Ref.TH.2549-CERN

SUPERCritical FIELDS AND DYNAMIC CONFINEMENT

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ABSTRACT

The mechanism leading to the dynamic confinement of heavy Fermions for strong coupling constants in quantum electrodynamics of two charged light and heavy Fermion fields is described

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21 August 1978

The quark confinement puzzle has been with us for the last decade and still no accepted solution to this problem exists. While there is a good candidate, QCD, for a theory of quarks and gluons, we do not know how to resolve the confinement problem within this framework. Unlike in QED we have to face several complications simultaneously: the non-perturbative treatment of relatively strongly coupled non-Abelian gauge theory. Therefore we regress here to study the relevant aspects of an Abelian gauge theory in the limit of strong coupling. Clearly this theory must be QED, but with an "unphysical" value of the coupling constant α .

Let us consider the quantum field theory of two charged Fermi fields, hence also called μ and e , e being much lighter than μ :

$$m_\mu / m_e \gg 1 \quad (1)$$

Our objective is to show that when the coupling e^2 becomes large

$$\alpha = e^2 / 4\pi \gtrsim 1 \quad (2)$$

it is impossible to create a free heavy Fermion. This will be the case when "muonium", the bound state of (μe) , and a free e are energetically a more favourable state than the free μ particle^{*)}.

It is the condition (1) which allows us to find a simple approximate solution to the difficult mathematical problem posed by Eq. (2); therefore we cannot derive similar results for the light Fermion confinement.

In the limit indicated by Eq. (1), up to corrections of the order of $(m_e/m_\mu)^2$, the transition amplitude

$$\psi_e = \langle \mu e | \hat{\Psi}_e | \mu \rangle \quad (3)$$

of the light Fermion field $\hat{\Psi}_e$ from the state consisting of one heavy Fermion to the (possibly bound) state of μ and e satisfies the Dirac equation ($\bar{m}_e$ is the reduced mass of the light Fermion):

^{*)} All that follows now is a (trivial) elaboration of this observation. From this point of view confinement seems to be more the rule than an exception in strongly coupled theories; confinement $\hat{=}$ absence of Lagrangian field particles from asymptotic states.

$$[\gamma \cdot (p - eA) - \bar{m}_e] \psi_e = 0 \quad (4)$$

Here "A" is the electromagnetic potential generated by the heavy Fermion:

$$\partial_\nu F^{\nu\alpha} = \langle \mu | e \hat{j}^\alpha | \mu \rangle \quad (5.a)$$

$$F^{\nu\alpha} = \partial^\nu A^\alpha - \partial^\alpha A^\nu \quad (5.b)$$

$$\hat{j}^\alpha = -\frac{e}{2} [\hat{\bar{\Psi}}_\mu, \gamma^\alpha \hat{\Psi}_\mu] - \frac{e}{2} [\hat{\bar{\Psi}}_e, \gamma^\alpha \hat{\Psi}_e] \quad (5.c)$$

Let us approximate a solution of Eq. (5) by the Coulomb potential

$$eA_\alpha = \delta_{\alpha 0} V \quad (6.a)$$

$$V = \begin{cases} -e^2/4\pi r & r > a \\ -e^2/4\pi a & r \leq a \end{cases}, \quad r = |\vec{x}| \quad (6.b)$$

with a suitable cut-off radius

$$a \ll m_e^{-1} \quad (7)$$

We will justify this ansatz further below.

Now we turn to the description of solutions of Eq. (4), of which the details can already be found in the literature¹⁾. In particular, we consider the stationary ansatz

$$\psi_e(\vec{x}, t) = e^{-i\epsilon t} \psi'_e(\vec{x}) \quad (8)$$

The lowest discrete eigenvalue ϵ is shown schematically in Fig. (1). As indicated there, for α not much larger than 1 and called the critical strength α_{cr} , the eigenvalue disappears from the discrete spectrum and becomes a

resonance in the continuum with $\epsilon < -m$ ^{*)}. Beyond this point, a naked μ will have higher energy than the state of a bound muonium (μ^+e^-) and a free e^+ . As was shown in Ref. 1), the associated decay time is governed by the width of the resonance in the continuum and is in general a very small fraction of m_e^{-1} when we are not too close to the critical coupling constant. Thus if $\alpha > \alpha_{cr}$ a naked heavy Fermion would quickly be neutralised by spontaneous production of a light lepton pair.

Since the heavy leptons would be produced in pairs, the self-screening described above must be now generalised to the case of an initial state consisting of a heavy Fermion pair. Let us first assume that the relative velocity v_r of the $\mu^+\mu^-$ pair in its centre of mass system is small as compared to the velocity of light c . Then the previous considerations are fully applicable to this case, with some changes in the potential V . It must now be that of two charges fixed at a distance $2R$. Thus, instead of Eq. (6.b) we now have

$$V = \frac{\alpha}{|\vec{x} - \vec{R}|} f_- - \frac{\alpha}{|\vec{x} + \vec{R}|} f_+ \quad (9)$$

where $f_{\pm}$ are the suitable cut-off factors which ensure that when $|\vec{x} \pm \vec{R}| < a$ the singularity of Eq. (9) is cut off in a similar fashion as in Eq. (6.b). A cut of the two-centre potential (9) along the vector $\vec{R}$ and going through the centre of mass is shown schematically in Fig. 2. As indicated there we can have simultaneously e^+ and e^- bound mainly to μ^- and, respectively, μ^+ . For a given $\alpha > \alpha_0$ (α_0 is as defined in Fig. 1, the point at which the eigenvalue ϵ descends below 0) we can draw the eigenvalue spectrum of the Dirac equation as a function of R as shown in Fig. 3. For some R , called R_{cr} , the eigenvalue of e^+ and e^- solutions join. Since the difference between the curves so denoted is the energy required to make a e^+ , e^- pair bound to μ^- , μ^+ ; beyond R_{cr} the naked μ^+ , μ^- state is unstable and makes a spontaneous transition to the state with a muonium-antimuonium pair. Thus as the distance between a pair of heavy Fermions increases, there is a point beyond which a light Fermion

