QUARK PAIR CONDENSATION AND CONFINEMENTA.J.G. Hey ^{*)},⁺)

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A B S T R A C T

We discuss the possibility of achieving quark confinement in QCD even if the $q\bar{q}$ potential falls with separation. The necessary condition is that the coupling exceeds a critical value of order 1. We study spontaneous pair creation due to a Coulomb interaction, and argue that in such a situation the true vacuum becomes a condensate of $q\bar{q}$ pairs. We argue heuristically that in QCD, if the true vacuum is such a condensate, it allows free quark propagation over limited distances only, and we point out possible excitations of the condensate that have the properties of hadrons.

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The Yang-Mills gauge theory of colored gluons and quarks (QCD) is generally regarded as the field theory underlying the strong interactions. It has the property of asymptotic freedom, i.e., the color electric interaction is logarithmically weaker than Coulomb at short distances. The gluon sector of QCD is responsible for this behavior.[1] Much effort has been devoted to the search for a proof that in QCD, the potential between a quark and an antiquark grows like the distance between them, thus making it impossible to separate them.¹⁾ Such a proof is also typically sought within the gluon sector. [2]

In this letter we propose an alternative to this picture of confinement. We argue that confinement may occur even if the potential between quarks falls with distance, provided that the coupling exceeds a critical value of order 1. Our picture is based on the observation that a strong Coulomb potential causes instabilities which result in pair production. This process can screen the color of a quark, and if it is quark quanta that are pair-produced, the quark's fractional flavor quantum numbers will be hidden as well.

The result that is the start of our considerations is that above a critical coupling constant there are bound states of e^+e^- pairs in massless scalar electrodynamics

with negative total energy (including rest masses) relative to the no-particle state (the naive vacuum). We show this in a variational calculation (without renormalization). For many years it has been known that above a critical coupling there are negative-total-energy bound states of particles in a fixed, external Coulomb potential.[3] They occur in non-Abelian as well as in Abelian gauge theories.[4] Physically, they lead to screening of the Coulomb potential.[5] Our result shows that negative energy Coulomb bound states of two massless particles exist as well. We expect that negative energy particle-antiparticle Coulomb bound states will form in non-Abelian gauge theories as well as in electrodynamics, so long as the coupling constant is large enough. In particular, there should be a critical coupling above which $q\bar{q}$ negative energy bound states will form in the attractive, color singlet, channel.

These field theoretic pair-formation instabilities imply that above the critical coupling the naive vacuum will be unstable to pair production. If a bound pair has negative energy, the lowest energy state, the true vacuum, will be filled with such pairs. These e^+e^- or $q\bar{q}$ pairs are not physical particles; they are merely a way of describing the true ground state in terms of the particle excitations of the naive one.

It seems plausible that if quark pair production instabilities result in the vacuum state being a condensate of $q\bar{q}$ pairs, their observational effect will be to confine quarks. The argument for this assertion is simply that color screening is "harder" to achieve in the naive vacuum, where it requires a pair production process, than in the condensate vacuum, where it only involves rearranging the quarks already present. Thus if an isolated quark would have its color spontaneously screened in the naive vacuum, which is the condition for the quark pair production instability, then color screening would seem very likely to occur in the condensate vacuum.

The following simple argument illustrates the origin of a critical coupling constant for Coulomb instabilities: Let us consider an electrostatic potential $-\alpha/r$, and ask whether is energetically favorable to produce from the vacuum a pair of massless particles with opposite charge, binding one to the central potential and thus screening it. If the characteristic radius of the bound state is R , then the kinetic energy of the bound massless particle is of order R^{-1} . This can be overcome by the binding potential if $\alpha \gtrsim 1$, which is our condition for the critical coupling constant.

This argument describes conditions under which a fixed charged source, the electrodynamic analogue of a heavy quark, would be screened by massless particles. However, a more significant question for an understanding of confinement is: Under what conditions will massless particles form negative energy bound states?

