Ref.TH.2519-CERN

HIERARCHY OF QUARK MASSES, CABIBBO ANGLES AND CP VIOLATION

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A B S T R A C T

Some ideas are put forward towards understanding the fact that mass ratios of quarks or leptons are usually rather large. These speculations are based on the assumption that the basic (bare) couplings at small distances are all of the same order of magnitude but otherwise unknown or random. The most promising explanation requires the bare left- and right-handed components of a quark to have different values for some almost conserved quantum number. These differences determine the order of magnitude of the corresponding quark mass. The generalized Cabibbo mixing angle connecting two types of quarks is then predicted to be given order of magnitude wise by the square root of the corresponding quark mass ratio. However, it is difficult to understand why CP is so well conserved.

Ref.TH.2519-CERN

28 June 1978

1. - INTRODUCTION

It is a striking feature of the quark and lepton spectra that the masses of successive particles usually increase by large factors. The purpose of the present article is to speculate on possible origins of these large mass ratios. Our basic philosophy here is that the mass ratios are not just put in by hand as bare couplings of quarks to Higgs particles, say, but that they arise naturally in the sense of Ref. 1). It may well be that the fundamental bare couplings of nature can never be calculated. However even though we should then not be able to calculate precisely the quark or lepton mass ratios, it should still be possible to understand why these ratios are large numbers.

From our considerations of the quark mass matrix, we obtain predictions for the decomposition of the quark weak eigenstates in terms of the quark mass eigenstates. This decomposition can be expressed in terms of generalized Cabibbo angles for which we get order of magnitude estimates. Also we obtain an estimate of the strength of CP violation which, as might be expected, turns out in general to be much bigger than the observed rate. When making explicit calculations, it is assumed that the standard $SU_2 \times U_1$ gauge model ²⁾ is the appropriate description of weak interactions at present energies.

Neutrinos could of course be exactly massless in the Weinberg-Salam scheme, although this would violate naive quark lepton symmetry. Anyway, in as far as the neutrino masses are negligible compared to the charged lepton masses, the observable effects of leptonic mixing angles are limited to fairly exotic effects such as neutrino oscillations. For this reason we shall concentrate on the quark mass matrix in this article, although similar considerations should apply at least to the charged leptonic sector.

Many attempts at calculating or estimating weak mixing angles can be found in the literature ^{3),4)}. Recently several authors ⁵⁾⁻⁷⁾ have rederived the phenomenologically successful relation $\theta_c^2 = m_d/m_s \approx M_\pi^2/2M_K^2$ for the Cabibbo angle, by imposing a discrete symmetry on a gauge model (usually $SU_2 \times SU_2 \times U_1$) having spontaneous breakdown of parity. In particular Fritzsch has suggested that the above-mentioned formula for the Cabibbo angle is, with minor corrections ⁷⁾, an exact expression. However, it is only possible to check this formula phenomenologically as an order of magnitude relation and we shall be satisfied in deriving it as such, viz. $\theta_c \sim \sqrt{m_d/m_s}$.

We start in Section 2 with a discussion of the determination of quark masses and state in what sense the quark mass ratios are large. In Section 3 we put forward our first idea for explaining the presence of large quark mass ratios. Namely, we ask whether the renormalization group equations for the Yukawa type couplings of Higgs particles to quarks can lead to an exponentiation of some bare (or short distance) couplings. It turns out to be difficult to generate large quark mass ratios in this way unless special selection rules are imposed. But then we are approaching the mechanism discussed in Section 4, where we consider what mass ratios can be expected if approximate selection rules are present. The quark mass matrix for this case is discussed in more detail in Section 5, where we derive the behaviour of the generalized Cabibbo angles. In Section 6 we discuss the phenomenological consistency of the predicted behaviour of these weak mixing angles and also the expected size of CP violating effects. The weakness of CP violation remains a puzzle for our model and we propose a rather unattractive way out in Section 7, which also contains the main conclusions and final remarks of the paper.

2. - THE BASIC FERMION MASS SPECTRUM

Lepton masses are well-defined concepts as they correspond to the physical masses of real particles. However, since quarks are believed not to exist as freely separated particles, quark masses are a priori not defined. Rather one has to take a specific model of confinement and then estimate the quark mass parameters needed within that model to generate observed properties of hadron states. For example, in the naïve non-relativistic quark model one is led to masses for the three lightest quarks of the order of 400 MeV. However, within the bag model ⁸⁾, one is led to u and d quarks having almost zero mass while the s quark has a mass $m_s \sim 300$ MeV. Confinement of the quarks inside a "bag" of extension ~ 1 fm then leads to an effective mass similar to that of the naïve constituent quark model.

In this article we shall use light quark masses obtained from estimates of their chiral symmetry breaking effects ^{9),10)} in the pseudo-scalar (Goldstone) boson mass spectrum. We feel that this method comes closest to determining the quark-Higgs coupling or the coefficient m_q of the mass term $m_q \bar{q}q$ in an effective Lagrangian. This approach leads to

very small u and d quark masses and then spontaneous symmetry breaking effects provide the dominant contribution to their "constituent" quark masses, as in the original Nambu-Jona Lasinio model ¹¹⁾. However, the current algebra PCAC techniques are only applicable to quarks having a "current" quark mass small compared to the typical strong interaction mass scale of 1 GeV. For the heavier quarks (c, t, b , etc.), the spontaneous effects should not be a dominant factor and the "current" to "constituent" quark mass ratio should tend to unity. In this case the non-relativistic picture of a heavy quark inside pseudoscalar mesons is more appropriate than the almost Goldstone boson considerations.

In the quark model, $SU_3 \times SU_3$ chiral invariance is broken by the quark mass terms

$$\mathcal{H}_m = m_u \bar{u}u + m_d \bar{d}d + m_s \bar{s}s \quad (2.1)$$

and the PCAC hypothesis leads to the following mass formula ^{9),10)} for the pseudoscalar (Goldstone) boson P consisting of a quark q_1 and an antiquark $\bar{q}_2$

$$\begin{aligned} M_P^2 &= \frac{\sqrt{2}i}{F_P} (m_1 + m_2) \langle 0 | \bar{q}_2 \gamma_5 q_1 | P \rangle \\ &= \frac{2}{F_P^2} (m_1 + m_2) (\langle 0 | \bar{q}_1 q_1 | 0 \rangle + \langle 0 | \bar{q}_2 q_2 | 0 \rangle) \end{aligned} \quad (2.2)$$

where F_P is the meson decay constant. The assumption of SU_3 symmetry for the quark matrix elements then gives the standard current algebra mass ratios ⁵⁾

$$m_u : m_d : m_s = 1 : 1.8 : 36 \quad (2.3)$$

after the cancellation of photon self-energy effects between the pion and the kaon according to Dashen's theorem ¹⁰⁾. There is some theoretical uncertainty ^{5),7),12),13)} in the derivation of these ratios but they should be sufficiently accurate for our present order of magnitude investigations.

