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THE SYSTEMATICS OF POSSIBLY NARROW QUARK STATES
WITH BARYON NUMBER ONE

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ABSTRACT

We discuss the systematics of hadronic colour singlets consisting of four quarks and an anti-quark. For the configurations that could give narrow colour isomer states we compute relative mass splittings between different flavour multiplets, discuss production possibilities and cross-sections. Such states have, perhaps, already been observed.

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1. INTRODUCTION

There has recently been renewed theoretical interest in hadronic states composed of more exotic quark configurations than the usual $q\bar{q}$ for mesons and qqq for baryons.

This interest has been motivated by the experimental¹⁻³⁾ discovery of mesonic states with a lifetime 10-50 times longer than usual hadronic resonant states. Their theoretical interpretation is quite a challenge and many proposals have been put forward. The most prosaic is an interpretation as quasibound baryon-antibaryon⁴⁾ which would mean that they consist of a $3q3\bar{q}$ state in a "molecular"-like configuration. For many theoreticians the same states have been looked upon as the long sought after $qqq\bar{q}$ states forced upon nature by dual models^{5,6)}. The narrow widths are here somewhat non-trivial to explain -- a new type of Zweig rule (for colour junctions) has been invoked as an explanation. An analysis by M. Pennington has however shown that duality constraints most probably are fulfilled by the $qqq\bar{q}$ states that form the meson continuum⁷⁾. Another viewpoint where colour⁸⁻¹⁰⁾ is the key to an understanding of the long lifetime, is that in the multi-quark states more complicated than the $q\bar{q}$ and qqq states the colour degree of freedom is unfrozen and shows itself in a greater variety of states than one would expect with colourless quarks. We get colour isomers. As quarks are triplets in colour and free particles must by law be colour singlets, it follows that the building of $q\bar{q}$ and qqq to form colour singlets is unique and there is no difference in the number of states that can distinguish coloured quarks from uncoloured ones. As soon as the mesonic states have significant components of $2q2\bar{q}$ states or the baryonic states of $4q1\bar{q}$ states, the situation is different -- for the diquonium (short for diquarkonium) state $2q2\bar{q}$, the two quarks can be either in a colour $\bar{3}$ or 6 state combining with the two antiquarks in a colour 3 or $\bar{6}$ state to give a singlet.

Similarly in the $4q1\bar{q}$ configuration there is more than one way of constructing a colour singlet, and we would expect different physical properties of the baryonic states corresponding to the different colour couplings between the quark subsystems, corresponding to different colour isomers.

Much work⁸⁻¹⁰⁾ has already been done in exploring this point of view for the diquonium configurations $2q2\bar{q}$. In this paper we shall address ourselves to the classification of states when the dominant quark content is four quarks and one antiquark and therefore carries baryon number one. Some work has been done for the s-wave configurations¹¹⁾ of these mesobaryon states; in the same philosophy that was explained in earlier papers by Chan and one of us, we shall try to find quark configurations that possibly could be rather stable. Their flavour multiplets will be catalogued as will be their production mechanism and dominant decay modes.

We shall furthermore give some estimates of their expected masses. There exists also another recent study of the properties of a subclass of five quark states¹²⁾. All group theoretical results required to follow this study are collected in the Appendix.

2. THE INTERESTING CONFIGURATIONS

It seems foolhardy to try to establish the properties of $4q\bar{q}$ states as long as there is little or no experimental evidence for any of them. Such states will in general dissociate extremely easily into one baryon with three quarks and a meson containing the additional $q\bar{q}$ configuration. However, some configurations with high angular momenta between subsystems of quarks can have fundamentally different properties. Consider as an example the case when the three-quark system Θ has its minimal spatial extension with relative angular momentum zero and the remaining $q\bar{q}$ state D is also in its lowest angular momentum state.

D can then be a colour singlet or an octet, whereas Θ can have colour $3 \otimes 3 \otimes 3 = 1 \oplus 8 \oplus 8 \oplus 10$.

If Θ and D are separately colour singlets they will interact only via the Van der Waal forces of QCD and will dissociate immediately into a baryon and a meson. However, if the ΘD system is a colour singlet with Θ and D being separately colour octets and in addition the relative angular momentum $\ell_{\Theta D}$ is so big that the antiquark in D is prevented by the centrifugal barrier from tunnelling into Θ where it can combine with one of the quarks to make a (colourless) meson, the state would have a much enhanced stability. In this case we call the bond between Θ and D an octet bond⁹⁾.

For the colour singlets where D is composed of two quarks in a colour sextet 6 (or a colour triplet $\bar{3}$) and Θ of two quarks and one antiquark coupled to colour $\bar{6}$ (3) we will speak of a colour 6 (3) bond. As remarked before, colour singlet bonds are very weak as they are of a Van der Waal nature ("molecular").

Colour triplet bonds are also relatively easily broken as the creation of a single $q\bar{q}$ pair permits the system to break up into two separate states which are colour singlets. Colour triplet bonds will therefore lead to hadronic states that have a lifetime comparable to excited $q\bar{q}$ (meson) states or excited qqq baryon states (as these are also bound together by triplet bonds).

In the particular case of five-quark systems colour triplet bonds are particularly uninteresting. If two quarks and one antiquark are in a colour triplet state there is always a component of the state corresponding to a $q\bar{q}$ configuration where $q\bar{q}$ is in a colour singlet. This singlet would immediately leave the prison of the colour confining "potential" and the $q\bar{q}qqq$ state would probably not show any resonance structure.

The colour octet and sextet bonds have a nature that is never observed in $q\bar{q}$ and qqq spectroscopy -- however there might be some experimental evidence for the existence of qq and $\bar{q}\bar{q}$ systems bound together by sextet bonds in states that have been called mock-baryonium states⁸⁾. We believe that such types of binding are inherent in any non-Abelian theory based on the existence of coloured quarks and that they therefore are of fundamental importance.

The simplest examples of colour octet and sextet bonds in multi-quark states with baryon number one will be considered in the following two sections.

Readers not interested in details can pass directly to the end of Section 5.

3. COLOUR OCTET BONDS

Suppose that we have a quark-antiquark system $D(N_c, N_s)$ which forms a flavour nonet in a relative s-state. The transformation properties of the state under $SU_3^{\text{colour}} \times SU_2^{\text{spin}}$ can be of the following type where we shall use the notation (N_c, N_s) ; N_c denotes the multiplicity of the colour representation, N_s the spin multiplicity. As a quark (antiquark) is a $(3, 2)$ $[(\bar{3}, 2)]$ state the $q\bar{q}$ state will be $(3, 2) \otimes (\bar{3}, 2) = (1 + 8, 3 + 1)$.

The configurations of the colour singlet states $(1, 3)$ and $(1, 1)$ are the well-known vector meson and pseudoscalar meson flavour nonets, respectively; the colour octets with spin one and zero, respectively, are $(8, 3)$ and $(8, 1)$ and these can never be observed as free particles. They can however be linked together with other colour octets to make observable free colour singlet systems.

The $2q2\bar{q}$ system which has been called M-mesonium⁸⁾ is the system $D(8, N_s) D(8, N_s)$. Here we shall look at the system $D(8, N_s) \Theta(8, N_s)$ where Θ denotes a qqq state and $\Theta(8, N_s)$ is the colour octet configuration of three quarks with spin multiplicity N_s .

As Θ consists of three quarks in a relative s-state, the Pauli principle determines the flavour multiplicity of the configuration with given colour and spin multiplicity.

