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PROCEEDINGS OF THE LEP SUMMER STUDY

Les Houches and CERN
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document.
These Proceedings are dedicated to the memory of Bruno Touschek, whose name is so closely associated with the early developments of electron storage rings.
ABSTRACT

Options for the future major high-energy accelerator in Europe have converged on LEP, a large electron-positron storage-ring facility. The purpose of the LEP Summer Study was to discuss $e^+e^-$ physics beyond PETRA energy ($2 \times 19$ GeV) and to assess the LEP design which had just evolved from work conducted at CERN. Apart from the overall summary report, a set of specialized reports deal with present theoretical expectations, anticipated rates and experimental conditions, and with the design of the machine and its interaction areas. Volume 1 contains summary reports presented at CERN as conclusions to the Summer Study. Volume 2 contains transcripts of review talks presented at Les Houches, sometimes combined into reports providing a global view of specific topics. Some additional specialized notes are also included. Material on machine design is incomplete, to avoid duplication with the design report CERN/ISR-LEP/78-17.
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FOREWORD

The LEP Summer Study consisted of two parts. The first and longest part was a meeting at Les Houches, France, on the premises of the Summer School of Theoretical Physics; it extended over 10 days and brought together close to 80 participants. The second part was a two-day meeting at CERN, immediately following the Les Houches meeting, during which the conclusions were presented to and debated by a very wide audience of physicists and engineers.

The purpose of the Summer Study was: to discuss $e^+e^-$ physics beyond PETRA energies in the most general sense, and to assess the particular LEP design which had originated from the studies conducted at CERN. The Les Houches discussions on physics at LEP energies were prepared by a few specialized Working Groups which met throughout the summer. Work on the machine design was conducted at CERN and resulted in a report, now widely known as the "Blue Book", CERN/ISR-LEP 78-17, "Design Study of a 15 to 100 GeV $e^+e^-$ Colliding Beam Machine (LEP)", which was presented and discussed in detail at Les Houches.

The present proceedings are presented in two Volumes. The first contains the summary reports presented during the CERN part of the Summer Study by the Conveners responsible for the five specialized Working Groups at Les Houches. In addition to these, there is a special report on polarization at LEP energies and the transcripts of the talks given by J.B. Adams, S.L. Glashow and M. Vivargent. Also included are a few ECFA/LEP Notes, adjacent to the relevant reports, which the rapporteurs found appropriate for inclusion since they provide important information not reported in sufficient detail in the summary reports.

Volume 2 comprises the reports corresponding to the plenary talks presented at Les Houches. The purpose of these talks was to summarize the work carried out in preparation for the Summer Study and to provoke and add sparkle to the discussion within the specialized Working Groups. This Volume begins with reports on topical questions connected with the LEP machine design. These include, in particular, a discussion on superconducting accelerating cavities. Next follows a review of the PETRA and PEP programmes: this topic actually opened the meeting at Les Houches. Following these reports are discussions covering experimental and theoretical aspects of physics at LEP energies: these cover in greater detail some of the points to be found in Volume 1. Present ideas about detectors are an important input to the discussions of physics at LEP and two specialized reports on detectors conclude the Volume. Volume 2 also includes some ECFA/LEP Notes covering specific experimental questions.
Since it was intended that these proceedings should avoid duplicating material previously presented in the "Blue Book", it therefore needs to be consulted in conjunction with that report. As a result, the contributions from H. Hoffmann, E. Keil and W. Schnell at Les Houches are not included in these proceedings, whereas those from W. Bauer and J. Le Duff, being largely complementary to the contents of the "Blue Book", are included. It should also be mentioned that, since these proceedings combine the summary talks presented at CERN and the talks presented at Les Houches which surveyed specific topics prior to discussions within the specialized Working Groups, they are bound to contain some overlap. Nevertheless, it was thought appropriate to present some of the important questions at different levels of detail and technicality, rather than merely try to summarize them only once, as was in fact brilliantly achieved by the Conveners in their CERN summary talks.

Mention is made throughout the proceedings of the ECFA/LEP Notes. Altogether close to 50 specialized ECFA/LEP Notes have been written in connection with the Summer Study, the valuable information contained therein being incorporated in the different reports. The complete list of Notes written up to 1st January, 1979 may be found in Appendix D of the Summary Report by M. Jacob, LEP Summer Study/1-1. Finally, a few photographs are included which may help to recreate the congenial and studious atmosphere at Les Houches.

Maurice Jacob and Christine Redman would like to take this opportunity to express their thanks to all those inside and outside CERN whose work and help have been instrumental in organizing the Summer Study and in bringing these proceedings to their present form.
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LOOKING BACK ON LES HOUCHES
Debating Machine design in small and extended circles.
At lunch and coffee time
Three-body interactions on LEP issues
As S. L. Glashow put it
"There is no conceivable scenario in which LEP is anything but maddeningly exciting."
MACHINE DESIGN*

Comments on the Machine Design Study at Les Houches
S. Tazzari**

Design Constraints in Large $e^+e^-$ Storage Rings
J. Le Duff***

Superconducting Accelerating Cavities for High-Energy $e^+e^-$ Storage Rings
W. Bauer****

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* This report contains material presented at Les Houches covered neither by the Blue Book (CERN/ISR-LEP/78-17) nor by the specialized report on Site and Buildings (LEP Summer Study/1-12).

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Copies available upon request from Ch. Redman, CERN/ISR, LEP Summer Study Secretariat.
The energy, luminosity and the sheer size of LEP -- a project conceived at a time when the previous generation of machines are still in the testing or construction stage -- have confronted the designers with a number of new technical and cost problems. At the same time, LEP being one of the few machines ever designed to investigate very specific and exciting theoretical predictions, the need for all interested physicists to participate in the definition of an optimized design was especially strongly felt. This is why a distinctive feature of the Les Houches Summer Study has been the large amount of time devoted to machine physics and techniques.

On the machine side, the greatest part of the work had been done in advance and was available in the form of a very detailed design study by the LEP Study Group, the "Blue Book"; five plenary talks by E. Keil, W. Schnell, H. Hoffmann, B. Montague and C.J. Zilverschoon were devoted to the presentation of its contents. In addition, the difficult task of reviewing for the non-specialists (and the specialists alike) the methods by which an optimized design is arrived at, the experimental evidence on which the main assumptions are based, and the constraints to which the designer is subject when trying to reconcile high luminosity with high energy, reliability, efficiency, and minimum cost, was undertaken by Le Duff in his review talk on Machine Constraints presented here.

From the very lively discussions that followed all papers (so lively that the schedule had to be reshuffled more than once) the 70 GeV conventional RF machine, together with its proposed extension to 100 GeV by gradual conversion to a superconducting RF system, emerged as a sound starting point. The questions raised provided valuable input to the machine working group and are well reflected in the Summary Report.

Two issues stood out, in our opinion, as dominant: the need for experimental data from machines nearer in size to LEP in order to better assess the potential of the proposed design; and the request for the proposed extension to 100 GeV to become an essential part of the project.

To the former, the seminar given by G. Voss on the very first results from PETRA, coming into operation with a smoothness that exceeded all expectation, gave promise of quick and definite answers. Also PEP being scheduled to start operating next year, the results of two slightly different designs will soon be

*) CERN/ISR-LEP/78-17 (1978)
available for comparison, providing a very solid experimental basis for extrapolation to higher energies.

As far as energy is concerned, the exhaustive review by W. Bauer on the state of the art of superconducting cavities, based on experimental results from prototype cavities in actual operation, gave good reason to believe that, as far as one can see at the present time, the extension of the LEP-70 design by the use of a superconducting RF system can reasonably be proposed, thus effectively making LEP into an optimized 100 GeV per beam machine.
1. INTRODUCTION

The design of large $e^+e^-$ storage rings at present looks very conventional. This fact can be roughly summarized bearing in mind that such a storage ring will consist of
- strong focusing, separated functions, FODO channels,
- low-beta insertions,
- sextupoles for chromaticity corrections,
- classical RF structures,
- a design luminosity of $10^{32} \text{ cm}^{-2} \text{ s}^{-1}$.

A natural thought, then, is that the building of a large storage ring will be mainly a question of size and cost. This is indeed true for LEP. However, very stringent theoretical and technical difficulties arise which show that energies far beyond the present LEP value will be difficult to achieve (although it is still hard to set a precise limit) unless new accelerator techniques are shown to work.

The present report deals with some of these difficulties and makes an extensive use of scaling laws. However, the first aim has been to introduce non-experts to the basic principles which enter in the design of $e^+e^-$ storage rings.

A selection of useful formulae presented in the first part will help in following the remainder.

2. A BRIEF SUMMARY OF SINGLE-PARTICLE DYNAMICS

2.1 Betatron oscillation

In a storage ring the particle follows a guide field which mainly consists of a sequence of bending magnets and quadrupoles. When the bending magnets have no gradient (parallel pole tips), one has a so-called separated function machine. Generally the design trajectory passes through the centre of the quadrupoles where there is no field, while the magnets are designed to bend the particle in the horizontal plane, although it is sometimes necessary to have vertical bending too. The total horizontal bending angle is $2\pi$ while the total vertical one, when such bending exists, is zero. The bending angle from each individual magnet need not necessarily be equal; if it is not, the machine is non-isomagnetic.

A particle not moving on the design orbit will be focused in one plane and defocused in the other when going through a given quadrupole. This is the reason why two types of quadrupoles (F and D) must alternate in order to obtain a total
focusing effect in both planes. Such a sequence is called a FODO channel (or cell, or lattice), where "0" stands for a straight section or a bending magnet.

The motion of a particle which has the design energy $E_0$ is described, to first order, by Hill's equation:

$$y'' + K_y(s)y = 0,$$

where:

- $s$ is the azimuthal coordinate ($y'' = d^2y/ds^2$) along the design orbit,
- $y$ is the transverse coordinate (x or z) around the design orbit,
- $K = \left[1/(p/e)\right](\partial B_z/\partial x)$ is the normalized gradient.

The general solution for the previous equation is Floquet's solution, which can be written in the following way:

$$y(s) = a\sqrt{B_y(s)} \cos \left[\phi_y(s) + \vartheta\right]$$

with

$$\phi_y(s) = \int_0^s \frac{ds}{B_y(s)} \quad \text{(betatron phase)}.$$ 

This formulation shows that the betatron motion is in fact a pseudo-harmonic oscillation. The $\beta$ function, often called the envelope function, has the periodicity and the symmetry of the magnetic structure, and characterizes this structure. At a given azimuth, the discrete motion is purely sinusoidal:

$$y(s_i) = a\sqrt{B_y(s_i)} \cos \left[2\pi n Q_y + \vartheta\right] \quad (n = \text{integer}),$$

with the definition

$$\int_0^s \frac{ds}{B_y(s)} = 2\pi Q_y = \mu,$$

where $Q$ is called the betatron wave number or the betatron tune, and $\mu$ is the total betatron phase over one revolution.

As is well known, the equation for an harmonic oscillator can be expressed in a matrix form. In the present case, expressing the angle $y'(s_i)$, it can easily be shown with a little algebra that, at the location $s_i$, the motion from one turn to the next is given by

$$\begin{bmatrix} y(s_i) \\ y'(s_i) \end{bmatrix}_{n+1} = \begin{bmatrix} \cos \mu + \frac{\beta_y'(s_i)}{2} \sin \mu & \beta_y(s_i) \sin \mu \\ \frac{1 + \beta_y^2(s_i)}{2} \sin \mu & \cos \mu - \frac{\beta_y'(s_i)}{2} \sin \mu \end{bmatrix} \begin{bmatrix} y(s_i) \\ y'(s_i) \end{bmatrix}_n.$$
where the $2 \times 2$ matrix is called the Twiss matrix. It must obviously be equal to the total transfer matrix over one turn which can be obtained by multiplying the individual sector matrices of the magnetic structure. Identification of the two matrices permits $\beta$ to be determined at a given azimuth, for instance at a symmetry point where $\beta' = 0$. The $\beta$ function elsewhere is then obtained by solving the following differential equation in each sector with $K = \text{const}$:

$$\frac{1}{2} \beta'' - \frac{1}{4} \beta'^2 + K(s) \beta^2 = 1,$$

which satisfies Hill's equation.

Let us now look at what happens to a particle which has an energy deviation $\varepsilon = E - E_0$. A first effect comes from the fact that such a particle has a different path in the magnets so that it can no longer follow the design orbit. The equation of motion now becomes

$$y'' + K(s)y = \frac{1}{\rho(s)} \frac{\varepsilon}{E_0},$$

where $\rho(s)$ is the radius of curvature of the bending magnets. A particular solution which satisfies this equation is a closed trajectory with deviation from the design orbit. It is expressed in the following way:

$$y_{\varepsilon}(s) = \eta_y(s) \frac{\varepsilon}{E_0},$$

where $\eta$ is called the dispersion function. It is now clear that the betatron oscillation as defined earlier will occur around this new closed orbit. The $\eta$ function, in addition to the $\beta$ function, characterizes the magnetic structure, and similar techniques are used to compute it.

A second effect related to an energy deviation is the linear chromaticity. This is due to the energy dependence of the normalized gradient:

$$K = \frac{1}{p/e} \left. \frac{\partial E}{\partial x} \right|_{s} \approx K_0 \left(1 - \frac{\varepsilon}{E_0}\right).$$

As a consequence, the wave numbers will be energy-dependent. In separated function machines the chromaticities are negative in both planes, and for large storage rings they become prohibitive because the effect of each individual quadrupole adds up. Large tune shifts must be avoided because of the resonant condition

$$mQ_x \pm nQ_z = p \quad (m, n, p = \text{integers})$$

which limits the operating diagram $Q_x, Q_z$ to small discrete areas. The resonances occur for non-ideal machines which include field errors. The effect of these defects on the closed orbit or the betatron oscillation can be strongly amplified for some values of the tunes.
2.2 Synchrotron oscillation

Electrons and positrons circulating along the guide field lose energy, mainly in the bending magnets. A particle of nominal energy $E_0$ will radiate $U_0$ in each revolution:

$$U_0 \text{ (GeV)} = 88.454 \times 10^{-6} \frac{E_0 \text{ (GeV)}}{\rho \text{ (m)}},$$

where $\rho$ is the bending radius of the dipole magnets. If there are many types of bending magnets the formula is still valid if one uses

$$\frac{1}{\rho} = R \left\langle \frac{1}{\rho^2(s)} \right\rangle,$$

where $R$ is the mean radius of the ring. The mean value is defined as follows:

$$\langle F \rangle = \frac{1}{2\pi R} \int F(s) \, ds.$$

In order to compensate for these losses, the storage ring will include a radio-frequency system with accelerating cavities distributed around the circumference.