In order to fix the quark mass scale, we assume that m_s is given approximately by the mass splittings between strange and non-strange particles. We then obtain ⁵⁾

$$\begin{aligned} m_s &= 150 \text{ MeV} \\ m_d &= 7.5 \text{ MeV} \\ m_u &= 4.2 \text{ MeV} \end{aligned} \quad (2.4)$$

These values correspond to effective quark masses appropriate to the typical hadronic energy scale of $\mu \simeq 1 \text{ GeV}$. We shall discuss the dependence of the quark masses on the energy scale in the next section. From the masses of charmed particles and the $T(9.5)$ particle we arrive in the same way at the values

$$\begin{aligned} m_c &= 1200 \text{ MeV} \\ m_b &= 4400 \text{ MeV} \end{aligned} \quad (2.5)$$

for the heavy quark masses. It should be remarked that the constituent quark masses M_q of the naïve quark model include spontaneous effects so that crudely we have

$$M_q = m_q + \Delta \quad (2.6)$$

where

$$\Delta \approx 350 \text{ MeV} \quad (2.7)$$

It would be very useful to have a general formula expressing the quark masses in terms of the pseudoscalar meson masses valid in both the relativistic (Goldstone boson-like) regime where

$$M_p^2 \simeq M_0 \cdot (m_1 + m_2) \quad (2.8)$$

and the non-relativistic regime where

$$M_p^2 \simeq (M_1 + M_2)^2 \quad (2.9)$$

A simple interpolation formula, motivated by the Nambu-Jona Lasinio model ¹¹⁾ and satisfying these requirements, is

$$\begin{aligned} M_p^2 &= (m_1 + m_2)(m_1 + m_2 + 4\Delta) \\ &= (M_1 + M_2)^2 - 4\Delta^2 \end{aligned} \quad (2.10)$$

It corresponds to taking

$$\langle 0 | \bar{q}_i q_i | 0 \rangle = \frac{F^2}{2} (M_i + \Delta) \quad (2.11)$$

in the current algebra formula (2.2) and phenomenologically reproduces the quark masses given above within an accuracy of about 10%.

As we have already emphasized, the striking feature of the quark and also the lepton spectrum is the appearance of large mass ratios. This is especially evident if we restrict ourselves to comparing quarks with the same electric charge. For instance, assuming the T(9.5) particle to be associated with a new $Q = -\frac{1}{3}$ quark, the bottom type quarks have mass ratios

$$m_b : m_s : m_d = 590 : 20 : 1 \quad (2.12)$$

corresponding to a geometric mean of $\sqrt{590} \simeq 24$ per quark flavour. Similarly for the $Q = \frac{2}{3}$ or top type quarks we have

$$m_c : m_u = 290 : 1 \quad (2.13)$$

The total geometric mean per quark step of the same charge is then $\sqrt[3]{590 \times 290} = 56$. Similarly the ratios of the charged lepton masses are

$$m_\tau : m_\mu : m_e = 3600 : 200 : 1 \quad (2.14)$$

giving a geometrical mean of 60 per lepton type. Thus the mass ratios of the leptons and quarks are qualitatively rather similar corresponding to an overall geometrical mean of 58. It is essentially this number 58 that we call large, when we say that the basic fermion mass ratios are large. We, of course, know nothing yet about the mass ratios of neutrinos.

If quarks of the same charge are ordered according to their mass, then the $Q = -\frac{1}{3}$ and $Q = \frac{2}{3}$ quarks at the same position in the hierarchy do not have such large mass ratios. In fact we have

$$\begin{aligned} m_d : m_u &= 1.8 : 1 \\ m_c : m_s &= 8 : 1 \end{aligned} \tag{2.15}$$

giving a geometrical average which is approximately the cube root of 58. As emphasized by Fritzsche ⁷⁾ and Weinberg ⁵⁾, isospin invariance in strong interactions is not due to the degeneracy in mass of the u and d quarks but rather to their both being very small compared to the typical strong interaction mass scale. It is thus a consequence of the above-mentioned large variation of quark masses with two of them very close to zero rather than of a geometrical symmetry. The purpose of the present article is to seek explanations for the very large mass ratio of the order of 60.

3. - LARGE MASS RATIOS FROM THE RENORMALIZATION GROUP ?

Our first idea for explaining the large mass ratios of quarks and leptons at our present energy scale was to generate them by the renormalization group from the effective masses ("bare" Higgs couplings) at a more fundamental high energy scale. At this high energy scale the effective masses of the basic fermions are all supposed to be of the same order of magnitude. So we are interested here in the renormalization group equations for the Yukawa couplings of Higgs particles to fermions ¹⁴⁾. It is namely those couplings that in the conventional Weinberg-Salam model are proportional to the masses of fermions such as quarks and leptons. We must therefore show that the ratios of these Yukawa couplings increase on the average by the order of 50 or 60 from particle to particle, in order to agree with the fermion spectrum discussed in the previous section. It is of course the effective Yukawa couplings at the present energy scale $\mu \sim 1$ GeV (where we find the quarks and measure their masses) that are relevant for obtaining this large mass ratio. Since we do not have strict conservation of "horizontal flavour" the Yukawa couplings of the Higgs field are in fact matrices in which the indices denote quark or lepton flavours (of the same electric charge). Indeed an arbitrary coupling matrix is allowed by the gauge symmetries.

For simplicity let us consider the standard Weinberg-Salam model with fermions classified as left-handed weak doublets and right-handed weak singlets in the usual manner. Masses are then generated by the neutral component of a single complex $Y=1$ Higgs field doublet ϕ taking on a non-zero vacuum expectation value $\langle \phi^0 \rangle \neq 0$. The Yukawa couplings of the quarks to the Higgs scalar may be written

$$\begin{aligned} \mathcal{L}_{\text{Yukawa}} = & \bar{\Psi}_t G_t \phi^\dagger \psi_L + \bar{\Psi}_L \phi G_t^\dagger \psi_t + \\ & + \bar{\Psi}_b G_b \tilde{\phi}^\dagger \psi_L + \bar{\Psi}_L \tilde{\phi} G_b^\dagger \psi_b \end{aligned} \quad (3.1)$$

where $\tilde{\phi} = i\sigma_2 \phi^*$ is the conjugate doublet having weak hypercharge $Y = -1$. Also ψ_L , ψ_t and ψ_b denote columns of fields of left-handed, top type ($Q = \frac{2}{3}$) right-handed and bottom type ($Q = -\frac{1}{3}$) right-handed quarks, respectively. These fields are two-component fields in the sense that for each type of horizontal flavour there are two complex components of these fields transforming into each other under Lorentz transformations. In order to avoid massless (neutrino-like) quarks and ensure anomaly free vector gauge currents for colour and electromagnetism, we must have equally many right- and left-handed quarks¹⁵⁾. So the Yukawa coupling matrix G_t (G_b) becomes square in the space of top (bottom) quarks.

