to rename the bump at 3.77 GeV (baptized ψ'' by DESY) which by all models is interpreted as a D wave $c\bar{c}$ state, and utilize the notation ψ'' for radial excitation.

bound states which can be described by static wave functions. This approximation is certainly reliable as long as we restrict ourselves to mesons which include at least one charmed quark^{*)}. The coupling of the three mesons is achieved by the effective creation of a $q\bar{q}$ pair out of the hadronic vacuum whereas the initial particles act as spectators. Following old tradition⁷⁾ we take the $q\bar{q}$ pair being created with the vacuum quantum numbers, i.e., in a $SU(4)$ singlet 3P_0 state. Actually, this 3P_0 is the only possibility in view of the spectator condition. This picture has led not only to a satisfying description of the detailed helicity structure of the even parity meson decay $B \rightarrow \omega\pi$, (e.g.,) but has also proved fairly successful in describing all interesting three-meson vertices involving light mesons⁶⁾ (at least as long as one can trust in the use of wave functions at all). Of course, we could relax the spectator condition and choose a different way of quark-pair creation (for instance, via one or more gluon emission). This would change the details of the predictions but not the gross features of the decay amplitudes. Since the afore-mentioned model is not only very simple but has also yielded such good results for all other meson vertices, we prefer to use it in the following.

The non-relativistic transition matrix element is proportional to the product of two overlap functions: the overlap of the $SU(4) \times SU(2)$ (spin) parts of the wave functions

$$S^{s_z} = \langle \Phi_B \Phi_C | \Phi_A \Phi_0^{s_z} \rangle \quad (2)$$

[Φ_0 is the $SU(4) \times SU(2)$ part of the created $q\bar{q}$ pair of and s_z is its third spin component] and the spatial overlap integral I^{l_z} which can be written (using momentum space wave functions $\tilde{\Psi}$) in the rest frame of A as

$$I^{l_z}(\vec{k}_B) = \int (\vec{k}_B + \vec{k}_C) \left(d\vec{k} \gamma_1^{l_z}(\vec{k}_B - \vec{k}) \tilde{\Psi}_B^*(-\vec{k}) \tilde{\Psi}_C^*(\vec{k}) \tilde{\Psi}_A(\vec{k} + \vec{k}_B) \right) \quad (3)$$

Here both the spectator- and the 3P_0 -condition has been obeyed: spin (s) and angular momentum (l) of the created pair are coupled to give a total 3P_0 state. Thus we get

$$\langle B C | A \rangle = \gamma \sum_{l_z, s_z} \langle 1 l_z 1 s_z | 0 0 \rangle S^{s_z} I^{l_z}(\vec{k}_B) \quad (4)$$

*) For a detailed discussion of the non-relativistic approximation within the charmonium potential model, see Ref. 5).

where γ is an unknown universal normalization constant which reflects the strength of the quark-pair-creation mechanism. This general formula will be applied to different decay modes of the ψ''' (4.414). The spin parts of the wave functions are the usual static ones, whereas for the spatial parts of the wave functions we take the numerical solutions of the Schrödinger equation with the correct potential (1) ⁵⁾. Two remarks are in order:

- i) The model implies SU(4) (flavour) symmetry breaking via the different quark masses entering the Schrödinger equation.
- ii) Equation (1) does not contain a spin-dependent part since for non-relativistic configurations spin effects are of higher order in a relativistic expansion. Therefore the (unperturbed) wave functions are the same for all states having the same quark content, orbital angular momentum and main quantum number.

Let us now apply the model to the different decay modes of the ψ''' (4.414). There are three main types of final states which are allowed by phase space:

