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THE POMERON AND THE f : AN IDENTITY CRISIS

M.R. Pennington

CERN - Geneva

B. Schrempp *)

University of Geneva

and

F. Schrempp

CERN - Geneva

A B S T R A C T

We show that the seemingly independent and plausible properties of Pomeron- f identity, asymptotic planarity and standard analyticity and unitarity constraints on trajectories are not compatible. This has strong implications for current realizations of the topological expansion.

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Ever since the concept of a vacuum trajectory, the Pomeron, was introduced the question of whether the f and its recurrences lie on this leading trajectory has been discussed. The question has recently come into focus once again as a result of calculations ¹⁾⁻⁵⁾ in the framework of the topological expansion ⁶⁾, which demonstrate the identity ^{*)} of the f and this leading vacuum trajectory (at the cylinder level, i.e., at the first unitarity correction).

The aim of this paper is to investigate whether this P - f identity is consistent with a given set of simple and widely accepted assumptions. Let us imagine the following simple world.

- 1) P - f identity : $\alpha_P(t) \equiv \alpha_f(t)$. This leading vacuum trajectory is responsible for roughly constant total cross-sections with $\alpha_P(0) \approx 1$. Near $t=0$ it has considerably flatter slope than the universal Regge slope, $\alpha' \simeq 1 \text{ GeV}^{-2}$. Moreover it has the particles $f, h, \dots$ lying on it as recurrences (Fig. 1).
- 2) Approximate ρ - f exchange degeneracy ^{**) 4), 9)} for $t \gtrsim m_f^2$ ("asymptotic planarity" ^{4), 9)}).

$$\text{Re } \Delta\alpha(t) = \text{Re } (\alpha_P(t) - \alpha_f(t)) \rightarrow 0 \text{ as } t \rightarrow +\infty, \quad (1)$$

with $\text{Re } \Delta\alpha(t) \ll 1$ for $t \gtrsim m_f^2$, as sketched in Fig. 1. This rather vague definition will be made more precise at a later stage.

- 3) Analyticity : The trajectories considered are analytic functions of t with only a right-hand cut, beginning at the lowest particle threshold. This property is true for simple non-colliding poles in the j plane of an S matrix with Mandelstam analyticity or in potential theory ¹⁰⁾. We assume $|\Delta\alpha(t)|$ is polynomially bounded in the t plane. Asymptotically linear trajectories are, of course, a specific simple realization of this.

^{*)} Not to be confused with the concept of f dominance of the Pomeron ⁷⁾.

^{**) For definiteness we take the ρ as the reference trajectory, with which to compare the Pomeron, but we could equally well choose the A_2 , for example ⁸⁾.}

- 4) Positivity : $\text{Im } \alpha(t) \geq 0$ for t above threshold. This follows from unitarity for trajectories satisfying 3) ¹⁰⁾. At the particle positions $\text{Im } \alpha$ is related to the resonance widths by $\text{Im } \alpha(m^2) \simeq \alpha' m \Gamma \geq 0$.

The concepts defined by statements 1)-4), and the physical relevance of our results, only become precise in specific schemes. Two such are topological approaches at the cylinder level and a simple data-oriented view of the Pomeron and the ρ . These we call Cases I, II, which we now introduce in some detail.

Case I

The topological expansion provides a systematic perturbative approach to the strong interactions. The dual unitarization of the Oxford-Rutherford Laboratory group ^{3),11)}, as well as the Berkeley version of the topological expansion ¹²⁾, are specific realizations of this classification. Practical calculations have so far only reached the level of the first unitarity corrections (the so-called cylinder level). However, already at this level, this scheme is able to correlate a surprisingly large number of hadronic phenomena ^{11),12)}.

Requirements 2), 3), 4) are trivially satisfied at the planar, input, level by the ρ, f being one (exchange-degenerate) linear trajectory. The cylinder correction leaves the ρ (and A_2) unaffected ¹³⁾, while the Pomeron emerges as a strong deformation of the planar f ^{3),1)}. P - f identity [as defined in 1)], asymptotic planarity, 2), and the P/f being a simple pole in the j plane arise as basic results ^{1),2),4),5),11),12)}. Asymptotic planarity is, in fact, realized by an inverse power behaviour, $\text{Re } \Delta\alpha(t) \sim t^{-\epsilon}$, $\epsilon > 0$, for $t \rightarrow +\infty$ ^{4),9),14)}. Within Case I, the aim of this paper is then to ask whether the standard analyticity and positivity properties 3), 4) remain satisfied despite this strong deformation due to the cylinder correction.

