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ANOMALIES, FIELD TRANSFORMATIONS, AND THE RELATION
BETWEEN $SU(4)$ AND $SO(4)$ SUPERGRAVITY

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A B S T R A C T

We examine at the one-loop level the equivalence between $SU(4)$ and $SO(4)$ supergravity by calculating some amplitudes which are zero in $SO(4)$ theory but may not be zero in $SU(4)$ theory. The results of the calculation show that the theories are not equivalent and global supersymmetry is violated unless certain finite counter terms are introduced.

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There exist two models of extended supergravity which exhibit $SU(4)$ invariance. One of them is $SO(4)$ supergravity ¹⁾ in which the action is $SO(4)$ invariant and only the equations of motion have $SU(4)$ invariance. The second one is the recently constructed $SU(4)$ extended supergravity ²⁾ in which the action itself has the invariance. The equations of motion of these two theories can be transformed into each other by a combination of point field transformations and duality transformations of vector fields ²⁾, so that the theories are equivalent at the classical level. At the quantum level the equivalence is not so obvious. There exist theorems ³⁾ proving the equivalence (equality of S matrices) of two theories whose actions are obtained one from the other by point field transformations $\varphi \rightarrow \varphi + F(\varphi)$, $F(0) = F'(0) = 0$. The present situation, involving duality transformations and massless particles is not covered by these theorems. It is therefore interesting to investigate the situation by an explicit one-loop calculation.

In general, one-loop calculations in supergravity of (finite) S matrix elements appear exceedingly difficult to perform. However, in $SU(4)$ theory it is possible to look at some special amplitudes. The corresponding amplitudes are zero in $SO(4)$ theory and their vanishing in $SU(4)$ theory would be an indication of the equivalence of the two theories. Furthermore these amplitudes should be related to each other if global supersymmetry is not violated at the one-loop level ^{4),5)}. One can check if such violation does occur. The calculation of these amplitudes is also interesting, because it involves matrix elements of currents with anomalous behaviour, and because it brings up questions about regularization methods in supergravity.

Our results are as follows.

- 1) A straightforward calculation of the amplitudes reveals that the two theories are not equivalent at the one-loop level and that global supersymmetry is violated. It is conceivable that this result is a consequence of a bad (non-supersymmetric) regularization method.
- 2) Both the equivalence and the global supersymmetry can be restored by adding to the $SU(4)$ Lagrangian finite one-loop counter terms. However, the resulting action is then not explicitly supersymmetric.

We should perhaps emphasize that we do not, in our one-loop calculation, perform any renormalization; the theory is unrenormalizable. We simply compute S matrix elements which are finite without any subtraction.

The Lagrangians of the two theories ^{1), 2)} are

$$\begin{aligned}
 \mathcal{L}(SO(4)) = & -\frac{e}{2\chi^2} R - \frac{1}{2} \varepsilon^{\mu\nu\rho\sigma} \bar{\Psi}_\mu^i \gamma_5 \gamma_\nu D_\rho \Psi_\sigma^i - \frac{e}{8} (F_{\mu\nu}^{ij})^2 \\
 & - \frac{e}{2} \bar{\chi}^i \gamma^\mu D_\mu \chi^i - \frac{e}{2} [(\partial_\mu A)^2 + (\partial_\mu B)^2] \\
 & + \frac{\chi}{2\sqrt{2}} \bar{\Psi}_\mu^i (e F^{\mu\nu ij} + i \gamma_5 \tilde{F}^{\mu\nu ij}) \Psi_\nu^j \\
 & + \frac{\chi}{4} e \varepsilon^{ijkl} \bar{\Psi}_\lambda^i \sigma^{\mu\nu} F_{\mu\nu}^{kl} \gamma^\lambda \chi^j \\
 & + \frac{\chi}{2} e \bar{\Psi}_\mu^i \partial_\nu (A + i \gamma_5 B) \gamma^\nu \gamma^\mu \chi^i \\
 & - \frac{\chi}{8} e \varepsilon^{ijkl} A F_{\mu\nu}^{ij} F^{\mu\nu kl} + \frac{\chi}{8} \varepsilon^{ijkl} \tilde{F}_{\mu\nu}^{ij} F^{\mu\nu kl} \\
 & + \text{terms containing at least four fields}
 \end{aligned} \tag{1}$$

