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ON THE CHALLENGE OF MEASURING THE COLOUR CHARGE OF GLUONS

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ABSTRACT

We propose two ways of measuring "directly" the chromodynamic gluon self-couplings. We argue that the measurements would serve to establish (or disprove) the gauge and non-Abelian nature of the candidate strong interaction theory: QCD. Both experiments concern the production in e^+e^- annihilation of a heavy vector meson resonance, followed by its hadronic decay into three "gluon jets". The asymmetry of the decay plane relative to the electron axis is a measure of $\alpha_s = g^2/4\pi$, with g the coupling for gluon self-interactions. If QCD is right, this value of g should coincide with the measured coupling of quarks to gluons. The two proposed experiments are difficult: they both demand very large statistics, one requires longitudinally polarized beams, the other the detection of weak-electromagnetic interferences. But the stakes are high.

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1. INTRODUCTION

The QCD Lagrangian for n quark flavours:

$$\mathcal{L} = \sum_{f=1}^n \bar{q}_f (i \not{D} - m_f) q_f - \frac{1}{4} F_{\mu\nu}^a F_a^{\mu\nu} \quad (1a)$$

$$D_\mu \equiv \partial_\mu - ig A_\mu^a \frac{\lambda^a}{2} \quad (1b)$$

$$F_{\mu\nu}^a \equiv \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + \tilde{g} f_{bc}^a A_\mu^b A_\nu^c \quad (1c)$$

contains, besides the coupling of quarks to gluons, characteristically non-Abelian cubic (and quartic) gluon self-interactions. Gauge invariance demands that these couplings have the same strength g (or g^2) as the quark-gluon vertex: $\tilde{g} = g$ in Eq. (1). Many of QCD's most welcome properties (i.e., renormalizability, asymptotic freedom) and dogmatic hopes (i.e., quark confinement) depend on the existence of these very specific gluon self-couplings. The quark-gluon coupling constant has been "directly" measured in studies of the breakdown of scaling in electroproduction¹⁾ and other reactions^{2,3)}. The non-Abelian gluon couplings enter this analysis to higher orders of perturbation theory. In practice they are reflected in the assumed logarithmically decreasing form (as a function of momentum transfer) of the renormalization group improved effective quark-gluon coupling constant:

$$\alpha_s(Q^2) \equiv \frac{g^2}{4\pi} \simeq \frac{12\pi}{(33-2n) \ln Q^2/\Lambda^2} \quad (2)$$

where n is the number of quark flavours with mass $m_f \lesssim Q/2$ ²⁾ and Λ is a parameter whose measured value is $\Lambda \sim 600 \text{ MeV}$ ¹⁾. To date, the most precise determination of Λ or, equivalently, α_s comes from the analysis of scaling deviations in electroproduction at moderate energies. To observe the logarithmic variation of the coupling constant itself, as a way to "measure" the gluon self-couplings, would require experiments of unprecedented precision. Thus, we have to date a very indirect access to gluon-gluon interactions: they have been "observed" only in as much as they enter the justification of a predictive and successful use of QCD perturbation theory. It is not difficult to imagine processes where the interactions of gluons among themselves play a less remote role than in the study of deviations from scaling. An example is the observation of high p_T jets in

hadron-hadron collisions⁴⁾, for which a diagram containing a cubic gluon coupling is shown in Fig. 1a. To disentangle the gluon self-couplings from measurements in such processes would pose at least two problems: the gluon momentum distributions within hadrons are not theoretically known, there are competing diagrams, as the ones in Fig. 1b that lead to the same effects. Another example, where gluon self-couplings may be relevant, is the "four jet" cross-section in e^+e^- annihilation (see Fig. 1c). Here, one would again face the problem of competing gauge-invariance related diagrams (Fig. 1d) besides the necessity of enormous statistics and large energies.