Case II

Alternatively we can adopt a more general view of the properties 1)-4), the compatibility of which is in question, by giving up the notion of a specific dynamical scheme. While now these properties enter solely as plausible assumptions, the implications of our results take on a more general nature.

The ρ and P/f are heuristically defined as trajectories which smoothly interpolate, in both their real and imaginary parts, the masses and widths of their observed particle recurrences for $t > 0$, while passing through the relevant scattering data for $t < 0$ (as they are displayed in Fig. 1). Though in our sketch asymptotic planarity appears to be realized monotonically, our proofs will, in fact, allow a certain class of oscillations.

Despite the plausibility of each of the properties 1)-4) we shall demonstrate that these are in fact strongly inconsistent. The essence of the argument is as follows : asymptotic planarity and P/f identity prescribe the behaviour of $\text{Re } \Delta\alpha(t)$ [see Eq. (1)] for $t > 0$, typically as in Fig. 1. This, together with right-hand cut analyticity in t , strongly constrains the behaviour of both $\text{Im } \Delta\alpha(t)$ for $t > 0$ and $\Delta\alpha(t)$ for $t < 0$, but in a way not necessarily consistent with the properties listed in 1) and/or 4).

To implement explicitly right-hand cut analyticity of the trajectory difference $\Delta\alpha(t)$, it is convenient to use dispersion relations. We start from a standard N times subtracted dispersion relation for $\Delta\alpha(t)/\sqrt{4\mu^2-t}$, where $4\mu^2$ is the lowest threshold of the ρ and f trajectories, viz. the 2π threshold ^{*}). Let us specify the important constraint of asymptotic planarity by the following two conditions :

$$\begin{aligned} \text{i)} \quad \text{Im} \left(\frac{\Delta\alpha(t)}{\sqrt{4\mu^2-t}} \right) &=_{t>4\mu^2} \frac{\text{Re } \Delta\alpha(t)}{\sqrt{t-4\mu^2}} \\ &= O(t^{-\frac{1}{2}-\epsilon}) \quad \text{for } t \rightarrow +\infty \end{aligned} \quad (2)$$

with $\epsilon \geq 0$,

- ii) this limit is approached in a "non-oscillatory" fashion as defined in Ref. 15) ; this criterion does, in fact, allow for oscillating functions as long as they are "well-behaved" for $t \rightarrow +\infty$ by either dying out or their period becoming large asymptotically, for example.

^{*}) It seems most natural to identify the threshold with $4m_\pi^2$. However, all our future arguments will not significantly change if the effective threshold is pushed up even as high as m_ρ^2 .

We can then rewrite the dispersion relation in the following transparent form

$$\frac{\Delta\alpha(t)}{\sqrt{4\mu^2-t}} = \sum_{n=0}^{N-1} c_n (4\mu^2-t)^n + \frac{1}{\pi} \int_{4\mu^2}^{\infty} \frac{dt'}{\sqrt{t'-4\mu^2}} \frac{\text{Re } \Delta\alpha(t')}{(t'-t)} \quad (3)$$

where

$$\sum_{n=0}^N c_n \equiv 0 \quad \text{for } N = \sigma.$$

Moreover conditions (i) and (ii) can be used to show that the integral I in representation, Eq. (3), is bounded as

$$I = O(|t|^{-\frac{1}{2}-\epsilon}) \quad \text{for } |t| \rightarrow \infty \quad (4)$$

as for example demonstrated in Ref. 15). To discuss the strong consequences of this representation, Eqs. (3), (4), let us consider separately the two cases :

- (a) no subtractions : $N = 0$
- (b) subtracted : $N \geq 1$.