In order to set up the renormalization group equations we require the β functions for the Yukawa coupling matrices G_t and G_b . So first we must define the effective coupling matrix $G_b(\mu)$ at a renormalization point mass scale μ by, say, the value of a three-point function where the three relevant fields ψ_b^+ , ϕ^+ and ψ_L have momenta of order of magnitude μ . There is some arbitrariness in this definition but it should at least be chosen so that in the weak coupling approximation the bare three-point functions regularized with a cut-off Λ are equal to $G_b(\Lambda)$. The β functions are then defined by

$$\beta_b = \frac{\partial G_b(\mu)}{\partial \log \mu} = - \frac{\partial G_b(\mu)}{\partial \log \Lambda} \quad (3.2)$$

and

$$\beta_A = \frac{\partial G_A(\mu)}{\partial \log \mu} = - \frac{\partial G_A(\mu)}{\partial \log \Lambda} \quad (3.3)$$

The point of the renormalization group ¹⁶⁾ is that one can use a set of effective couplings at one scale, μ_1 say, to specify the free parameters of a renormalizable theory and then calculate all the other couplings at another scale μ_2 . This means that the β functions can be calculated in terms of the effective (renormalized) couplings at the relevant scale μ . If the renormalizable theory is scale invariant in the classical approximation, all the coupling constants are dimensionless and the β functions have no explicit dependence on the scale μ , provided they are expressed in terms of the effective couplings at that scale μ .

Now the basic philosophy of the present article is that the fundamental couplings of nature can be considered to be chosen at random. Thus they should at least neither be extremely large nor extremely small. With this assumption of course the elements of the coupling matrices G_b and G_t should all be of order one. It then becomes a mystery why the mass ratios of leptons and quarks of different horizontal flavours should be so large. However, the scale at which the fundamental couplings are defined must be specified since the couplings vary with μ according to the renormalization group equation. As simple examples we note that for pure QED the scale μ_0 at which the coupling is of order unity has the enormous value ¹⁷⁾

$$m_e \exp\left(\frac{3\pi}{2\alpha}\right) = 1.2 \cdot 10^{277} \text{ GeV}$$

where α is the electromagnetic fine structure constant, while for pure asymptotically free QCD it has the value ¹⁸⁾ $\mu_0 \simeq 0.5 \text{ GeV}$. Phenomenologically it is clear that if all particle interactions are to become of the same order of magnitude it must occur at some high mass scale. Various authors ^{17), 19), 20)} have suggested that the Planck mass $G_{\text{Newton}}^{-1/2} = 1.2 \times 10^{19} \text{ GeV}$ provides the fundamental mass scale. Here we simply assume the existence of some high mass scale μ_0 at which the couplings are all of the order unity. We do not attempt to specify the dynamics in any more detail and in particular do not require the existence of a simple grand unified model ²¹⁾ of all interactions. Grand unified theories would correspond to the special case where the couplings are related to each other by Clebsch-Gordon coefficients at the fundamental mass scale.

The lowest order renormalization group equations for the Yukawa coupling matrices take the following form ¹⁴⁾

$$16\pi^2 \frac{dG_A}{d\log\mu} = -\alpha_A G_A + a(G_A G_A^\dagger + G_b G_b^\dagger) G_A^\dagger + b [\text{Tr}(G_A G_A^\dagger) + \text{Tr}(G_b G_b^\dagger)] G_A \quad (3.4)$$

$$16\pi^2 \frac{dG_b}{d\log\mu} = -\alpha_b G_b + a(G_A G_A^\dagger + G_b G_b^\dagger) G_b^\dagger + b [\text{Tr}(G_A G_A^\dagger) + \text{Tr}(G_b G_b^\dagger)] G_b \quad (3.5)$$

Here a and b are constants and α_t and α_b are the appropriate sum ²⁰⁾ of gauge coupling constants squared evaluated at the scale μ . These equations have recently been studied by Maiani et al. ²⁰⁾. However, we do not follow their philosophy of assuming the absence of new interactions above the energy scale set by the Weinberg-Salam model $G_F^{-1/2} \simeq 300$ GeV until gravitational forces become effective at the Planck mass. We would rather take the view that the elementary particle mass spectrum extends up to much higher energies. As mentioned above, our aim is not to guess the correct dynamical structure at these energies but just to investigate under what circumstances the renormalization group can generate large mass ratios. So we are only interested in the general properties of the renormalization group equations and their solutions and not in their specific form. For example, it is for this reason that we do not bother to include the couplings to the lepton sector.

We now assume that the Yukawa couplings are all of the same order of magnitude at some high mass scale μ_0 and ask whether the renormalization group equations can generate large ratios between them at low energy. When the couplings become of order unity, higher order terms in the renormalization group equations can become important and ultimately the couplings are expected to tend to a stable fixed point or a limit cycle ¹⁶⁾. In order to demonstrate the effect of the renormalization group equations we therefore consider two possible descriptions of the Yukawa couplings at the fundamental mass scale. These are : (a) fixed point behaviour, and (b) lowest order renormalization group behaviour.

Since all the right-handed fields are classified as weak singlets, only the unitary transformations on the left-handed fields in the diagonalization of the quark mass matrix are physically relevant. It is therefore sufficient to consider the structure of the positive definite Hermitian matrices $H_t = G_t^+ G_t$ and $H_b = G_b^+ G_b$. In case (a) we use the linearized form of the renormalization group equations about the assumed fixed point $H = H^*$ as $\mu \rightarrow 0$,

$$\frac{d(H - H^*)}{d \log \mu} = T \cdot (H - H^*) \quad (3.6)$$

where T is a fourth order tensor. We can expand the solution in terms of the eigenmatrices E^n of the tensor T

$$T \cdot E^n = \lambda_n E^n \quad (3.7)$$

to obtain

$$H - H^* = \sum_n c_n E^n \left(\frac{\mu}{\mu_0} \right)^{\lambda_n} \quad (3.8)$$

The stability of the fixed point requires that all the eigenvalues satisfy $\lambda_n \geq 0$. As the energy scale μ is reduced the deviation from the fixed point H^* will be dominated by the smallest eigenvalue λ_n . The ratio of these contributions to the elements of H are given by the ratio of the corresponding matrix elements of E^n . A random eigenmatrix will therefore not generate mass ratios in a natural way for an infra-red free fixed point. It is necessary to build them directly into the structure of the fixed point eigenmatrices E^n , which effectively corresponds to the introduction of selection rules. Also for $H^* \neq 0$, we do not obtain many successive large mass ratios.