- a) $D\bar{D}, D\bar{D}^* + D^*\bar{D}, D^*\bar{D}^*$ and also $FF, \bar{F}\bar{F}^* + F^*\bar{F}, F^*\bar{F}^*$. The corresponding mass values are $m_D = 1.865$, $m_{D^*} = 2.007$, $m_F = 2.030$, $m_{F^*} = 2.140$.
- b) $D_A\bar{D} + D\bar{D}_A, D_B\bar{D} + D\bar{D}_B, D_T\bar{D} + D\bar{D}_T$, where $D_{A,B,T}$ are the even parity partners of the D mesons in the configurations $^3P_1, ^1P_1, ^3P_2$, respectively. Their masses have been predicted ⁵⁾ from the potential (1) enriched by the usual Breit-Fermi type spin-spin and spin-orbit Hamiltonian. Taking these predictions ^{*)} and our experience with the P_c states around 3.45 GeV, we consider the following mass values as most reasonable: $m_{D_A} = 2.250$ GeV, $m_{D_B} = 2.300$ GeV, $m_{D_T} = 2.350$ GeV.
- c) $D\bar{D}' + D\bar{D}', D^*\bar{D}' + D^*\bar{D} + \dots, D^*\bar{D}^* + D^*\bar{D}^*$, where D' and D^* are the radial excitations of the D mesons which are expected to exist anyway. Their masses are predicted ^{5)*)} from (1) to be $m_{D'} = 2.160$ GeV and $m_{D^*} = 2.200$ GeV.

There might be additional allowed decay channels, for instance, $D_A\bar{D}^* + D^*\bar{D}_A, \dots; \bar{F}\bar{F}' + F'\bar{F}, \dots; D\bar{D}\pi, \dots$. But they are (expected to be) very much suppressed by phase space, if they are open at all; so we neglect them in the following.

Decays of type (a) and (c) include only s wave mesons. The overlap integral (3) can, therefore, easily be computed by going over to configuration space wave functions. We get

*) We are aware of the fact that predictions concerning $c\bar{q}$ systems (where q is a u or a d quark) rest on a weaker footing because of the more relativistic nature of these states. On the other hand, the excellent agreement of the original predictions of the D mass (1.842 GeV) with the actual data (1.865 GeV) motivates us to believe that the non-relativistic treatment is appropriate even for these states.

$$I^{\ell_z}(\vec{k}_B) = -\sqrt{\frac{3}{2}} \frac{1}{\pi} \delta(\vec{k}_B + \vec{k}_C) \int d\vec{x} d\vec{y} \Psi_B^*(|\vec{y}|) \cdot \Psi_C^*(|\vec{y}-\vec{x}|) \Xi(\ell_z) (i\vec{\nabla}_x - \vec{k}_B) \Psi_A(|\vec{x}|) e^{\frac{i}{2} \vec{k}_B \cdot \vec{x}}. \quad (5)$$

Here the dependence of the decay amplitude on k_B , the CM momentum of the decay products, is clearly exhibited. The main k_B dependence is implied by the exponential $e^{\frac{i}{2} \vec{k}_B \cdot \vec{x}}$ which leads to zeros of I^{ℓ_z} , and whose origin can be traced back to the Fourier transform of the wave functions. A first rough estimate⁶⁾ shows that for ground state wave functions, the first node lies somewhere around $4\pi/R$ where R is the smallest of the radii of the three wave functions involved. For charmonium states $R \sim 1.5 \text{ GeV}^{-1}$ and we therefore expect the first node to lie somewhere above 7 GeV. Consequently this nodal structure will have practically no influence on decay amplitudes of realistic decays with momentum release of typically 1 GeV or less. But if one state in the vertex is replaced by its higher radial excitation, the corresponding wave function itself shows a sort of oscillatory behaviour (in r) which superimposes to the exponential oscillation and implies that the nodes (in k_B) of I^{ℓ_z} migrate towards lower values of k . A more thorough discussion of these effects is given in Ref. 6). Here we are concerned with the third radial excitation ψ''' . We expect therefore a very drastic variation of the corresponding decay amplitude with k , the first nodes being at fairly small values of k .

We have calculated the amplitudes for the decays of ψ''' into the channels of types (a) and (c) explicitly by means of charmonium wave functions. The resulting values as depending on k_B are plotted in Fig. 1. The nodal structure shows up very clearly. We have only quoted the amplitudes for pseudoscalar-pseudoscalar channels but it should be clear that they are identical for the other channels, up to the notorious spin factors.

Also indicated in the figure are the on-shell CM momenta for the various physical decay modes.