$$\begin{aligned}
 \mathcal{L}(SU(4)) = & -\frac{e}{2\chi^2} R - \frac{1}{2} \varepsilon^{\mu\nu\rho\sigma} \bar{\Psi}_\mu^i \gamma_5 \gamma_\nu D_\rho \Psi_\sigma^i \\
 & - \frac{e}{4} e^{-\sqrt{2}\chi\varphi} [A_{\mu\nu}^n A^{\mu\nu n} + B_{\mu\nu}^n B^{\mu\nu n}] \\
 & - \frac{e}{2} \bar{\chi}^i \gamma^\mu D_\mu \chi^i - \frac{e}{2} [(\partial_\mu \varphi)^2 + e^{2\sqrt{2}\chi\varphi} (\partial_\mu B)^2] \\
 & - \frac{\chi}{2\sqrt{2}} B [\tilde{A}^{\mu\nu m} A_{\mu\nu}^n + \tilde{B}^{\mu\nu m} B_{\mu\nu}^n] \\
 & + \frac{\chi e}{8} e^{-\frac{\chi}{\sqrt{2}}\varphi} [\bar{\Psi}_\mu^i (C^{\mu\nu ij} + \hat{C}^{\mu\nu ij}) \Psi_\nu^j - i \bar{\Psi}_\mu^i \gamma_5 (\tilde{C}^{\mu\nu ij} + \hat{\tilde{C}}^{\mu\nu ij}) \Psi_\nu^j] \\
 & - \frac{\chi e}{4\sqrt{2}} e^{-\frac{\chi}{\sqrt{2}}\varphi} \bar{\Psi}_\mu^i C^{\mu\nu ij} \sigma_{\mu\nu} \gamma^\rho \chi^j \\
 & - \frac{i\chi}{4\sqrt{2}} e^{\sqrt{2}\chi\varphi} \partial_\sigma B \varepsilon^{\mu\nu\rho\sigma} \bar{\Psi}_\mu^i \gamma_\rho \Psi_\nu^i - \frac{3i\chi e}{4\sqrt{2}} e^{\sqrt{2}\chi\varphi} D_\mu B \bar{\chi}^i \gamma_5 \gamma^\mu \chi^i \\
 & + \frac{\chi}{4} e \bar{\Psi}_\mu^i (\partial_\nu \varphi + e^{\sqrt{2}\chi\varphi} \partial_\nu B i \gamma_5) \gamma^\nu \gamma^\mu \chi^i
 \end{aligned} \tag{2}$$

where $A_{\mu\nu}^n = \partial_\mu A_\nu^n - \partial_\nu A_\mu^n$, $C_{\mu\nu}^{ij}$ are certain combinations of $A_{\mu\nu}$ and $B_{\mu\nu}$, $\tilde{A}_{\mu\nu}^n = \frac{1}{2} \epsilon^{\rho\sigma} A_{\rho\sigma}^n$, and $\hat{D}_\mu$ is the supercovariant derivative. The supersymmetry transformations of SU(4) theory that we shall need are

$$\begin{aligned}\delta\varphi &= \bar{\epsilon}^i \chi^i & \delta B &= i e^{-\sqrt{2}\chi\varphi} \bar{\epsilon}^i \gamma_5 \chi^i \\ \delta\chi^i &= \gamma_\mu (\hat{D}_\mu \varphi + i \gamma_5 e^{\sqrt{2}\chi\varphi} \hat{D}_\mu B) \epsilon^i + \dots \\ \delta\psi_\mu^i &= \frac{2}{\chi} D_\mu \epsilon^i - \frac{i}{\sqrt{2}} \hat{D}_\mu B e^{\sqrt{2}\chi\varphi} \gamma_5 \epsilon^i + \dots\end{aligned}\quad (3)$$