So we now turn to case (b) and suppose that the lowest order equations (3.4), (3.5) are valid up to the fundamental mass scale, where the elements of the squared Higgs coupling matrices $H_t(\mu_0)$ and $H_b(\mu_0)$ are all of the same order of magnitude. It is convenient to introduce the auxiliary functions $\rho_t(x)$ and $\rho_b(x)$ of the variable $x = \log \mu$ defined by the integrals

$$\rho_i(x) = \int_x^L dx' \exp\left(\frac{1}{8\pi^2} \int_{x'}^L dx'' \alpha_i(x'')\right) \quad (3.9)$$

where $L = \log \frac{\mu}{\mu_0}$ and α_t and α_b are the appropriate sums of gauge coupling constants. Also we use the notation

$$\rho'_i = \frac{d\rho_i}{dx} \quad (3.10)$$

for the derivatives of ρ_t and ρ_b . We note that $\rho(L) = 0$, $\rho'(L) = 1$ and that in the approximation of a constant α_i

$$\rho_i = \frac{8\pi^2}{\alpha_i} \left[\left(\frac{\mu_0}{\mu} \right)^{\frac{\alpha_i}{8\pi^2}} - 1 \right] \quad (3.11)$$

and

$$\rho'_i = - \left(\frac{\mu_0}{\mu} \right)^{\frac{\alpha_i}{8\pi^2}} \quad (3.12)$$

It is readily seen from the lowest order renormalization group equations (3.4), (3.5) that

$$16\pi^2 \frac{d(G_t^{-1} G_b)}{dx} = (\alpha_t - \alpha_b) G_t^{-1} G_b \quad (3.13)$$

and hence that

$$G_b = \left(\frac{\rho'_b}{\rho'_t} \right)^{\frac{1}{2}} G_t A \quad (3.14)$$

where the constant matrix A is given by

$$A = G_t(\mu_0)^{-1} G_b(\mu_0) \quad (3.15)$$

The Hermitian matrices $H_i = G_i^\dagger G_i$ are thus related by

$$H_b = \frac{\rho'_b}{\rho'_t} A^\dagger H_t A \quad (3.16)$$

It follows that the renormalization group equations for H_t can be written in the form

$$8\pi^2 \frac{dH_t^{-1}}{dx} = -\alpha_t H_t^{-1} + a \left(I + \frac{\rho_b'}{\rho_t'} AA^+ \right) + b \left[\text{Tr} H_t + \frac{\rho_b'}{\rho_t'} \text{Tr} (A^+ H_t A) \right] H_t^{-1} \quad (3.17)$$

We see immediately from Eq. (3.14) that, on the average, the ratio of a bottom quark mass to a top quark mass is renormalized by a factor $(\rho_b'/\rho_t')^{\frac{1}{2}}$. This power dependence on the energy scale μ arises directly from the difference in gauge group quantum numbers between top and bottom quarks. Our aim is now to show that any power dependence on μ generated by the renormalization group equations (3.17) is essentially the same for each top (bottom) quark mass eigenvalue. Unfortunately it is not possible to write down the general solution of Eq. (3.17) in an explicit form. However, it is possible to give approximate solutions which demonstrate that such energy dependence always appears in the form of coefficient functions multiplying matrices and that the Higgs coupling matrices themselves are not exponentiated.

For example we can treat top and bottom quarks identically, by neglecting the weak and electromagnetic gauge coupling constants and setting $\alpha_t = \alpha_b$. Then $\rho_t = \rho_b$ and we obtain

$$H_t(\mu) = H_t(\mu_0) \cdot \left(\frac{-\rho_t'}{d(x)} \right) \quad (3.18)$$

and a similar expression for $H_b(\mu)$, where the function

$$d(x) = 1 + \frac{h_0}{8\pi^2} \cdot (a + kb) \rho_t(x) \quad (3.19)$$

Here the constant h_0 and the transformation matrix A are defined in terms of the Higgs coupling matrices at the fundamental energy scale μ_0 by

$$H_t(\mu_0) = h_0 (I + AA^+)^{-1} \quad (3.20)$$

$$H_b(\mu) = h_o (I + (A^\dagger A)^{-1})^{-1} \quad (3.21)$$

and

$$k = \text{Tr} (H_t(\mu) + H_b(\mu)) \quad (3.22)$$

Maiani et al.²⁰⁾ have also obtained a solution of the form (3.18). However, they kept $\alpha_t \neq \alpha_b$ but neglected the second term in Eq. (3.17) (i.e., set $a=0$). In that case, one must take

$$d(x) = 1 + \frac{b}{8\pi^2} [\rho_t \text{Tr} H_t(\mu) + \rho_b \text{Tr} H_b(\mu)] \quad (3.23)$$

The solution (3.18) illustrates explicitly the fact that the energy dependence resulting from the renormalization group equations occurs in the form of ordinary functions multiplying matrices. In general the mass eigenvalues will all be renormalized by the same order of magnitude but not by exactly the same amount. Similarly, in general, there will be a finite renormalization of the weak interaction mixing angles. Again this is best seen by giving another explicit solution of Eq. (3.17). In this case we neglect the trace terms (i.e., set $b=0$) and obtain

$$H_t(\mu)^{-1} = [H_t(\mu_o)^{-1} + \frac{a}{8\pi^2} (\rho_t I + \rho_b A A^\dagger)] \cdot \begin{pmatrix} -1 \\ \rho_t' \end{pmatrix} \quad (3.24)$$

This solution provides a good example of the general form of the energy dependence generated by the renormalization group. There can be a large overall renormalization of the top (bottom) quark mass matrix by many orders of magnitude but there is only a finite renormalization of the mass ratios. It is only possible to obtain a large renormalization of the mass ratio between a top quark and a bottom quark (or a lepton)²²⁾ where the difference in gauge quantum numbers ensures that they are treated differently by the dynamics (i.e., $\alpha_t \neq \alpha_b$).