A "synchronous particle" is a particle which gains exactly the energy lost. This particle has the nominal energy because the RF frequency is chosen to be an harmonic of the revolution frequency corresponding to the reference orbit ($f_{RF} = h f_r$). For the other particles, either entering the machine at a different time or having an energy deviation $\varepsilon$, the gain is different. This leads to the synchrotron oscillation which is schematically described in Fig. 1.

In this figure, $t$ represents the time the particles enter the cavities, while $V$ is the corresponding voltage in the cavity gap; $t_1$ and $t_2$ represent the synchronous particle. Assume, now, that a particle enters at time $t_1'$, which means that this particle comes in earlier than the synchronous particle $t_2$; it gets more energy than it has lost and will then have a different revolution frequency characterized by

$$\frac{\Delta f_{RF}}{f_{RF}} = -\alpha \frac{\varepsilon}{E_0},$$

where $\alpha$, called "the momentum compaction factor", is a property of the guide field and describes the change in path length corresponding to an energy deviation. To first order, the path length changes only in the magnets because of the curvature, and then we get

$$\alpha = \langle \eta \rangle_{mag}/R.$$

In electron storage rings where particles travel with the velocity of light, there is no supplementary effect relating the revolution frequency to the velocity.

According to this effect the particle will be delayed and will come closer to the synchronous particle. The reverse will happen for particle $t_2''$ and it will also come closer to the former particle. The result is that the particle tends to oscillate around the synchronous particle.
The same analysis done for particles $t'_1$ and $t''_2$ shows that the deviation from $t_1$ is increased so that point can be considered as unstable.

Finally, particles will be trapped around synchronous particles located at $t_2$, $t_2 + T_{\text{rev}}/h$, etc., and this represents the bunching effect.

Notice that particle $t'_1$ will not be captured by another bucket and will slide continuously, losing more and more energy until it strikes the vacuum chamber, while particle $t''_2$ will be captured around $t_2$. Then it appears that the maximum phase amplitude for the synchrotron oscillation is represented by $t_1$, to which corresponds a maximum energy deviation when passing at time $t_2$ (in between there is only a positive energy gain). This maximum energy deviation, which limits the stable region, is called the "acceptance" of the RF system, and is given by

$$
\left( \frac{e}{E_0} \right)_{\text{max}} = \frac{1}{h\alpha} \frac{F_s}{f_r} \sqrt{\frac{2G(\phi_s)}{\cos \phi_s}} ,
$$

$$G(\phi_s) = 2 \cos \phi_s - (\pi - 2\phi_s) \sin \phi_s ,
$$

where $F_s$ is the synchrotron oscillation frequency for small amplitudes (linear motion) and $f_r$ the revolution frequency:

$$
\frac{F_s}{f_r} = \sqrt{\frac{h\alpha U_0}{2\pi E_0}} \cot \phi_s ,
$$

the synchronous phase angle being defined by

$$eV \sin \phi_s = U_0 .
$$

The harmonic number $h$ represents the maximum number of stable buckets one can get along the circumference. However, injection techniques allow a small number of buckets to be filled depending on the required number of bunches.

2.3 Beam dimensions

In $e^+e^-$ storage rings, beam dimensions result from two effects related to the synchrotron radiation: damping of the oscillation, and quantum excitation.

A particle moving in a magnetic field perpendicular to its velocity radiates energy at the rate

$$P_{\gamma} = \frac{2}{3} \frac{e^2 c^3 r}{(mc^2)^3} e^2 \times B^2 .
$$

This formula, integrated over one revolution, gives the energy loss per turn which has been used previously. It is extracted from the classical theory of electromagnetic radiation by relativistic particles, and for our purpose can be considered as the basic expression for the radiation damping mechanism.
A particle which has an energy deviation from the nominal one will radiate at a different rate. Moreover, as explained earlier, such a particle has a different orbit (and a different path length) and then moves in a different magnetic field. If this particle radiates more than the synchronous one, it can be either damped if the initial energy deviation is positive or antidamped if the initial energy deviation is negative.

By definition the damping coefficient $\alpha_x$ is the one entering in the following equation of motion:

$$\frac{d^2x}{dt^2} + 2\alpha \frac{dx}{dt} + \omega^2 x = 0,$$

and the damping time is

$$\tau_x = \frac{1}{\alpha_x}.$$

For the energy oscillation (or the related phase oscillation) the damping time can be written:

$$\tau_\varepsilon = \frac{2}{x} \frac{E_0}{U_0} \frac{1}{J_\varepsilon},$$

$$J_\varepsilon = 2 + D_x + D_z,$$

where $D$ is a property of the guide field:

$$D = \frac{\int n_y \rho_y \left( \frac{1}{\rho_y^2} + 2 K_y \right) ds}{\int \rho_x^2 ds} (y = x \text{ or } z).$$

The damping of betatron oscillations is quite different as it is not a direct effect from the radiation process but rather the effect of the energy gain from the RF. As a matter of fact the radiation is emitted tangentially to the particle trajectory (see Fig. 2a) and then gives a reactive impulse $\delta p$ opposite to the momentum $p$ of the particle. This, when considered over a differential path length $ds$, changes neither the position nor the angle of the particle, which means that the betatron amplitude is not affected.

When crossing a cavity gap the particles get back the total energy loss, but for all values of the betatron phase the corresponding impulse $\delta p$ is parallel to the design orbit. As shown in Fig. 2b, this gives an angular kick leading to a new betatron amplitude. When averaging over all betatron phases, the final amplitude is shown to be less; the betatron oscillations are damped.

However, the previous result is true only when the dispersion function is zero, otherwise the energy lost by radiation moves the closed orbit so that the betatron motion gets a new amplitude. As for energy oscillation, the damping effect is now affected by the guide-field structure. The damping time being again defined by
Fig. 1

Fig. 2
\[ \frac{1}{\tau} = \alpha = - \frac{1}{a} \frac{da}{dt} \quad (a = \text{oscillation amplitude}), \]

one gets

\[ \tau_y = \frac{2}{r} \frac{E_y}{U_0} \frac{1}{J_y} \quad (y = x \text{ or } z) \]

which finally implies that

\[ J_x + J_z + J_\varepsilon = 4, \]

where the J numbers are called the "partition numbers".

For an ideal separated-function lattice the three motions are damped. However, in the presence of closed-orbit distortions antidamping may occur either for the transverse motions or the longitudinal one, as will be seen later.

In spite of damping, the beam sizes never become infinitely small because particles are always subjected to noise and mainly to energy fluctuations. Although there are many types of energy fluctuations, the most important one comes from the synchrotron radiation itself, which is emitted as quanta of discrete energies.

It is known that damped harmonic oscillators, whose positions are disturbed by random fluctuations \( \delta x \) occurring at a rate per second \( \dot{\alpha} \), with a mean square \( \langle \delta x^2 \rangle \), have a Gaussian distribution with a standard deviation:

\[ \sigma^2_x = \frac{1}{4} \dot{\alpha} \langle \delta x^2 \rangle \tau_x. \]

Then in the particular case of energy oscillations,

\[ \sigma^2_\varepsilon = \frac{1}{4} \dot{\alpha} \langle \varepsilon^2 \rangle \tau_\varepsilon, \]

where \( \varepsilon \) stands for a quantum of energy. A detailed analysis of the synchrotron radiation spectrum shows that

\[ \dot{\alpha} \langle \varepsilon^2 \rangle = 2C_q \frac{E^3}{(mc^2)^2} \langle P_Y \rangle \frac{\langle 1/\rho^3 \rangle}{\langle 1/\rho^2 \rangle}, \]

where

\[ C_q = 3.84 \times 10^{-13} \quad (\text{m}), \]

\[ \langle P_Y \rangle = \text{mean power radiated}. \]

Finally it comes out that the energy spread of the beam is

\[ \sigma_\varepsilon = \sqrt{C_q/\rho J_\varepsilon} \frac{E^2}{mc^2}, \]

where \( \rho \) is averaged over the whole circumference:

\[ \frac{1}{\rho} = \frac{\langle 1/\rho^3 \rangle}{\langle 1/\rho^2 \rangle}. \]
In the synchrotron motion the phase oscillation is associated with the energy oscillation and represents the longitudinal position of the particle as compared to the synchronous particle. The corresponding bunch length is then

\[ \sigma_L = \frac{2}{\pi} \frac{\sigma_E}{E_0} R \alpha. \]

The quantization of the synchrotron radiation also affects the betatron motion since energy fluctuations lead to closed-orbit jumps if the dispersion function is non-zero. The mechanism is then quite similar to the one discussed before, apart from some complications related to pseudo-harmonic oscillation. The results in terms of standard deviations of the betatron distributions are:

\[ \sigma_y = \sqrt{\frac{\beta y}{\bar{U}_y}} \frac{\sigma_E}{E_0} \ (y = x \text{ or } z), \]

where \( \bar{U}_y \) is an invariant which characterizes the magnetic structure:

\[ \bar{U}_y = \frac{\langle \frac{J_y}{J_y} \rangle}{\langle \rho^{-3} \rangle}, \]

\[ U_y(s) = \frac{\eta^2_y + \left[ \eta_y' \beta_y - \eta_y \beta_y' / 2 \right]^2}{\beta_y}. \]

The mean values are taken over the whole circumference. It can be seen that if \( \eta \) vanishes in one plane for instance, the corresponding transverse beam size is zero, unless some extra noise has to be considered. This is what generally happens in the vertical plane with no bending. In this case the vertical size is usually given by coupling between the two transverse betatron motions due to magnet imperfections (e.g. tilted quadrupoles). Such a coupling effect will lead to betatron energy transfer from the horizontal to the vertical plane. In this case conservation implies that the sum of the two invariants \( \bar{U}_x \) and \( \bar{U}_z \) must be equal to \( \bar{U}_0 \) obtained without coupling. The coupling coefficient is then \( k = \frac{\bar{U}_z}{\bar{U}_x} \).

By definition, the transverse emittance is

\[ \varepsilon_y = \frac{\sigma^2}{\beta_y}, \]

and it appears also as an invariant for the betatron motion.

If we ignore wigglers, the horizontal emittance in a large electron storage ring is virtually determined by the arcs, where periodic regular cells with identical bending magnets are located and where the \( n_x \) function follows approximately \( \sqrt{E_x} \). Then we get

\[ \varepsilon_{x_0} = \frac{J_x}{\beta_x} \langle \frac{n_x^2}{\beta_x} \rangle \left( \frac{\sigma_x}{E_0} \right)^2. \]
A reasonably good approximation is

\[ E_{x_0} = \frac{J}{J_x} R_{\text{arc}} \frac{\mu^3}{8Q^3} \frac{1}{\tan(\mu/2) \sin^2(\mu/2)} \left( \frac{\sigma_E}{E} \right)^2, \]

where \( R_{\text{arc}} \) is the mean radius of the arcs, \( Q \) is the arc contribution to the tune, and \( \mu \) is the betatron phase advance per cell.

3. THE BEAM-BEAM INTERACTION

3.1 The parameter \( \xi \) and the linear tune shift

When a particle crosses the opposite bunch in an interaction region, it sees the macroscopic electromagnetic field of that bunch and gets a transverse impulse which disturbs its betatron motion.

If the test particle stays close to the bunch axis (small betatron oscillations) it sees a linear force, which is focusing in both planes. This force can be obtained by expanding the electromagnetic field generated by an elliptical beam with a Gaussian particle distribution.

Now, if the bunch is short enough, this force will lead to an angular deviation, the particle position being almost unchanged. This is equivalent to a thin-lens quadrupole, keeping in mind that the gradient must have the same sign in both planes. It can be shown that the integrated gradient over the bunch length is

\[ (K\xi)_{x,z} = \frac{2N}{\gamma(\sigma_x + \sigma_z)} \frac{r_e}{\sigma_x \sigma_z}, \]

with

\[ N = \text{number of particles in the bunch} \]
\[ \gamma = \frac{E}{mc^2} \]
\[ r_e = \frac{e^2}{4\pi\varepsilon_0 mc^2} \]
\[ \sigma = \text{transverse r.m.s. of the particle distributions}. \]

The new betatron motion, including the perturbation from the space-charge forces, will be represented by the product of two matrices:

\[ \begin{pmatrix} 1 & 0 \\ K\xi & 1 \end{pmatrix} \times \begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix}, \]

where the first one is the space-charge thin-lens approximation, while the second one is either the transfer matrix over one turn or the Twiss matrix (in that case the envelope function is taken at the crossing point).
The unperturbed tune is given by
\[ \cos 2\pi Q_0 = \frac{1}{2} (T_{11} + T_{22}) , \]
while the final tune becomes
\[ \cos 2\pi (Q_0 + \Delta Q) = \cos 2\pi Q_0 + \frac{1}{2} T_{12} K \]
with
\[ T_{12} = -\beta_0 \sin 2\pi Q_0 . \]

Let us now define the parameter \( \xi \):
\[ \xi_{x,z} = \frac{N r_e}{2\pi \gamma (\sigma_x + \sigma_z)} \left( \frac{\beta_0}{\sigma_{x,z}} \right) , \]
and then
\[ \cos 2\pi (Q_0 + \Delta Q)_{x,z} = \cos 2\pi Q_{0_{x,z}} - 2\pi \xi_{x,z} \sin 2\pi Q_{0_{x,z}} , \]
where \( \Delta Q \) is the linear tune shift corresponding to the linearized space-charge force. The parameter \( \xi \) characterizes the space-charge strength; it also includes the value of the envelope function at the crossing point, which acts as an amplification factor.

One can also write
\[ \xi_{x,z} = \frac{\sin 2\pi \Delta Q_{x,z}}{2\pi} \left[ 1 + \tan (\pi \Delta Q_{x,z}) \cotg (2\pi Q_{0_{x,z}}) \right] \]
\[ = \Delta Q_{x,z} \left[ 1 + \frac{\pi \Delta Q_{x,z}}{\cotg (2\pi Q_{0_{x,z}})} \right] . \]

This formula shows that when \( Q_0 \) is not too close to an integer, \( \xi \) is a good approximation for the linear tune shift. When there are several interaction points the formula stays valid if \( Q_0 \) and \( \Delta Q \) express the tune and the tune shift between two crossing points, assuming that all the crossing points are located at homologous points around the ring. However, in that case the parameter \( \xi \) still represents the space-charge strength at each crossing point, while the linear tune shift is the sum of the contributions of each crossing point.

In addition to the tune shift, the linear approximation of the space-charge force modifies the optics. In particular, the envelope function \( \beta \) will change, and from the previous matrix formalism it can be shown that the corresponding change at the crossing points is given by
\[ \beta_{x,z} \sin 2\pi Q_{x,z} = \beta_{0_{x,z}} \sin 2\pi Q_{0_{x,z}} , \]
where the index "0" still characterizes the unperturbed values.
Moreover, if the dispersion function \( \eta \) is not zero at the crossing point it will be changed at the same point in the following way:

\[
\eta = \frac{\eta_0}{1 + 2\pi \xi \ctg \pi Q_0}.
\]

This formula is also valid for representing the final orbit displacement between the two beams, at the crossing point, which would result from an initial beam separation.