Under the group $SU_{18} \supset SU_6^{\text{colour spin}} \times SU_3^{\text{flavour}} \supset SU_3^{\text{colour}} \times SU_2^{\text{spin}} \times SU_3^{\text{flavour}}$ Θ must be a completely antisymmetric state of three quarks and it must therefore transform under the $(18 \cdot 17 \cdot 16)/3! = 816$ -dimensional representation of SU_{18} . The enlargement of the flavour group to SU_4^f to contain charmed quarks or to SU_6^f to contain also top and bottom quarks is trivial.

This representation is easily seen to decompose as follows under $SU_3^{\text{colour}} \times SU_2^{\text{spin}} \times SU_3^{\text{flavour}}$:

A flavour singlet Θ^1 which contains $\Theta(10,4)$ and $\Theta(8,2)$
 decuplet Θ^{10} $\Theta(1,4)$ and $\Theta(8,2)$
 octet Θ^8 $\Theta(10,2)$, $\Theta(8,4)$, $\Theta(8,2)$ and $\Theta(1,2)$ ^(3.1)

Furthermore

the singlet Θ^1 transforms under the 56 dimensional representation of $SU_6^{\text{colour spin}}$
 Θ^{10} " 20 "
 Θ^8 " 70 "

The colour singlets $\Theta^{10}(1,4)$ and $\Theta^8(1,2)$ are the familiar $J^P = \frac{3}{2}^+$ decuplet and $J^P = \frac{1}{2}^+$ octet baryons, respectively.

We shall shortly proceed to combine to colour singlets the states $\Theta^F(N_c, N_s)$ and $D^{F'}(N'_c, N'_s)$ where F (F') denotes the flavour multiplicity of the three-quark states Θ^F or the $q\bar{q}$ state $D^{F'}$ assuming the relative angular momentum of the qqq system and the $q\bar{q}$ to be large. However, before this we will briefly discuss the relative effective mass splittings between the different substates $\Theta^F(N_c, N_s)$ and $D^{F'}(N'_c, N'_s)$.

This does not mean that we look upon the Θ or D system as being free -- that would be nonsense. What we always suppose is that Θ or D are part of a total system which has total colour zero, where the complicated and non-linear QCD interaction gives confinement. What we then calculate is some effective mass of Θ and D inside this colour singlet.

As has been demonstrated by De Rújula, Georgi and Glashow, a perturbative treatment of colour magnetic hyperfine splitting provides a very good description of mass splittings in s-wave $q\bar{q}$ and qqq systems which are colour singlets. This approach has been adopted in the discussion of more complicated quark configurations ^{8-10,14)} and so we will continue to use it here.

Assuming then that the Hamiltonian responsible for the mass splitting of a multiquark state is

$$H' = -\frac{8\pi}{3} \alpha_s \sum_{i,j} \lambda^a(i) \lambda^a(j) \underline{\sigma}(i) \cdot \underline{\sigma}(j) \quad (3.2)$$

where the summation is over all quark-quark colour magnetic interactions and α_s is the strong coupling constant of gluons to quarks, we can compute mass splittings. α_s is evidently dependent on the separation between the two quarks -- it is just the small value of α_s at small distances that makes perturbation theory valid in the asymptotically free theory of quantum chromodynamics.

The Hamiltonian given above is diagonal in the states $\Theta^F(N_c, N_s)$ and $D^F(N_c, N_s)$ and so we can easily calculate its expectation values:

State	Expectation value of H'	
$D^9(1,3)$	$16/3 c$	(3.3)
$D^9(1,1)$	$-16 c$	
$D^9(8,3)$	$-2/3 c$	
$D^9(8,1)$	$2 c$	

$\Theta^1(10,4)$	$-4c$	(3.4)
$\Theta^1(8,2)$	$-14c$	
$\Theta^{10}(1,4)$	$8c$	
$\Theta^{10}(8,2)$	$10c$	
$\Theta^8(8,4)$	$2c$	
$\Theta^8(8,2)$	$-2c$	
$\Theta^8(1,2)$	$-8c$	
$\Theta^8(10,2)$	$4c$	

The "constant" c contains the effect of the value of the quark wave function at the origin and could in principle vary from state to state.

If we stay in the flavour sector containing no strange quarks there are arguments^{8,15)} making it reasonable to use the same $c \approx 20$ MeV in all states. A great number of $qq\bar{q}q$ states contain, however, strange quarks, often as hidden strangeness via $s\bar{s}$ pairs.

In the simplest cases of qq pairs or qqq coupled to a colour singlet Tsou¹⁵⁾ has taken into account SU_3^{flavour} splitting of the hyperfine colour magnetic Hamiltonian (3.2). She shows that when one strange quark interacts with a non-strange quark, c is closer to 12.5 MeV than 20 MeV. As it is very difficult to include SU_3^{flavour} splitting in H' when the number of quarks is large (5 when we treat the over-all s-wave qqqq $\bar{q}$ system) we shall perform all calculations in the symmetry limit of (3.2). SU_3^{flavour} will be broken through the quark masses and whenever there is a strange quark in the system we shall use a value of 15 MeV for c. We think this is a reasonable approximation to make at this time since there is very little experimental data with which to compare the result. As has been noted before, in (3.3) and (3.4) we recognize the familiar hadrons which are the ground states of the usual quark model as $D^9(1,1)$ and $D^9(1,3)$ for mesons, and $\Theta^8(1,2)$ and $\Theta^{10}(1,4)$ for baryons.

We remark in (3.4) the very great binding energy in the flavour singlet state of three quarks $\Theta^1(8,2)$. This is the basic state that can couple with itself to give Jaffe's possible $\Lambda\Lambda$ bound state¹⁶⁻¹⁸⁾ in the 6q sector of the quark model. Such states in which we have two Θ 's coupled together carry baryon number two and will not be considered further in this paper but in a forthcoming one. The flavour singlets are the completely antisymmetric states in flavour and will therefore contain at least one strange or charmed quark (or top or bottom ...). With their big mass defect due to the hyperfine interaction they will appear in the configurations which produce the lightest five-quark states; this is the reason why hidden strangeness is so important even in the non-strange sector.

The mesobaryon states are now obtained by combining the basic states with baryon number zero (3.3) with the basic states with baryon number one (3.4) such that the total colour is zero. Two different couplings are possible -- colour singlet bonds as between $D^9(1,1)$ and $D^8(1,2)$, and colour octet bonds. Colour singlet bonds are, as noted before, supposed to be so weak that the decay of the product state is so fast (superalowed) that the states will probably never be observed as resonances. The colour octet bonds on the contrary can build up states that will have great stability when the angular momentum between D and Θ is reasonably big.

These states are $D^9(8, N_s) \otimes \Theta^F(8, N'_s)$ and they will have a flavour multiplicity of $9 \cdot F$ where F is the flavour multiplicity of the three-quark state with colour 8 and spin multiplicity N'_s .

We list the types of states we then find with colour octet bonds, together with their flavour content in terms of SU_3 representations, spin contents and accumulated binding energy defect from the spin-spin interaction inside D and Θ as computed in (3.3) and (3.4).