We conclude that large mass ratios are not generated naturally by the renormalization group without the aid of special selection rules or quantum numbers. Therefore we now turn to a discussion of the rôle of

approximate conservation laws in understanding the quark mass spectrum. We shall not explicitly pursue the renormalization group mechanism of generating large mass ratios with the aid of new quark gauge quantum numbers here. However, it should be possible to reformulate the discussion of Section 4 in such terms.

4. - APPROXIMATE SELECTION RULES AND QUARK MASSES

It is natural ^{4)-7), 23)} to try to explain the occurrence of large mass ratios in terms of approximate selection rules. The usual approach ²³⁾ is to assume the existence of a symmetry which requires some quark (lepton) masses to be zero and then generate a finite mass at some order in a symmetry breaking interaction. Recently there have been many attempts ⁵⁾⁻⁷⁾ to understand the quark masses in terms of discrete symmetries. Unfortunately there are many possible discrete symmetries and the choice of the appropriate one seems to be almost completely arbitrary. Also in many cases the large quark mass ratios appear to be accidental.

In this paper we prefer to consider the more familiar continuous symmetries and consequently, if we gauge the symmetry, have to accept the existence of very heavy gauge particles which mediate completely new types of interaction. For convenience we shall restrict ourselves to the simplest continuous symmetry, i.e., we shall consider the case of a single almost conserved Abelian integrally quantized charge R . We also assume the existence of many very heavy fundamental fermions having various values of this new charge R . So the natural mass scale is that of these heavy particles, which is much higher than that presently available at machine energies. From this point of view, all of the presently observed quarks and leptons are effectively massless and we are only observing the low energy tail of some more fundamental physics. We impose the requirement that the known basic fermions are massless in the limit of exact R conservation, by allowing their left- and right-handed components to have different values of the charge R .

We generate super heavy particle masses by the standard method of giving a non-zero vacuum expectation value $\langle \phi_0 \rangle$ to a neutral ($R=0$) Higgs scalar. Similarly we assume that the symmetry breaking mechanism responsible for the masses of the light (known) fermions is provided by the vacuum expectation value $\langle \phi_1 \rangle$ of some singly charged ($R=1$) Higgs

boson ^{*)}. The symmetry breaking parameter is then

$$\epsilon = \frac{g(\mu \sim 1 \text{ GeV})}{g(\mu_0)} \cdot \frac{\langle \phi_1 \rangle}{\langle \phi_0 \rangle}$$

where $g(\mu)$ is essentially any running ϕ_1 Yukawa coupling constant.

We can take over the general form of the quark mass term in the Lagrangian

$$\mathcal{L}_m = - \bar{\Psi}_t M_t \Psi_L - \bar{\Psi}_b M_b \Psi_L + \text{h.c.} \quad (4.1)$$

from the Yukawa interaction Lagrangian of Eq. (3.1). The mass matrix M_t for the top quarks is then given by Feynman diagrams of the type shown in the Figure, corresponding to a multiple breaking of R conservation. The number of ϕ_1 tadpoles contributing to the mass matrix element $M_{t,ij}$ is given by the difference in R quantum numbers between the left- and right-handed top quarks L_j and t_i . It follows that the quark mass matrix is of the form

$$M_{ij} = g_{ij} \epsilon^{a_i + b_j} \quad (4.2)$$

where we have dropped the subscript t for convenience here and in the rest of this section. The power $a_i + b_j$ gives the difference in R quantum numbers between the corresponding left- and right-handed quarks, $R_{Lj} = b_j$ and $R_{t_i} = -a_i$. Since, by assumption, all observed quarks are not super-heavy particles, we can choose positive integers $a_i > 0$ and $b_j \geq 0$. Also we order the quarks so that $a_{i+1} \geq a_i$ and $b_{j+1} \geq b_j$. As we are only concerned with mass ratios, we can choose our unit of mass so that $\langle \phi_0 \rangle = 1$ and the coefficients g_{ij} are then complex numbers of order unity. We assume the existence of a sufficient number of super heavy particles, which can be exchanged in the Figure, that the coefficient matrix

$$G = (g_{ij}) \quad (4.3)$$

^{*)} The Higgs boson ϕ_2 responsible for the weak intermediate vector boson masses does not contribute to the fermion masses due to the R selection rules.

is non-singular. Thus we obtain an order of magnitude factorizing mass matrix equation (4.2) with effectively random complex coefficients equation (4.3) of order unity.

Let us now verify that the above form equation (4.2) of the mass matrix M leads to large quark mass ratios. Since ϵ is a small number, we can consistently evaluate all the mass eigenvalues to leading order. The product of the n largest eigenvalues of M is clearly given ^{*)} to leading order by

$$\prod_{i=1}^n \lambda_i = \det G^n \cdot \epsilon^{k_n} \quad (4.4)$$

where

$$k_n = \sum_{i=1}^n a_i + b_i \quad (4.5)$$

and G^n is the $n \times n$ submatrix formed by the first n rows and n columns of G . The unitary transformation, explicitly constructed in the next section, which diagonalizes the mass squared matrix $H = M^\dagger M$ to leading order also brings the mass matrix M into triangular form. Hence the n^{th} largest quark mass is given to leading order by

$$m_n = \frac{|\det G^n|}{|\det G^{n-1}|} \epsilon^{a_n + b_n} \quad (4.6)$$

Thus we obtain large (for $i < j$) quark mass ratios

$$\frac{m_i}{m_j} \sim \epsilon^{a_i - a_j + b_i - b_j} \quad (4.7)$$

The order of magnitude factorization ansatz of Eq. (4.2) seems to be the natural way of generating large quark mass ratios. The interpretation of ϵ as a small symmetry breaking parameter is then also natural. Recently discrete symmetry models ^{6), 7), 24)} for weak interaction mixing angles have been studied in which the quark masses are taken as input. The resulting quark mass matrices effectively satisfy our ansatz but with strong correlations between the various matrix elements. In particular they require $a_i = b_i$ and several of the coefficients g_{ij} to be zero. However, these discrete

*) Here we assume that $a_n > a_{n-1}$ and $b_n > b_{n-1}$. In the case of degenerate values $a_n = a_{n-1}$ or $b_n = b_{n-1}$ the equation (4.4) is only order of magnitude-wise correct.

symmetry models do not naturally require large quark mass ratios. Of course these models make precise predictions for the generalized Cabibbo angles in terms of the exact quark masses. Our ansatz presumes only that the quark mass ratios are large and then relates the order of magnitude of the weak mixing angles to these ratios as we shall now demonstrate.