As a consequence of this linear effect of the beam-beam interaction, the invariance of the betatron motion will be modified (and then the emittance) and the momentum compaction too. If the latter reaches zero, we get a limit for the stability of the synchrotron oscillation.

3.2 The beam-beam limit

On all the existing \( e^+e^- \) storage rings a current limitation has been observed when the two beams collide. This limitation, which appears either as a growth of the beam transverse emittance or as a bad lifetime, is obviously due to the space-charge forces. In the earlier days it was thought that such a limit could be related to a maximum permissible tune shift. However, a tune shift, in itself, is not very dangerous as it can be compensated by a different initial tuning of the ring. In fact a deeper analysis of the space-charge forces shows that the corresponding non-linearity gives a tune shift which depends on the betatron amplitude. The maximum tune shift occurs for particles having small amplitudes, while particles with large amplitudes are not perturbed. Then it is clear that the linear tune shift represents, in fact, the width of a tune spread. A maximum permissible value for the linear tune shift may now correspond to the fact that particles cannot spread over dangerous resonances, these resonances being due either to the natural defects of the ring structure or to a space-charge non-linearity.

Table 1 shows that the maximum linear tune shift \( (\Delta Q_{\text{max}})_{\text{total}} \) is different for different rings, which does not confirm the previous assumptions unless the dangerous resonances are different from one ring to another. This is, however, doubtful, as it has been shown -- at least on ACO and ADONE -- that the maximum total linear tune shift depends on the number of bunches per beam (or on the number of crossing points). The linear tune shift per crossing also depends on that number and decreases roughly like the square root of it.

However, if the beam-beam limit was due to the excitation of resonances through the non-linearity of the space-charge force, all these observations would probably be less surprising, the width of the resonances depending on the strength of the space-charge force and on the periodicity of the corresponding perturbations. Moreover, the tune spread will then be meaningless as it can no longer be separated
from the resonant effect. Unfortunately no good model is yet available to give a correct understanding of the beam-beam limit.

A new look at Table 1 shows that ADONE and SPEAR have reached the same \( \xi_{\text{max}} = 0.06 \). They both operate above an integer. However, ADONE operates with three bunches per beam (six crossing points) and very close to the integer, while SPEAR operates in the one-bunch mode (two crossing points), with lower \( \beta \) values at the crossings and less close to an integer.

A mixing of these two types of operation is used in the design of the new storage rings: low-\( \beta \) at the crossing, many bunches per beam. The hope is still to get \( \xi_{\text{max}} = 0.06 \). How good is such a scaling? A partial answer will soon come from PETRA operations.

Among the uncertainties which one has in manipulating the parameter \( \xi_{\text{max}} \) is the fact that for a given machine it may depend on the operating energy. However, apart from ADONE the energy dependence on the other storage rings is either very small or non-existent. This is perhaps a good reason to ignore it in new designs?

### 3.3 Optimization of the luminosity

When two bunches interact, the rate of events which occurs at a given interaction region is:

\[
\dot{n} = f_r \frac{N^+ N^-}{bS} \sigma_{\text{tot}},
\]

where \( \sigma_{\text{tot}} \) is the total cross-section of the process under study, \( N^\pm \) are the number of particles in each beam, \( S \) the effective beam area, \( f_r \) the revolution frequency of the ring, and \( b \) the number of bunches per beam.

This can also be written:

\[
\dot{n} = L \sigma_{\text{tot}},
\]

where

\[
L = f_r \frac{N^+ N^-}{bS} \left( S = 4\pi \sigma_0 \frac{k}{\Delta} \right)
\]
represents the luminosity which depends only on the ring parameters. The cross-section is generally purely betatron, as the dispersion function is made to vanish at the interaction point in order to avoid coupling effects with the longitudinal motion which may come from the beam-beam interaction itself.

According to the previous section, the maximum current per bunch which can be stored in the collision mode is

\[
\left( \frac{N}{b} \right)_{\text{max}} = \frac{2\pi \gamma (\sigma_x + \sigma_z)}{r_e} \left( \frac{\sigma_y}{\beta_{x,z}} \right) \xi_{\text{max}},
\]

where obviously the plane which first limits the current must be taken into consideration. Without transverse coupling, the vertical dimension is so small that

\[
\frac{\sigma_z}{\beta_z} < \frac{\sigma_x}{\beta_x}
\]

(all these values obviously being taken at the crossing point), and the maximum expected luminosity is then

\[
L = \frac{\pi b \xi_{\text{max}}^2}{r_e^2} \left( \gamma^2 + \frac{\sigma_z^2}{\sigma_x^2} \right) \frac{\sigma_z}{\beta_z}.
\]

Introducing now the transverse coupling coefficient \( k \) as defined in Section 2.3:

\[
L = \frac{\pi b \xi_{\text{max}}^2}{r_e^2} \left( \gamma^2 + \sqrt{k} \frac{\beta_z}{\beta_x} \right) \frac{\sqrt{k}}{1 + k} \frac{\beta_z}{\beta_x} \frac{E_x}{\gamma E_0},
\]

it is seen that a higher luminosity is obtained by increasing the transverse coupling. However, above a certain amount of coupling the limit will fall into the horizontal plane:

\[
k_{\text{max}} = \beta_z/\beta_x.
\]

Then the optimum luminosity becomes

\[
L = \frac{\pi b \xi_{\text{max}}^2}{r_e^2} \left( \frac{1}{\beta_x} + \frac{1}{\beta_z} \right) E_x \gamma E_0,
\]

which corresponds to the optimum current per beam:

\[
N = \frac{2\pi b \xi_{\text{max}}}{r_e} \gamma E_0.
\]

From these formulae it is obvious that more luminosity is obtained if the radial emittance is larger and \( \beta \) smaller. For a given ring the luminosity will increase like \( \gamma^4 \), remembering that the natural emittance goes like \( \gamma^2 \).
Present designs include a maximum luminosity $L = 10^{32} \text{ cm}^{-2} \text{ s}^{-1}$ at the design energy $E$. The most flexible parameter to adjust in order to get this luminosity is the emittance $\varepsilon_x$, the other parameters being more or less fixed by other considerations, e.g.

- the natural value for $b$ is half the number of intersection regions;
- the revolution frequency depends directly on the design energy from power dissipation and cost considerations;
- both $\beta$ cannot be very small at the same time, because the first quadrupole at the end of the interaction region is focusing in one plane and defocusing in the other. Generally $\beta_z$ is made small and then the value of $\beta_x$ does not matter very much. The minimum value for $\beta_z$ which one can reasonably expect is of the order of the bunch length; it directly results from the choice of RF frequency.

The total RF power must now provide for the beam power at the design energy and for the cavity losses corresponding to the required accelerating voltage:

$$P_{\text{total}} = P_b + P_d,$$

with

$$P_b = 2IU_0 \quad (I = N e f_r)$$

$$P_d = \frac{V^2}{2R_{sh}}.$$ 

The steep rise in cavity losses makes the luminosity drop very rapidly beyond design energy so that the design energy is almost the maximum energy of the ring.

Above and below the design energy the slopes for the luminosity curves can be improved by some manipulation of the emittances.

A new kind of limitation which occurs in very high energy storage rings, and which is purely technical, comes from the maximum radiated power which the vacuum chamber can dissipate per unit length. If the previous optimization for the maximum luminosity reaches this limit it is not very easy to maintain both $E_{\text{max}}$ and $L_{\text{max}}$ by adjustment of the other parameters.

### 3.4 Scaling laws

An important question for users is the following: Can we go further in designing high-energy storage rings while still having a maximum luminosity of $10^{32} \text{ cm}^{-2} \text{ s}^{-1}$?

From the previous section, assuming no further improvement will come from $\beta$ and keeping the same number of interaction regions, the scaling law is

$$f_r \gamma^2 \varepsilon_x = \text{const}, \quad (f_r \propto 1/R).$$
Moreover, if one starts from a machine which has reached the maximum power that the vacuum chamber can dissipate, the scaling law for the beam power is

\[ P_b = N \int E \delta \rho \propto \rho. \]

Expressing N and \( \delta E \) leads to

\[ \rho \text{ and } P_b \propto \gamma^{3/2}; \quad \text{and} \quad I \propto \gamma^{-1}, \quad N \propto \gamma^{1/2}. \]

Assuming that R follows the same law as \( \rho \), the scaling law for the emittance becomes

\[ E_{x0} \propto \gamma^{-1/2}. \]

As seen in Section 2, the emittance is given by the arcs and

\[ E_{x0} \propto \frac{R_{\text{arc}}}{\rho} \frac{\gamma^2}{Q_{\text{arc}}^3} \propto \gamma^2. \]

This is true for a constant betatron phase shift per cell. The wave number \( Q_{\text{arc}} \) is only the contribution from the arc and is proportional to the number of cells \( n \). Then, if \( L \) is the length of one cell,

\[ L = \frac{2\pi R_{\text{arc}}}{n} \propto \gamma^{2/3}. \]

A constant betatron phase per cell means

\[ K\ell = \text{const}, \]

where \( K \) is the normalized gradient of the quadrupole (for F and D) and \( \ell \) its length.

Finally, in order to maintain a constant luminosity for higher design energies, the focal length of the regular quadrupoles must scale like

\[ K\ell \propto \gamma^{-2/3}. \]

If the maximum tolerable gradient is reached, the quadrupole length will be increased slightly (\( \ell \propto \gamma^{1/3} \)) but slowly as compared to \( L \). From the optics point of view this looks quite reasonable. However, it must be pointed out that the maximum of the \( \beta \) function in the arc will go like \( L \) (or \( \gamma^{2/3} \)), and that this will make the lattice more sensitive to field errors as discussed in the next section.

One constraint which comes from the scaling law \( \rho \propto \gamma^{3/2} \) is that the bending field at the design energy scales like

\[ B \propto 1/\sqrt{\gamma}. \]

In order to operate the distributed pumps inside the magnets, this field must be above a certain value \( B_{\text{min}} \). The consequences are that the injection energy will increase faster than the design energy and that the range of energy operation will be smaller and smaller. A design alternative to cure this effect is to develop new types of bending magnets or to find new ways of pumping the arcs.
Notice that cost optimization gives a different scaling, namely $\rho = E^2$, which means that a larger storage ring may have less vacuum chamber heating problems. On the other hand, the pumping limitation will appear earlier.

4. DISTURBED CLOSED-ORBIT EFFECTS

4.1 Misalignment and field errors

A machine is never perfect. The main defects come from the tolerances that can be technically accepted for the positioning of the elements; from the limited field quality; and from the approximate identity of the machine elements. Although at the manufacturing stage the accuracy is better, it is quite common by now to assemble the magnetic elements with positioning errors of the order of 0.1 mm. The field errors are of the order of $10^{-4}$ while the power supplies stability is $10^{-5}$.

An obvious consequence of these errors is that the design orbit becomes a broken line which can no longer be followed by a particle. The particle which was supposed to follow it will be deviated and will start a betatron oscillation now defined through a new function $K(s)$ which includes eventual gradient errors.

These errors are randomly distributed around the ring, and the corresponding periodicity is one turn. They cannot be compensated locally because either they come from an unknown position inside the alignment tolerance or they are not detected by magnetic measurements. A number of correction methods exist using the disturbed particle motion due to these errors. Then the main problem is to get a stable disturbed motion in order to observe it.

4.2 Disturbed closed orbit

In the present section the dipole field errors distributed all around the ring are considered. A field error $\Delta B$ located in an azimuthal interval $\Delta s$ will cause the slope of the betatron motion to change by an amount

$$\Delta y' = \frac{\Delta B}{p/e} \Delta s = \theta_1,$$

neglecting the change in position (thin-lens approximation).

The equation of motion then becomes

$$y'' + K(s)y = \sum \theta_1 \delta(s - s_i),$$

to which a particular solution, which closes on itself (single-valued trajectory), can be found:

$$y_{c.o.}(s) = \frac{\sqrt{B(s)}}{2 \sin \frac{\pi}{Q_y} \sum \theta_1 \sqrt{B(s_i)}} \cos \left[ \phi_y(s_i) - \phi_y(s) - \pi Q_y \right].$$

This is called the disturbed closed orbit. The betatron motion as defined earlier will occur around this particular orbit.
The errors being randomly distributed, and in view of the large number of elements around the ring, one can write

\[ \langle y^2 \rangle_{\text{c.o.}} = \frac{\beta}{8 \sin^2 \pi Q_y} \sum_i \frac{\beta_i}{\langle \theta_i^2 \rangle} . \]

Applying this formula to LEP (LEP-70/42) in the case where dipole field errors come from displaced quadrupoles \((\theta_i = K_i \ell_i^2 \delta y_i, \delta y_i\) representing the displacement of the quadrupole axis), it was found that in spite of their relatively small number, the effect of the big insertion quadrupoles was about three times larger than the effect of all the other quadrupoles. Moreover, the maximum closed-orbit displacement will occur in these quadrupoles following the maxima of the \(\beta\) function. A few centimetres closed-orbit displacement are expected for LEP, which of course could be reduced by using higher tolerances for the alignment of these peculiar quadrupoles.

Notice that horizontal closed-orbit distortions inside the sextupole magnets will lead to gradient errors:

\[ \Delta K = k_s x_{\text{c.o.}} \quad \left( k_s = \frac{1}{p/e} \frac{3^2 B}{3x^2} \right), \]

giving a tune shift. If the total tune approaches an integer it is seen that the closed-orbit amplitude will be strongly amplified. This is perhaps an effect which may discourage putting sextupoles close to the insertion quadrupoles, unless they appear necessary for good chromaticity correction.