It is of interest to find observables that "directly" measure the coupling of gluons to themselves, that do not crucially depend on unknown hadron "wave functions" and that do not require the singling-out of a process from amongst competing ones. In this note we propose two measurements that would do just that. The corresponding experiments are painstaking and difficult, but feasible. A non-zero result of these experiments would constitute a strong indication of the non-Abelian nature of colour charge. A result that establishes the identity of quark-gluon and gluon-gluon coupling strengths would be a strong indication of the gauge principle underlying the QCD Lagrangian.

The observables we discuss concern the production and hadronic decay of heavy quark-antiquark vector meson states (V) in e^+e^- annihilation. In leading order of QED and QCD perturbation theory, the process is

$$e^+(p_+) e^-(p_-) \rightarrow \gamma^*(Q) \rightarrow V \rightarrow G(k_1) G(k_2) G(k_3)$$

where the G 's denote gluons and the quantities in parentheses are four-momenta. Define the customary scaled energies

$$x_i \equiv 2Q \cdot k_i / Q^2 \quad (3a)$$

$$x_1 + x_2 + x_3 = 2 \quad (3b)$$

and the differential decay rate of the vector meson $d\Gamma/dx_1 dx_2$. In today's state of development of QCD, we can only reliably calculate quantities that are insensitive to its incompletely understood non-perturbative aspects. The differential decay rate of a vector meson is, in this sense, not calculable for, even in a non-relativistic treatment of the quark bound state, it depends on an obviously non-perturbative piece of information: the quark bound state wave function. It has been argued⁵⁾ that, much the same as in QED, our non-perturbative

ignorance drops from some suitably defined ratios. An example is $\Gamma^{-1} d\Gamma/dx_1 dx_2$, where Γ is the total decay rate into gluons. At this point one faces another blow from our ignorance of non-perturbative QCD: gluons are not particles actually observed in V decays. It is generally hoped that for sufficiently heavy V 's, the resulting gluons, not unlike quarks, will materialize in the form of jets. These jets may be observable on an event-by-event basis for very heavy V 's and are expected to be observable⁵⁾ on a statistical basis for V 's as light as the T . The details of particle distributions within jets obviously depend on the non-perturbative effects that give rise to the observed bound states of quarks. But!, the argument goes^{6), 5)}, one may trade in a jet for a single over-all information: a directed energy flow. This directional energy can allegedly be identified with the momentum of the hypothetical QCD quantum that gave rise to the jet⁵⁾. The perturbative prediction of $\Gamma^{-1} d\Gamma/dx_1 dx_2$ would thus be a reliable prediction of a normalized three-jet cross-section. The quoted arguments are not proofs, for there may be unforeseen surprises in the non-perturbative properties of QCD. To proceed, we accept the arguments and treat jet cross-sections as calculable in terms of QCD quanta.

2. DEFINITION OF THE NORMAL TO THE HADRON DECAY PLANE OF A HEAVY VECTOR MESON V

To specify the observables that will concern us, we need to define the normal to the three-gluon decay plane and give it a meaning in terms of the particles actually observed. Our convention for the normal is as follows: we distinguish the gluons by ordering their fractional momenta $x_1 > x_2 > x_3$ and we define the normal to the decay plane in the direction of $\vec{k}_1 \times \vec{k}_2$, with the right-hand screw rule. In Fig. 2a the normal sinks into the paper. For a particular event $V \rightarrow$ hadrons, we define an event axis, an event plane, and three jet-candidate momenta as follows (watch Fig. 2b). Define a plane dividing space into two hemispheres and an axis normal to this plane. The event axis is defined by maximizing the sum $\sum_i |\vec{p}_{\parallel}^i|$ of momenta parallel to the axis. By convention, the axis points into the hemisphere (labelled "hard" in the figure) where the total momentum $\sum_i |\vec{p}_{TA}^i|$ transverse to the axis is minimum. This oriented axis will be the direction of the "hardest jet". The event plane contains the axis and is defined by minimizing the total momentum $\sum_i |\vec{p}_{TP}^i|$ transverse to the plane. There remains a two-fold ambiguity: the sign of the normal $\vec{n}$ to the decay plane. To remove it, we need to identify an "intermediate" and a "softest" jet candidate. This is done by requiring $|\sum_i \vec{p}_i(\text{INTERMEDIATE})| > |\sum_i \vec{p}_i(\text{SOFT})|$ as in Fig. 1b. The momenta of the three massless gluons that allegedly underlie the event are identified as sums of the momenta of the real particles in the corresponding parts of the decay plane