5. - THE WEAK COUPLING MATRIX

The hadronic charged weak current is given by

$$J_\mu = \bar{\psi}_L^t \gamma_\mu \psi_L^b \quad (5.1)$$

where the superscripts t and b indicate the top and bottom components respectively of the left-handed quark doublets ψ_L . When the current J_μ is expressed in terms of the quark mass eigenstates, it is necessary to introduce a unitary matrix A in flavour space connecting the top and bottom quarks,

$$J_\mu = \bar{q}^t A \gamma_\mu (1 - \gamma_5) q^b \quad (5.2)$$

This weak coupling matrix is given by

$$A = U_t^\dagger U_b \quad (5.3)$$

where U_t and U_b are the unitary transformations which diagonalize the mass squared matrices $H_t = M_t^\dagger M_t$ and $H_b = M_b^\dagger M_b$.

For the moment, let us drop the subscripts t and b and consider the unitary transformation U which diagonalizes the positive definite Hermitian matrix

$$H = M^\dagger M \quad (5.4)$$

where M is a quark mass matrix satisfying Eq. (4.2). The columns of the unitary matrix U are of course just the eigenvectors of H . Starting with the largest mass quark, it is readily seen that the components of the first eigenvector are given to leading order by

$$U_{i1} = \frac{g_{1i}^*}{g_{11}^*} \cdot \varepsilon^{b_i - b_1} \quad (5.5)$$

Similarly the eigenvector for the next most massive quark is given to leading order by

$$U_{i2}^* = \frac{g_{11} g_{2i} - g_{21} g_{1i}}{g_{11} g_{22} - g_{21} g_{12}} \cdot \varepsilon^{b_i - b_2}, \quad i \gg 2 \quad (5.6)$$

and

$$U_{12} = -U_{21}^* \quad (5.7)$$

It is possible to proceed systematically in this way to show that the unitary matrix U is of the form

$$U_{ij} = \omega_{ij} \varepsilon^{|b_i - b_j|} \quad (5.8)$$

The coefficients ω_{ij} are all complex numbers of order unity which are expressed in terms of the original random numbers g_{ij} of Eq. (4.2) to leading order by ^{*)}

$$\omega_{ij}^* = \frac{\det G_{ji}^{j-1}}{\det G^j}, \quad i \gg j \quad (5.9)$$

and

$$\omega_{ij} = - \sum_{k=i+1}^j \omega_{ki}^* \omega_{kj}, \quad j > i \quad (5.10)$$

Here G_{lm}^n is the $(n+1) \times (n+1)$ matrix

$$G_{lm}^n = \begin{pmatrix} G^n & (g_{rm}) \\ (g_{rs}) & g_{lm} \end{pmatrix} \quad \begin{matrix} r, s = 1, 2 \dots n \\ l, m > n \end{matrix} \quad (5.11)$$

^{*)} Degenerate values of a_i or b_i again require special treatment, although the order of magnitude estimate of Eq. (5.12) is still correct.

formed by bordering the matrix G^n of Eq. (4.4) with the first n elements of the ℓ 'th row ($g_{\ell s}$) and the m 'th column (g_{rm}) of G and setting the $(n+1, n+1)$ matrix element equal to $g_{\ell m}$.

We have therefore demonstrated that the order of magnitude of the transformation matrix element

$$U_{ij} \sim \varepsilon^{|b_i - b_j|} \quad (5.12)$$

is given by the difference in R quantum numbers between the left-handed components of the i and j quarks. On the other hand, the order of magnitude of the quark mass ratio

$$\frac{m_i}{m_j} \sim \varepsilon^{a_i - a_j + b_i - b_j} \quad (5.13)$$

is determined by the sum of the R quantum number differences for both the left- and the right-handed components. So, in general, we have (for $i > j$)

$$U_{ij} \sim \left(\frac{m_i}{m_j} \right)^{C_{ij}} \gg \frac{m_i}{m_j} \quad (5.14)$$

where

$$C_{ij} = \left(1 + \frac{a_i - a_j}{b_i - b_j} \right)^{-1} \ll 1 \quad (5.15)$$

Typically we would expect the distribution of quantum number differences to be similar for the left- and right-handed components of the observed quarks so that, on the average $C_{ij} \sim \frac{1}{2}$ and (for $i > j$)

$$U_{ij} \sim \left(\frac{m_i}{m_j} \right)^{\frac{1}{2}} \quad (5.16)$$

At this stage we must specify more precisely the transformation properties of the weak current J_μ under our Abelian charge R . We assume that J_μ commutes with R , so that the top and bottom components of each left-handed weak doublet ψ_{Li} have the same value of the Abelian charge $R_{Li} = b_i$ as already implicitly assumed in Section 4. This guarantees that

both the top and bottom quark mass matrices M_t and M_b can be written in the form of Eq. (4.2) with $a_{i+1} \geq a_i$ and $b_{j+1} \geq b_j$. The powers b_j will be the same but the powers a_i will in general be different for M_t and M_b . We see from Eqs. (4.6) and (5.12) that, apart from possible degeneracies, the ordering of top (bottom) quarks according to increasing b_i corresponds in first approximation to the ordering according to decreasing mass. Thus we have naturally introduced the fact that the weak current mainly couples a top quark to the bottom quark of the same generation, i.e., at the corresponding point in the mass hierarchy, so that except for degenerate b_i values

$$A_{ij} = \delta_{ij} + O(\epsilon) \quad (5.17)$$

Let us now examine the structure of the weak coupling matrix A using Eq. (5.7) for U_t and U_b . It immediately follows from Eq. (5.3) that A is of the same form

$$A_{ij} = \alpha_{ij} \epsilon^{|b_i - b_j|} \quad (5.18)$$

where to leading order the coefficients α_{ij} can be expressed in terms of the corresponding coefficients w_{ij}^t and w_{ij}^b for U_t and U_b ,

$$\alpha_{ij} = \sum_{k=i}^j w_{ki}^t * w_{kj}^b, \quad i \gg j \quad (5.19)$$

So the weak coupling matrix elements A_{ij} also satisfy the order of magnitude relations Eqs. (5.14) and (5.15) for both top and bottom quark masses. If top and bottom quarks of the same generation do not have masses of the same order of magnitude, a natural generalization of the statistical considerations leading to Eq. (5.16) gives

$$A_{ij} \sim \left(\frac{m_i^t m_i^b}{m_j^t m_j^b} \right)^{\frac{1}{4}}, \quad i \gg j \quad (5.20)$$

as our best a priori estimate of the weak coupling matrix in terms of quark masses. The phenomenological implications of the above form of the weak coupling matrix are the subject of the next section.