There is another interesting formulation for the disturbed closed orbit which is obtained by Fourier analysis of the distributed errors. The result is a Fourier expansion of the disturbed closed orbit in terms of the harmonics of the revolution frequency:

\[ y_{\text{c.o.}}(s) = \sqrt{B(s)} \sum_{k=1}^{\infty} \frac{1}{1 - (k/Q)^2} \left[ b_k \cos k\phi + a_k \sin k\phi \right], \]

with

\[ a_k = \frac{1}{\pi Q} \int \beta^{1/2} \sum_i \theta_i \delta(s - s_i) \sin k\phi \, ds, \]

\[ b_k = \frac{1}{\pi Q} \int \beta^{1/2} \sum_i \theta_i \delta(s - s_i) \cos k\phi \, ds, \]

\[ \phi = \int \frac{ds}{Q \delta} \]

It appears that the most important harmonic corresponds to the integer \(k\) close to the tune value. Theory as well as practice shows that such an harmonic can be corrected by a set of two correcting dipole coils. However, defining the correct location for these elements requires first a measurement of the closed orbit; this is only possible if the expected closed orbit stays inside reasonable tolerances.
4.3 Vertical dispersion

As seen in a previous section, the dispersion function is a result of the energy dependence of the curvature effects. Ideally, if bending occurs in the horizontal plane there is no vertical dispersion. For a non-ideal machine, field errors are energy-dependent; moreover, as the resulting disturbed closed orbit goes off axis through the quadrupoles (and sextupoles), it sees a dipole field which is also energy-dependent. The dispersion function will now represent the disturbed closed orbit for an off-momentum particle:

\[ \eta_z'' + K \eta_z = (K - k_x) z_{c.o.} - \sum_i \theta_i \delta(s - s_i). \]

The solution, which closes on itself, has the same aspect as the solution for the disturbed closed orbit, and we get

\[ \eta_z(s) = \frac{\sqrt{\beta_z(s)}}{2 \sin \pi Q_z} \left\{ \int \sqrt{\beta_z(s_1)} (K - k_x) z_{c.o.} \cos \left[ \phi_z(s_1) - \phi_z(s) - \pi Q_z \right] ds_1 \right. \]

\[ \left. - \sum_i \theta_i \sqrt{\beta_z(s_1)} \cos \left[ \phi_z(s_1) - \phi_z(s) - \pi Q_z \right] \right\}. \]

The second part of the bracket is the disturbed closed orbit, and this can be neglected since large values of \( \eta_z \) are of interest (\( \eta_z \gg 1 \text{ cm} \)).

A local correction of the chromaticities will cancel the first part of the bracket leaving the dispersion function equal to the disturbed closed orbit. However, this is not always possible since the natural horizontal dispersion is zero in the insertion quadrupoles.

It must be pointed out that what happens in the vertical plane will also happen in the horizontal one, giving a modified horizontal dispersion function.

As seen before, the disturbed closed orbit will have its maxima in the insertion regions. Moreover, at these places the integrated gradients \( KL \) are large, so it is quite easy to see that here again the insertions will be the major source. However, the function \( K - k_x \) has a complicated azimuthal dependence due to chromaticity corrections, and it is then hard to see how the rest of the machine contributes to building up vertical dispersion. This can be looked at differently, as follows. The dependence of the dispersion function on the closed orbit can be described in terms of a Fourier expansion according to the similarity of the two corresponding differential equations. From this study it appears that a good approximation for the dispersion function is (LEP-70/92):
where \( k \) and \( n \) are integers such that \( k \equiv Q_z \) and \( n \equiv 2Q_z \). The first part of the bracket is the product of the chromaticity and the main harmonic of the closed orbit:

\[
\frac{\Delta Q}{\Delta p/p} = -2 \frac{\Delta Q}{\Delta p/p}.
\]

This term cancels out when the chromaticity is corrected, otherwise it will become quite large for large storage rings.

The second part of the bracket correspond to an oscillation at the same frequency as that of the main harmonic of the closed orbit, although it may include slightly different frequencies. The coefficient \( B_n \) which enters there has to do with the Fourier expansion of the chromatic variation of the \( \beta \) function; effectively it can be shown that

\[
\frac{d\beta_z/\beta_z}{dp/p} = \frac{1}{2} \left\{ B_0 + \sum_{n=1}^{\infty} \frac{1}{1 - \left( \frac{n}{2Q_z} \right)^2} \left[ B_n \cos n\phi_z + A_n \sin n\phi_z \right] \right\},
\]

with

\[
\phi_z = \int_0^{s} \frac{ds}{Q_z \beta_z}, \quad B_0 = \frac{1}{2nQ_z} \int \beta_z (K - k, \eta_x) \, ds, \quad B_n = \frac{1}{\pi Q_z} \int \beta_z (K - k, \eta_x) \cos n\phi_z \, ds, \quad A_n = \frac{1}{\pi Q_z} \int \beta_z (K - k, \eta_x) \sin n\phi_z \, ds.
\]

Notice that for a machine with \( N \) superperiods, \( n \) must be a multiple of \( N \). Moreover if \( \phi \) is chosen to be zero at a symmetry point, \( A_n \) will cancel out.

It now follows that a chromaticity correction done with little care on the chromatic variation of \( \beta \) may lead to high amplification of the dispersion function through the disturbed closed orbit.
As said before, for a storage ring the size of LEP the insertions are mainly responsible for closed-orbit distortions and antidamping. For the last effect an amplification factor comes from the disturbed dispersion function, which depends on both the closed orbit and the chromatic properties of the ring. In that case the contribution from the arcs and from the insertions does not appear clearly when sextupolar corrections are applied. A scaling in that case is then difficult.

For the disturbed closed orbit it was seen that the contribution from the arcs scale like $\sqrt{N}$, where $N$ is the number of regular cells, which means roughly like $\sqrt{\gamma}$ for a constant phase advance per cell. If this is true, then even for much larger storage rings the main contribution will still come from the insertions and will increase if the number of insertions increases (anyway the effect will increase because the length of the insertion quadrupoles increases if the $\beta$ at the crossing points are kept constant).

From these effects the main constraints will appear when storing the first beam because
- the closed orbits and the dispersion functions must be compatible with the aperture;
- the particle motion must be damped;
- the tune shift due to closed-orbit displacement in the sextupoles must be small enough to avoid bad resonances.

However, for first operation the $\beta$ at the crossing points can be brought to a level such that the contribution from the insertions becomes of the order of the
contribution from the arcs. This technique should first permit closed-orbit corrections. Later a path, at constant tunes, should allow the beam to be kept stable, with perhaps additional corrections. This will of course require quite sophisticated hardware and software.

A first limit in machine size (apart from cost) may be reached when the contribution from the arcs is such that an increase in \( \beta \) at the crossing points does not help any more, unless tolerances or field errors can be much improved, or else correction methods can be applied to unstable beams over a fraction of a revolution only.

5. BEAM CAVITY INTERACTION

5.1 The normal beam loading

This effect takes place in the more general context of beam interaction with the surroundings. Specific features, however, come from the fact that the cavities are active circuits fed by transmitters. The usual model for studying this interaction is shown on Fig. 3, where the resonant circuit represents the cavity, \( i_g \) is an ideal current source for the generator, \( i_f \) is the beam current.

The generator operates at a frequency \( \omega_g \), which is an harmonic of the revolution frequency (\( \omega_g = h \omega_r \)).

In electron rings the stored beam looks like a periodic \( \delta \) function (short bunches) which can be expanded into a Fourier series. Usually the Fourier component at frequency \( \omega_g \) is taken into account, the other being too far away from the resonant curve. This Fourier component has a well-defined phase as compared to the cavity voltage \( V \), which is in fact the synchronous phase angle as has been explained for the synchrotron motion (Fig. 4).

An analysis of the equivalent circuit (Fig. 3), including the properties of synchrotron motion, gives a steady state which of course depends on the beam-induced voltage and the cavity tuning. This state can be described by the following formulae:

\[
\begin{align*}
\tan \phi_v &= \frac{V_b \cos \phi_s - V \tan \phi_y}{V_b \sin \phi_s + V}, \\
V_g &= \frac{V_b \sin \phi_s + V}{\cos \phi_v}
\end{align*}
\]

where

- \( \phi_s \) = synchronous phase angle
- \( \phi_y \) = cavity tuning angle
- \( V \) = cavity voltage
- \( V_g = R I_g \)
- \( V_f = R I_f \quad (I_f = 2I_{av}) \)
- \( \phi_v \) = phase angle between \( I_g \) and \( V \).
The reactive component from the induced voltage can be compensated by a proper tuning of the cavity (tg $\phi_v = 0$). The resistive component then remains. It represents the energy lost by the beam to the fundamental mode which has of course to be provided by the RF generator. Let us call $P_b$ the corresponding power:

$$P_b = i_{av} U_0.$$  

The required voltage $V$ for stable synchrotron oscillation must be provided also by the RF generator, and correspond to Joule losses in the cavity:

$$P_d = \frac{V^2}{2R_s},$$  

where $R_s$ is the shunt impedance. The total RF power provided by the generator is then

$$P_g = P_b + P_d + P_{\text{reflected}}.$$  

Note that $R$ which appears in the equivalent circuit is related to $R_s$ through the coupling factor $\beta$ which corresponds to the coupling device between the transmitter and the cavity (coupling loop):

$$R = \frac{R_s}{1 + \beta}.$$  

The total power provided by the generator can be optimized. For instance it is quite clear that the reactance of the loaded cavity must be compensated (tg $\phi_v = 0$). Moreover, a good matching will suppress the reflected power. This corresponds to a suitable adjustment of the coupling factor

$$\beta = 1 + \frac{P_b}{P_d}.$$  

It has been pointed out by K. Robinson that the steady state may be an unstable state under slow perturbations applied to the system. However, the corresponding limit in the stored current is avoided by the proper matching described above.

5.2 The heavy transient beam loading

This case happens when the bunch spacing is large and the distance between the current spectrum lines becomes small enough for more Fourier components to interact with the cavity. This will change the beam-induced voltage, and it is also clear that a perfect matching is no longer possible for all these harmonics. However, a new optimum matching can be obtained, as pointed out by P. Wilson, leaving some reflected power which increases as the bunch spacing increases. These matching conditions also give a stable steady state, as in the previous case.
Finally, in the presence of heavy transient beam loading on the fundamental mode, the total power from the generator will be
\[ P_g = P_b + P_d + P_r, \]
where \( P_r \) denotes the reflected power, now different from zero.

**Remark**

If for a given power the distance between bunches is large, it follows that the instantaneous energy removed by the beam at each single passage in the cavities is quite large. The cavity must refill before the next bunch passes, and this gives the kind of modulation shown in Fig. 5.

5.3 **Higher-order mode losses**

There is now concern with higher-order eigenmodes (parasitic modes) in the cavities. Again it is clear that some Fourier components of the current may interact with these higher modes, leading to an additional beam loading. This interaction may be resonant for some modes, but on the average it is incoherent, in which case a good approximation consists of looking at the energy loss for a single bunch passage. For this effect the bunch length is quite significant since for very short bunches the current spectrum extends to higher frequencies.

Each cavity will contribute to the energy loss and, since for a large machine the number of cavities increases very rapidly, the effect becomes non-negligible and is very soon comparable with synchrotron radiation losses.

The generator will eventually have to compensate for these additional losses in two ways:

i) The total voltage in the cavities must be increased because higher-order mode losses appear as an energy loss for each particle in the bunch, which adds to the energy loss by synchrotron radiation:

\[ V_{\text{total}} = V \left( 1 + \frac{U_{\text{hm}}}{U_0} \right), \]

where \( V \) is the required voltage when the additional energy loss \( U_{\text{hm}} \) is neglected.

ii) The higher-mode effect requires an additional amount of beam power:

\[ P_{b\text{ hm}} = U_{\text{hm}} I_{\text{av}}. \]

Now the total generator power becomes

\[ P_g = P_b + P_{b\text{ hm}} + P_d \left( 1 + \frac{U_{\text{hm}}}{U_0} \right)^2 + P_r, \]
Fig. 3

Fig. 4

VARIATION OF CAVITY STORED ENERGY WITH TIME

- 1.0 -

- 0.8 -

- 0.6 -

- 0.4 -

- 0.2 -

0 2 4 6 8 10 12 14 16 18 20

TIME MICRO SECONDS

BUNCH LENGTH \( \sim 3 \times 10^4 \) \( \mu s \)

\( \sim 0.5 - 1.4 \) \( \mu s \)

BUNCH SPACING \( 18.5 \) \( \mu s \)

CAVITY FILLING TIME \( \sim 25 \) \( \mu s \)

Fig. 5
and it is seen that the contribution to wall losses from the higher modes will be more significant than the additional beam power as soon as the energy loss on these modes becomes of the same order of magnitude as the synchrotron radiation loss.

It can be noticed that another contribution comes from the vacuum chamber itself where parasitic modes are also excited. However it appears that for large storage rings this component is less important than the one that has been discussed.

5.4 Scaling laws

As the general formulae for the generator power requirement are quite complicated, when including all the effects we mentioned earlier, the optimization is done by computer.

Qualitative features, however, seem to be easy enough to understand. First of all if the higher modes are neglected, together with the transient effect on the fundamental mode, the beam power will then scale roughly with top energy as $E^{3/2}$ (see Section 3); the Joule losses depend very much on the choice of the total length of the cavity, which determines the total shunt impedance, but, if the maximum power per metre that the cavities can handle is used, then this length will only depend on the total voltage and will increase much faster than $E^{3/2}$. An increase in length keeping clear of the maximum power per metre will make it increase roughly as $E^{3/2}(L_c \propto E^{3/2})$, so that a reasonable approximation is $P_g \propto E^{3/2}$.

If we now include the parasitic-mode losses, assuming a constant bunch length and a constant number of bunches, they will scale like the product of the cavity length ($\propto E^{7/2}$) with the number of particles per bunch ($E^{3/2}$), namely like $E^4$. Certainly this can be reduced a little bit at the expense of the cavity losses by reducing the total cavity length, but anyway this effect will increase the slope of $P_g$ with energy.

As to the last effect, which corresponds to a voltage modulation (reflected power), it is essentially related to bunch spacing. If the number of bunches stays constant it will increase rapidly, and to keep this power at a low level it is necessary to increase the number of bunches; then there is concern with the number of crossing points and with electrostatic beam separation at some unwanted crossing points.

Quantitatively speaking, a reasonable scaling with a constant number of bunches is shown in Fig. 6 where the power has been optimized for several energies by E. Keil. An increase in bunch number will decrease the generator power.

Finally, although no clear limitation appears, either the power consumption will increase drastically (technical constraint) when going to a larger radius, or dynamical constraints will appear when increasing the number of bunches.
Fig. 6
An alternative solution, when many bunches are necessary, would be a double ring with the required amount of common straight sections.

The use of superconducting cavities will reduce all these effects considerably.

6. COMMENTS ON DESIGN ALTERNATIVES

Twenty years ago storage rings appeared as an alternative to synchrotrons on the way to higher energies, although they could not replace them completely.

Nowadays we begin to think about alternatives to "conventional" e⁺e⁻ storage rings such as PEP, PETRA, and LEP (they are quite close to simple extrapolations from SPEAR). There are several reasons for this:
- size of a conventional very high energy storage ring;
- constraints due to luminosity, power loss, and tolerances;
- capital and running costs.

However, the present alternatives appear more as possible improvements starting from a conventional machine, than as actual design alternatives which at the beginning may require small-scale models.

Among possible improvements, the designers are certainly thinking very much along the lines of using superconducting cavities either to save power or to increase the top energy, but there are still problems to be solved before these can be used in a storage ring. However, if it could be shown very soon that a set of such cavities was reliable and stable enough in the presence of high beam current, it could be possible to build a smaller storage ring for the same top energy. The same conclusions are true also for the magnetic structure, where possible improvement may come from new types of magnets, leading to a smaller storage ring for a given energy. Designers have been reconsidering combined-function magnets, but at present these cause as many problems as they solve, apart from the fact that they may give less flexibility in machine operation. In any case it seems that developing new techniques may require the construction of smaller-size models before they can be used in very high energy storage rings. Small existing storage rings can also be used to check new types of components.