$\vec{k}_1 = \sum_i \vec{P}_i$ (HARD), $\vec{k}_2 = \sum_i \vec{P}_i$ (INTERMEDIATE), $\vec{k}_3 = \sum_i \vec{P}_i$ (SOFT). All momenta have been projected into the decay plane. The x_i 's are defined as in Eq. (3). Notice that by construction $x_1 > x_2 > x_3$. The normal to the decay plane is defined along the direction $\vec{k}_1 \times \vec{k}_2$. The "thrust" is defined as the x value of the leading jet, $T = x_1$ ⁷⁾, and is kinematically constrained to the range $2/3 < T < 1$. For $T = 1$ the gluons are aligned or one is soft: events will look like two-jet events. For $T = 2/3$ the gluons are 120° apart but the cross-section vanishes⁵⁾. The interesting region is in between. The explicit calculations of Ref. 5) indicate that at the mass of the T and for $T < 0.9$ the normal to the hadron decay plane will be definable in practice. For $1 > T > 0.9$ events are too similar to two-jet events and the $e^+e^- \rightarrow \gamma \rightarrow q\bar{q}$ background is too important for the normal to be unambiguously defined. For hypothetical vector mesons heavier than the T , the relative number of events with a well-defined normal increases.

3. THE ASYMMETRY OF THE HADRON DECAY PLANE FOR LONGITUDINALLY POLARIZED e^+e^- BEAMS

We are now in a position to discuss our first proposed measurement. Let the colliding electron and positron be longitudinally polarized with opposite helicities (same "spin" direction $\hat{s}$) as in Fig. 2c, where $\hat{s}$, for the sake of definiteness, has been chosen to lie along the e^- momentum. The leptons in e^+e^- facilities naturally tend to polarize themselves in the direction of the magnetic field; transversally, that is. But the feasibility and potential advantages of rotating the natural polarization have been discussed in the literature in optimistic terms⁸⁾. For beams polarized with opposite helicities the resulting vector meson V is polarized. Is this polarization information transmitted to the V decay products as an orientation of the normal to the decay plane? It is on the affirmative answer to this leading question that our results are based.

Let θ , as in Fig. 2c, be the angle between $\vec{k}_1 \times \vec{k}_2$ and $\hat{s}$ and define the asymmetry:

$$A(x_1, x_2, \cos\theta) = \frac{\frac{d\sigma(\cos\theta = |\cos\theta|)}{dx_1 dx_2 d\cos\theta} - \frac{d\sigma(\cos\theta = -|\cos\theta|)}{dx_1 dx_2 d\cos\theta}}{\frac{d\sigma(\cos\theta = |\cos\theta|)}{dx_1 dx_2 d\cos\theta} + \frac{d\sigma(\cos\theta = -|\cos\theta|)}{dx_1 dx_2 d\cos\theta}} \quad (4)$$