State	SU_3^{flavour} content	Spin content	Mass defect (negative for attraction)	
$D^9(8,3) \quad \Theta^1(8,2)$	$1 + 8$	$\frac{1}{2}, \frac{1}{2}$	$-44/3 \text{ c}$	(3.5)
$D^9(8,1) \quad \Theta^1(8,2)$	$1 + 8$	$\frac{1}{2}$	-12 c	
$D^9(8,3) \quad \Theta^{10}(8,2)$	$10 + 10 + 8 + 27 + 35$	$\frac{3}{2}, \frac{1}{2}$	$28/3 \text{ c}$	
$D^9(8,1) \quad \Theta^{10}(8,2)$	$10 + 10 + 8 + 27 + 35$	$\frac{1}{2}$	$32/3 \text{ c}$	
$D^9(8,3) \quad \Theta^8(8,4)$	$1 + 8 + 8 + 8 + 10 + \bar{10} + 27$	$\frac{5}{2}, \frac{3}{2}, \frac{1}{2}$	$4/3 \text{ c}$	
$D^9(8,1) \quad \Theta^8(8,4)$	$1 + 8 + 8 + 8 + 10 + \bar{10} + 27$	$\frac{3}{2}$	4 c	
$D^9(8,3) \quad \Theta^8(8,2)$	$1 + 8 + 8 + 8 + 10 + \bar{10} + 27$	$\frac{3}{2}, \frac{1}{2}$	$-8/3 \text{ c}$	
$D^9(8,1) \quad \Theta^8(8,2)$	$1 + 8 + 8 + 8 + 10 + \bar{10} + 27$	$\frac{1}{2}$	0	

For a given orbital angular momentum L between D and Θ the computed values of the colour magnetic force should give the mass splittings between the different flavour multiplets (as long as we compare states containing the same number of strange quarks). One observation of a mesobaryonium state will therefore fix the masses of the rest.

Moreover, for each value of L there is a series of possible total angular momentum states corresponding to $J = L + S, \dots, |L - S|$. For each total spin of the two clusters D and Θ we expect that the leading trajectory with $J = L + S$ will be by far the easiest to locate, as is the case in ordinary hadron spectroscopy. Now some of this multitude of states will be spurious; to undertake the sorting out of states corresponding to an oscillating centre of mass or (and) wrong symmetry in the exchange of identical particles is no small task for five-particle states, and we will postpone this until there is some experimental verifications of the crudest aspects of the model.

We expect the states of $D\Theta$ to fall on a Regge trajectory with a slope determined from the fact that they are bound with colour octet bonds. The work of Johnson and Thorn¹⁹⁾ from the MIT bag gives

$$\alpha'_8 = \frac{2}{3} \alpha'_3$$

where α'_8 is the slope of trajectories with octet bonds $\alpha'_3 \approx 0.9 \text{ GeV}^{-2}$ is the slope of trajectories with the (normal) triplet bonds.

The trajectory which fixes the centre of mass M_L for an L-excited multiplet is then

$$L = \alpha_0 + \alpha'_8 M_L^2$$

One observed member of one multiplet with given L will then approximately determine the mass of all states when we take the colour magnetic splitting (3.2) into account. It could be possible to make a guess at the masses of the states by using a mass formula as has been proposed by Han and Nambu^{20,12,9)} where the colour electric energy is approximately taken into account. This we shall not do in this paper, rather we are going to establish lower limits on the masses of five-quark states from experiment, both from the spectroscopic knowledge we have and from the knowledge of limits on the intercept of baryonic Regge trajectories. As curvature of Regge trajectories for physical states is expected owing to the different size of the colour magnetic interaction when $L = 0$ and L is great, we shall postpone the discussion of masses to a later section where it will be shown from experimental knowledge that $\alpha_0 \lesssim -1.5$ to -2 .

4. COLOUR SEXTET BONDS

The configurations studied in this section will be of the type where the system Θ consists of two quarks and one antiquark and D of a quark-quark pair with no angular momentum excitation inside Θ or D but with possibly large orbital momenta between Θ and D ¹²⁾.

Our basic states will then be

$$\begin{aligned} D^{\bar{3}}(6,3) \\ D^{\bar{3}}(\bar{3},1) \\ D^6(6,1) \\ D^6(\bar{3},3) \end{aligned} \tag{4.1}$$

for the quark-quark states.

For the $qq\bar{q}$ state Θ we shall distinguish between the states $\mathcal{A}$ where the two quarks are antisymmetric ($\bar{3}$) in flavour and so in a 21-dimensional representation of colour spin, and the states $\mathcal{S}$ where the two quarks are symmetric in flavour (6) and in a 15-dimensional representation of colour spin. The flavour content of the states $\mathcal{A}$ will then belong to the $\bar{3} \times \bar{3} = 3 + \bar{6}$ representation of SU_3^{flavour} , the states $\mathcal{S}$ will belong to $6 \times \bar{3} = 15 + 3$ of SU_3^{flavour} .

The basic states will then be

$$\mathcal{A}(3,4), \mathcal{A}(3,2), \mathcal{A}(15,4), \mathcal{A}(15,2), \mathcal{A}(\bar{6},2), \mathcal{A}(3,2) \quad (4.2)$$

and $\mathcal{S}(3,2), \mathcal{S}(15,2), \mathcal{S}(3,4), \mathcal{S}(\bar{6},4), \mathcal{S}(\bar{6},2)$ and $\mathcal{S}(3,2)$.

where we again have used the notation

$$\mathcal{A}(\text{colour, spin}) \text{ and } \mathcal{S}(\text{colour, spin})$$

The colour sextet bonds will then be between the systems $D^{\bar{3}}(6,3)$ and $D^6(6,1)$ on one side and $\mathcal{A}(\bar{6},2)$, $\mathcal{S}(\bar{6},4)$ and $\mathcal{S}(\bar{6},2)$ on the other side.

As before we compute the mass splitting due to the spin-spin interaction inside each of the systems D , $\mathcal{A}$ and $\mathcal{S}$ separately, and combine them to give the mass defect of the system where the $q\bar{q}$ and $qq\bar{q}$ systems are separated by a large centrifugal barrier caused by the orbital angular momentum L . We then get:

State	Flavour content	Mass defect	Maximal J
$D^{\bar{3}}(6,3) \mathcal{A}(\bar{6},2)$	$(3+\bar{6}) \times \bar{3} = 1+8+8+10$	$-\frac{4}{3}c - 8c = -\frac{28}{3}c$	$L + \frac{3}{2}$
$D^{\bar{3}}(6,3) \mathcal{S}(\bar{6},4)$	$(3+15) \times \bar{3} = 1+8+8+10+27$	$-\frac{4}{3}c + \frac{4}{3}c = 0$	$L + \frac{5}{2}$
$D^{\bar{3}}(6,3) \mathcal{S}(\bar{6},2)$	$(3+15) \times \bar{3} = 1+8+8+10+27$	$-\frac{4}{3}c + \frac{16}{3}c = 4c$	$L + \frac{3}{2}$
$D^6(6,1) \mathcal{A}(\bar{6},2)$	$(3+\bar{6}) \times 6 = 8+10+1+8+27$	$4c - 8c = -4c$	$L + \frac{1}{2}$
$D^6(6,1) \mathcal{S}(\bar{6},4)$	$(3+15) \times 6 = 8+8+8+10$	$4c + \frac{4}{3}c = \frac{16}{3}c$	$L + \frac{3}{2}$
$D^6(6,1) \mathcal{S}(\bar{6},2)$	$(3+15) \times 6 = 8+8+8+10$ $+10+10+27+35$	$4c + \frac{16}{3}c = \frac{28}{3}c$	$L + \frac{1}{2}$

(4.3)

These states are again expected to fall on Regge trajectories according to orbital angular momentum excitations but this time with a slope

$$\alpha'_6 = \sqrt{\frac{2}{5}} \alpha'_3 = 0.63 \alpha'_3 \approx 0.57 (Gw)^2$$

such that the orbital excitations give rise to mean masses given by

$$M = \sqrt{\frac{L - \alpha_0}{\alpha'_6}}$$

The colour magnetic spin interaction then splits these states by the amount given by (4.3).