6. - CP VIOLATION AND MIXING ANGLE PHENOMENOLOGY

The weak coupling matrix for a model with n doublets of quarks can be parametrized in terms of $\frac{1}{2}n(n-1)$ generalized Cabibbo angles and $\frac{1}{2}(n-1)(n-2)$ phase parameters. In the previous section we have obtained order of magnitude estimates for the generalized Cabibbo angles but the phases, which are responsible for CP violation, are not constrained and can thus be treated as random in our approach, as we show explicitly below from the three doublet model. Therefore in general we expect large CP violation effects in weak interaction processes. The phenomenology of CP violation and quark mixing angles has already been studied in some detail by Ellis et al.²⁵⁾ in the context of a six-quark model. We shall use their results to deduce the phenomenological consequences of our estimates of the weak mixing parameters.

Firstly, let us consider the case of the one truly measured Cabibbo angle for which it is well known that the relation (5.16) is very successful phenomenologically for bottom rather than top quark masses,

$$\theta_c \sim \left(\frac{m_d}{m_s} \right)^{\frac{1}{2}} \gg \left(\frac{m_u}{m_c} \right)^{\frac{1}{2}} \quad (6.1)$$

Since the top and bottom quark mass ratios are not of the same order of magnitude in this case, our prediction would rather be

$$\theta_c \sim \left(\frac{m_u m_d}{m_c m_s} \right)^{\frac{1}{4}} = 0.12 \quad (6.2)$$

This is tolerable as an order of magnitude prediction of $\theta_c = 0.23$ but is not quite such a good estimate as

$$\theta_c \sim \left(\frac{m_d}{m_s} \right)^{\frac{1}{2}} - \left(\frac{m_u}{m_c} \right)^{\frac{1}{2}} = 0.16 \quad (6.3)$$

obtained in discrete symmetry models based on $SU_2 \times SU_2 \times U_1$.

The fact that mass ratios of top and bottom quarks of the same generation are not quite of order one (e.g., $m_c/m_s \simeq 8$) can be attributed to their right-handed components having different R quantum numbers. In order to reproduce Eq. (6.1) we might, for example, have $a_s - a_d = b_s - b_d = -b_c - b_u = 3$ and $a_c - a_u = 5$.

We have chosen rather large quantum number differences as an example, since the absence of order of magnitude quark mass degeneracies suggests a low density on the R axis of bare massless left- and right-handed quarks in the exact R symmetry limit. Approximate mass degeneracies between quarks i and j will only occur when both $a_i = a_j$ and $b_i = b_j$. However, it follows from Eq. (5.18) that the latter condition is sufficient to obtain large mixing between quarks i and j . The assumption of large R quantum number differences between successive generations implies that these conditions are unlikely to be satisfied. It also means that the symmetry breaking parameter ϵ is not a very small number so that the R symmetry is rather badly broken.

Another consequence of large R quantum number differences between successive generations is that the averaging procedure ($a_i - a_j \approx b_i - b_j$) leading to Eq. (5.16) is statistically somewhat more reliable. Also it should be quite accurate for quarks separated by many generations. A weak mixing angle of the form given by Eq. (5.20) is consistent with the known phenomenological constraints²⁵⁾ on the coupling between low and high mass quarks²⁶⁾. In order to verify this and to discuss CP violation, let us consider the weak coupling matrix for the case of six quarks^{*)} explicitly. By a suitable redefinition of the phases of the quark states, the weak coupling matrix of Eq. (5.18) can be transformed into the form

$$A' = \begin{pmatrix} e^{i\delta} & s_2 + s_3 e^{i\delta} & s_1 s_2 \\ s_3 + s_2 e^{i\delta} & 1 & s_1 \\ -s_1 s_3 & -s_1 & 1 \end{pmatrix} \quad (6.4)$$

which is the standard six-quark model parametrization of Kobayashi and Maskawa^{25),27)} in the small angle limit $\cos\theta_1 \rightarrow 1$, $\sin\theta_1 \rightarrow s_1$.

*) In our model the effective three-quark mass matrix is of our general form $\bar{M}_{ij} = \bar{g}_{ij} e^{a_i + b_j}$ and the coefficients $\bar{g}_{ij}$ can be expressed in terms of the coefficient matrix of the complete quark mass matrix Eq. (4.2). For an $(n+3) \times (n+3)$ mass matrix M , we have $\bar{g}_{ij} = \det G_{i',j'}^n / \det G^n$, $a_i = a_{i'}$, and $b_j = b_{j'}$, where $i' = n+i$ and $j' = n+j$.

In the approximation of Eq. (5.20) we can express the mixing angles s_i in terms of the quark masses,

$$\begin{aligned} s_1 &= |A_{32}| \sim \left(\frac{m_d m_u}{m_s m_c} \right)^{\frac{1}{4}} = 0.12 \\ s_2 &= \left| A_{21} - \frac{A_{31}}{A_{32}} \right| \sim \left(\frac{m_s m_c}{m_b m_t} \right)^{\frac{1}{4}} = \left(\frac{41 \text{ MeV}}{m_t} \right)^{\frac{1}{4}} \\ s_3 &= \left| \frac{A_{31}}{A_{32}} \right| \sim \left(\frac{m_s m_c}{m_b m_t} \right)^{\frac{1}{4}} = \left(\frac{41 \text{ MeV}}{m_t} \right)^{\frac{1}{4}} \end{aligned} \quad (6.5)$$

The limit $s_3^2 < 0.06$ set by the experimental validity of universality²⁵⁾ then requires the t quark mass to satisfy the order of magnitude limit

$$m_t \gtrsim 11 \text{ GeV} \quad (6.6)$$

The "leakage" to higher mass quarks will of course be even more suppressed.

For the mixing angle s_2 , Ellis et al.²⁵⁾ give the restriction

$$s_2^2 < \eta \ln \eta + \sqrt{(\eta \ln \eta)^2 + \eta} \quad (6.7)$$

from the $K_L^0 - K_S^0$ mass difference, where

$$\eta = \frac{m_c^2}{m_t^2} \quad (6.8)$$

In the limit of small η , this gives

$$s_2^2 < \eta^{\frac{1}{2}} = \frac{m_c}{m_t} \quad (6.9)$$

which should be compared with our estimate (6.5). Consistency requires

$$m_t \gtrsim m_b \cdot \frac{m_c}{m_s} = 35 \text{ GeV} \quad (6.10)$$

For higher generation quarks, the analogous result is

$$m_{t'} \gtrsim 8 m_{b'} \quad (6.11)$$

So we have verified that our generalized Cabibbo angle estimates lead to very reasonable constraints on the masses of any yet to be discovered quarks.