More details can be found in Refs. 1), 2). Note that to obtain our form one should make there the replacements $\kappa \rightarrow \kappa/\sqrt{2}$, $\gamma_\mu \rightarrow i\gamma_\mu$, $g_{\mu\nu} \rightarrow -g_{\mu\nu}$. We note that for quantum calculations the above Lagrangians have to be supplemented by gauge fixing and ghost terms. In particular the supersymmetry gauge fixing term $\bar{\psi} \cdot \gamma \not{\partial} \gamma \cdot \psi$ is accompanied by the spinor ghost Lagrangian

$$\mathcal{L}_g = - \bar{c}^* \gamma^\mu D_\mu c + \frac{i\chi}{2\sqrt{2}} e^{\sqrt{2}\chi\varphi} \hat{D}_\mu B \bar{c}^* \gamma^\mu \gamma_5 c + \dots \quad (4)$$

as follows from the transformation rule in Eq. (3).

Both theories contain the graviton e_μ^a , four gravitinos ψ_μ^i , four "neutrinos" X^i , and scalar and pseudoscalar fields A (or φ) and B . SO(4) theory contains six vector fields A_μ^{ij} while SU(4) theory contains three vector and three axial vector fields A_μ^n, B_μ^n . The fact which apparently may distinguish these two theories at the quantum level is the coupling, in SU(4) theory, of the B pseudoscalar meson to what looks like the divergence of axial vector currents. At the classical level, after use of the equations of motion, this coupling does not play a rôle and the two theories are equivalent. At the one-loop level, because of anomalies, this is no longer the case and it would appear that the B meson can decay into several gravitons via a loop of fermions or vector mesons. In SO(4) theory the same decay (through a vector loop) cannot take place because of the factor ϵ^{ijkl} in the coupling $B \epsilon^{ijkl} F_{\mu\nu}^i F_{\mu\nu}^j$. Similarly in SU(4) theory the scalar meson φ has couplings which would allow it to decay into gravitons and finally the process $X \rightarrow \text{gravitino} + \text{gravitons}$ appears possible. In SO(4) theory these processes are not allowed.

The three processes above are formally related to each other by global supersymmetry. The methods of Ref. 5) lead to relations such as

$$\langle +2, -2, -2 | S | \varphi \rangle = \langle +2, -2, -2 | S | B \rangle = \lambda \langle +2, -2, -3/2 | S | \chi \rangle \quad (5)$$

where λ is a kinematical factor, and by crossing symmetry similar relations for amplitudes such as $\langle +2, -2 | S | \varphi, +2 \rangle$, etc. (here $+2$ denotes a graviton of positive helicity, etc.).

We begin with the B decay process which is given by the amplitude $i \langle \text{grav} | J(0) B(0) | B \rangle$ where

$$\begin{aligned} J = & -\frac{\kappa}{2\sqrt{2}} \left[\tilde{A}_{\mu\nu}^{\sim} A^{\mu\nu} + \tilde{B}_{\mu\nu}^{\sim} B^{\mu\nu} \right] \\ & - \frac{i\kappa}{2\sqrt{2}} D_{\sigma} \left[\frac{1}{2} \varepsilon^{\mu\nu\sigma} \bar{\Psi}_{\mu}^i \gamma_{\rho} \Psi_{\nu}^i + \bar{c}^{*i} \gamma^{\sigma} \gamma_5 c^i \right] \\ & - \frac{3i\kappa}{2\sqrt{2}} D_{\sigma} \left[\frac{1}{2} \bar{\chi}^i \gamma^{\sigma} \gamma_5 \chi^i \right] \end{aligned} \quad (6)$$

obtained from $\mathcal{L}(\text{SU}(4))$ by keeping zeroth order terms in φ , and including a contribution from $\mathcal{L}_g$. Obviously it suffices to compute the matrix element $\langle \text{gravitons} | J(0) | 0 \rangle$. The contributions of the second and third terms in J are known, being just the anomalous divergences of the spin $\frac{3}{2}$ (and its ghost) ^(6),7) and spin $\frac{1}{2}$ ⁽⁸⁾ chiral currents. We have