This asymmetry or the related expected value $\langle \vec{k}_1 \times \vec{k}_2 \cdot \vec{s} \rangle$ are "T-odd" observables: quantities that change sign under the operation of reversing spins and momenta. Denote this operation by $i \rightarrow \tilde{i}$, $f \rightarrow \tilde{f}$ for initial and final states. Let T_{fi} be the scattering amplitude. T-odd observables are proportional to $|T_{fi}|^2 - |T_{\tilde{f}\tilde{i}}|^2$. In a theory that respects time reversal invariance $|T_{fi}|^2 = |T_{\tilde{f}\tilde{i}}|^2$, but T-odd observables, in the definition of which the rôle of initial and final states is not interchanged, need not vanish. It is generally well known that in a time-reversal invariant theory T-odd observables are linear and quadratic in the absorptive part $\mathcal{A}$ of the final state interaction⁹⁾

$$|T_{fi}|^2 - |T_{\tilde{f}\tilde{i}}|^2 = -2 \operatorname{Im}(T_{fi}^+ \mathcal{A}_{fi}) - |\mathcal{A}_{fi}|^2 \quad (5a)$$

$$\mathcal{A}_{fi} \equiv \sum_n T_{nf}^+ T_{ni} \quad (5b)$$

In QCD perturbation theory the T-odd asymmetry a of Eq. (4) is of order $g^2/4\pi = \alpha_s$. To this order it comes entirely from the first term in the right-hand side of Eq. (3a) and the relevant diagrams are shown in Fig. 3. Notice that g^2 or α_s refer to the gluon self coupling, the quark-gluon coupling drops from the ratio that defines the asymmetry. It is in this sense that the asymmetry directly measures the gluon self-interactions. The asymmetry arises from a final-state interaction between the three gluons in the V decay.

Before we proceed to compute a explicitly in QCD perturbation theory we must pause to comment on why the perturbative calculation should be a sensible prediction of the effect. A purist may be satisfied with a brief answer: the result is infra-red insensitive^{6),10)}. But we attempt to answer, this time in detail, two questions that already surfaced in the introductory discussion on the calculation of jet cross-sections.

First, why do the details of the vector meson wave function (the sombre boxes of Fig. 3) not influence the result? To answer this question we first assume that the quark-antiquark bound state is non-relativistic. To first non-trivial order of perturbation theory, both the numerator and the denominator in the definition of the asymmetry Eq. (4) are proportional to the fourth power of the wave function at the origin $|\psi(0)|^4$. Knowledge of this quantity would require the solution to the bound state problem: a long-distance non-perturbative problem that we are still unable to solve in QCD. But $|\psi(0)|^4$ drops from the ratio that defines the asymmetry. The rest of the calculation concerns the annihilation cross-section $q\bar{q} \rightarrow G_1 G_2 G_3$ at a mass scale of order Q . This short-distance problem is tractable

in perturbation theory and will constitute our result. One may wonder whether the factorization of the scattering amplitude into a yet incalculable long-distance part and a perturbatively calculable short-distance part survives to the first non-trivial order of perturbation theory. This is known to be the case in positronium annihilation, the analogous QED problem. We assume factorization also occurs in QCD. Our result for the asymmetry is then correct to $O(\alpha_s)$. Non-relativistic corrections to the quark-antiquark bound state wave function are of order $\langle p^2 \rangle / 2m_q^2 \sim (Rm_q)^{-1}$, with R the mean radius of the states. These corrections are presumably tiny for heavy V 's and we neglect them.

Second, why do the final state interactions of real outgoing particles (or, equivalently, higher order and non-perturbative effects in QCD) not affect the asymmetry dramatically? Higher orders in QCD only introduce corrections that are down by powers of α_s , since the jet cross-section on which the calculation is based is infra-red finite, provided the jets are defined à la Sterman-Weinberg⁶⁾ as fractions of energy flowing in certain finite aperture cones. In a complementary language, for the interactions between real outgoing particles to affect the asymmetry, they would have to tilt the hadron decay plane. This plane is defined in terms of jet momenta of order Q . To tilt it requires exchanges of hard quanta (momentum of order Q). The QCD calculation took care of these, any remaining effects in the conversion of QCD quanta into real particles are expected to be soft. Otherwise, the whole predictive power of perturbative QCD would collapse. Quark jets in e^+e^- annihilation (away from resonances) with a distribution $1 + \cos^2 \theta$, would not have been observed; just to give an example. Let N be the particle multiplicity of a certain value of Q . The interactions between real outgoing particles, if the corresponding phase shifts add randomly, would only produce an asymmetry a of the rough order of magnitude:

$$a(\text{non perturbative}) \sim a(\text{perturbative}) \frac{1}{\alpha_s N^2} \sqrt{\frac{N(N-3)}{6}} \quad (6)$$

This non-perturbative asymmetry is at the level of 10-20% of the perturbative QCD asymmetry for V 's as heavy as the T , or heavier.