In the small mixing angle limit, the CP violating phase

$$\delta = \arg \left(\frac{A_{32} A_{21}}{A_{31}} - 1 \right) \quad (6.12)$$

is expected to be random in our model. We can see this by noting that, for small ϵ , the ratio $A_{32}A_{21}/A_{31}$ depends only on the complex coefficients g_{ij} and not on ϵ itself. Furthermore note that this ratio can have an imaginary part, which is in general of order unity.

However, CP violation in the $K_0 - \bar{K}_0$ system is very weak and we have ²⁵⁾ the experimental constraint

$$S_2 S_3 \sin \delta \left[-\ln \eta - 1 + \frac{S_2^2}{\eta} \right] \approx 10^{-3} \quad (6.13)$$

For small η this gives, using Eq. (6.5),

$$\sin \delta \approx 10^{-3} \frac{\eta}{S_2^3 S_3} \sim 0.03 \cdot \frac{m_c}{m_t} \quad (6.14)$$

Assuming $m_t > m_b$, this requires

$$|\sin \delta| \lesssim 8 \cdot 10^{-3} \quad (6.15)$$

whereas our above order of magnitude limits, Eqs. (6.6) and (6.10), on m_t would require

$$10^{-3} \lesssim |\sin \delta| \lesssim 3 \cdot 10^{-3} \quad (6.16)$$

This result is clearly very unlikely in our model where δ should be random. As in most other models, we have to unnaturally introduce a small CP violating parameter but this is completely against the spirit of our approach.

7. - CONCLUSIONS AND DISCUSSION

In this article, we have attempted to understand the occurrence of large quark and lepton mass ratios. Our discussion has been based on the philosophical assumption of strong naturalness. By this we mean that, at some fundamental energy scale μ_0 , all coupling constants are of order unity and can be considered as essentially random.

Initially we tried to generate the masses from running Higgs coupling constants, which were taken to be of order one and random at some very high energy scale μ_0 . However, large mass ratios do not develop naturally out of the renormalization group equations by exponentiation of the bare Higgs couplings. In order for this approach to work, it seems to be necessary to introduce a conservation law which distinguishes horizontal quark or lepton flavours dynamically, e.g., a gauge charge would do.

Since we were led to introduce new conservation laws anyway, we decided it was simpler to postulate directly the existence of quantum numbers which separate the left- and right-handed components of the fundamental fermions. A crude model of this type, based on an approximately conserved "chiral charge" R , was discussed in Section 4. Our basic ansatz Eq. (4.2) for the quark mass matrix, with random complex coefficients g_{ij} of order unity, can then be obtained from Feynman diagrams of the type shown in the figure. It is necessary to assume the existence of a large number of super heavy particles but otherwise their properties are not specified. The R symmetry breaking parameter ϵ is subject to renormalization effects and may be small not only by accident but also due to the scale dependence of the corresponding Higgs couplings ^{*)}.

*) The parameter ϵ may become large or small in the infra-red region depending on whether the gauge or the Yukawa and scalar couplings dominate.

The ansatz of Eq. (4.2) automatically gives large quark mass ratios and predicts the order of magnitude of the weak coupling matrix element A_{ij} to be given by the geometric mean of the square roots of the corresponding top and bottom quark mass ratios. This prediction is rather successful for the one measured Cabibbo angle and is not in conflict with any other weak interaction phenomenology, provided that the t quark mass lies in the range $11 \lesssim m_t \lesssim 35$ GeV. However, our ansatz also implies that the CP violating phase parameter δ of Eq. (6.2) should be of order unity. The combination of this result for δ with our estimates of the generalized Cabibbo angles is incompatible with the smallness of CP violation observed in K_L decay.

Our chiral quantum number (R symmetry) model is thus in conflict with the observed almost exact CP conservation. Does this mean that our basic philosophy of naturality or random couplings should be abandoned? We feel that it would be wrong to draw such a strong conclusion on the basis of this simple model. Therefore we now give a brief discussion of some possible modifications of the model which could rescue our philosophy. Our prediction for CP violation can be reduced by modifying our estimates of either 1) the generalized Cabibbo mixing angles or 2) the phase angle δ .

A rather obvious proposal for a mechanism of type 1) is to introduce more approximately conserved quantum numbers than just R . One may thereby obtain selection rules that separate the quarks into different groups, between which the weak mixing angles are suppressed. For example, we could imagine that the t and b quarks are strongly separated from the four lowest mass quarks u, d, s and c . Then CP violation would also be strongly suppressed. Preliminary investigations suggest that such a situation could arise quite naturally when new quantum numbers are introduced with random values.

A possible mechanism of type 2) is to introduce quark substructure. Thus we could imagine that the quarks are made up of pre-quarks which have a similar type of dynamics to the quarks themselves. Then the pre-quarks would similarly have large mass ratios, weak mixing angles and CP violating phases as predicted by our model. Even though the CP violating phases are of order unity, the lowest bound states of the pre-quarks will obey approximate CP conservation due to the mixing angles being small. These lowest bound states contain the familiar quarks, Higgs particles, etc. We may still consider the

interaction between these bound states to be random apart from the above-mentioned approximate CP conservation. This implies that we would still have a quark mass matrix of the form given in Eq. (4.2) but the otherwise random coefficients g_{ij} are restricted to be approximately real (with an appropriate choice of quark state phases). Now the Kobayashi-Maskawa CP violating phase δ is predicted to be small.

There are other possibilities of suppressing CP violation if discrete symmetries are introduced ^{7),24)}.

In conclusion we do not have to abandon our strong naturality assumption in spite of the initial difficulties with reconciling large mass ratios and the experimental size of CP violation.

ACKNOWLEDGEMENTS

It is a pleasure to thank our colleagues in Glasgow, CERN and Copenhagen for helpful discussions. We would especially like to thank N. Brene, M.K. Gaillard, R.G. Moorhouse, C. Sachrajda, D. Sutherland and G. Zoupanos. One of us (H.B.N.) wants to acknowledge financial support from the UK Science Research Council for a visit to Glasgow University.

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FIGURE CAPTION

Feynman diagram which generates the quark mass matrix element $M_{t,ij}$. Full lines represent quarks, wavy lines represent super heavy particles and dashed lines represent ϕ_1 tadpoles. The Yukawa couplings are assumed to be of order unity (and random) at the fundamental scale μ_0 .