$$\begin{aligned} D_{\sigma} \langle \text{grav} | \frac{i}{2} \varepsilon^{\mu\nu\sigma} \bar{\Psi}_{\mu} \gamma_{\rho} \Psi_{\nu} - i \bar{c}^{*} \gamma^{\sigma} \gamma_5 c | 0 \rangle^{\text{one loop}} \\ = -i \frac{21}{(4\pi)^2 48} \langle \text{grav} | \varepsilon^{\alpha\beta\rho\sigma} R_{\mu\nu\alpha\beta} R^{\mu\nu}_{\rho\sigma} | 0 \rangle^{\text{trees}} \end{aligned} \quad (7)$$

$$\begin{aligned} D_{\sigma} \langle \text{grav} | \frac{i}{2} \bar{\chi} \gamma^{\sigma} \gamma_5 \chi | 0 \rangle^{\text{one loop}} \\ = i \frac{1}{(4\pi)^2 48} \langle \text{grav} | \varepsilon^{\alpha\beta\rho\sigma} R_{\mu\nu\alpha\beta} R^{\mu\nu}_{\rho\sigma} | 0 \rangle^{\text{trees}} \end{aligned} \quad (8)$$

The matrix element of the operator $A_{\mu\nu} \tilde{A}^{\mu\nu}$ can also be determined. The easiest way perhaps is to recognize that it can be written as the divergence of a quantity $j^\sigma = \epsilon^{\mu\nu\rho\sigma} A_{\mu\nu} A_\rho$ and to compute its matrix element to two gravitons

$$M = \partial_\sigma \langle p, p' | j^\sigma | 0 \rangle \quad (9)$$

then generalize by using general covariance. One can use the Adler-Rosenberg method ⁹⁾ which extracts the whole contribution to M from the finite part of a triangle diagram with two on-shell gravitons attached to a loop of vector mesons and the operator j^σ at the third vertex. We find

$$\begin{aligned} & \langle \text{grav} | A_{\mu\nu} \tilde{A}^{\mu\nu} | 0 \rangle^{\text{one loop}} \\ &= i \frac{12}{(4\pi)^2 48} \langle \text{grav} | \epsilon^{\alpha\beta\rho\sigma} R_{\mu\nu\alpha\beta} R^{\mu\nu\rho\sigma} | 0 \rangle^{\text{trees}} \end{aligned} \quad (10)$$

The total one-loop contribution is obtained by adding Eqs. (7), (8), (10) with appropriate factors (note that there are four gravitinos, four "neutrinos" and six vector fields). The result is zero ! Remarkably enough this comes about because of just the right cancellation of "anomalous" contributions.

In principle this should be the end of the story. If we could invoke the global supersymmetry relations Eq. (7), we would conclude that the φ and χ decay amplitudes vanish as well and, in this sector at least, $SU(4)$ and $SO(4)$ theories are one-loop equivalent. In order to verify this we look now at the φ decay amplitude.

It is sufficient to compute the matrix element $\langle \text{gravitons} | T(0) | 0 \rangle$ where

$$\begin{aligned} T = & \frac{\sqrt{2}\chi e}{4} [A_{\mu\nu}^{\sim} A^{\mu\nu\sim} + B_{\mu\nu}^{\sim} B^{\mu\nu\sim}] \\ & - \sqrt{2}\chi e \partial_\mu B \partial^\mu B \end{aligned} \quad (11)$$

obtained from $\mathcal{L}(SU(4))$ by keeping terms linear in φ which could possibly contribute to the process of interest.