The important point for our purpose here is the fact that these physical momenta for many decay channels lie very near to zeros of the decay amplitudes which means that the corresponding partial widths are very much suppressed. Before we give the resulting numerical values for the decay branching fractions we still have to consider the channels of type (b). Here even parity mesons are involved and therefore the wave functions contain spherical harmonics. The overlap integrals are much more difficult to evaluate. We avoid writing down the resulting expression here, because the gross feature of the couplings is similar to the cases

(a) and (c) -- at least what concerns the over-all nodal structure. The full formulae are contained in our extensive analysis [cf. Eqs. (B.1), (B.3), (B.5) and (A.29) of Ref. 6)]. As an example for the resulting structure we have drawn in Fig. 2 the decay amplitudes for $\psi''' \rightarrow D_A D$ (which are actually almost equal to the ones for $\psi''' \rightarrow D_B D$). There are two amplitudes corresponding to the two possible couplings (longitudinal and transversal). Here the suppression is not as big as before, at least as long as the $D_{A,B}$ masses are as low as we have assumed. Furthermore, we predict that the couplings will be predominantly transversal. Now we are in a state to predict the ratios of the various decay widths without disposing of the unknown normalization γ . Assuming equal creation strength for a $u\bar{u}$, $d\bar{d}$ and $s\bar{s}$ pair (which assumption is advocated by an analysis of light meson decays within the same picture⁶⁾) we get the following ratios for the partial widths of ψ''' *):

$$D\bar{D} : (D\bar{D}^* + D^*\bar{D}) : D^*\bar{D}^* = 1 : 1.50 : \sim 0 \quad (6a)$$

$$F\bar{F} : (F\bar{F}^* + F^*\bar{F}) : F^*\bar{F}^* = \sim 0 : 0.29 : 1 \quad (6b)$$

$$D'\bar{D} : D'\bar{D}^* : D^{*'}\bar{D} : D^{*'}\bar{D}^* = \sim 0 : 0.25 : \sim 0 : 1 \quad (6c)$$

$$D_A\bar{D} : D_B\bar{D} : D_\tau\bar{D} = 1 : 1.61 : 1.81 \quad (6d)$$

$$D\bar{D} : (D_A\bar{D} + D\bar{D}_A) = 1 : 2.04 \quad (7a)$$

$$D\bar{D} : (D^{*'}\bar{D}^* + D^*\bar{D}^{*'}) = 1 : 9.15 \quad (7b)$$

$$D\bar{D} : F^*\bar{F}^* = 1 : 1.01 \quad (7c)$$

*) Remember that these are also the ratios for the production rates of the corresponding channels at 4.414 GeV.

If we drop the assumption of equal creation strength of light and strange quarks, respectively, we would lose only relation (7c).

Thus we predict a very interesting situation: The dominant decay modes of ψ''' (4.414) are (in decreasing order):

$$D'^* D^*, D_T D, D_B D, D' D^*, D_A D, F^* \bar{F}^*, D \bar{D}.$$

ψ''' should not (or very little) decay into $D' \bar{D}$, $D'^* \bar{D}$ and especially not into $D^* \bar{D}^*$ and $F \bar{F}$ channels which would be expected to be dominant from naïve (phase space) arguments. There seems to be a first experimental indication for the suppression of $D^* \bar{D}^*$ and probably also of $F \bar{F}$ ⁸⁾. If this is confirmed and, even more, if the dominance of the other indicated modes is confirmed experimentally, it would not only be a strong argument for our specific model but also in general for the mesons being composed of two constituents.

If we want to find absolute values for the partial widths we have to dispose of γ . This can be done, e.g., by calculating (within the model) the width of the ψ'' (4.028) which decays exclusively into $D \bar{D}$, $D \bar{D}^* + D^* \bar{D}$ and $D^* \bar{D}^*$, and compare the results with the experimental width of ψ'' of 52 ± 10 MeV. We get ⁶⁾ for ψ''

$$\Gamma(D \bar{D}) : \Gamma(D \bar{D}^* + D^* \bar{D}) : \Gamma(D^* \bar{D}^*) = 1 : 6.7 : 0.85 \quad (8)$$

and find from it $\gamma = 1.12$. Using this for the ψ''' decays we find the partial widths as quoted (together with the momentum values used) in Table 1. The sum of these widths gives 68 MeV -- in very good agreement with the experimental value ⁹⁾

$$\Gamma(\psi'''(4.414)) = 66 \pm 15 \text{ MeV}.$$

Thus the nodal structure of the decay amplitudes does not only imply very unorthodox branching fractions for the different modes but helps to understand the extremely small total width for the radial excitation ψ''' .