We address this question in massless scalar electrodynamics by finding the condition for a particle-anti-particle bound state to have a lower energy than the (naive) vacuum. The field theory is defined by the Lagrangian

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F_{\mu\nu} + (\partial_\mu + ieA_\mu)\phi^* (\partial_\mu - ieA_\mu)\phi. \quad (1)$$

In the radiation gauge, $\vec{\nabla} \cdot \vec{A} = 0$, the Hamiltonian is

$$\mathcal{H} = \Pi^* \Pi + (\vec{\nabla} - ie\vec{A})\phi^* (\vec{\nabla} + ie\vec{A})\phi + \frac{1}{2}\vec{E}^2 + \frac{1}{2}\vec{B}^2, \quad (2)$$

where the longitudinal electric field is constrained by

$$\vec{\nabla} \cdot \vec{E} = -ie(\phi^* \Pi - \Pi^* \phi) \quad (3)$$

Restricting the Hamiltonian to the subspace of states

with one zero-total-momentum particle-antiparticle pair and no radiation, states of the form

$$\int d^3p \psi(\vec{p}) a_{\vec{p}}^+ b_{-\vec{p}}^+ |0\rangle \quad (4)$$

where a^+ and b^+ are creation operators for electrons and positrons, defined at a fixed time, and minimizing the normalized expectation value of $\mathcal{H}$ over this space of states, we find the relative momentum wave equation²⁾

$$(\vec{p}|-E)\psi(\vec{p}) = \frac{\alpha}{8\pi^2} \int d^3q \left[\frac{|\vec{q}| + |\vec{p}|}{(\vec{p}-\vec{q})^2} + 2 - \frac{\vec{p} \cdot \vec{q}}{\vec{p}^2 \vec{q}^2} - \frac{1}{|\vec{p}||\vec{q}|} \right] \psi(\vec{q}). \quad (5)$$

For s-wave states, this reduces to the radial momentum-space wave equation

$$(E-2p)\psi(p) = -\frac{\alpha}{4\pi p^2} \int_0^\infty dq \psi(q) [(q+p)^2 \ln \left| \frac{p+q}{p-q} \right| - 2pq]. \quad (6)$$

The question that we are interested in is: For what values of α can there be solutions of Eq. (6) with negative E ? We approach this problem by solving the Mellin transform of Eq. (6) (which is a finite difference equation). Requiring that $\psi(p)$ be a square-integrable

function and $E < 0$, we find that these two conditions can be fulfilled only if

$$\alpha > \alpha_c = 3/2 . \quad (7)$$

The Mellin transform analysis of this problem will be presented in detail in a separate paper.[6]

This critical coupling is larger than that for spontaneous pair production in an external Coulomb potential ($\alpha_c = 1/2$). The difference can be understood if we imagine giving the electron a small mass m ; it is due to the different reduced masses, which set the dynamical energy scales. For a system of a static (infinite mass) source and a mass m particle, binding energies are proportional to m , while for a system of two mass m particles, they are proportional to $\frac{m}{2}$. However, for an instability, the binding energy of the lowest energy level must exceed $2m$ in each case. Hence a larger α_c is needed in the system of two equal-mass particles.

There is more to demonstrating confinement than showing that isolated quarks would be unstable excitations of the naive vacuum. If α exceeds α_c , the pair formation instability will change the vacuum radically, because the lowest energy state will be filled with bound particle-antiparticle pairs. The relevant question is whether or not this new vacuum

supports charged excitations. To develop some intuition about the new vacuum state, we make an analogy with the BCS theory of superconductivity. The particle-antiparticle bound state with lower energy than the (naive) vacuum is the analogue of a Cooper pair. Following the analogy, we should picture the new vacuum as a coherent state of these bound pairs. A simple example of such a coherent state is

$$\exp\left\{\int \tanh \theta(k) a_{\vec{k}}^+ b_{-\vec{k}}^+ d^3k\right\} |0\rangle. \quad (8)$$

where a^+ and b^+ are the creation operators of the particle and antiparticle. A parametrization of θ that gives a Lorentz invariant mass for the charged excitations (the pseudoparticles) is

$$-2 \theta(k) = \ln \frac{\sqrt{\vec{k}^2 + m^2}}{|\vec{k}|}. \quad (9)$$

For $\alpha > \pi$, this state has lower energy than the original vacuum. In fact, its energy density is proportional to m^4 ; hence it can be made lower by increasing m -- the mass of the pseudoparticle -- indefinitely.

The example studied above is completely scale invariant because it is based on a Coulomb binding force. The analogous force in QCD is different. At short distances, the coupling vanishes logarithmically and so we expect no small scale instabilities. However, if for some R_0 , the running coupling constant $\alpha(R)$ exceeds α_c for $R > R_0$, the necessary condition exists for a quark-antiquark pair-formation instability to occur. This distance R_0 is then an intrinsic length scale for QCD; it is roughly the minimum internal size of a $\bar{q}q$ state which has lower energy than the naive vacuum, and is presumably the characteristic correlation length between quarks in the condensate of $\bar{q}q$ pairs that we picture as the true vacuum. In this picture, the vacuum looks roughly like the naive vacuum at distance scales much smaller than R_0 , but the effects of the condensate become important at distances larger than R_0 . Specifically, the short-distance singularities of Green's functions should be the same as in the naive vacuum, although the condensate interferes with long distance quark propagation.