Looking to the near future it seems that (even if this does not appear clearly throughout this paper) e⁺e⁻ storage rings may reach theoretical and technical limitations, and that new types of machine may then be necessary in order to reach higher and higher energies even with less luminosity. Along these lines we can mention the following research:
- beam-beam collisions with two linacs;
- collective acceleration;
- laser acceleration.
Let us conclude by mentioning another possible design alternative that is somewhat clearer for designers. This is the possibility of reaching a given energy by steps at lower energies (and perhaps lower luminosity). In the perspective of conventional storage rings this means that in the beginning only the tunnel is matched to the final top energy.

Bibliography

The present report made extensive use of the following writings:
- LEP 70-(1 to 100) notes.
- The LEP Study Group, Design study of a 15 to 100 GeV $e^+e^-$ colliding beam machine (LEP), CERN/ISR-LEP/78-17, in which more detailed references can be found.
SUPERCONDUCTING ACCELERATING CAVITIES
FOR HIGH-ENERGY e⁺e⁻ STORAGE RINGS

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I. INTRODUCTION

Reasons for the interest in superconductivity

In high-energy electron storage rings, the RF power dissipated in the normal conducting cavity walls in the ultimate limitation on reaching higher luminosities or higher energies. Owing to the increase of the RF dissipation with the eighth power of the particle energy, both construction and operation costs for the whole RF system play a determinant role.

Superconducting cavities offer the possibility of reducing the cavity losses to almost zero, thus leaving nearly all RF power available for the beam. In addition, as the accelerating field is no longer determined by cooling problems and cost considerations, one might hope to reduce the length and number of the cavities and so not only make up for the higher cost/m of superconducting systems but even lower the total construction costs.

In the past there has been much enthusiasm about using superconductivity in RF accelerators, and also many drawbacks and objections. In the meantime, the RF separator has been completed in our Institute and has been operating at CERN for 2000 hours without any trouble\(^1\) -- so confidence in superconductivity has reasons to rise at present. This is also due to the fact that the limitations, especially on the field strength, are now better understood. Let me talk a little about the state of the art, about some specific problems arising in storage rings, and how we try to overcome them. First, however, I want to present an extension of Burt Richter's cost optimization procedure\(^2\) for the case of superconducting cavities.

2. COST CONSIDERATIONS

I will briefly repeat Richter's procedure, and then I can show what has to be changed to include superconductivity.

He introduces unit costs \(k_1\) (SF/m) circumference in the arcs; \(k_2\) (SF/m) installed RF power; \(k_3\) (SF/m) active cavity length; and \(k_4/\epsilon\) (SF/W) RF operating costs, based on a 10-year operation with 6000 h on-time each year. So the costs \(C\) of construction and operation are approximated by the following equation:

\[
C = 2\pi R k_1 + (P_b + P_D) (k_2 + k_4/\epsilon) + L k_3 + F
\]
(R = radius in the arcs; \( P_b, P_D \) beam and cavity RF power; \( L \) = cavity length; 
\( F \) = fixed costs that do not enter the cost optimization; \( \varepsilon \) = klystron efficiency).

Then he uses an equation connecting luminosity \( \mathcal{L} \), beam power \( P_b \), particle energy \( E \), bending radius \( \rho \), maximum tune shift \( \Delta v \), and amplitude function \( \beta_y \) at the interaction point:

\[
\mathcal{L} = 1.23 \times 10^{33} \frac{\Delta v P_b \text{ (MW)} \rho \text{ (m)}}{E^3 \text{ (GeV)}^3 \beta_y \text{ (m)}} ,
\]

and the definition of shunt impedance \( Z \):

\[
Z = \frac{V_{RF}^2}{P_D L} ,
\]

where \( V_{RF} \) is proportional to the energy loss per turn by synchrotron radiation

\[
U \text{ (MV)} = \frac{88.5 \times 10^{-3} \times E^3 \text{ (GeV)}^3}{\rho \text{ (m)}} .
\]

By differentiation, we get the minimum of \( C-F \) with respect to \( L \) and \( R \) at

\[
\rho^* = \sqrt{\frac{A_1 \delta^2 Y k_2^4 + 2 A_2 \eta \delta^h \sqrt{k_3 k_2^2 / Z}}{2\pi k_1 \xi}} ,
\]

\[
L^* = \frac{A_2 \eta \delta^h}{\rho^* \sqrt{Z k_3}} ,
\]

where

\[
A_1 = 0.8 \times 10^3 \text{ (V} \cdot \text{m)} \text{ from the luminosity equation;}
\]
\[
A_2 = 0.885 \times 10^{13} \text{ (V} \cdot \text{m)} \text{ from the energy loss per turn;}
\]
\[
\gamma = \frac{\beta_y}{0.05} ; \quad \eta = \frac{V_{RF}}{U} ; \quad \xi = R/\rho ; \quad \delta = E/100 \text{ GeV} ; \quad k_2^4 = k_2 + k_4 / \varepsilon .
\]

In superconducting cavities \( Z \) becomes very high, so that \( L^* \) becomes extremely short. This makes necessary an accelerating field which cannot be realized. So for superconducting (SC) cavities \( L \) is determined by the achievable accelerating field \( E_{acc} \) rather than by a cost optimization:

\[
L^*_{SC} = \frac{A_2 \eta \delta^h}{E_{acc} \rho^*_{SC}} .
\]
and therefore

\[ \rho_{SC}^* = \sqrt{\frac{(A_1 \delta^3 \gamma k_2' + k_3 A_2 \eta \delta^b)}{2\pi k_1 \xi}} \quad (8) \]

Here the cavity dissipation \( P_D \) has been neglected, and the additional costs for cooling etc. are included in \( k_3 \), which is now higher than in the normal conducting case.

As the beam tube of LEP is designed to withstand not more than 25 MW of synchrotron radiation, for constant \( P_b \) the following equation applies instead of Eq. (8):

\[ \rho^* (P_b = \text{const}) = \sqrt{\frac{k_3 A_2 \eta \delta^b}{E_{acc} 2\pi \xi k_1}} \quad (9) \]

We now put a set of numbers into these equations, for instance the unit costs that Richter uses:

- \( k_1 = 25.6 \times 10^3 \) SF/m
- \( k_2' = 5.96 \) SF/W
- \( k_3^{NL} = 162 \times 10^3 \) SF/m
- \( k_3^{SC} = 324 \times 10^3 \) SF/m

and \( R/\rho = 1.5 \), \( V_{RF}/U = 1.4 \) from the LEP parameter list. For \( Z \), I take 24 MN/m and for \( \gamma = 2 \) (\( \beta^* = 0.1 \) m), \( E_{acc} = 3 \) MV/m.

I am not able to discuss these numbers here; for instance, SF 160,000 for a 1 m cavity seems to me rather high already, and when I double this amount for niobium cavities, including cryostat, surface treatment, refrigerator, and so on, it is almost certainly too pessimistic. However, the general result, shown in Table 1, is rather independent of the specific numbers.

A superconducting machine of 70 GeV would be smaller and less expensive than a normal one\(^3\). A smaller machine even would allow a higher frequency to be used, with consequently higher accelerating field, cheaper cavities, etc. Unfortunately, as was pointed out by Keil\(^4\), this machine could not be extended to 100 GeV. On the other hand, if we compare a normal LEP-70 with a superconducting LEP-100 we notice some similarity: the radius becomes almost the same, the cavity length is comparable, the RF power installed is more than sufficient. One has therefore to defray only the costs of the superconducting items, which amounts to about SF 568 (probably less!), to convert LEP-70 into an optimized LEP-100.
Comparing the total costs for both alternatives one might ask, Why not build a superconducting LEP-100 right away? This brings us to the question of the superconductivity in general: What has been achieved? What are the limitations? What open questions are to be answered?

Table 1
Cost comparison of normal (NC) and superconducting (SC) LEP 70-100.

<table>
<thead>
<tr>
<th></th>
<th>70 GeV (NC)</th>
<th>70 GeV (SC)</th>
<th>100 GeV (SC)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bending radius ( \rho ) (m)</td>
<td>2469</td>
<td>1576</td>
<td>2355</td>
</tr>
<tr>
<td>Cavity length ( L ) (m)</td>
<td>1491</td>
<td>629</td>
<td>1753</td>
</tr>
<tr>
<td>Beam power ( P_b ) (MW)</td>
<td>18.9</td>
<td>29.6</td>
<td>25</td>
</tr>
<tr>
<td>Cavity power ( P_D ) (MW)</td>
<td>40.5</td>
<td>0.005</td>
<td>0.013</td>
</tr>
<tr>
<td>Total power ( P_{\text{tot}} ) (MW)</td>
<td>59.4</td>
<td>29.6</td>
<td>25</td>
</tr>
<tr>
<td>Luminosity ( \mathcal{L} ) (cm(^{-2}) sec(^{-1}))</td>
<td>(10^{32})</td>
<td>(10^{32})</td>
<td>(4 \times 10^{31})</td>
</tr>
<tr>
<td>Cost of ring (MSF)</td>
<td>596</td>
<td>380</td>
<td>(568)</td>
</tr>
<tr>
<td>Cost of RF (MSF)</td>
<td>354</td>
<td>176</td>
<td>(149)</td>
</tr>
<tr>
<td>Cost of cavities (MSF)</td>
<td>242</td>
<td>204</td>
<td>568</td>
</tr>
<tr>
<td>Total cost (MSF)</td>
<td>1192</td>
<td>760</td>
<td>(1285)</td>
</tr>
</tbody>
</table>

3. RF SUPERCONDUCTIVITY. STATE OF THE ART.

3.1 Surface resistance

The \( Q \)-value of a cavity can be split into

\[
Q = \frac{C}{R_s},
\]

where \( C \) is a geometrical constant of the order of 200–300 \( \Omega \) for accelerating cavities, and \( R_s \) is the surface resistance.

We call

\[
I = \frac{R_s(300 \text{ K, Cu})}{R_s(T, \text{ Nb})}
\]

the improvement factor. \( I \) can take values up to \( 10^6 \), depending on material, temperature, frequency, and surface quality.
The surface resistance can be computed by theory; it is proportional to \( \omega^2 \exp (-\Delta/kT) \). Experimentally one observes a deviation from the exponential law at very low temperatures:

\[
R_s = R_{\text{theory}} + R_{\text{res}}
\]

For niobium, in all practical cases at 4 K the theoretical surface resistance (Table 2) can easily be achieved; somewhere below 4 K the residual resistance, which is independent of temperature, begins to dominate (Fig. 1).

### Table 2
Theoretical surface resistance and improvement factor of Nb and Nb₃Sn at 4 K

<table>
<thead>
<tr>
<th>Frequency (MHz)</th>
<th>Niobium</th>
<th>Nb₃Sn</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rₛ (Ω)</td>
<td>I</td>
<td>Rₛ (Ω)</td>
</tr>
<tr>
<td>8665</td>
<td>1.85 \times 10^{-5}</td>
<td>1.3 \times 10^{3}</td>
</tr>
<tr>
<td>2860</td>
<td>2.02 \times 10^{-6}</td>
<td>6.9 \times 10^{3}</td>
</tr>
<tr>
<td>700</td>
<td>1.21 \times 10^{-7}</td>
<td>5.7 \times 10^{4}</td>
</tr>
<tr>
<td>500</td>
<td>6.17 \times 10^{-8}</td>
<td>9.4 \times 10^{4}</td>
</tr>
<tr>
<td>350</td>
<td>3.02 \times 10^{-8}</td>
<td>1.6 \times 10^{5}</td>
</tr>
</tbody>
</table>

Fig. 1: Surface resistance \( R_s(f,T) \) of different niobium cavities as a function of \( T_c/T \).
It can be seen that a lower frequency gives lower $R_\text{res}$ ($\propto \omega^2$), but because $R_\text{res}$ is not decreasing with frequency, it begins to dominate earlier at lower frequency. This means that at high frequencies an operating temperature below 4 K, i.e. at 2 K, reduces $R_\text{s}$ by a factor larger than 10, whereas at low frequencies it does not pay to work below 4 K. This has positive and negative consequences: 4 K means simple refrigerators, less stringent requirements for vacuum seals (no superleaks), but much less heat conductivity of the helium; care has to be taken that parts having a complicated shape, such as coupling loops etc., are properly cooled. If, in the future, Nb$_3$Sn cavities would be available, this argument would shift to higher frequencies. With Nb$_3$Sn it would be possible to operate at 4 K up to 10 GHz; with Nb only up to 1 GHz, roughly. Unfortunately, in Nb$_3$Sn cavities $R_\text{res}$ is still fairly high, so that the theoretical $R_\text{s}$ has not been achieved so far.

In conclusion, it can be said of the surface resistance of Nb that for low-frequency applications, where operation at 4 K is the best choice, achieving improvement (factors of $10^4$-$10^5$) is no problem.

This is true for low fields. What happens if the field strength is increased to practicable values?

### 3.2 Field limitations

It is well known that the superconductivity breaks down when an applied d.c. magnetic field $B_{dc}$ becomes larger than a critical field $B_c (T)$.  

For niobium, $B_c (T=0)$ is about 1930 G, $T_c = 9.2$ K. Unfortunately, in almost all measurements with RF, the obtainable fields are much lower than this d.c. value.

For Nb we can classify these field limitations into three types:

a) **Thermal breakdown.** This occurs when a large fraction of the surface is heated by the RF to such an extent that $T_c$ is reached. In general, this limitation can be avoided by proper cooling.

b) **Magnetic breakdown.** At certain spots on the surface, say at a microscopic peak, hole, or slit, or at local dirt inclusions, a lowered $B_{sc}^* < B_c (Nb)$ leads to a
local normal-conducting spot which thermally explodes and spreads over a macroscopic area within a few microseconds, and the field breaks down owing to the increased losses. This breakdown is typical for many cavities. The limitation can be shifted to higher fields only by careful surface preparations, very clean assembly, avoidance of dust, and so on.

Employing all these methods, field values close to that of the d.c. critical field have been achieved in TE and TM cavities at 10 GHz\(^5\), and about half that value in accelerating or deflecting cavities at 10 GHz\(^6,7\).

c) **Electron loading**\(^8\). In accelerating cavities an electric field perpendicular to the surface is present. This field leads to electron field emission.

The electrons gain energy in the electric field and then hit a surface. Electron multiplication takes place if the secondary emission corresponds to an output ratio larger than 1 and when there are trajectories where accelerated electrons hit a surface where secondaries see favourable electric fields for acceleration.