To compute a explicitly in QCD perturbation theory, we approximate the vector meson V as a non-relativistic bound state of quarks. The amplitude for $\gamma^* \rightarrow V \rightarrow G_1^a G_2^b G_3^c$ is then the classical Ore-Powell amplitude for orthopositronium decay¹¹⁾, times a colour factor d^{abc} . Let V have a polarization vector ϵ , let the gluons have a colour index and polarization vector ϵ^a , and momenta k_i , $i = 1, 3$; $k_1 + k_2 + k_3 = Q$. The decay amplitude is¹¹⁾

$$M \propto \frac{d^{abc}}{Q \cdot K_1 Q \cdot K_2 Q \cdot K_3} \cdot [F \cdot F_1^a F_2^b F_3^c + \tilde{F} \cdot F_1^a \tilde{F}_2^b F_3^c + \text{cyclic}(1,2,3)] \quad (7)$$

where

$$F \cdot F_i \equiv F_{\mu\nu} \cdot F_i^{\mu\nu} \quad (8a)$$

$$F_{\mu\nu} = Q_\mu \epsilon_\nu - Q_\nu \epsilon_\mu \quad (8b)$$

$$F_{i,\mu\nu}^a = K_\mu^i \epsilon_\nu^a - K_\nu^i \epsilon_\mu^a \quad (8c)$$

$$\tilde{F}_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} F^{\rho\sigma} \quad (8d)$$

$$\tilde{F}_{i,\mu\nu}^a = \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} F_i^{a,\rho\sigma} \quad (8e)$$

Insertion of Eqs. (7) and (8) into the graphs of Fig. 3, followed by a straightforward but somewhat lengthy calculation¹²⁾ gives the result:

$$a(x_1, x_2, \cos\theta) = -\alpha_s \frac{\cos\theta}{2 + \sin^2\theta} \cdot \frac{n(x_1, x_2)}{d(x_1, x_2)} \quad (9)$$

where, if the QCD prediction for the gluon self-couplings is right, α_s is given by Eq. (2) and

$$\begin{aligned} n(x_1, x_2) = & \frac{6 [(1-x_1)(1-x_2)(1-x_3)]^{1/2}}{x_1 x_2 x_3} \left\{ \frac{x_1 - x_2}{x_3^2} + \frac{(x_1 - x_2)(1-x_3)(2+x_3^2)}{(2-x_3) x_3^4} \ln(1-x_3) \right. \\ & \left. - \frac{(1-x_3)^2}{(1-x_1)(1-x_2)x_3^2} [x_3(\ln x_1 - \ln x_2) + (x_1 - x_2) \ln(1-x_3)] \right\} + \text{cyclic}(1,2,3) \end{aligned} \quad (10a)$$

$$d(x_1, x_2) = \frac{(1-x_1)^2}{x_2^2 x_3^2} + \text{cyclic}(1,2,3) \quad (10b)$$