The pair condensate resembles somewhat the chirality-breaking ground state of Nambu and Jona-Lasinio [7]. However, since quarks would interact with the condensate via long range forces, rather than the point interactions of the Nambu-Jona-Lasinio model, its physical effects can be quite different in QCD. Rather than just giving the quark

a finite mass, we think it plausible that the polarizability of the condensate would cause an isolated quark to be an intrinsically unstable object. The two lightest flavors of quarks, which are abundant in the condensate, have the quantum numbers necessary to screen fractional charge and hypercharge when they screen color.

To calculate the excitation spectrum of the $q\bar{q}$ condensate requires a much more thorough understanding of its structure than we presently have available. For example, the fermionic nature of quarks presumably tends to limit the density of a $q\bar{q}$ condensate, but the precise effect is unknown. Also, the response of the condensate to external charges has not been calculated, so that our remarks indicating that the condensate will screen color are just a plausibility argument and not a proof. Finally, while we have guessed at the effect of asymptotic freedom on the condensate's structure, a quantitative calculation is lacking.

Because of all these uncertainties, we cannot be at all sure about the excitation spectrum of the condensate. However, we should point out that there are systems with the characteristics of hadrons that are plausible candidates for excitations. By this we mean that we can argue heuristically that the expected value of the Hamiltonian in such states lies a finite amount above the ground state, and that the excitation will not strongly disturb the surrounding condensate, so that it has a fair chance of being stable.

The systems which are our candidates for excitations are collections of quarks and gluons which, together with

all their associated fields, are confined to a region smaller than R_0 . The primary example would be a compact $q\bar{q}$ system in a color singlet state. Such a system would not set up a long range color field, and so would not affect, and in turn be affected by, the condensate. The $q\bar{q}$ state could be pictured as residing in the interstices of the condensate. It would have a finite, positive energy: finite, because it is a state of two massless quanta, confined to a finite size region, attracting each other with a force weaker than a Coulomb interaction; positive, because at distances less than R_0 the negative potential energy from the attractive color interaction is less than the positive kinetic energy associated with their confinement to a region smaller than characteristic pair separation in the condensate. We would associate these excitations with physical mesons. Other small (relative to R_0) color singlet systems, for example, qqq , or $qqq\bar{q}$ or $q\bar{q} +$ gluons, would also be finite energy excitations, corresponding to baryons or exotic mesons, respectively.

Finally, let us describe several alternatives for the $q\bar{q}$ potential, to indicate the conditions under which we expect the scenario described here to occur. Fig. 1 shows an $\frac{\alpha_c}{r}$ Coulomb potential compared with three curves, all of which are logarithmically weaker near the origin. Curve A corresponds to the common lore of an asymptotically rising potential leading to quark confinement. Curve B designates a situation where quark confinement does not exist. Curve C describes a situation where the

phenomenon that we propose will take place: The vacuum will become a condensate of light $q\bar{q}$ pairs whose characteristic correlation length is of the order of the value of the point $r = R_0$ at which the curve C crosses α_0/r . Within this new vacuum quarks may propagate freely for short distances, but the condensate will inhibit their long distance propagation as free, stable particles.

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FOOTNOTES

1. The notion of a linearly rising potential is often expressed in field theory by Wilson's gauge-invariant criterion that $\ln \langle \exp i e \oint A_\mu dx_\mu \rangle$ grow asymptotically like the area enclosed by the integration path.
2. Scalar QED requires a $(\phi^* \phi)^2$ counterterm to be renormalizable. This counterterm is needed to control the high energy behavior in the two particle sector; a finite part of it is included in Eqs. (5) and (6) to make these bound state equations finite.

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FIGURE CAPTION

Fig. 1: The curve α_c/r (dashed line) compared with three possibilities for the $q\bar{q}$ effective potential.

A) An asymptotically rising potential. B) A potential weaker than α_c/r , which would not be expected to confine.

C) A potential stronger than α_c/r at large distances, which is expected to lead to the effects described here.

Figure 1.