We have not only the classical two-side multipacting, where electrons oscillate back and forth in resonance with the electric field in the gap of the cavity, but also one-side multipacting exists in the corners, where the electric field goes through zero. In TM\(_{015}\) modes this develops by the bending of electron trajectories on closed loops as shown in Fig. 2\(^8\). In a stable potential well, the electrons oscillate on cyclotron resonance-like trajectories. Some \(E_T\) is needed to gain energy for the production of secondaries.

All multipactor levels scale like \(E_B E \sim f/n\) and have an impact energy \(\varepsilon \sim (E/f)^2 \sim (1/n)^2\), where \(f\) is the resonance frequency and \(n\) the order of the resonance.

What do these electrons do to the cavity?

a) They cause surface damage, i.e. irreversibly lowering \(Q\) and the achievable field. Very often this can be cured only by a subsequent surface treatment, e.g. anodizing, chemically polishing, or electropolishing.

b) They reversibly load the cavity down to such an extent that \(Q\) decreases by more than a factor of 10.

c) They heat the cavity wall at certain spots, especially in corners, where the magnetic field and therefore the cavity dissipation (\(\sim R_B B^2\)) is high. If there is a local bad spot where \(B_C\) is lowered, we have a magnetic breakdown, which is triggered by the electrons.

It is possible that more than one of these effects occur in one cavity, and that the behaviour differs from run to run. This makes the analysis of measurements rather troublesome and an improved diagnosis of the effects very important.
Fig. 2: High field trajectories in a TM$_{11}$ mode at 0.5 GHz with $d_0 = 225$ mm. The field is chosen so that between the parallel plates ($R \geq 110$ mm), two-side multipacting of first order is approached; this is shown to drift quickly to the outside wall. The field emission trajectories, starting in the high-field region around $R = 81$ mm, drift to the outside wall. With a retardation of $20^\circ$ relative to the E-field variations, the same surface is hit only far outside at $R = 180$ mm. The trajectory of the secondary electron hits the outside wall and there feeds quasi-resonant one-side multipacting. In this cavity the dangerous one-side multipacting of first order would already occur at about 10 mT, which corresponds to about $E_{\text{max}} = 5$ MV/m (from Ref. 8).

There are two ways of working on this electron problem. First, one has to understand the effects which drive the electrons out of the surface, both the primary field emission and the secondary emission. We know that the niobium-oxide layers on the surface play a predominant role there, and understanding of the enhancement of the $e^-$ emission in the oxide seems to be very important. Perhaps a method will be found for improving the surface in such a way that the electron emission is reduced.

The second possibility is to compute the electron trajectories and try to find a geometry which suppresses as many electron paths as possible.

Both ways are being pursued at Karlsruhe and elsewhere$^{9,12}$. As long as this electron problem is not solved, we have to live with the electrons. This means that it is necessary to find a design where the field is sufficiently below the actual threshold for severe electron loading. This is indeed possible, so that
an application for storage rings is attractive even with the electron problem as it stands today.

Which accelerating field gradient can we expect? As we do not have measurements for a storage-ring cavity, we can only guess from results on similar cavities.

We have measured two types of cavities, one of which is shown in Fig. 3 (the other has a gap of 210 mm).

Fig. 3: 700 MHz single-cell cavity with a gap length of 112 mm, a beam hole diameter of 40 mm, and rounded corners
It differs from a storage-ring cavity in the following way:
a) the operating frequency is 700 MHz,
b) the beam hole is smaller by more than a factor of 2,
c) the outer corner is rounded by a radius of 10 mm.

These two cavities gave repeatedly the result shown in Fig. 4. Please note the peak field at the surface, 8 MV/m, and the Q-values above $10^9$ at 4 K. However, a few remarks are necessary about the acceleration fields of 4 MV/m shown on the graph.

The long cavity makes a cell distance of $\lambda$ necessary, which means a very poor ratio of active/physical length. The short cavity would allow a cell distance of $\lambda/2$.

If we scale the result of the long cavity down to 350 MHz, assuming that the multipacting level which is responsible for the field limit scales linearly with the frequency, we would expect only 1 MV/m as accelerating field, because of the $\lambda$-cell distance. For the short cavity, with a cell distance of $\lambda/2$, this would be 2 MV/m. It is this linear scaling of multipactor levels that urges people to think earnestly about higher frequencies for storage rings\(^3\).

On the other hand, we learned in the meantime that the outside corners ought to be sharp in order to minimize $E_T$ and hence the impact energy; so we expect better results from the cavities we are building right now for DORIS, about which I will report in a moment.

At this point let me add a few remarks about using a higher frequency. What could be gained if a higher frequency would be feasible from the standpoint of the storage ring designer? We think that the cavities would become cheaper; the material costs scale with $1/f$. The saving in fabricating costs might go linearly, or less than that, with frequency. All handling, surface treatment, furnace treatment, and assembling would be simpler. The smaller cryostat with less helium consumption would need less space in the tunnel. All this should result in a lowered cost/m figure. In any case, by achieving higher fields the total cavity length is reduced.

To achieve the necessary aperture, comparatively large iris diameters would be chosen -- therefore the cell-to-cell coupling becomes large and a very simple iris-coupled structure would become feasible.

Among the disadvantages of higher frequencies, it is found that $Q_s$, the number of synchrotron oscillations per turn, becomes higher, and the power going into higher modes increases. The overvoltage ratio $V_{RF}/U_0$ has to increase, thus eating up some of the increase gained in the accelerating field.
Fig. 4: $Q_0$-value of a 700 MHz single-cell iris cavity with gap length 210 mm (other dimensions as shown in Fig. 3) as a function of peak electric field at the surface. The accelerating field shown is valid when this geometry would be used with a cell-to-cell distance of $\lambda/2 = 214$ mm.
Also, because of the larger beam hole, another reduction of fields by an increased peak field/accelerating field ratio cannot be avoided. As was mentioned earlier, operation at 1.8 K, or Nb₃Sn (which is not yet available), would be necessary. If a multicell structure is chosen, the output coupling of higher modes is not yet solved, whereas it is available for single cells; single cells, on the other hand, would be just too many at higher frequencies.

The replacement of all readily installed RF equipment would increase the costs of the conversion, but it seems to me that the reduction in cavity costs might account for that.

I have tried to express all these arguments in numbers; the result is shown in the Appendix.

4. PROBLEMS TYPICAL FOR STORAGE RINGS

So far, I have mentioned mainly the facts that are common to all applications of superconducting cavities. In storage rings, some additional problems have to be considered:

a) Vacuum. The cavities are connected to a vacuum pipe, several kilometres in length, part of which is at room temperature. How long can the cold cavity be allowed to pump gases from the warm parts of the system down to its cold surface? What happens when a vacuum failure occurs? Can the cavity be operated for appreciably long periods of time?

b) Synchrotron radiation. Although the cavities are situated in straight sections, not all radiation can be shielded from its surface. How much harm is then done to the superconducting properties?

It was measured at DORIS\(^{13}\) that about 1 W/m of scattered synchrotron radiation hits the cavity surface. This is quite tolerable from cooling considerations, but it generates about \(10^{14}\) photoelectrons per second, i.e. 16 mA! Assuming we could completely eliminate the electrons produced by field and secondary emission from the cavity, we get approximately the same amount from the storage ring itself!

c) Higher-order modes. It is necessary to couple the higher-order mode energy out to a room temperature load without dissipating too much in the helium. This coupling has to reject the fundamental mode.

We calculated the power going into helium for several operating modes; for instance, for a superconducting cavity and for a cavity that has gone normal, say, by a field breakdown\(^{14,15}\). In the superconducting case, one needs a \(Q_{\text{ext}}\) of the output coupling of less than \(10^6\) only when one mode happens to resonate with
the beam. For the normal conducting case, \( Q_{ext} = 10^3 \) is necessary for many modes. If a cavity is normal, the RF source is switched off and the beam remains on; also, the fundamental mode is excited and has to be coupled out or very quickly tuned away.

d) Very high RF power is needed for the beam. The design of an input coupling system for a superconducting cavity which couples some 100 kW from outside of the cryostat into the niobium cavity without excessive heat losses is not a trivial task.

e) Cavity fabrication, refrigeration technique, realistic cost estimates.

Some of these questions can only be answered by testing in a real operating storage ring. We therefore started to build two cavities which will be tested in DORIS; the last section will give an outline of the progress of this experiment as it stands now.

5. SUPERCONDUCTING DORIS TEST CAVITIES

5.1 Cavity design and fabrication

Two test cavities are being built right now: one of them, with a gap of 27 cm, is shown in Fig. 5; the other has a shorter gap of 22.5 cm. The geometry is chosen according to our best present knowledge in order to reduce electron loading. Many trajectory calculations suggested this shape, especially the gap distance, the sharp corners, and the elliptic roundings at the beam holes. As we have vacuum inside and atmospheric pressure outside, we had to reinforce the end plates, because 1 \( \mu \text{m} \) deformation gives a frequency shift of 1 kHz. The bandwidth of the cavity loaded by beam and input coupling is 5 kHz. The figure shows also the flanges for input coupling, higher-mode output, and tuner. The cavities are made of sheet niobium, which is argon arc-welded. Only the coupling parts are machined from solid material. (One cavity costs DM 15,000 for fabrication + DM 15,000 for material.) Both cavities are due at the end of this year.

5.2 RF components

a) In this particular experiment the input coupling has to bring about 100 kW of beam power into the cavity; this is shown in Fig. 6. The main features are the following: a field transformer increases the coupling strength and keeps the field inside the cavity undistorted. The coupling loop is flooded by liquid helium. The inner and outer conductors are separated capacitively at two temperature levels in order to reduce heat influx from outside to the cold parts. The cavity window is separated from the outside world by a ceramic window at 80 K and a second window at room temperature. Tests on this system at high power are at present under way. Cryotests of critical components have been successfully completed.
Fig. 5: Niobium test cavity for DORIS
Fig. 6: Input coupling system for the DORIS cavity. The coupling hole in the cavity wall (e) is partially closed by the field transformer (a). This has the purpose of reducing field distortions and field enhancement, and it increases the coupling strength. The coupling loop (b) is cooled by liquid helium. The inner and outer conductors of the coaxial line are capacitively separated (c) to reduce heat influx and to separate parts having different temperatures. All parts marked black are made of niobium. The ceramic window (d) is cooled by liquid nitrogen. (Not shown in the picture: a similar separation and a water-cooled window at room temperature, and the coaxial waveguide connection outside the cryostat.)

b) The output couplings for higher modes are shown in Fig. 7. Two of them are needed to cope with an azimuthal asymmetric mode. The exponential line is used to reject the fundamental mode. This system has been tested on a full-size, room-temperature, model cavity in our laboratory. It couples out all modes up to 2 GHz with a coupling $Q_{ext} \leq 10^6$. (Modes above this frequency disappear through the beam hole.) This system is described in detail elsewhere\textsuperscript{16}.)
Fig. 7: Cross-section of one higher-mode output coupling with exponential coaxial line to reject the fundamental mode
c) The tuner deforms the end plates slightly, as shown in Fig. 8.

![Fig. 8: Tuning system for DORIS cavity. The cavity (a) is mounted in a supporting frame (b) which holds the movable bars (c). By pressing the connecting band (d) downward, the bars pull the beam tubes (e) outward. The system acts against the atmospheric pressure of the helium bath that presses the end plates inward.](image)

5.3 Cryostat

The cavity will be cooled in a helium bath at 4 K. Beam pipes will be cooled before reaching the cavity and will act as a baffle. The evaporating cold He gas is used to precool the 80 K shield, the beam tubes, and the couplings. The cryostat design has been completed; it will be available early in 1979.

6. CONCLUSIONS

Superconducting accelerating cavities have the potential of reducing both construction and operating costs of e+e− storage rings. Q-values achieved so far are sufficient; accelerating fields approach values which make an application interesting. A higher operating frequency would be much preferred from the standpoint of superconductivity. Typical problems for storage rings have to be studied in an experiment at a real storage ring. The construction of two cavities to be tested at DORIS is under way.
Acknowledgements

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P. Kneisel O. Stoltz


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13) G. Bathow, private communication.

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16) L. Szecsi, Breitbandige Auskopplung für parasitäre Modes mit Hochpass-Filter-Charakteristik für Speicherring-Resonatoren, Primärbericht 08.02.02.F02L (KfK, Karlsruhe, 1978).

1) Overvoltage ratio: 

\[ q = \frac{V_{RF}}{U_0} \]

\[ F(q) = 2 \left[ \sqrt{q^2 - 1} - \cos^{-1} \left( \frac{1}{q} \right) \right] \]  
Eq. (3.61)

\[ F(q) = \frac{\xi \cdot \alpha \cdot k \cdot E_i}{J \cdot E_0} \]  
Eq. (5.141)

\[ \frac{2 \tau}{\tau_C} = e^\psi / \xi \]  
Eq. (5.137)

\[ \tau_C = \frac{1}{\alpha} = \frac{2 T_0 E_0}{U_0} \]  
Eq. (4.20)

\[ U_0 = \frac{C \cdot \tau^b}{\rho_0} \]  
Eq. (4.8)

\[ \tau_C = \frac{2 T_0 \rho_0}{C \cdot E_0^3} \]

\( V_{RF} \) = peak cavity voltage; \( U_0 \) = energy loss/turn; \( U_0 \) includes energy loss by synchrotron radiation and higher-order modes, but for the first approximation the loss by higher modes is omitted.

\( I_\xi = 2 + D \) [Eq. (4.51)]

\[ D = (\alpha R / \rho_0) << 1 \text{ therefore omitted} \]

\( \alpha \) = momentum compaction factor = \( (2\pi R / C) \cdot (1 / Q^2) \)

\( R \) = mean radius in the arcs

\( \rho_0 \) = bending radius

\( C \) = circumference

\( E_0 \) = nominal energy of stored beam (eV)

\( E_1 = 1.08 \times 10^8 \text{ eV} \)

\( k \) = harmonic number \( C \cdot RF / c \)
\[ \tau_q = \text{quantum lifetime of stored beam} \]
\[ \tau_\varepsilon = 1/a_\varepsilon \text{ damping constant of energy oscillations} \]
\[ T_0 = C/c = \text{revolution time} \]
\[ C_\gamma = 8.85 \times 10^{-5} \text{ m(GeV)}^{-3} \text{ [Eq. (4.2)].} \]

I have used the LEP-70 parameter list (CERN/ISR-LEP/78-17) as a basis for all further calculations. The procedure to evaluate \( q \) is as follows: \( C = 22208 \text{ m,} \)
\( E_0 = 70 \text{ GeV, and } \tau_q = 24 \text{ h} \) are fixed numbers; \( \xi \) is determined by Eq. (5.137); then \( q \) is given by Eqs. (5.141) and (3.61).

