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SPECIAL SHELL STRUCTURE FOR NUCLEI AT HIGH ANGULAR MOMENTA

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A B S T R A C T

The strong shell effects observed experimentally and theoretically at high spins and moderate deformations are explained in terms of the interference between two effects. Thus, the effect of the quadrupole deformation to destroy the spherical shell degeneracy can be largely compensated by the orbital separation due to rotational motion. In the resulting coupling scheme, the subshell degeneracy of the spherical pure oscillator is partly regained at a finite deformation and still stronger shell effects are predicted at very high and today unobserved spins.

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Some different regions of nuclei with long-lived high-spin isomers have recently been identified [1-4]. Such "yrast traps" are generally expected for axially symmetric nuclei where the high spin is formed by a few particles being aligned along the symmetry axis (nuclei rotating around a symmetry axis) [5]. A generalised macroscopic-microscopic method to treat nuclear high spin states in the yrast region has recently been worked out [6,7]. The low energy of the aligned configurations is then the combined effect of a low macroscopic energy and a low shell energy. As the macroscopic energy is straight-forward to estimate and has a smooth particle number dependence, it emerges as an interesting problem to study systematic effects in the shell energy.

The shell energy is a function of the distribution of single-particle states around the Fermi surface. It is obtained from a diagonalisation of the cranked single-particle Hamiltonian, $h^\omega = h^0 - \omega j_x$, the corresponding eigenvalues being denoted by e_i^ω [6]. Subsequently, the energies in the laboratory system e_i are calculated as expectation values of the static single-particle Hamiltonian h^0 and the spin values m_i as expectation values of j_x . The single-particle sum is then given by $E_{sp} = \sum e_i$ and the total spin by $I = \sum m_i$. Following ref. [6], the shell energy is defined by

$$E_{\text{shell}}(I) = E_{sp}(I) - \langle E_{sp} \rangle(I) \quad (1)$$

i.e. as the difference between the Strutinsky smoothed and **discrete** single-particle sums, both quantities calculated for the same I -value.

A numerically similar definition of the shell energy is [8]

$$E_{\text{quasi-shell}}(\omega) = \sum e_i^\omega(\omega) - \langle \sum e_i^\omega(\omega) \rangle \quad (2)$$

where the different "sums" are evaluated for the same rotational frequency, ω . This latter definition is a direct measure of the level density (of the e_i^ω 's) at the Fermi surface. It is thus more straight-

forward to use when systematic effects of the shell energy as a function of deformation, spin and particle number are studied. Thus, we will use this definition below and simply refer to it as the shell energy.

Consider now a deformed potential that is cranked around its symmetry axis. The projection of j on the symmetry axis, m , is then a good quantum number and the single-particle energies in the rotating and non-rotating systems are simply related by $e_i^\omega = e_i - \hbar\omega m$. We now want to show that within each j -shell, the splitting of the different orbitals due to rotation can be approximately compensated by the splitting due to deformation. This can of course only be achieved for one half of the orbitals in the shell as the rotational splitting depends on the sign of m while the deformational does not. To get a numerical estimate, we use for $\hbar^0$ the modified oscillator potential with ϵ measuring the deformation in the usual way [8]. The pure j -shell is then split according to $e_i(\epsilon) = e_i(\epsilon=0) + \epsilon\hbar\omega_0(N+3/2)(3m^2 - j(j+1))/(6j(j+1))$. The distance between two orbitals with consecutive values of m is

$$\Delta_\epsilon = \epsilon\hbar\omega_0 \cdot \frac{N+3/2}{j+1} \cdot \frac{\langle m \rangle}{j} \quad (3)$$

where $\langle m \rangle$ is the mean m -value of the two orbitals. Furthermore, with ϵ positive, the orbitals of lowest absolute m are lowest in energy. The splitting due to rotation is for the same orbitals trivially calculated as

$$\Delta_\omega = \hbar\omega \quad (4)$$

independent of $\langle m \rangle$. In this case, the orbitals of highest m come lowest in energy. The equality of Δ_ϵ and Δ_ω then leads to

$$\frac{(\omega/\omega_0)}{\epsilon} = \frac{N+3/2}{j+1} \frac{\langle m \rangle}{j} \quad (5)$$

which thus depends on the ratio of j to N and of $\langle m \rangle$ to j . Furthermore, the high- j subshells are most pure and have the highest degeneracy. The present coupling scheme is thus most interesting for such subshells. The

largest possible value of j is $N + \frac{1}{2}$ and the above ratio then comes out independent of N .