To begin with let us calculate the two-graviton matrix element

$$\langle p, p' | e \partial_\mu B \partial^\mu B | 0 \rangle = \epsilon^{*\alpha\beta}(p) \epsilon^{*\gamma\delta}(p') M_{\alpha\beta\gamma\delta} \quad (12)$$

with $\epsilon_{\alpha\beta} = \epsilon_{\beta\alpha}$, $p^\alpha \epsilon_{\alpha\beta}(p) = \epsilon_{\alpha\alpha} = 0$. The relevant part of M has the structure

$$\begin{aligned} M_{\alpha\beta\gamma\delta} = & (\eta_{\alpha\gamma} \eta_{\beta\delta} + \eta_{\alpha\delta} \eta_{\beta\gamma}) M_1 \\ & + (p'_\alpha p_\gamma \eta_{\beta\delta} + \text{symmetrization}) M_2 \\ & + p'_\alpha p'_\beta p_\gamma p_\delta M_3 \end{aligned} \quad (13)$$

The conservation equation $p^\alpha M_{\alpha\beta\gamma\delta} = 0$ relates M_1 and M_2 to M_3 after which one recognizes that, as might be expected

$$\begin{aligned} \langle p, p' | e \partial_\mu B \partial^\mu B | 0 \rangle = \\ = \frac{1}{2} M_3 \langle p, p' | R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta} | 0 \rangle^{\text{trees}} \end{aligned} \quad (14)$$

M_3 receives contributions only from a triangle graph which gives

$$\begin{aligned} I = & -4i \int_0^1 dy \int_0^y dx \int d^4 q [q + p(1-x) + p'(1-y)]^2 \\ & \frac{[q + p'(1-y)]_\alpha [q + p'(1-y)]_\beta [q - px]_\gamma [q - px]_\delta}{[q^2 + 2 p \cdot p' x(1-y) + m^2]^3} \end{aligned} \quad (15)$$

and only terms a priori logarithmically divergent or finite contribute to M_3 . We have added a mass term m^2 in the B propagator to take care of possible infra-red divergences. We use dimensional regularization and find

$$\begin{aligned} M_3^{10\delta} = & \frac{1}{(4\pi)^2} \int_0^1 dy \int_0^y dx \Gamma(2 - \frac{n}{2}) [2 p \cdot p' x(1-y) + m^2]^{\frac{n}{2}-2} \\ & \cdot [m x^2(1-y)^2 + 2(1-y)(1-2y)x^2 - 2(1-y)^2 x(1-2x)] \end{aligned} \quad (16)$$

$$M_3^{\text{fin}} = -\frac{1}{(4\pi)^2} \int_0^1 d\gamma \int_0^\gamma dx \frac{2 p \cdot p' x^2 (1-\gamma)^2 (x+\gamma-2xy)}{2 p \cdot p' x (1-\gamma) + m^2} \quad (17)$$

Actually M_3^{log} is finite as well. Furthermore, there is in fact no infra-red divergence so one can set $m^2 = 0$ in which case we obtain $(M_3(B) = M_3^{\text{log}} + M_3^{\text{fin}})$

$$M_3(B) = -\frac{1}{(4\pi)^2} \frac{1}{24} \quad (18)$$

independent of $p \cdot p'$. (We note in passing that a calculation which takes the limit $p \cdot p' / m^2 \rightarrow 0$ or equivalently looks only at M_3^{log} gives the value of the trace anomaly for a spin zero field ¹⁰.)

A similar calculation for the vector field contribution leads to

$$M_3(A_\mu) = -\frac{1}{(4\pi)^2} \frac{53}{720} \quad (19)$$

Adding the contributions with appropriate factors we obtain a non-zero result for the ϕ decay amplitude, which can be represented by an effective Lagrangian

$$\mathcal{L}_{\text{eff}} = -\frac{\sqrt{2} \kappa e}{2} M_3(\text{tot}) \phi R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta} \quad (20)$$

Thus taken at face value both the equivalence between $SO(4)$ and $SU(4)$ theories is violated and the global supersymmetry relation (5) does not hold.

In looking for possible reasons for the discrepancy one can of course blame it on the use of a non-supersymmetric regularization method. We note that the B decay calculation, carried out using the Adler-Rosenberg method appears to be independent of the regularization method since it only requires extracting from a Feynman diagram a manifestly finite part. Thus one might believe that this calculation has given the right result. On the other hand the ϕ decay calculation using dimensional regularization is somewhat suspect. However, we should note that the B decay result came about because of cancellations of boson and fermion contributions, a typical supersymmetry effect. The ϕ decay receives no fermion contributions so cancellation seems unlikely.