The asymmetry $a(x_1, x_2, \cos \theta)$ may be too "exclusive" a quantity to be measured with adequate statistics. Thus, we define an asymmetry $A(T, \cos \theta)$ for events with fixed thrust $T = x_1$ and x_2 integrated over its kinematical range:

$$A(T, \cos \theta) \equiv \frac{\frac{d\sigma(\cos \theta = |\cos \theta|)}{dT d\cos \theta} - \frac{d\sigma(\cos \theta = -|\cos \theta|)}{dT d\cos \theta}}{d\sigma/dT d\cos \theta} = \quad (11)$$

$$= \alpha_s \frac{\cos \theta}{2 + \sin^2 \theta} \cdot \frac{n(T)}{d(T)}$$

An even more inclusive quantity is a "forward-backward" asymmetry of the normal to the decay plane in which the angular dependence has been integrated in the appropriate hemispheres:

$$A_{FB}(T) = \frac{d\sigma(\cos \theta > 0)/dT - d\sigma(\cos \theta < 0)/dT}{d\sigma/dT} = \alpha_s \frac{3}{16} \frac{n(T)}{d(T)} \quad (12)$$

The analytic expression for $n(T)$ is too cumbersome to be useful. We have thus plotted $n(T)$, $d(T)$ and $n(T)/d(T)$ in Figs. 4a, b, and c, respectively. The solid lines are the QCD predictions. The process that converts QCD quanta into jets endows the outgoing particles with a small but finite p_T relative to the jet axis. This tends to shift the observed thrust distribution towards smaller thrust values⁵⁾ by an amount of the order $\sim \langle N(Q) \rangle 400 \text{ MeV}/2Q$. We do not expect this effect to be important for the ratio $n(T)/d(T)$.

A word on numerics is now in order. Let $V = T$ be the chosen vector meson. Suppose one were to select events in the range $0.8 < T < 0.9$ for which the hadron decay plane can be determined and the cross-section is a relatively large (20%) fraction of the total cross-section. Substituting $T \approx 0.87$ in Eq. (12) and $Q = 6$ in Eq. (2) we find $A_{FB} \sim 0.3\%$. Thus the measurement of a total of ten T decays is needed to establish the effect at the level of 2σ away from zero. To establish the non-Abelian nature of QCD is no easy task! On the other hand, the measured asymmetry may turn out to be much bigger than predicted, thus invalidating perturbative calculation. In this sense, the smallness of the predicted effect is no reason for despair: it may be very difficult to check that the prediction is right, but it may be very simple to check that it is wrong. The reason why the QCD perturbative asymmetry turns out to be a small fraction of α_s is kinematical. By construction of the observable at hand, the numerator function Eq. (10a) vanishes on the edges of the (x_1, x_2, x_3) "Dalitz plot",

as well as on its medians ($x_1 = x_2$, $x_1 = x_3$, $x_2 = x_3$), while the denominator function Eq. (10b) does not.

For a long time now, the hypothetical reader may have been wondering why we have not been considering transversally polarized lepton beams, a strong transverse polarization being provided for free in colliding ring facilities. We now proceed to dissipate her (his) doubts: the T-odd asymmetries for transversally polarized beams vanish as m_e/Q .

4. THE ASYMMETRY OF THE HADRON DECAY PLANE FOR e^+e^- BEAMS NOT LONGITUDINALLY POLARIZED

We now propose a different observable that may be used to measure the gluon self-couplings with unpolarized or transversally polarized electron beams. This is the asymmetry of the hadron decay plane on a V resonance relative to the electron momentum, related to a non-zero expectation value $\langle \vec{k}_1 \times \vec{k}_2 \cdot \vec{p}_e \rangle$. This observable, besides being T-odd, is parity-odd. It has the advantage of not requiring beam polarization gymnastics and the disadvantage of relying on the interference between weak and electromagnetic interactions. For the effect to be large it is necessary that weak neutral currents violate parity and that Nature be nice enough to provide us with a quark bound state heavier than an intermediate vector boson Z (or lighter within a few dozen GeV). For the sake of illustration we compute the effect in a one Z theory. The $e^+e^- \rightarrow q\bar{q}$ amplitude is