To make the orbitals approximate degenerate over the largest possible range of m , it seems reasonable to make eq. (5) exact for some intermediate value of $\langle m \rangle$, say $\langle m \rangle = 0.5j$. For $j = N + 1/2$ this then leads to $(\omega/\omega_0):e = 0.5$ while for example for $N = 4$ and $j = 7/2$, we get $(\omega/\omega_0):e = 0.61$. In addition, the couplings between the j -shells on the prolate side, where the present coupling scheme is best developed, will tend to make the distances between the low- m orbitals larger than for a pure j -shell. As a consequence, the shell structure will be maximal on the prolate side for a somewhat larger value of $(\omega/\omega_0):e$ than suggested from eq. (5) and in the numerical calculations below, we will use $(\omega/\omega_0):e = 0.75$.

In the case of $(\omega/\omega_0):e = 0.75$, the neutron single-particle levels are shown in fig. 1. The general features outlined above are very clearly exhibited. About half of the levels emerging from each subshell are closely bunched. This is especially true on the prolate side, where due to the deformational couplings between the different subshells, the orbitals are more nearly equidistant than for a pure shell. Thus, with rotation added, there will be an approximate degeneracy over a larger region of m . On the other hand, the couplings on the oblate side generally have the opposite effect.

A second important effect which is observed in fig. 1 is that the bunches of near-degenerate levels from the different j -shells intersect at a prolate deformation. For example, the $g_{7/2}$ orbitals intersect with the $h_{11/2}$ orbitals and in addition, the positive m orbitals of $d_{5/2}$ show a tendency to come closer to these bunches. With similar calculations as above for pure j -shells, we find that if the intruder shell with $N = N^*$ and $j = N^* + 1/2$ (e.g. $h_{11/2}$) is for spherical shape at a distance of d (in units of $\hbar\omega_0$) above the subshell with $N = N^* - 1$ and $j = N^* - 3/2$ (e.g. $g_{7/2}$), the bunches of orbitals from these different subshells will intersect at $e=d$ (independent of N^*). For the $h_{11/2}$ and

$g_{7/2}$ subshells, this would give an ϵ -value of 0.20 which is indeed very close to the intersection seen in fig. 1. The similar intersection between the $i_{13/2}$ and $h_{9/2}$ orbitals is also well described by $\epsilon=d$. Thus, the general features of fig. 1 with regions of low and high level density are well understood.

The shell energy diagram, based on the single-particle levels of fig. 1, is shown in fig. 2. As expected, the whole figure, covering neutron numbers 40-190, shows a very regular structure with valleys and ridges. Starting from spherical shape, the valleys bend towards smaller particle numbers on the prolate side and towards larger particle numbers on the oblate side. Let us study in some more detail the case $\epsilon = 0.20$ ($\omega/\omega_0 = 0.15$) where the shell structure discussed in this paper is fully developed. The fact that the strong shell structure depends on the interplay between deformation and rotation can easily be verified by comparing with fig. 2 of ref. [9]. From there it is evident that the shell structure is much weaker if either $\omega/\omega_0 = 0$ at $\epsilon = 0.20$ or $\epsilon = 0$ at $\omega/\omega_0 = 0.15$. Indeed, the shell structure at $\epsilon = 0.20$, $\omega/\omega_0 = 0.15$ appears to be stronger than that for any clearly deformed shape without rotation. For example, the variation in fig. 2, with $E_{\text{shell}} = 4.4, -3.2, 6.2, -5.0$ and 5.4 MeV for $N = 54, 66, 84, 100$ and 120 respectively is more pronounced than any variation observed for deformed shapes exhibiting special shell structure as shown in figs. 8 and 11 of ref. [8]. This result is somewhat surprising as, due to the two-fold time-reversal degeneracy of the orbitals, one would a priori expect stronger shell structure in the non-rotational case. The very strong shell structure in the rotational case is apparently due to the fact that the subshell degeneracy of the pure spherical oscillator is partly regained at a finite deformation.

The numbers given within the contour plot of fig. 2 refer to the total spin for the corresponding deformation (rotational frequency) and particle number. If we again concentrate at $\epsilon = 0.20$ ($\omega/\omega_0 = 0.15$) we

find that also the variation of the total spin is very regular. Thus, the total spin is close to its smooth value when E_{shell} is max. or min. while it has a maximum when the shell energy is decreasing and a minimum when the shell energy increases. These features are easily understood from fig. 1 as in the regions of low level density, the orbitals are generally up-sloping ($m < 0$) while they are down-sloping ($m > 0$) in regions of high level density. The difference ΔI between the actual value and the smooth value may be 15-20% which is considerable. As the difference term between the two shell energy definitions (eq. (1) and (2)) is proportional to $(\Delta I)^2$ [8], this correction term is of some importance. However, ΔI is close to zero for the extremal points of the shell energy and the general structure of fig. 2 would thus be unchanged even with a different shell energy definition.