2) \( Q_s \) [From the note LEP-70/76 (E. Keil)]:
\[ Q_s = \left( -\frac{k \alpha \cos \phi_s V_{RF}}{2\pi E_0} \right)^{1/2}. \]

3) r.m.s. bunch length \( \sigma_z \):
\[ \sigma_z = \frac{\alpha C (\sigma_\varepsilon/E_0)}{2\pi Q_s} \text{ (E. Keil, private communication).} \]

This gives \( \sigma_z (I=0) \); I multiply this value by the bunch lengthening factor \( B \):
\[ B = \left\{ 0.0915 \times 0.06 \times 5 \left[ E_0 \text{ (GeV)} k \cos \phi_s \right]^{1/2} \left( \frac{C}{2\pi R} \right)^{1/2} \right\} \text{ (E. Keil, private comm.).} \]

\[ \left( \frac{\sigma_\varepsilon}{E_0} \right)_0 = \frac{C_q \gamma_0^2}{J_{\overline{\varepsilon}} \rho_0} \text{ [M. Sands, Eq. (5.48)]; } C_q = 3.84 \times 10^{-13}; \quad \gamma_0 = \frac{E_0}{m_0 c^2}. \]

4) For conversion of LEP-70 into LEP-100 by switching from normal conducting cavities to superconducting ones, the following assumptions should be used: \( C, \rho_0, \tau_q \), and the number of bunches are kept constant. The tune \( Q \) is scaled like \( (\gamma^0)^{1/2} \), and the momentum compaction factor \( \alpha \) is therefore changed by
\[ \alpha = (2\pi R/C) \frac{1}{Q^2}. \]

5) Higher-order mode losses:
I use E. Keil, C. Pellegrini, A. Turrin and A.M. Sessler, Nuclear Instrum. Methods 127, 475 (1975), for the evaluation of the higher-mode losses in the cavities. For the losses in the beam pipe, I assume \( Z_{pipe} = 0.37 \times 10^{-10} \Omega/\text{m} \) and scale according to \( \sqrt{F} \) (E. Keil, private comm.).

The beam current of LEP-100 is calculated according to the restriction that \( P_b \) has to be kept constant at 25 MW. Therefore I (one beam) = 3.35 mA.
Now I have to consider different accelerating structures for different frequencies. In this way the beam-hole radius is kept constant at $a = 0.06 \text{ m}$ (except at 1500 GHz, where $a = 0.05 \text{ m}$ is used because of $E_p/E_{acc}$ and because of the shunt impedance). At frequencies from 700 MHz onwards this beam hole gives high enough cell-to-cell coupling to use a multicell iris-coupled structure, whereas for 357 and 500 MHz a single-cell structure with individual input couplings for each cell would be preferred. The achievable peak electric field at the surface is scaled from our measurements at 700 MHz ($E_p = 8 \text{ MV/m}$), and the corresponding accelerating field gradient $E_{acc}$ is given by LALA computations.

The values arrived at are collected in Table A1, which shows that for superconducting cavities a frequency higher than 360 MHz would be preferable, if the problems connected with the reduced bunch length and the increased $Q_s$ could be solved. The last line of the table shows an estimated $Q_s$ for the case where the converted machine is to be operated at full luminosity and at 70 GeV.

### Table A1

Conversion of LEP-70 into LEP-100 using superconducting cavities of different frequencies. Beam power $P_b = 25 \text{ MW}$.

<table>
<thead>
<tr>
<th>Frequency (MHz)</th>
<th>350</th>
<th>500</th>
<th>700</th>
<th>1000</th>
<th>1500</th>
</tr>
</thead>
<tbody>
<tr>
<td>Overvoltage ratio</td>
<td>1.14</td>
<td>1.18</td>
<td>1.22</td>
<td>1.29</td>
<td>1.38</td>
</tr>
<tr>
<td>Bunch length (mm)</td>
<td>17</td>
<td>15</td>
<td>13</td>
<td>11</td>
<td>10</td>
</tr>
<tr>
<td>Energy into H.O.M. (MeV)</td>
<td>75</td>
<td>84</td>
<td>80</td>
<td>108</td>
<td>132</td>
</tr>
<tr>
<td>Power into H.O.M. (MW)</td>
<td>0.5</td>
<td>0.6</td>
<td>0.5</td>
<td>0.7</td>
<td>0.9</td>
</tr>
<tr>
<td>Total RF voltage (MV)</td>
<td>4334</td>
<td>4490</td>
<td>4652</td>
<td>4956</td>
<td>5336</td>
</tr>
<tr>
<td>Accelerating gradient (MV/m)</td>
<td>2</td>
<td>2.7</td>
<td>3.2</td>
<td>3.2</td>
<td>4.1</td>
</tr>
<tr>
<td>Cavity length (m)</td>
<td>2167</td>
<td>1663</td>
<td>1454</td>
<td>1549</td>
<td>1302</td>
</tr>
<tr>
<td>Synchrotron frequency $Q_s$</td>
<td>0.072</td>
<td>0.091</td>
<td>0.11</td>
<td>0.15</td>
<td>0.20</td>
</tr>
<tr>
<td>Synchr. freq. $Q_s$ for operation at 70 GeV and full luminosity</td>
<td>0.107</td>
<td>0.14</td>
<td>0.17</td>
<td>0.23</td>
<td>0.30</td>
</tr>
</tbody>
</table>
LEP SUMMER STUDY

Organized under the Joint Sponsorship of

ECFA and CERN

Les Houches and CERN
10 to 22 September, 1978

SITE AND BUILDINGS

C.J. Zilverschoon
CERN, Geneva

Copies available upon request from Ch. Redman, CERN/ISR
LEP Summer Study Secretariat
Like many other parts of the machine, the buildings described in our report should not be considered as the final design, but rather as the result of a technical feasibility study. We believe that the buildings proposed are reasonable and can be constructed, but we know that alternative solutions are possible and that a number of improvements can be made and must be studied. Next year, our Site and Buildings design group will study a number of points in the LEP project aimed at better exploitation and lower cost.

As for the position of the machine, indicated in the report, this is also an example of a possible place rather than our final proposal. ECFA has recommended that we should study a site close to the present CERN site in the first place, and the one described is a typical example. For next year we plan a number of geotechnical studies to help us in making a more definite proposal. Today's talk will be on the following items:

1. **Site characteristics**
   a) geology,
   b) topography and position of machine,
   c) available services.

2. **Underground civil engineering**
   a) tunnels,
   b) access shafts.

3. **Programme for further studies**
   a) site studies,
   b) civil engineering.

(The most important underground works, the experimental areas, are not on this list since they were already discussed in an earlier session.)

The first figure (Fig. 1) is a geological cross-section of the Geneva region, about 10 km south of the site proposed, but very similar to it. Starting at the bottom we can see the following geological formations:

1) The old crystalline rock that existed already more than 600 M years ago in the Precambrian era. This is granite, which reaches the surface in the Mont Blanc massif and on the other side in the Beaujolais and further on in the Massif Central. Very old earth movements have shaped it into a dish of several km depth with even deeper local depressions.
Figure 1: Geological Profile through the "Pays de Genève" by Danilo Rigassi
2) These depressions are filled with deposits of the Carboniferous period of the Paleozoic era (about 300 M years ago) when there was a tropical vegetation. This same layer is found closer to the surface at the coal mines of St. Etienne.

3) Of the next layer, Trias (i.e. the oldest period of the Mesozoic era about 200 M years ago) the lowest part consists of sandstone formed in a desert climate that existed in our region since the end of the Permian period. The upper part of this layer contains salt, deposited somewhat later when the climate had changed and shallow lagunes covered the region.

Up to here the geology is of little practical interest to LEP but we now come to the layers in which LEP might be built.

4) During the rest of the Mesozoic era (i.e. the Jurassic and Cretaceous periods, 200 to 70 M years ago) the region was covered with warm seas in which corals and other small shellfish constructed their skeletons layer after layer of limestone, in total 2 km thick.

Ground movements during this era (early Alpine formation) had started to shape the Salève and the Jura by folding, but towards the end there was a major orogenic event (laramian) which in America created the Rocky Mountains and in Europe caused a general rising of the ground. From then on the region was covered with a rather shallow fresh-water lake and the Jura and the Salève were sticking out, so that they were not covered with the next layer; on the contrary, erosion took away the upper cretaceous on them.

5) This next layer, molasse, was formed in the lake from deposits that were eroded in the Alps and carried along by rivers. This happened in the Cainozoic era about 30 M years ago and a layer of up to 300 m thickness was formed. Since the Alpine formation continues this layer is gently folded and its level rises towards the Jura.

6) The top layer, only a few tens of metres thick and therefore invisible on the section, was created in a very recent past some 10 to 20'000 years ago when the ice of the Würm glaciation withdrew. At the maximum the Rhône and Arve glaciers had reached all the way down to Lyon and all the way up (in altitude) to the top of the Salève and almost the top of the Jura. When they withdrew, they left behind a layer of stones, gravel, sand and clay called moraine. In the region considered, the thickness may vary between zero and 25 m.

As a result of these geological formations, if one makes a borehole in our region, one will typically find: a few decimetres of top soil (terre végétale), then a few tens of metres of moraine, a few hundred metres of molasse, a few thousand metres of limestone and then the very old stone and finally the granite. To complete the geological picture, one can look at a very much simplified geological
map of the region where the type of rock one finds immediately under the top soil (say 50 cm deep) is indicated (Fig. 2). It can be seen that in the proposed location the machine will touch the area where the limestone reaches the surface.

After about 15 km of tunnel work, CERN has by now a large experience of constructing in moraine and molasse. We know that moraine can give problems because it is inhomogeneous with a rather low module of elasticity so that it may move in a non-uniform way. Where it contains clay, there may be water tables that can move and cause instabilities.

The molasse contains very little water and has turned out to be very stable, if treated properly. Caution is only needed where it contains a specific type of clay (montmorillonite) that swells when absorbing water. This instability can be avoided by keeping the molasse under pressure and avoiding intake of water.

We have no experience in limestone. From tunnel work elsewhere in the Jura that we have seen, we do not expect too much trouble. But some types of limestone are soluble and contain caves and underground water veins and we know that these exist also in some places in the Jura. A careful study of borings to be made in the limestone and in particular also in the interface molasse-limestone is therefore necessary.

The next aspect, the topography of the site, is of course more directly visible and in Fig. 3 we can see the position of the machine and the surface level lines 450, 500 and 550 m. At the circumference the highest point of the surface is 588 m and the lowest 428 m. In our proposal the machine would be built in a tilted plane in such a way that where it almost touches the SPS it is at the same level as the SPS, i.e. 400 m, and the whole LEP tunnel lies between 380 and 490 m above sea level. In this way, we stay as much as we can in the molasse and keep the depths of the eight access pits between 35 m and 90 m. Without the constraint to touch practically the SPS tunnel we could have done better and, of course, bending the machine up and down can also help, but that needs more detailed studies. The topology of the injection complex of the machine can be explained from the next picture (Fig. 4).

The linac building is at ground level and contains a trench, made by cut-and-fill a few metres down, just enough for shielding. The booster synchrotron is at the same level as the linac, just below the ground level, in a tunnel made by cut-and-fill. From this synchrotron we go through a sloping tunnel to the injector storage ring, the tunnel of which is bored in the molasse, some 17 m lower than the linac, then we go down again through sloping tunnels to the main LEP tunnel, 40 m lower than the injector storage ring. Visible also are the access shaft to the surface building and the SPS tunnel. A top view of the area is shown in Fig. 5.
Figure 2
Figure 4: Artist's impression and schematic view of the injector area. Each major construction is followed by its height above sea level (in metres)
Building underground in this way seems to imply no legal problems to construct even under built-up areas, only the ground around the auxiliary buildings where the access shafts reach the surface will have to be put at CERN's disposal. Another great advantage is that the environmental disturbance is extremely small as can be realized when flying over the present SPS area.

Coming now to the point of services: in its proposed position, the LEP programme would profit from all existing CERN services such as buildings, technical and scientific services and even 400 GeV protons would be available for future options, such as e-p collisions or further acceleration in a large accelerator that could be built in the LEP tunnel.

Also the general facilities that the Geneva area offers to CERN (airport, housing, schools ....) would be available to the LEP programme.

The electrical 400 kV line, built for the SPS is strong enough to deliver, in addition, the power required for LEP.

The capacity of the pipe that brings the cooling water from the lake to the SPS and back to the river is sufficient to cover, in addition, the needs of LEP.

We come now to the underground building work and Fig. 6 shows a horizontal section through the region around an experimental area of which there are eight along the circumference. We see the main tunnel, the klystron tunnels, the access tunnels and the vertical access shaft. From the goods lift in the access shaft we have a direct access to the klystron tunnels (which will be shielded from the main tunnel during operation) for maintenance and repairs and also the main tunnel can be reached without passing through the experimental area. Cables and pipes also avoid the experimental area by way of the access tunnels.

A cross-section of the normal part of the main tunnel (Fig. 7) shows its dimensions: they are the same as those of the SPS tunnel, but as compared with that tunnel, we have left out a metal sheet and a layer of concrete, so that the hole to be bored in the rock is 40 cm smaller. We foresee only a single layer of concrete made from prefabricated slabs ("voussoirs"). We shall not try to keep out the water, but drain it away in order to avoid building up of pressure. The waterflow expected, at least in the molasse, is very small. By adopting a very low level of the beam (80 cm) we provide ample space for a possible future second machine.

In Fig. 8 we see a section through the RF and klystron tunnels. Both are somewhat bigger than the magnet section of the main tunnel, viz. 4.40 and 5 m diameter, respectively. Finally, we can see a section of an access pit (Fig. 9), leading from an auxiliary building to an experimental hall. It is 10 x 4 m with
Figure 7
space for cables, pipes, emergency stairs, a personnel lift and a very large loading platform to bring down even the largest machine components and experimental equipment with a maximum weight of 60 tons.

The action of tunnelling will be done by means of a tunnel boring machine similar to the one which was used for the SPS tunnel. These machines are being developed and improved continuously and there is one type that interests us very much. In the normal machines the boring element consists of a round shield, covered with cutters. Since behind the boring machine the tunnel is covered with concrete slabs on the inside, the machine cannot go back, its boring shield is too big. But in a type that is in use in a few places (for tunnels of a smaller cross-section) the shield is replaced by a set of three arms with cutters and these arms can be moved radially. This has two large advantages: it can make tunnels with a varying diameter (which is interesting for our RF tunnels) and it can go backwards (which may be an interesting way to make our klystron tunnels). If this machine would not become available in time, we would make the non-standard tunnels like the experimental hall with an Alpine-type machine, i.e. a borer with a mobile head as is used in mining. The injector tunnel and transfer tunnels will be made in the same way as the main tunnel, but the linac and booster synchrotron tunnels, which are very close to the surface, will be made by the cut-and-fill method.

For the vertical shafts explosives will be used. I shall not say anything about surface buildings (they are conventional) but rather go on to the last point on our list, i.e. the programme for studies from now on.