*) In the limit that "a", the cut-off radius, is very small α_{cr} approaches 1 and this eigenvalue dives more steeply into the continuum. However, it is essential that $a \neq 0$, otherwise the bound state cannot descend below $\epsilon = 0$ as a function of α .

pair can be created^{*)} in order to screen the charges of the heavy Fermion field. Thus in the adiabatic approximation, when the heavy Fermions move sufficiently slowly, only muonium is seen asymptotically. Even when the relative momentum of the μ^+ , μ^- pair is large, the neutralisation will proceed more likely without creation of free light Fermions. However, it can then occur before or after R_{cr} , the e pair energy being shared with the kinetic energy of the μ pair. Still, there is always a (very) small chance that the μ 's will separate before being neutralised. Then the process described in the first part of this paper leads to the required neutralisation. The fate of the free light Fermions is unclear at the present time, we will return to this point further below. Thus, although ultimately confined, in the initial stage of the creation process a heavy Fermion pair is present, quite in the manner required by recent experiments³⁾.

It remains to justify the validity of Eqs. (4,6,9). In the limit given in Eq. (1), it has been shown⁴⁾ that the Dirac equation (4) is the proper limit of the Bethe Salpeter equation when the kernel includes the ladder and crossed ladder diagram. However, when $\alpha \geq 1$ we must also consider further contributions. This can be done more easily taking as the starting point the basis provided by the solutions of the Dirac equation. In particular, we have to consider the vertex corrections of the light Fermion as well as its self-energy. Scaling the results of QED we conclude that these contributions cannot alter the qualitative character of our discussion. Clearly, all quantitative work demands extensive computation of all these contributions. This is the reason why at present we can only estimate: $\alpha_{cr} \sim 1$, $R_{cr} \sim m_e^{-1}$. Since we take the heavy Fermion as the source of a static Coulomb field, we had to simulate the relevant effect of recoil, e field vacuum polarisation as well as vertex corrections introducing the cut-off radius "a" in Eq. (6). Both recoil and vertex corrections tempt to smear out the point-like character of the source. We note that in QED (unlike QCD) the singularity of a point-like source increases when vacuum polarisation is considered, however for already smeared-out source it only leads to further smearing.

Let us return now to the discussion of the fate of the light Fermions. In the spirit of our discussion they would be confined if the bound particle-anti-particle state would lie below the empty vacuum state. This seems to be the case for $\alpha > 3/2$ in scalar electrodynamics⁵⁾. However, to solve the confinement

*) It is possible to view the pair creation process as if originating in the strong constant electric fields: an expression non-analytic at $\alpha = 0$ has been first given by Schwinger²⁾.

problem here one has to construct a new global vacuum state which would provide a lower bound for the Hamiltonian. In this language our muonium state is the local ground state in the presence of a heavy Fermion.

We hope that the dynamic confinement presented here is in some sense an analog of the behaviour of QCD, with the exception that QCD may be supercritical already for quite small coupling constants. While the instability of QED vacuum appears for $\alpha_{cr} > 1$, the work on effective Lagrangians⁶⁾ in the presence of Yang-Mills fields indicates an instability already at $\alpha_{QCD} = 0$, but the work on classical solutions of Yang-Mills equations with fixed sources indicates an instability at $\alpha_{cr} \approx 3/2$ ⁷⁾.

Finally, we emphasize the difference in our concept of dynamic confinement and the static confinement, where a steeply rising "confining" potential is considered. It is not clear at present how the connection between these two descriptions can be established, in particular also in view of the phenomenological successes of the bag models (for light quarks) and the Schroedinger equation with the confining potentials (for heavy quarks).

In summary, we have presented a model of dynamic confinement following from the possible supercritical coupling of charged particles. Taking a heavy Fermion as a source of the electromagnetic field, we have considered the fate of this state as the strength of the electric charge is increased. We have found that beyond a certain critical value α_{cr} a heavy Fermion can decay into a bound "onium" state of heavy and light Fermion and a free light Fermion. Furthermore, we show that when a heavy Fermion pair is created, it will soon become a pair of confined heavy Fermions (and possibly, some other stuff), spontaneous decay being possible beyond a certain distance of separation.

I would like to thank V. Weisskopf for encouragement and D. Horn for a fruitful discussion.

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FIGURE CAPTIONS

- Figure 1 : Lowest bound state of the Dirac equation for a cut-off Coulomb potential as a function of α . At α_0 the binding equals m_e and at α_{cr} it is $2m_e$; beyond the naked muon becomes unstable and is degenerate with muonium and a free positron.
- Figure 2 : Two-centre potential cut along the axis going through the centre of mass and connecting a heavy particle-antiparticle pair separated by the distance $2R$.
- Figure 3 : Lowest bound states for particles and antiparticles of the Dirac equation in the potential of a heavy particle-antiparticle pair. At R_{cr} the spontaneous neutralisation of the heavy particles begins.

FIG.1

FIG. 2

FIG.3