$$\begin{aligned} \mathcal{M} = & \frac{C e^2}{Q^2} \bar{e} \gamma_\mu e \bar{q} \gamma^\mu q + \frac{e^2}{Q^2 - M_Z^2 + i M_Z \Gamma_Z} \bar{e} (g_V \gamma_\mu + g_A \gamma_\mu \gamma_5) e * \\ & * \bar{q} (G_V \gamma_\mu + G_A \gamma_\mu \gamma_5) q \end{aligned} \quad (13)$$

where C is the quark electric charge and the rest of the notation is self-explanatory. Define

$$V \equiv \frac{C}{Q^2} + \frac{g_V G_V}{Q^2 - M_Z^2 + i M_Z \Gamma_Z} \quad (14a)$$

$$A \equiv \frac{G_V g_A}{Q^2 - M_Z^2 + i M_Z \Gamma_Z} \quad (14b)$$

$$R \equiv 2 \operatorname{Re} V A^* / (|A|^2 + |V|^2) \quad (14c)$$

and define asymmetries of the hadron decay plane exactly as in Eqs. (4), (10a) and (10b). The angle θ is now the angle between the normal to the hadron decay plane and the direction of flight of the electron. The prediction for this parity-odd, T-odd asymmetry is then R times the prediction for the corresponding T-odd asymmetry discussed before. To illustrate these predictions we use the standard model, for which:

$$g_A = G_A = -1/2 \quad (15a)$$

$$g_V = -\frac{1}{2} + 2\sin^2\theta \quad (15b)$$

$$G_V(c = \frac{2}{3}) = \frac{1}{2} - \frac{4}{3}\sin^2\theta \quad (15c)$$

$$G_V(c = -\frac{1}{3}) = -\frac{1}{2} + \frac{2}{3}\sin^2\theta \quad (15d)$$

$$M_Z = 37.3 \text{ GeV} / (\cos\theta \sin\theta) \quad (15e)$$

In Fig. 5 we show the function $R([M_V/M_Z]^2)$ for quarks of charge $C = 2/3, -1/3$, $\sin^2\theta = 0.25$, and $\Gamma_Z = 1.5 \text{ GeV}$. It can be seen in the figure that the T-odd, parity-odd asymmetry is comparable to the T-odd, parity-even asymmetry discussed in the previous section only if the quark bound state vector meson V has a mass comparable or higher than the mass of weak intermediary Z . Should such a heavy V exist, the interference between weak and electromagnetic interactions may help in studying the detailed structure of the strong interactions.

5. CONCLUSIONS

We set out to find an experimental indication of an extremely specific property of QCD: the vector bosons of the theory are coloured, and act as sources of their own field. We have attempted to find "clean" tests of this property of a non-Abelian theory, tests where we can argue that only the well-understood perturbative properties of QCD play a rôle. With such strictures it is not surprising that we turned out to find experimental tests that are very hard to carry out in practice. Fortunately, they are not unthinkable hard. This is our main conclusion.

If the tests we propose are carried out and agree with our predictions, we would no doubt triumphantly conclude that the colour "charge" of gluons has been measured, that the non-Abelian nature of QCD has been vindicated. Unfortunately,

at this point in time, we cannot state the contrary with similar confidence. Suppose the tests are carried out and they disagree with the perturbative QCD prediction. Should this be considered as a deadly blow to QCD? Unfortunately not. The point is that we have argued, but not proved, that the perturbative QCD prediction is insensitive to the details of the incompletely understood non-perturbative aspects of the theory. The observables we discuss are infra-red insensitive. This is clearly a necessary but perhaps not a sufficient criterion to disregard non-perturbative effects. It all depends on how peculiar and violent these are. Hopefully, by the time our proposed measurements are carried out, we will know much more on this subject. Even if theorists remain unable to fathom the non-perturbative aspects of QCD, experiment would have cleared out some crucial points. One of them is whether or not hard gluons manifest themselves as jets.