Again we shall show that an intercept $\alpha_0 \approx -2$ is not inconsistent with our knowledge of exotic exchange. This may appear paradoxical: for we see that the leading trajectory will occur when the states $D^3(6,3)$ $S(\bar{6},4)$ have $J = L + \frac{5}{2}$. These would have a trajectory

$$\alpha(M) = -2 + \alpha'_6 M^2 + \frac{5}{2} = \frac{1}{2} + \alpha'_6 M^2$$

if the trajectory was linear at low masses.

We shall now see that the colour magnetic interaction gives an additional repulsion when $L = 0$ so that the trajectories may be curved to give a real intercept far below -0.5 .

5. THE OVER-ALL s-WAVE SYSTEM

The ground state of the $4q1\bar{q}$ system should occur when all quarks are in relative s-waves. To determine the mass splitting due to spin-spin interaction in this case we have to diagonalize the Hamiltonian (3.2) when there is colour magnetic forces between all 5 particles.

As remarked earlier there is room for some, hopefully small, improvement in our calculations here. To use different interactions between the strange/non-strange, strange/strange and non-strange/non-strange quarks would demand the calculation of so many recoupling coefficients for SU_2 and SU_3 that we have decided for the moment to use the colour magnetic interaction in its symmetric form (3.2). For qqq this would correspond to neglecting the $\Lambda\Sigma$ mass difference of 75 MeV.

Let us start with the four quarks in a relative s-state. Because of the Pauli principle this state must be antisymmetric in the combined wave function of colour $\times$ spin $\times$ flavour. It must therefore transform under the 3060-dimensional

irreducible representation of $SU_{18} \supset SU_3^{\text{colour}} \times SU_2^{\text{spin}} \times SU_3^{\text{flavour}}$. This decomposes as follows into colour-spin CS and flavour F: writing the states $|\underline{CS}, \underline{F}\rangle$ the 3060 states of SU_{18} are

$$|210, \underline{3}\rangle, |105, \underline{6}\rangle, |105', \underline{15}\rangle \text{ and } |\overline{15}, \underline{15'}\rangle \quad (5.1)$$

Of these states most contain at least one strange quark, exceptions are one isoscalar in $|105, \underline{5}\rangle$, one isotriplet in $|105', \underline{15}\rangle$ and one isotensor in $|\overline{15}, \underline{15'}\rangle$. The techniques developed by Jaffe¹⁴⁾ make it easy to compute the action of the colour magnetic Hamiltonian on all these states.

Next we combine these states with the antiquark state which transforms as an antisextet under the colour-spin group and as an antitriplet under SU_3^{flavour} . We find

$$\begin{aligned} |210, \underline{3}\rangle \otimes |\bar{6}, \bar{3}\rangle &= |56, \underline{9}\rangle + |70, \underline{9}\rangle + |1134, \underline{9}\rangle \\ |105, \underline{6}\rangle \otimes |\bar{6}, \bar{3}\rangle &= |560; \underline{10} + \underline{8}\rangle + |70; \underline{10} + \underline{8}\rangle \quad (5.2) \\ |105', \underline{15}\rangle \otimes |\bar{6}, \bar{3}\rangle &= |540, \underline{8} + \underline{10} + \underline{27}\rangle + |70, \underline{8} + \underline{10} + \underline{27}\rangle \\ &\quad + |20, \underline{8} + \underline{10} + \underline{27}\rangle \\ |\overline{15}, \underline{15'}\rangle \otimes |\bar{6}, \bar{3}\rangle &= |20, \underline{10} + \underline{35}\rangle + |70, \underline{10} + \underline{35}\rangle \end{aligned}$$

None of these states are spurious.

The only possible colour singlets contained in these states come from the combinations of four quarks making up a colour triplet. Decomposing the colour spin multiplets into their colour and spin content unravels the following colour triplets (the notation is now |colour spin, colour, spin, flavour))

$$\begin{aligned}
 & |210; 3, 3; \underline{3}\rangle \quad |210; 3, 1; \underline{3}\rangle \\
 & |105; 3, 3; \underline{\bar{6}}\rangle \quad |105; 3, 1; \underline{\bar{6}}\rangle \\
 & |105'; 3, 5; \underline{15}\rangle \quad |105'; 3, 3; \underline{15}\rangle \quad |105'; 3, 1; \underline{15}\rangle \\
 & |\bar{15}; 3, 3; \underline{15'}\rangle
 \end{aligned} \tag{5.3}$$

Similarly the $qqqq\bar{q}$ states contain the colour singlets with the following spin multiplicities

$$\begin{aligned}
 & |70; 1, 2; \underline{9}\rangle \\
 & |1134; 1, 4; \underline{9}\rangle, \quad |1134; 1, 2; \underline{9}\rangle \quad \text{for the flavour nonet} \\
 & |560; 1, 4; \underline{18}\rangle, \quad |70; 1, 2; \underline{18}\rangle \quad \text{for the flavour} \\
 & \quad \text{octet + antidecuplet} \\
 & |540; 1, 6; \underline{45}\rangle, \quad |540; 1, 4; \underline{45}\rangle, \quad |540; 1, 2; \underline{45}\rangle \\
 & |70; 1, 2; \underline{45}\rangle, \quad |20; 1, 4; \underline{45}\rangle \quad \text{for the octet} \\
 & \quad \text{+ decuplet + 27-plet}
 \end{aligned} \tag{5.4}$$

$$\begin{aligned}
 & |20; 1, 4; \underline{45}\rangle, \quad |\bar{70}; 1, 2; \underline{45}\rangle \quad \text{for the 45-flavour states} \\
 & \quad \text{containing one decuplet and the 35-plet}
 \end{aligned}$$

These states arise from the coupling of the four-quark states with the antiquark state as follows

$$\begin{aligned}
 |I\rangle &= |1134; 1, 4; \underline{9}\rangle = |(210; 3, 3; \underline{3})(\bar{3}, 2, \bar{3}) 1, 4; \underline{9}\rangle \\
 |II\rangle &= |1134; 1, 2; \underline{9}\rangle = \left[\frac{1}{3}\right] |(210; 3, 3; \underline{3})(\bar{3}, 2; \bar{3}) 1, 2; \underline{9}\rangle \\
 &\quad - \left[\frac{2}{3}\right] |(210; 3, 1; \underline{3})(\bar{3}, 2; \bar{3}) 1, 2; \underline{9}\rangle \\
 |III\rangle &= |70; 1, 2; \underline{9}\rangle = \left[\frac{2}{3}\right] |(210; 3, 3; \underline{3})(\bar{3}, 2; \bar{3}) 1, 2; \underline{9}\rangle \\
 &\quad + \left[\frac{1}{3}\right] |(210; 3, 1; \underline{3})(\bar{3}, 2; \bar{3}) 1, 2; \underline{9}\rangle \\
 |IV\rangle &= |560; 1, 4; \underline{18}\rangle = |(105; 3, 3; \underline{\bar{6}})(\bar{3}, 2; \bar{3}) 1, 4; \underline{18}\rangle
 \end{aligned} \tag{5.5}$$