If we ignore for the moment the global supersymmetry problem, the non-equivalence between $SO(4)$ and $SU(4)$ theories can be easily understood and remedied. We remark first that the statement of the theorem on equality of S matrices for two Lagrangian theories related by point transformations has to be examined with some care. What it actually says ³⁾ is that the S matrices of the two theories can be made equal, provided a suitable set of local counter terms is added to one or both Lagrangians; and these counter terms need not be infinite. For example, consider

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^2 - \bar{\Psi}(\gamma \cdot D + m)\Psi - \frac{1}{2} (\partial_\mu \sigma)^2 - \frac{1}{2} \mu^2 \sigma^2 \quad (21)$$

where σ is a free pseudoscalar field, and the rest is the standard QED Lagrangian. Under the field transformation

$$\begin{aligned} \Psi &\rightarrow \Psi + \lambda \gamma_5 \sigma \Psi \\ \mathcal{L} &\rightarrow \mathcal{L}' = \mathcal{L} - \lambda \sigma \partial_\mu (\bar{\Psi} \gamma_5 \gamma^\mu \Psi) - 2m\lambda \sigma \bar{\Psi} \gamma_5 \Psi \\ &\quad + \lambda^2 \sigma \bar{\Psi} \gamma^\mu \partial_\mu (\sigma \Psi) \end{aligned} \quad (22)$$

and as it stands $\mathcal{L}'$ is not equivalent to $\mathcal{L}$ because in $\mathcal{L}'$ σ decay to two photons can occur via the axial anomaly. The two Lagrangians become equivalent only if $\mathcal{L}'$ is modified by the addition of a finite counter term $\sim \sigma F_{\mu\nu} \tilde{F}^{\mu\nu}$.

In our case clearly the equivalence between $SO(4)$ and $SU(4)$ theory can be re-established, as far as φ decay is concerned, by adding to $\mathcal{L}(SU(4))$ the counter term $\Delta\mathcal{L} = -\mathcal{L}_{\text{eff}}$, with $\mathcal{L}_{\text{eff}}$ given by Eq. (22). This will also make global supersymmetry hold, since the φ amplitude will be zero. We note, however, that $\mathcal{L} + \Delta\mathcal{L}$ will not be explicitly supersymmetric, a somewhat unsatisfactory feature. It is possible, of course, to define a new $\mathcal{L}(SU(4))$ Lagrangian by making point field transformations in the one of Eq. (2). This will lead to new couplings and possibly a vanishing decay amplitude for the scalar field, without the need to introduce counter terms. If so the new Lagrangian might be a better starting point for quantum calculations. (However, the obvious field transformations we have tried do not help.)

What can we conclude from our results ?

- a) Although the Lagrangians of Eqs. (1), (2) are equivalent classically this equivalence does not hold at the quantum level unless counter terms are introduced.
- b) Although formally the transformation laws of Eq. (3) should lead to equal decays for the scalar and pseudoscalar mesons this is not the case at the one-loop level without introducing counter terms, or perhaps using a different regularization scheme. As a consequence, one might also question global supersymmetry arguments that would be used for establishing renormalizability in this model.
- c) It is interesting that the vanishing of the B amplitude comes about because of cancellations of "anomalies" multiplied by particular coefficients. It is not the total chiral current which is divergenceless at the one-loop level but rather the quantity J of Eq. (6) which may be viewed as the divergence of a "current" associated with one of the $SU(1,1)$ invariances of Ref. 2).
- d) The φ field also couples to an object related to the $SU(1,1)$ invariance. What about the X coupling ? Is there an extension of the $SU(1,1)$ invariance to an additional fermionic symmetry ?

Finally we should note that our results only partially check the quantum equivalence of $SO(4)$ and $SU(4)$ theories. For the amplitudes we have looked at this equivalence can be achieved by adding local counter terms. But in general most one-loop processes cannot be represented by local expressions and such processes would provide a better test for the equivalence of the two theories. Unfortunately they are much harder to calculate.

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