From the discussion above, it is evident that for nuclei with a few particles outside closed shells the shell energy contribution will tend towards oblate shape for particle states and towards prolate shape for hole states. This has already been discussed in refs. [9,10] but the present coupling scheme, connecting deformation with rotational frequency, makes the quantitative comparison more straightforward and enables us to extend the analysis to higher spin states. Examples of experimentally observed high spin states with a few particles outside closed shells are those in the lead region [3,4] where also more detailed calculations [9,10] bear out that ^{204}Pb has a tendency to become prolate and ^{212}Rn oblate. The high spin isomers with $N \geq 82$ which were predicted in ref. [6] and which have now been experimentally observed [1] should be slightly oblate from the same arguments.

However, an even more exciting application of the present coupling scheme exists for the more deformed cases with larger ϵ -values. Of particular interest is then nuclei with $Z \approx 66-72$ and $N \approx 100-110$. Indeed, for $\epsilon = 0.20$ and $\omega/\omega_0 = 0.15$, a shell energy of around -8 MeV is obtained

for ^{168}Er (the proton shell energy landscape has a general structure very similar to that for the neutrons). The total spin, which is the sum of the proton and neutron spins, is then about 80% which is maybe too high a spin for experimental observation. Indeed, as the rotational part of the liquid drop energy increases as I^2 and favours oblate shapes, it seems, at this spin and deformation, unlikely with a real minimum in the total energy surface. On the other hand, the yrast isomers for Hf-nuclei with a maximum spin of 22% [2] represent certainly the beginning realisation of this coupling scheme and spin values of 40-60% do not seem unrealistic for prolate nuclei in this region. If not yrast isomers, such states might still be isomeric due, for example, to a secondary minimum in the potential energy surface at a high spin value (cf. fig. 14 of ref. [9]). Other combinations of Z and N with a similar prolate coupling scheme might be $Z \approx 46$ and $N \approx 70$ or both Z and N between 40 and 50.

Above it has been stressed and it is evident from the figures that the present coupling scheme is better developed on the prolate than on the oblate side. There are several reasons, for example, (i) the different j-shells are more pure on the prolate than on the oblate side, (ii) the deformational coupling between the j-shells tend to make the different m-orbitals of a specific shell more equidistant on the prolate side and less on the oblate side (iii) the ordering of the subshells with the intruder shell generally somewhat above one or two other high-j shells make the bunches of orbitals from the different j-shells converge and intersect on the prolate side while they diverge on the oblate side. On the other hand, the liquid drop energy favours oblate shapes and most yrast traps would be expected to be oblate. Indeed, $\omega/\omega_0:\epsilon = 0.75$ is probably not the optimal ratio on the oblate side as the couplings between the different j-shells tend to make the deformational splitting between orbitals of different m smaller. For a fixed ϵ , a smaller value of the rotational frequency would then be needed to compensate for this

splitting. Thus, by making the ratio $(\omega/\omega_0):\epsilon$ dependent on ϵ , the shell structure of fig. 2 could probably be even more pronounced.

As a final remark we want to add that the present coupling scheme essentially depends only on the deformational splitting of the subshells and of the ordering of the subshells for spherical shape. Consequently, the general features would certainly be the same also in for example a Woods-Saxon potential.

In summary, it has been shown that for one half of a spherical subshell, the splitting due to quadrupole deformation can to a large extent be compensated by the splitting due to rotation. This coupling scheme is applicable both for oblate and prolate nuclei rotating around their symmetry axes but is better developed on the prolate side. Indeed, the shell structure for $\epsilon = 0.20$ and $\omega/\omega_0 = 0.15$ appears to be stronger than for any deformed shape without rotation. The present coupling scheme is partly realised for near-spherical nuclei in the lead region where isomers with spin values of 20-30% have been experimentally observed. The detected high spin isomers for nuclei with $N \geq 82$ also appear to be a manifestation of the same effect. Even more exciting applications are however yrast traps at much higher spin values which might be formed for example for nuclei with $Z \approx 66-72$ and $N \approx 100-110$.

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Figure captions

- Fig. 1 Neutron single-particle levels calculated as a function of ϵ (ω/ω_0) for a deformed potential cranked around its symmetry axis. The rotational frequency is related to the quadrupole deformation through $\omega/\omega_0 = 0.75\epsilon$ (the sign of the rotational frequency is of no physical significance and has been chosen positive). The number on each orbital refers to $2m$ where m is the projection on the symmetry axis of the j quantum number. Furthermore, the numbers within rings refer to the neutron number and those within rectangles to the total spin.
- Fig. 2 The shell energy landscape for neutrons calculated for the same ratio of $(\omega/\omega_0):\epsilon$ as in fig. 1. The thick lines separate regions of positive and negative shell energy and in shaded regions the shell energy is below -2 MeV. The numbers within the contour map refer to the total spin for the corresponding deformation (rotational frequency) and particle number. In this figure, as in fig. 1, the κ - and μ -values are chosen different for the different N -shells to make the potential applicable all over the nuclear periodic table. Note the pronounced ridges and valleys and the very strong shell effects for large prolate deformations.

FIG. 1

FIG. 2