$$\begin{aligned}
|V\rangle &= |70; 1, 2; \underline{18}\rangle = |(105; 3, 1; \underline{6}) (\bar{3}, 2; \underline{3}) 1, 2; \underline{18}\rangle \\
|VI\rangle &= |540; 1, 6; \underline{45}\rangle = |(105'; 3, 5; \underline{15}) (\bar{3}, 2; \underline{3}) 1, 6; \underline{45}\rangle \\
|VII\rangle &= |540; 1, 4; \underline{45}\rangle = \sqrt{\frac{1}{10}} |(105'; 3, 5; \underline{15}) (\bar{3}, 2; \underline{3}) 1, 4; \underline{45}\rangle \\
&\quad + \sqrt{\frac{9}{10}} |(105'; 3, 3; \underline{15}) (\bar{3}, 2; \underline{3}) 1, 4; \underline{45}\rangle \\
|VIII\rangle &= |540; 1, 2; \underline{45}\rangle = \sqrt{\frac{13}{16}} |(105'; 3, 3; \underline{15}) (\bar{3}, 2; \underline{3}) 1, 2; \underline{45}\rangle \\
&\quad + \sqrt{\frac{3}{16}} |(105'; 3, 1; \underline{15}) (\bar{3}, 2; \underline{3}) 1, 2; \underline{45}\rangle \\
|IX\rangle &= |20; 1, 4; \underline{45}\rangle = -\sqrt{\frac{9}{10}} |(105'; 3, 5; \underline{15}) (\bar{3}, 2; \underline{3}) 1, 4; \underline{45}\rangle \\
&\quad + \sqrt{\frac{1}{10}} |(105'; 3, 3; \underline{15}) (\bar{3}, 2; \underline{3}) 1, 4; \underline{45}\rangle \\
|X\rangle &= |70; 1, 2; \underline{45}\rangle = -\sqrt{\frac{3}{16}} |(105'; 3, 3; \underline{15}) (\bar{3}, 2; \underline{3}) 1, 2; \underline{45}\rangle \\
&\quad + \sqrt{\frac{13}{16}} |(105'; 3, 1; \underline{15}) (\bar{3}, 2; \underline{3}) 1, 2; \underline{45}\rangle \\
|XI\rangle &= |20; 1, 4; \underline{45}\rangle = |(\bar{15}; 3, 3; \underline{15}') (\bar{3}, 2; \underline{3}) 1, 4; \underline{45}\rangle \\
|XII\rangle &= |\bar{70}; 1, 2; \underline{45}\rangle = |(\bar{15}; 3, 3; \underline{15}') (\bar{3}, 2; \underline{3}) 1, 2; \underline{45}\rangle
\end{aligned}$$

Among these states the colour magnetic interaction (3.2) is diagonal for $|I\rangle$, $|IV\rangle$, $|V\rangle$, $|VI\rangle$, $|XI\rangle$, and $|XII\rangle$ with eigenvalues $-2^{0/3} c$, $^{4/3} c$, $-24 c$, $^{40/3} c$, $^{40/3} c$ and $^{88/3} c$. It is to be noted that we have very strong attraction in the state $|V\rangle$ that has $J^P = \frac{1}{2}^-$ and contains a flavour octet and an antidecuplet. This opens the possibility that we can expect fairly low lying exotic states with $S = +1$, $I = 0$ and $S = -2$, $I = \frac{3}{2}$.

The other states are mixed by the colour magnetic interaction -- a straightforward calculation gives the following eigenstates and eigenvalues: For the flavour nonet spin $\frac{1}{2}^-$

$$\begin{aligned}
|II'\rangle &= \cos \varphi |II\rangle - \sin \varphi |III\rangle & -6,015 c \\
|III'\rangle &= \cos \varphi |III\rangle + \sin \varphi |II\rangle & -36,65 c
\end{aligned} \tag{5.6}$$

with eigenvalue

and mixing angle $\varphi = -4.72^\circ$. For the flavour 45 spin $\frac{3}{2}^-$

$$\begin{aligned}
|VII'\rangle &= \cos \psi |VII\rangle - \sin \psi |IX\rangle & 10,91 c \\
|IX'\rangle &= \cos \psi |IX\rangle + \sin \psi |VII\rangle & -9,58 c
\end{aligned} \tag{5.7}$$

with eigenvalue

and mixing angle $\psi = -9,1^\circ$. Finally for the flavour 45 spin $\frac{1}{2}^-$ we find

$$\begin{aligned} |\overline{\text{VIII}}'\rangle &= \cos \varepsilon |\overline{\text{VIII}}\rangle - \sin \varepsilon |X\rangle \\ |X'\rangle &= \cos \varepsilon |X\rangle + \sin \varepsilon |\overline{\text{VIII}}\rangle \end{aligned} \quad \begin{array}{l} \text{with eigenvalue } 12,55c \text{ (5.8)} \\ -7,22c \end{array}$$

and mixing angle $\varepsilon = -6,08^\circ$.

Is it possible to make any correspondence between these multiquark s-wave states and observed negative parity resonance states in meson-baryon systems? If we try we cannot avoid conflict with the particle assignments of the canonical three-quark model where negative parity states are orbital excited states of the three quarks. The situation is very similar to the problem of whether the 0^+ mesons should be p-wave quark-antiquark excitations or s-wave states¹⁴⁾ of two quarks and two antiquarks²¹⁾.

We can however use experimentally determined and well-known states to put lower limits on the mass of crypto-exotic $4q\bar{q}$ states and from that obtain lower limits both on exotic states and highly excited states bound with colour sextet and octet bonds.

Before making a comparison with experiment we ask, What would we naively expect for the masses of $4q\bar{q}$ states? By far the greatest binding energy due to the hyperfine colour magnetic splitting occurs for the states $|\text{III}'\rangle$ corresponding to a crypto-exotic nonet and for the states $|V\rangle$ which contain a crypto-exotic flavour octet and an exotic antidecuplet. Even if all these states contain at least one strange quark, their masses will be lower than states with only non-strange quarks: this is again an example of the "flavour antisymmetry" rule for masses^{14,22)}.

To estimate the (mean) value of c for states containing one strange quark we use the mass difference (16 c) between $\Sigma^*(1385)$ and the mean of the masses of $\Sigma(1190)$ and $\Lambda(1115)$ and find $c \approx 15$ MeV.

The effective mass of a non-strange quark in a baryon is one sixth of the mass of $\Delta(1236)$ and of the nucleon giving $\bar{m} \approx 360$ MeV, whereas the mass of the strange quark is around 175 MeV $\equiv \Delta m$ higher⁹⁾.

The mean mass $\bar{M}$ of the $I = 1$ (Σ') member of the lowest lying $J^P = \frac{1}{2}^-$ nonet and its lightest $I = 0$ member (Λ') should be

$$\bar{M} = 5 \bar{m} + \Delta m - 36,65c \approx 1440 \text{ MeV.} \quad (5.9)$$

Assuming the mass difference of Σ' and Λ' to be 75 MeV as in the $J^P = \frac{1}{2}^+$ octet we would find a mass of Λ' of 1403 MeV²³⁾. There is indeed an observed ($\frac{1}{2}^-$) state at this mass but no corresponding $\Sigma(\frac{1}{2}^-)$ at 1480 MeV.