Two final remarks are in order.

i) The nodal structure of coupling constants within non-relativistic quark models and its connection to the number of nodes of the wave functions is known since some time ^{2),3)} and has also been used for similar purposes. In fact, Le Yaouanc et al. ¹⁰⁾ even calculated the partial widths of ψ''' with the same mechanism of quark pair creation but they use harmonic oscillator wave functions (Gaussians). Now it is well known that an oscillator potential greatly fails to describe the heavy meson states as concerns both their spectrum and the leptonic widths. Since the detailed

structure of the k dependence of decay amplitudes (especially the exact position of the nodes) is fairly model-dependent, we consider the calculations of Ref. 10) rather a show-case for the general features than a detailed prediction. In fact, Le Yaouanc et al. predict completely different dominant decay modes of ψ'' , for instance, $D^* \bar{D}^*$ (which should be completely absent due to our predictions), $F^* \bar{F}$, $D_T D$; whereas $D \bar{D}^*$ (which we predict to be considerably active) should be suppressed. Furthermore, the channels of type (c) are completely neglected in Ref. 10) thus putting some question-mark on their value for the total width. On the other hand, Eichten et al.²⁾ calculate the decay amplitudes within a different model. There, the $q\bar{q}$ pair is also created with effectively constant strength but in a pseudoscalar state [cf. Eq. (3.33) of Ref. 2)] but this does not effect the existence of nodes. Unfortunately, for the wave functions of the charmed mesons they use again Gaussians, although they start from a charmonium (linear) potential. So it is hard to estimate the relevance of the details of their prediction. It is also fair to state at this point that the oscillating behaviour of vertex amplitudes involving composite systems has long been known to nuclear physicists. Good examples¹¹⁾ in this field are the excitation form factors which have their first zeros at much lower momentum transfers (typically some hundred MeV's) -- a fact which is clear from the larger extension of the nuclei.

ii) We have considered the state at 4.414 as a pure 4^3S_1 state, whereas there could be some mixture from the 3^3D_1 state expected to lie near to it. Such a S-D-mixture should definitely be present in the case of $\psi''(4.028)$ because there is a clear signal of a state, slightly higher in mass, which could very well be the orthogonal mixture. Although no such additional state is manifest beyond 4.414, we expect even here a slight admixture of a D state. This could also explain the small leptonic width of 4.414 (0.5 keV experimentally versus 1.7 keV from theoretical prediction for a pure $4S$ state). Such a small impurity would not affect our predictions considerably, whereas the big mixture of the $\psi''(4.028)$ casts doubts on our prediction (8) and it actually does not fit very well with the experimental results¹⁾. A decent correction for the indicated effects lies outside the scope of the present paper^{*)}.

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) Fortunately, the uncertainty about the fractions (8) concerns mostly the $D^ \bar{D}^*$ production, whose absolute value is small anyhow. Therefore, this does not affect our numerical value for γ very much.

Table 1

decay channel	k_{CM} (MeV)	partial width (MeV)
$D\bar{D}$	1181	2.81
$\overline{D^*} + D^* \bar{D}$	1060	4.21
$D^* \bar{D}^*$	919	~ 0
$F\bar{F}$	867	~ 0
$F\bar{F}^* + F^* \bar{F}$	725	0.81
$F^* \bar{F}^*$	542	2.81
$D\bar{D}' + \bar{D}D'$	905	~ 0
$D'\bar{D}^* + D^* \bar{D}'$	729	6.40
$D^* \bar{D}' + \bar{D}D'^*$	859	~ 0
$D^* \bar{D}^* + \bar{D}^* D'^*$	669	25.72
$D_A \bar{D} + \bar{D}D_A$	795	5.72
$D_B \bar{D} + \bar{D}D_B$	727	9.20
$D_T \bar{D} + \bar{D}D_T$	651	10.35

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FIGURE CAPTIONS

Fig. 1 : Decay amplitudes for $\psi''' \rightarrow D\bar{D}, F\bar{F}, D\bar{D}'$ versus k_{CM} (in arbitrary units).

Fig. 2 : Decay amplitudes for $\psi''' \rightarrow D_A D$. The full (dotted) line denotes the transversal (longitudinal) part of the amplitude.

FIG. 1

FIG.2