(a) No subtractions

Equations (3), (4) imply that the rapid decrease of $\text{Re } \Delta\alpha(t)$ for positive t is correlated with a similarly rapid decrease of $\Delta\alpha(t)$ for negative t . This is illustrated in Fig. 2 (curve A) for a typical monotonically decreasing input $\text{Re } \Delta\alpha(t)$ for $t > 4\mu^2$, as has been calculated by Tsou⁵⁾ within the dual unitarization scheme and displayed in Fig. 1. In particular, for $t \leq 0$,

$$\Delta\alpha'(t) \equiv \frac{d}{dt} (\alpha_P(t) - \alpha_S(t))$$

is strongly positive, i.e., the Pomeron is steeper than the ρ in contrast to all expectations [e.g., property 1]. In fact this result remains true for some interval of $t < 4\mu^2$ under considerably weaker conditions on $\text{Re } \Delta\alpha(t)$ than strict monotonicity. To see this, let us differentiate Eq. (3) with $N=0$ with respect to t , where $t < 4\mu^2$, and make the convenient change of variables in the integrand $t' = t'(\theta) = 4\mu^2 + (4\mu^2 - t) \times \tan^2 \theta/2$.

Then we obtain

$$\Delta\alpha'(t) = \frac{1}{2\pi(4\mu^2 - t)} \int_0^{\pi/2} d\theta \cos\theta \left[\operatorname{Re} \Delta\alpha(t'(\theta)) - \operatorname{Re} \Delta\alpha(t'(\pi - \theta)) \right] \quad (5)$$

where for $0 \leq \theta \leq \pi/2$: $0 \leq t'(\theta) - 4\mu^2 \leq 4\mu^2 - t$; $\infty > t'(\pi - \theta) - 4\mu^2 \geq 4\mu^2 - t$. Clearly $\Delta\alpha'(t)$ is trivially positive if the integrand is positive for all $\theta \leq \pi/2$. In particular, we deduce that the Pomeron slope is bigger than the ρ slope in a significant region, where scattering data exist, $-\tau \leq t - 4\mu^2 \leq 0$, if

- i) $\operatorname{Re} \Delta\alpha(t)$ decreases monotonically only in the small mirror symmetric interval ^{*)} $0 \leq t - 4\mu^2 \leq \tau$, and
- ii) $\operatorname{Re} \Delta\alpha(t) \leq \operatorname{Re} \Delta\alpha(4\mu^2 + \tau)$ for $t - 4\mu^2 > \tau$, which even allows wild oscillations to the negative side.

This result is clearly in strong conflict with the properties of the Pomeron as listed in 1) and illustrated in Fig. 1. Thus our simple world with properties 1)-4) is inconsistent with having no subtractions. This is particularly surprising for approaches based on a topological expansion (Case I), where the unsubtracted case is the most natural one : there one expects for consistency not only $\operatorname{Re} \Delta\alpha(t) \rightarrow 0$ for $t \rightarrow +\infty$, but also $\operatorname{Im} \Delta\alpha(t)/\sqrt{t}$ decreasing too (see also later discussion).

(b) $N \geq 1$ subtractions

From Eqs. (3), (4) we have

$$\frac{\Delta\alpha(t)}{\sqrt{4\mu^2 - t}} \underset{|t| \rightarrow \infty}{\sim} c_{N-1} |t|^{N-1} \quad \text{with } c_{N-1} \neq 0. \quad (6)$$

This implies the following correlations

$$\begin{aligned} \Delta\alpha(t) &\underset{t \rightarrow -\infty}{\sim} c_{N-1} (-t)^{N-\frac{1}{2}} \\ \operatorname{Im} \Delta\alpha(t) &\underset{t \rightarrow +\infty}{\sim} (-1)^N c_{N-1} t^{N-\frac{1}{2}}. \end{aligned} \quad (7)$$

^{*)} From Tsou's calculation ⁵⁾ we expect at least $\tau \gtrsim m_p^2$.

Let us emphasize that asymptotic planarity restricts the asymptotic growth to discrete powers :

$$N - \frac{1}{2} = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \dots \quad (8)$$

so that only very special behaviours are allowed. Moreover, the correction to the subtraction polynomial in Eq. (3) is exactly the same integral that gave us the entire result in the unsubtracted case. As seen in Fig. 2, this is typically rather small for all values of t with $|t| \gtrsim m_p^2$.