Moreover this would give for the exotic $I = 0$, $Y = 2$, $J^P = \frac{1}{2}^-$ member of the antidecuplet in states $|V\rangle M[Z^*(I = 0)] = 5 \bar{m} + \Delta m - 24 c \approx 1615$ MeV. There definitely is no $Z^*(I = 0)$ state at such a low mass.

If we want the ground state of $4q\bar{l}q$ to show up as resonant states we would have to reject these mass estimates; if on the other hand s-wave states are not resonant at all, nothing prevents us from using these masses formally to determine the intercept of the trajectories for L excitations. In the following section we shall make further studies of the possible masses for $qqqq\bar{q}$ states.

6. MASS ESTIMATES AND REGGE TRAJECTORIES

We now discuss the possible masses for five-quark states in the two cases where s-waves are resonant and where they are not.

It goes without saying that the nicest picture of five-quarks would emerge if the ground states would correspond to experimentally identified states. An obvious choice would then be to assign the $J^P = \frac{1}{2}^- \Lambda'$ and Σ' states at 1670 and 1750 MeV ²⁴⁾ to the lowest lying crypto-exotic nonet. This would then put the lowest lying (wide) exotic $J^P = \frac{1}{2}^- Z^*(I=0)$ at around 1900 MeV. At this time, most people would doubt that there can be an exotic $J^P = \frac{1}{2}^-$ at such a low mass although phase-shift analyses (which are very difficult in the KN system) have shown that a broad state of this type is not excluded altogether ²⁵⁾. Everybody will however agree (we hope) that in the case when the over-all s-wave of the five quarks corresponds to resonances our above assignment will give us reasonable lower limits on the mass of five-quark states.

With the calculated values of the hypersplitting then, we can calculate the mass of s-wave states and of all L excitations of the two spinning clusters corresponding to colour octet bonds by

$$L_8 = -2,6 + \frac{2}{3} \alpha' M_8^2 \quad (6.1)$$

and colour sextet bonds by

$$L_6 = -2,57 + \sqrt{\frac{2}{5}} \alpha' M_6^2 \quad (6.2)$$

where α' is the normal slope of Regge trajectories in ordinary mesons and hadrons coupled with colour triplet bonds, i.e. $\alpha' \approx 0.9 \text{ (GeV)}^{-2}$.

The Regge trajectory gives the "central mass" of an L excitation and the hyperfine interaction partially splits the mass degeneracy of different flavour multiplets. A further splitting inside each flavour multiplet is given by counting the number of strange quarks, adding 175 MeV for each.

As an example of our method of mass estimations let us take the lowest lying flavour multiplet that contains a real exotic: the antidecuplet with maximum angular momentum $J = L + \frac{1}{2}$.

The value of the hyperfine splitting when $L = 0$ is -24 c, for high L (and here we also use the same value for $L = 1$) it is zero for the colour octet bond states $D^9(8,1) \Theta^8(8,2)$. Before correcting for the strange quark content the mass of the antidecuplet will be

$$\sqrt{2,6 \cdot \frac{3}{2\alpha'}} - 24 \text{ c} \approx 1,72 \text{ GeV} \quad \text{for } L = 0$$

and

$$\sqrt{(2,6 + L) \cdot \frac{3}{2\alpha'}} \quad \text{for } L > 0.$$

In Fig. 1 we show the corresponding Regge trajectory for the $Y = 2, I = 0$ exotic whose quark content is $(ud-du)(ud-du)\bar{s}$.

It is tempting to speculate that the very strong additional attraction in the over-all s-state will induce a curvature of the Regge trajectory such that the exchange of Z^* ($I = 0$) baryon trajectory could be represented by a trajectory whose intercept would be much higher than a straight line extrapolation of high-mass states to negative mass squared would imply. Figure 1 shows this.

The non-existence of any K^-n backward scattering data makes it impossible to test such a speculation against observation, but it might be a spectacular result if experiment shows an energy dependence for this particular exotic exchange comparable to what is observed in nucleon exchange.

It should be noted that the situation discussed here, with curvature of the trajectory, is of the same kind as the expected curvature of the lowest lying nonet in the $qq\bar{q}\bar{q}$ sector^{8,10)}. Nicolescu²⁹⁾ has wondered if the effective ρ' trajectory used to explain pion-nucleon charge-exchange data is a baryonium trajectory of this kind.

Our approach is not in conflict with data on reactions where $Z^*(I = 1)$ exchanges should be of relevance, as in $K^+p \rightarrow pK^+$, as one can show that all reasonable trajectories one can draw through the $Z^*(I = 1)$ states have intercepts α_0 below -2 . The reason is that this exotic is in the flavour 27 representation and the dominant trajectory is the one corresponding to L excitations of the state $D^9(8,3) \Theta^8(8,4)$ in (3.5). Its s-wave state is the state $|VI$ of (4.4) where the hyperfine splitting gives a repulsion of $4/3$ c. As the hyperfine splitting for large L is less repulsive ($4/3$ c), the leading Regge trajectory for $Y = 2, I = 1$ states will be as depicted in Fig. 2. It has the opposite curvature to the $Z^*(I = 0)$ trajectory and although it is higher for positive M^2 , it has an extrapolated intercept at $M^2 = 0$

that for any sensible extrapolation would fall below $J = -2$. It is therefore lower than the K^*N cut in the exchange region. Its shape is shown on Fig. 2.

As our model does not seem to contradict known facts either in exchanges or in spectroscopy, we now examine where we could find the lightest narrow five-quark states. It is then necessary that the angular momentum between the quark clusters is at least one. From (3.5) it is obvious that the lightest states will belong to crypto-exotic nonets with excitations of $D^9(8,3)$ $\Theta^1(8,2)$ which has, in our approximations for the forces, degenerate $J = L + \frac{3}{2}$ and $J = L + \frac{1}{2}$ states. The states Λ' and Σ' are the lightest, they contain only one strange quark and their masses M_1 for $L = 1$ would [from (6.1) and (3.5)] be given by

$$M_1 = \sqrt{3,6 \cdot \frac{3}{2\alpha'}} - \frac{44}{3} c + \Delta m \approx 2.4 \text{ GeV}$$

and for $L = 2$

$$M_2 \approx 2.7 \text{ GeV.}$$

There are two narrow states reported, decaying into $\Lambda + n\pi$ above 2 GeV, one with mass 2.13 GeV ²⁶⁾ decaying into $\Lambda\pi^+\pi^-$ which could be a Λ' and one at 2.26 GeV ²⁷⁾ decaying into $\Lambda\pi^+\pi^+\pi^-$ which could be a Σ' . Our mass estimates are sufficiently rough to admit 0.2 GeV errors and if these states are on the trajectory of $D^9(8,3)$ $\Theta^1(8,2)$ we would predict their spin to be $\frac{3}{2}$ or $\frac{5}{2}$.

As these two states are the only ones we know which could possibly be mesobaryon states, we could use them to normalize our Regge trajectories. Instead of (6.1) we would then get

$$L_g \approx -1,94 + 0,6 M_L^2 \quad (6.3)$$

This would place the Λ' and Σ' $J^P = \frac{1}{2}^-$ states (with all quarks in an s-state) at a mass of 1.44 GeV and would be consistent with our first guess for the masses in (5.10). It appears that if Eq. (6.3) does describe the trajectory we would -- as explained at the end of Section 5 -- have to abandon any correspondence between most of the ground states of the five-quark system and accepted resonances. It would be nice if the 2.13 and 2.26 GeV states would be confirmed by future experiments. We would then believe that their interpretation as five-quark states was feasible. We shall come back to this later in our discussion of possible production mechanisms of narrow five-quark states.