On the practical side, we have two conclusions to offer. First "turning" the beam polarization in e^+e^- colliding rings from transverse to longitudinal has other advantages besides the well-advertised ones concerning the study of weak interactions. Second, if very heavy quark-antiquark bound states exist, parity violation by the weak interactions may serve as a tool to establish or disprove the non-Abelian nature of the strong interactions, a very perverse thought.

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REFERENCES

- 1) For a detailed study of scaling breakdown in electroproduction see, for instance : A. De Rújula, H. Georgi and H.D. Politzer - Ann.Phys.(NY) 103, 315 (1977).
For a recent measurement of Λ see : H.L. Anderson, H.S. Matis and L.C. Myriantopoulos - Phys.Rev.Letters 40, 1061 (1978).
- 2) A. De Rújula and H. Georgi - Phys.Rev. D13, 1296 (1976) ;
E.C. Poggio, H.R. Quinn and S. Weinberg - Phys.Rev. D13, 1958 (1976) ;
O. Nachtmann and W. Wetzel - Heidelberg Preprint HD-THEP-78-3 (1978).
- 3) P.C. Bosetti et al. - Oxford University Preprint 16/78 (1978).
- 4) R.P. Feynman, R.D. Field and G.C. Fox - Caltech Preprint CALT 68-651 (1978).
- 5) A. De Rújula, J. Ellis, E.G. Floratos and M.K. Gaillard - CERN Preprint TH.2455 (1978), to be published in Nuclear Phys. ;
G. Parisi - Ecole Normale Supérieure, Paris LPTENS 78/9 (1978).
- 6) G. Sterman and S. Weinberg - Phys.Rev.Letters 39, 1436 (1977) ;
G. Sterman - Stony Brook Preprints ITP-SB-77-69 (1977) and ITP-SB-77-72 (1977) ;
H. Georgi and M. Machacek - Phys.Rev.Letters 39, 1237 (1977).
- 7) E. Farhi - Phys.Rev.Letters 39, 1587 (1977).
- 8) R.F. Schwitters - PEP Note 75.
- 9) A. De Rújula, J.M. Kaplan and E. De Rafael - Nuclear Phys. B35, 365 (1971).
- 10) H.D. Politzer - Nuclear Phys. B129, 301 (1977) ;
C.T. Sachrajda - Phys.Letters 73B, 185 (1978) and 76B, 100 (1978) ;
D. Amati, R. Petronzio and G. Veneziano - CERN Preprint TH. 2470 (1978), to be published in Nuclear Phys. B., and in preparation ;
R.K. Ellis, H. Georgi, M. Machacek, H.D. Politzer and G.G. Ross - MIT Preprint CTP-768 (June 1978).
- 11) A. Ore and J.L. Powell - Phys.Rev. 75, 1696 (1949).
- 12) The QCD Feynman rules for gluon self-couplings contain many terms and so does the amplitude of Eq. (6). The number of terms in a brute force calculation (after the colour sums have been performed) is of the order of half a million. We did the calculation with the help of the algebraic program SCHOONSCHIP, written by M. Veltman. We checked it by hand for the four-gluon coupling insertion of Fig. 3a.

FIGURE CAPTIONS

Figure 1 a) and c). Examples of diagrams where triple-gluon couplings play a rôle. b) and d). Examples of competing diagrams.

Figure 2 a) Three gluons as decay products of a quark-antiquark vector meson resonance. b) The hadron decay plane with hard, intermediate and soft jet-candidates. c) The hadron decay plane and its normal, and their position relative to the e^+e^- beam axis.

Figure 3 Diagrams contributing to the numerator and denominator in the asymmetry of the normal to the hadron decay plane relative to the electron helicity or momentum.

Figure 4 a) The numerator $n(T)$ of Eqs. (11) and (12). b) The corresponding denominator of $d(T)$. c) Their ratio. T is thrust.

Figure 5 The ratio R of Eq. (14c) of the pseudoscalar T -odd asymmetry to the scalar T -odd asymmetry, as a function of the ratio of V mass to Z mass. See text.

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5