N = 1

The additional constant c_0 in representation, Eq. (3), - if chosen to be positive - allows for a positive increase in $\Delta\alpha(t)$ for negative t , as property 1) requires. However, Eq. (7) then forces $\text{Im } \Delta\alpha(t)$ to become equally strongly negative, i.e., like $-c_0\sqrt{t}$, in the positive t direction. Typical situations are displayed in Fig. 2, curves B, C. If $\Delta\alpha(t)$ for negative t increases as rapidly as the data require, it is obvious from curve C (Fig. 2) that $\text{Im } \Delta\alpha(t)$ is strongly negative right from threshold. This grossly violates the positivity property 4), $\text{Im } \alpha_P(t) \geq 0$, both in the topological approaches (Case I), where $\text{Im } \alpha_p \approx 0$, and in Case II where $\text{Im } \alpha_p$ and $\text{Im } \alpha_P$ are prescribed from the particle widths at their respective particle positions (see the bounds in Fig. 2). If we barely satisfy these positivity bounds (curve B) we are clearly still far from agreeing with the data, i.e., with property 1).

So we must also rule out one subtraction. Because of the discrete possibility of powers, Eq. (8), we must now face a growth of at least $t^{\frac{3}{2}}$ for $\text{Im } \Delta\alpha(t)$. For the topological approaches (Case I) such growth seems unacceptable. An important aspect of this approach is its truly perturbative nature for large positive t ¹²⁾ ("asymptotic planarity"). However, we have shown : starting from zero imaginary parts at the planar, input level, analyticity [as specified in property 3)] generates a growth of $\text{Im } \alpha_P$ at least as strong as $\sim t^{\frac{3}{2}}$ at the next, cylinder, level. This effect is clearly non-perturbative. Our arguments apply equally well to the specific solution for the Pomeron proposed by Veneziano ¹⁶⁾ (even though this Pomeron is not identical to the "cylinder f ").

So far we have discussed the implications of our results only for the topological approaches (Case I). In the data-oriented view of the world, Case II, we cannot a priori rule out higher numbers of subtractions. However, we verified that the scattering data for $t \leq 0$, the widths of the ρ , f , g and h and the positivity condition on $\text{Im } \alpha_{\rho}$ and $\text{Im } \alpha_g$ require at least $N=5$ subtractions. This is an appallingly high number, implying a growth of $|t|^{9/2}$ for both $\text{Im } \alpha(t)$ for $t \rightarrow +\infty$ and $\alpha(t)$ for $t \rightarrow -\infty$ of at least one of the two trajectories. If one wants to save all of the properties 1)-4), then one has to live with at least such a rapid growth. This seems unphysical to us.

We conclude : the simple $\mathbb{P}/f$ identity, asymptotic planarity, right-hand cut analyticity with reasonable asymptotic boundedness and positivity are surprisingly in conflict with each other.

This has strong implications for the current realizations of the topological expansion ^{11),12)} which automatically embody the first two of these properties at the cylinder level. Within these schemes right-hand cut analyticity ^{*)} or polynomial boundedness will have to be abandoned, or pathologically wild or persistent oscillations admitted in the attainment of asymptotic planarity. In our alternative more general data-oriented view, the choice of which of the properties to give up is a matter of taste.

Our purpose has been to point out that these seemingly independent and very plausible properties 1)-4) cannot all be elements of a consistent picture of strong interactions.

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*) For example, two colliding trajectories naturally give rise to left-hand cuts.

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FIGURE CAPTIONS

- Figure 1 Chew-Frautschi plot of the $\mathbb{P}/f$ and ρ trajectories in both the resonance and scattering regions. For $0 \leq t \leq m_f^2$, the $\mathbb{P}/f$ trajectory represents the calculation of Tsou⁵⁾.
- Figure 2 Plot of $\Delta\alpha(t)$, the difference between the $\mathbb{P}/f$ and ρ trajectory functions. The input curve is $\text{Re } \Delta\alpha(t)$ as calculated by Tsou⁵⁾ (Fig. 1). Curves A, B, C are the output $\Delta\alpha(t)$ for $t \leq 4\mu^2$ and $\text{Im } \Delta\alpha(t)$ for $t \geq 4\mu^2$, where A follows from an unsubtracted dispersion relation and B, C from a once subtracted relation with two different values of the subtraction constant. The data points correspond to the scattering data of Fig. 1.

FIG.1

FIG. 2