While the colour octet bonded states are expected to decay into one baryon and many mesons, their isomers, which are the colour sextet bonded states, can have an additional type of decay, which provides a clear signature for their existence. This is the decay into a baryon and an M-diquonium state^{8,12)}. This M-diquonium state will (eventually after cascading to a lower excitation) decay into $B\bar{B}$ through mixing with T-diquonium. The spectroscopy of the colour sextet bonded states has been studied by Fukugita, Konishi and Hansson¹²⁾ who call these states pseudobaryons. We shall therefore not repeat their discussion here, but just note a relatively trivial (although experimentally important) difference in our attitude to the mass scale. They use a theoretical mass formula (which may be approximately correct) and deduce a normalizing trajectory for L excitations

$$L = -3,5 + 0,57 M_L^2$$

instead of our (6.2).

As we have been able to solve the over-all s-wave problem we have tried to find experimental states to guide us in the normalization of trajectories. The consequence of this is that (for relatively low L excitations) we find states around 400 MeV lower than predicted by Ref. 12. If the Han-Nambu type²⁰⁾ of mass formula is correct, the masses of all states we have considered for octet bonds would be 400 MeV greater than we have guessed, making their identification more difficult.

7. PRODUCTION MECHANISMS FOR FIVE-QUARK STATES

This section will be very short because most of what we say has been said before. The prominent exchanges that can lead to five-quark states being produced in the forward direction relative to an incoming baryon will be either diquonium ($qq\bar{q}\bar{q}$) exchanges⁶⁾ or the exchanges of ordinary Regge poles -- pomeron included^{8,12)}. The coupling of an ordinary Regge pole to the baryon-mesobaryon vertex is however suppressed with respect to its coupling to a baryon-baryon vertex. This is because the mixing of $q\bar{q}$ and $qq\bar{q}\bar{q}$ through dual unitarization effects (at negative mass squared) is rather small. As this mixing exists even in the pomeron, it opens the possibility that meso- and pseudobaryons can be diffractively produced.

With 20% of $qq\bar{q}\bar{q}$ states in the pomeron²⁸⁾ the diffractive production of mesobaryons in proton-proton collisions should be around 5% of the production of three-quark states. If we take the cross-section for the diffractively produced N(2190)

of around 50 μb as an example, we would expect 1-2 μb as the total production cross-section of a mesobaryon. It is a pity that such a cross-section is not much greater than that expected for charmed baryon production; this will allow a greater possibility of confusion.

There is one possibility for an enhancement of the production of some particular states however. Let us imagine that we have diffractive production of five-quark states with a cross-section of one microbarn for each. Now as these decay (by cascade) many different states $|k\rangle$ may have a common decay product which is a five-quark state $|X\rangle$ with rather low excitation. The total production cross-section will then be the sum $\sum_k \sigma_k \cdot B_{k \rightarrow X}$ where σ_k is $\approx 1 \mu\text{b}$ and $B_{k \rightarrow X}$ is the branching ratio for $|k\rangle \rightarrow |X\rangle + \text{anything}$. Production can then be big.

This might seem far-fetched but unless our understanding of production is too wrong it seems to us the only explanation permitting us to understand production of narrow states with cross-section of around 10 μb at ISR energies. These states are the ones we have mentioned before: they have strangeness -1 and masses 2.13 and 2.26 GeV.

If they are five-quark states they must be decay products of mother states with $S = 0$ (and containing $s\bar{s}$ pairs) which are diffractively produced and then emit at least one K meson. In an eventual repetition of the experiment we would like to mention that it would be most interesting to know the effective mass plots of the XK system when X is the 2.13 or 2.26 GeV state. This would then eventually unravel many other five-quark states without strangeness.

8. CONCLUSIONS

In this paper we have given a systematic study of colour isomers consisting of four quarks and an antiquark. The possibility of finding such states with longer lifetimes than ordinary excited hadronic states leads us to study the configurations with colour 8 and colour 6 bonds.

Our estimated lower limits for the masses of such states are different depending on whether we assume the over-all s-wave states to be resonant or not. In the former case, we find it would not be impossible for the $\Lambda(1670)$ and $\Sigma(1750)$ to consist mainly of five quarks. In both cases narrow states found at the ISR could be interpreted as excitations of five-quark states. Although it is too early yet to have an opinion about multiquark states based solely on experimental data, we find the model of colour isomers attractive. The SU_3 colour group is essential in both its global and local versions. As a global symmetry it shows itself in favouring certain quark configurations; as a local symmetry it gives mass defects calculable from the colour magnetic interaction.

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APPENDIX

REPRESENTATIONS AND CASIMIR OPERATORS OF SU_3 AND SU_6

In this appendix we have gathered some technical group theoretical results which have been used in the course of the paper. This concerns the decomposition into irreducible representations of the product of representations of SU_3 and SU_6 (see Tables A1 and A2), and the decomposition of irreducible SU_6 representations with respect to $SU_3 \times SU_2$ (see Table A3). Table A2 can be described with the help of the results of Ref. 30 or using the classical Young diagram techniques, while for Table A3, we can use the results of Ref. 31.

In particular, the knowledge of these decompositions simplifies the computing, in each interesting case, of the colour magnetic interaction potential proportional to¹⁴⁾:

$$\begin{aligned} - \sum_a \sum_{i,j} \lambda_i^a \lambda_j^a \underline{\sigma}_i \underline{\sigma}_j = 8N + \frac{1}{2} C_6(Tot) - \frac{4}{3} S_{tot}(S_{tot}+1) \\ - \frac{1}{2} C_3(tot) + C_3(Q) + \frac{8}{3} S_Q(S_Q+1) - C_6(Q) \\ + C_3(\bar{Q}) + \frac{8}{3} S_{\bar{Q}}(S_{\bar{Q}}+1) - C_6(\bar{Q}) \end{aligned} \quad (A.1)$$

where N is the total number of quarks and antiquarks (here: $N = 5$), C_6 , (C_3) the values of the second-degree Casimir operator of SU_6 (SU_3) corresponding to the SU_6 colour spin (SU_3 colour) representations under consideration, while S stands for the spin value.

The useful values of $C_6(R)$ and $C_3(R)$ have been computed from the formulas¹⁴⁾:

$$C_6(R) = \frac{280}{9N_R} \sum_j d_j (2s_j+1) s_j (s_j+1) \quad (A.2)$$

where the sum on j covers all $SU_3 \times SU_2$ representations (d_j, s_j) contained in the representation R of dimension N_R , d_j representing the dimension of SU_3 representation and s_j the spin value in the jth pair, and

$$C_3(R) = \frac{32}{3N_R} \sum_k (2I_k+1) I_k (I_k+1) \quad (A.3)$$

where the sum covers all the isospin multiplets in the R representation of SU_3 .

The relevant values of $C_3(R)$ and $C_6(R)$ are listed in Table A4.

Table A1

Multiplication of representations of SU_3 ,
with their corresponding Young tableaux

$$\begin{aligned}
 3 \times 3 &= \bar{3} + 6 \\
 (1) \quad (1) \quad (1^2) \quad (2) \\
 3 \times \bar{3} &= 1 + 8 \\
 (1) \quad (1^2) \quad (0) \quad (2,1) \\
 3 \times 8 &= 3 + \bar{6} + 15 \\
 (1) \quad (2,1) \quad (1) \quad (2^2) \quad (3,1) \\
 6 \times 3 &= 8 + 10 \\
 (2) \quad (1) \quad (2,1) \quad (3) \\
 6 \times \bar{3} &= 3 + 15 \\
 (2) \quad (1^2) \quad (1) \quad (3,1) \\
 6 \times 6 &= \bar{6} + 15 + 15' \\
 (2) \quad (2) \quad (2^2) \quad (3,1) \quad (4) \\
 6 \times \bar{6} &= 1 + 8 + 27 \\
 (2) \quad (2^2) \quad (0) \quad (2,1) \quad (4,2) \\
 6 \times 8 &= \bar{3} + 6 + \bar{15} + 24 \\
 (2) \quad (2,1) \quad (1^2) \quad (2) \quad (3,2) \quad (4,1) \\
 8 \times 8 &= 1 + 8 + 8 + 10 + \bar{10} + 27 \\
 (2,1) \quad (2,1) \quad (0) \quad (2,1) \quad (2,1) \quad (3) \quad (3^2) \quad (4,2) \\
 10 \times 3 &= 15 + 15' \\
 (3) \quad (1) \quad (3,1) \quad (4) \\
 10 \times \bar{3} &= 6 + 24 \\
 (3) \quad (1^2) \quad (2) \quad (4,1) \\
 10 \times 6 &= \bar{15} + 21 + 24 \\
 (3) \quad (2) \quad (3,2) \quad (5) \quad (4,1) \\
 10 \times \bar{6} &= 3 + 15 + 42 \\
 (3) \quad (2^2) \quad (1) \quad (3,1) \quad (5,2) \\
 10 \times 8 &= 8 + 10 + 27 + 35 \\
 (3) \quad (2,1) \quad (2,1) \quad (3) \quad (4,2) \quad (5,1) \\
 10 \times 10 &= \bar{10} + 27 + 28 + 35 \\
 (3) \quad (3) \quad (3^2) \quad (4,2) \quad (6) \quad (5,1) \\
 10 \times \bar{10} &= 1 + 8 + 27 + 64 \\
 (3) \quad (3^2) \quad (0) \quad (2,1) \quad (4,2) \quad (6,3)
 \end{aligned}$$

Table A2

Multiplication of representations of SU_6 ,
with their corresponding Young tableaux

6×6	$=$	21	$+$	15
(1) (1)		(2)		(1 ²)
$6 \times \bar{6}$	$=$	35	$+$	1
(1) (1 ⁵)		(21 ⁴)		(1)
6×15	$=$	20	$+$	70
(1) (1 ²)		(1 ³)		(21)
6×20	$=$	$\bar{15}$	$+$	$105'$
(1) (1 ³)		(1 ⁴)		(21 ²)
6×21	$=$	56	$+$	70
(1) (2)		(3)		(21)
6×56	$=$	210	$+$	126
(1) (3)		(31)		(4)
6×70	$=$	210	$+$	$105 + 105'$
(1) (21)		(31)		(2 ²) (21 ²)
$\bar{6} \times 15$	$=$	6	$+$	$\bar{84}$
(1 ⁵) (1 ²)		(1)		(2 ² 1 ³)
$\bar{6} \times 21$	$=$	6	$+$	120
(1 ⁵) (2)		(1)		(31 ⁴)
$\bar{6} \times 105$	$=$	560	$+$	70
(1 ⁵) (2 ²)		(3 ² 1 ³)		(21)
$\bar{6} \times 105'$	$=$	540	$+$	$70 + 20$
(1 ⁵) (21 ²)		(32 ² 1 ²)		(21) (1 ³)
$\bar{6} \times 126$	$=$	56	$+$	700
(1 ⁵) (4)		(3)		(51 ⁴)
$\bar{6} \times 210$	$=$	56	$+$	$70 + 1134$
(1 ⁵) (31)		(3)		(21) (421 ³)
35×56	$=$	1134	$+$	$700 + 70 + 56$
(21 ⁴) (3)		(421 ³)		(51 ⁴) (21) (3)
35×70	$=$	1134	$+$	$56 + 560 + 2(70) + 540 + 20$
(21 ⁴) (21)		(421 ³)		(3) (3 ² 1 ³) (21) (32 ² 1 ²) (1 ³)

Table A3

Reduction of SU_6 representation with respect to $SU_2 \times SU_3$

6	=	(2,3)
15	=	(1,6) + (3, $\bar{3}$)
20	=	(2,8) + (4,1)
21	=	(3,6) + (1, $\bar{3}$)
35	=	(1 + 3, 8) + (1,3)
56	=	(2,8) + (4,10)
84	=	(2,15) + (2 + 4, 3 + $\bar{6}$)
105	=	(1,15') + (3,15) + (5, $\bar{6}$) + (1, $\bar{6}$) + (3,3)
105'	=	(3,15) + (1,15) + (3, $\bar{6}$) + (5,3) + (3,3) + (1,3)
120	=	(2 + 4, 3 + 15) + (2, $\bar{6}$)
126	=	(5,15') + (3,15) + (1, $\bar{6}$)
210	=	(3,15') + (5,15) + (3,15) + (1,15) + (3, $\bar{6}$) + (3,3) + (1,3)
540	=	(2 + 2 + 4, 27) + (2 + 4, 10 + $\bar{10}$ + 8) + (2 + 2 + 4 + 4 + 6, 8) + (2 + 4 + 6, 1)
560	=	(2,35) + (2 + 4, 27 + 10) + (2 + 4 + 6, 8 + $\bar{10}$) + (2 + 4, 8) + (4,1)
700	=	(4 + 6, 35) + (6,10) + (2, $\bar{10}$) + (2 + 4, 27 + 10 + 8)
1134	=	(2 + 4, 35) + (2 + 4 + 6, 27 + 10) + (2 + 4, 27 + 10) + (4 + 6, 8) + (2 + 4, 1 + 8 + $\bar{10}$) + (2 + 4, 8) + (2,8)

Table A4

Values of the Casimir $C_6(R)$ and $C_3(R)$

$C_6(6)$	$=$	$\frac{70}{3}$	$C_3(3)$	$=$	$\frac{16}{3}$
$C_6(15)$	$=$	$\frac{112}{3}$	$C_3(6)$	$=$	$\frac{40}{3}$
$C_6(20)$	$=$	42	$C_3(8)$	$=$	12
$C_6(21)$	$=$	$\frac{160}{3}$	$C_3(10)$	$=$	24
$C_6(56)$	$=$	90	$C_3(15)$	$=$	$\frac{64}{3}$
$C_6(70)$	$=$	66	$C_3(15')$	$=$	$\frac{112}{3}$
$C_6(84)$	$=$	$\frac{190}{3}$	$C_3(27)$	$=$	32
$C_6(105)$	$=$	$\frac{256}{3}$	$C_3(35)$	$=$	48
$C_6(105')$	$=$	$\frac{208}{3}$			
$C_6(120)$	$=$	$\frac{238}{3}$			
$C_6(126)$	$=$	$\frac{400}{3}$			
$C_6(210)$	$=$	$\frac{304}{3}$			
$C_6(540)$	$=$	98			
$C_6(560)$	$=$	114			
$C_6(700)$	$=$	162			
$C_6(1134)$	$=$	130			

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Figure captions

Fig. 1 : The shape of the $Z^*(I = 0)$ trajectory.

Fig. 2 : The shape of the $Z^*(I = 1)$ trajectory of the lowest lying 27-plet.

FIG.1

FIG.2