On the point of sites, we want to look at other possible locations. One that seems of particular interest, would be where LEP would touch the SPS in the south, i.e. close to the old CERN site ("Lab I"). In this way we would keep the possibility of colliding with protons but have the additional advantage of being able to use existing ISR facilities: buildings, halls, workshops, control room, maybe ISR tunnel or even ISR itself as an injector. We did not try this before, because we wanted to stay away from built-up areas, but if there are no problems, we could drop this condition. The consequences of an increased diameter must then also be examined: how far would we go into the Jura. For a really big machine, say 15 km diameter, built in one of the positions mentioned, a sizable part would be inside the Jura. If this would turn out to be difficult, there are other possibilities further south in the Haute Savoie where it seems one could stay in the molasse for most of the circumference of even very large machines. But then we would have to give up the e-p collisions. For that reason, we intend to put the main emphasis next year on sites touching the SPS. The geological borings that we shall start next year will also be in that region.
Borings are expensive, costing about 15 to 20'000 SF each and we cannot make an infinite number of them. We intend to start at the interface molasse-limestone, then in the Jura itself, possibly followed by a reconnaissance tunnel to see what one really gets along the projected trajectory of the tunnel. But borings in the molasse will be required as well: one can have surprises such as valleys made by old rivers and now filled with moraine or maybe even underground water tables. In addition, we want to know exactly the topography of the molasse in order to determine the best way of getting at least a few experimental areas practically at surface level, which would mean at the top of the molasse. It might be possible to do this by a simple tilting of the machine plane or else vertical bends would be required, but it will depend on the precise topography of the molasse.

We must also find out exactly which are the limitations of tunnelling under built-up areas on the French as well as on the Swiss part of the site.

On the point of underground civil engineering we want in the first place to keep track of the developments in tunnelling machines and tunnelling techniques in general. I have mentioned the special boring machine with arms, there is also the question of constructing the walls of tunnels and experimental halls in such a way that water is properly drained away without building up a pressure on the walls. We shall maintain our contacts with specialized firms in order to follow these developments.

When looking at the cost estimate, we find that in the underground work the tunnels themselves are relatively cheap but the experimental halls and the access shafts are pretty expensive. The experimental areas are of course of first importance and therefore we have to be careful if we want to economise on them. It is probably out of question to make them smaller; there is on the contrary a wish to make them bigger. Putting some of them near the surface may help to bring the cost down.

We are, ourselves, not really content with the access pits as proposed. They are very large, 10 x 14 m., and allow even the largest equipment for the machine to be brought down in a horizontal position. We want to investigate:

i) if we cannot bring down the largest equipment (mainly vacuum chambers and excitation bars for the magnets) in more than one piece and assemble them downstairs;

ii) how we could bring down equipment in a vertical position which admittedly causes problems at the loading and unloading places.

More drastically, we want to study if we can replace these shafts by sloping access tunnels, possibly combined with shafts of relatively small diameter.
LEP SUMMER STUDY
Organized under the Joint Sponsorship of
ECFA and CERN
Les Houches and CERN
10 to 22 September, 1978

THE PETRA AND PEP PROGRAMMES
A write-up* based on the talks presented by
G. Wolf, DESY, and
R. Schwitters, SLAC

* Written from notes and transparencies by M. Jacob
Copies available upon request from Ch. Redman, CERN/ISR
LEP Summer Study Secretariat
It was deemed appropriate to start the Summer Study at Les Houches with extensive presentations of the PETRA and PEP programmes. These presentations were given by G. Wolf and R. Schwitters, respectively. This paper is written from their transparencies. The reviewer (M. Jacob) is entirely responsible for possible inaccuracies and omissions.

1. The PETRA Programme
   1.1 Foreword

   PETRA is an acrostic for Positron-Elektron-Tandem Ring Anlage. The circumference of the machine is 2.3 Km. The energy range extends from 10 to 38 GeV (5 to 19 GeV beam energy) but the beam energy could be increased to 23 GeV at a later stage. The design luminosity at 15 GeV is $10^{32}$ cm$^{-2}$ s$^{-1}$. Construction started in December 1975. Beams were stored for the first time in July 1978.

   Figure 1 gives the general layout on the DESY site.

   1.2 Physics with PETRA

   At present, one can classify physics research at PETRA along different lines. They are as follows:

   a) $T (b\bar{b})$ spectroscopy and study of the $b\bar{q} (5q)$ states. This corresponds to the lower end of the energy range.

   b) Search for new narrow states (new quarks) and related spectroscopy.

   c) Jet studies with a possible differentiation between quark jets and gluon jets.

   d) Search for heavy leptons.

   e) Study of $\gamma\gamma$ processes.

   f) Study of weak-electromagnetic interference with possible hints at the weak boson(s).

   The present review follows them sequentially.

   The $T$ Family

   In connection with $b$ quark physics a look at the present (experiments at DORIS) is worthwhile. Figure 2.a) gives the $T$ peak as observed in 3 different experiments: DESY-Dortmund-Heidelberg-Lund, in DASP, DESY-Heidelberg-Munich and Aachen-DESY-Hamburg-Seigen-Wuppertal in PLUTO. The value of $R$ off resonance is $5.2 \pm 1$.

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* G. Voss presented the machine and discussed its performance by September 1978 in his talk at Les Houches.
Figure 1: The PETRA machine. General layout.
The parameters of the resonance are obtained in a (by now) standard way. The three experiments report respectively:

<table>
<thead>
<tr>
<th></th>
<th>PLUTO</th>
<th>DASP 2</th>
<th>DESY–HD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M$(GeV)</td>
<td>9.46 ± 0.01</td>
<td>9.46 ± 0.01</td>
<td>9.46 ± 0.01</td>
</tr>
<tr>
<td>$\Gamma_{ee}$(KeV)</td>
<td>1.3 ± 0.4</td>
<td>1.5 ± 0.4</td>
<td>1.1 ± 0.3</td>
</tr>
<tr>
<td>$B_{\mu\mu}$(10$^{-2}$)</td>
<td>2.7 ± 2.0</td>
<td>2.5 ± 2.1</td>
<td>1.0 ± 3.4</td>
</tr>
<tr>
<td>$\Gamma_{tot}$(KeV)</td>
<td>&gt; 20</td>
<td>&gt; 20</td>
<td>&gt; 15</td>
</tr>
</tbody>
</table>

The mean values, as presented at the Tokyo Conference by G. Flügge are $\Gamma_{tot} = 50$ KeV, $\Gamma_{ee} = 1.3 ± 0.2$ KeV, $B_{\mu\mu} = (2.6 ± 1.4) \times 10^{-2}$.

A prominent and very recent result has been the observation of the $T'$. The variation of the cross-section over the resonance peak is shown in Figure 2.b). The parameters, as reported by the two different groups, are as follows:

<table>
<thead>
<tr>
<th></th>
<th>D–HD–M</th>
<th>DASP 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M$(GeV)</td>
<td>10.016 ± 0.01</td>
<td>10.012 ± 0.01</td>
</tr>
<tr>
<td>$\Delta M$(MeV)</td>
<td>557 ± 5</td>
<td>555 ± 3</td>
</tr>
<tr>
<td>$\Gamma_{ee}$(KeV)</td>
<td>0.32 ± 0.10</td>
<td>0.5 ± 0.2</td>
</tr>
<tr>
<td>$\Gamma_{ee}(T)/\Gamma_{ee}(T')$</td>
<td>3.4 ± 0.9</td>
<td>≈ 3</td>
</tr>
</tbody>
</table>

The mass difference $\Delta M$ between the $T'$ and the $T$ ($556 ± 3$) is definitely smaller than the one observed in the $\psi$ family, namely $591 ± 1$ MeV.

The observed values for the $e^+e^-$ partial width, for both the $T$ and the $T'$ clearly favours $Q = 1/3$, or the $b$ quark assignment. It is then interesting to note that this is an assignment which gives a constant value for the quantity $\Gamma_{ee} / |\Sigma c_i q_i|^2$ for all $1^3S_1$ vector states. This is shown in Figure 2.c).

The spectroscopy of the $J/\psi$ family is displayed in Figure 3.a). The established states and the still expected ones are shown as solid and dashed lines respectively. The $T$ family should show a similar pattern.

Figure 3.b) gives the excitation energies as measured for the $J/\psi$ family and calculated by the Cornell group for the $T$ family.

Next to the $T$ family, states with $b$ quarks should be produced in pairs. They correspond to the quark assignments

$$ B_s^- = b\bar{u} \quad B_s^0 = b\bar{d} \quad B_s^+ = b\bar{s} \quad B_c^- = b\bar{c} $$

They will decay weakly.
Figure 2.a) : The $T$ as observed at DORIS
Figure 2.b) : The $T'$ as observed at DESY
Figure 2.c): The quantity $\Gamma_{vee}/|\Sigma c_i Q_i|^2$ (in KeV) for the vector mesons corresponding to the fundamental $^3S_1$ states.
Figure 3.a: The J/ψ spectroscopy with established states (solid lines) and still expected ones (dashed lines)
Figure 3.b) : Excitation energies (in MeV) for a $q\bar{q}$ system as functions of the quark mass $M$
The weak coupling at the quark level is defined through a Cabibbo matrix which is written as follows:

\[
A = \begin{pmatrix}
  c_1 & s_1 c_3 & s_1 s_3 \\
-s_1 c_2 & c_2 c_3 - s_2 s_3 e^{i\delta} & c_1 s_2 s_3 + s_2 c_3 e^{i\delta} \\
  s_1 s_2 & -c_1 s_2 c_3 - c_2 s_3 e^{i\delta} & c_1 c_2 c_3 + c_2 s_3 e^{i\delta}
\end{pmatrix}
\]

with \(c_1 = \cos \theta_1\), \(s_1 = \sin \theta_1\).

The Cabibbo angle proper corresponds to \(\theta_c = \text{Arg} \cos \theta_1\). The values of \(\theta_2\) and \(\theta_3\) are bounded by 20° and 160° respectively, when \(\theta_c = 13^0\). The CP violating phase \(\delta\) is such that \(\sin \delta > 5 \times 10^{-3}\).

The weak coupling induces second order transitions between the \(B_0\) and \(\bar{B}_0\) states. As a result a \(B_0\bar{B}_0\) primordial configuration, which should lead through semi-leptonic decays to final states of the type \((e^+\nu\chi)(e^-\bar{\nu}\chi')\), with yields referred to as \(N^++(N^-+)\) should, through \(B_0\bar{B}_0\) mixing, also give final state configurations of the type \((e^+\nu\chi)(e^-\bar{\nu}\chi')\) and \((e^+\bar{\nu}\chi)(e^-\nu\chi')\), with yields referred to as \(N^+\) and \(N^-\). It will be important to study such a mixing, measuring

\[
\gamma_2 = (N^{++} + N^{--})/(N^{++} + N^{--} + N^{++} + N^{--})
\]

for which theoretical estimates (Ellis et al., Ali et al.,) are at the level of \(\gamma_2 = 0.1\) (non strange) and \(\gamma_2 = 0.5\) (strange).

The \(N^{++}\) and \(N^{--}\) yields should actually differ. This is a CP violating effect associated with the phase \(\delta\). The asymmetry is given by

\[
a = \frac{N^{++} - N^{--}}{N^{++} + N^{--}} \approx 4 \tan 2\delta
\]

Estimates for the asymmetry \(a\) are at the level of \(5 \times 10^{-3}\) (non strange) and \(2 \times 10^{-3}\) (strange).

Similar effects, but now associated with closed loops with Higgs bosons, could possibly be one order of magnitude larger. This is however very model dependent.

**Search for New Quarks**

The first approach is to look for narrow states. The cross-section at the peak depends directly on the energy resolution of the beam \(\Delta E\). It increases as \(S\) at fixed radius and decreases as \(\rho^{-\frac{1}{2}}\), where \(\rho\) is the magnetic radius. The cross-section at the peak is given in practice by \(\bar{\sigma} = \frac{12\pi}{3} \frac{\text{Tee}}{\Delta E}\)

One may then compare the resolution at PETRA and at DORIS, giving also the corresponding value for a LEP machine (at 100 GeV per beam).
where $S$ is in (GeV)$^2$.

The signal over noise ratio can then be estimated (taking $Q = 2/3$ and $\Gamma_{\text{vee}} = 5$ KeV) as a function of the mass of the resonance. This gives for the PETRA range

<table>
<thead>
<tr>
<th>$M_\nu$ (GeV)</th>
<th>Integrated cross section (nb GeV)</th>
<th>Signal over Noise ratio (PETRA)</th>
<th>Events/day ($L = 10^{31}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>0.44</td>
<td>16</td>
<td>$32 \times 10^3$</td>
</tr>
<tr>
<td>20</td>
<td>0.24</td>
<td>8</td>
<td>$8 \times 10^3$</td>
</tr>
<tr>
<td>30</td>
<td>0.11</td>
<td>4</td>
<td>$2 \times 10^3$</td>
</tr>
</tbody>
</table>

This is manageable but one is far below the values which are typical of $J/\Psi$ spectroscopy.

A schematic picture for the variation of parameter $R$ as a function of the centre-of-mass energy is given in Figure 4. Next to $b\bar{b}$ states one should expect $t\bar{t}$ states ($Q = 2/3$). There is however no solid prediction for the $t$ mass. The expected precision on $R$ should be at the level of 0.1. The increase associated with the $b\bar{b}$ threshold ($Q = 1/3$) may then be hardly detectable.

Jet Studies

As first shown by the SLAC-LBL study in the $3 - 7.4$ GeV range, the final state hadrons organized themselves into two jets, as expected from a primordial quark anti-quark system. This leads to a decrease of the sphericity with increasing energy as the jet structure becomes more and more pronounced. As is now well known, the jets follow the $1 + \cos^2 \theta$ distribution associated with the primordial quarks of spin $1/2$. PLUTO, at DORIS, has now extended this analysis up to $9.5$ GeV. The jet picture has thus been further confirmed. Also, the jet axis, whether defined from charged particles or neutrals, is found to be the same. The variation of the mean observed sphericity as a function of energy is shown in Figure 5. It combines SLAC results (open dots) and DORIS results (full dots).

There is an obvious departure from the phase space model which matches data as well as the jet model at 3 GeV, and full agreement with the jet model. At 5 GeV half of the energy is found within a cone of $33^\circ$. At 9.4 GeV it is within a cone of $28^\circ$ only. Going further in energy is of great potential interest. One may
Figure 4: An artist's impression of what could be the behaviour of $R$ as a function of energy. The shaded region is still terra incognita.
Figure 5: Variation of the observed sphericity as a function of energy.
then differentiate between the "naive" jet picture, whereby jet members have a fixed transverse momentum with respect to the jet axis, from expectations based on QCD according to which gluon emission leads to a widening of the jet with increasing centre-of-mass energy. A possible limiting behaviour could then correspond to a fixed opening angle within which a fixed and large fraction of the energy should be confined for a fixed and large fraction of all hadronic events. In the model of Sterman and Weinberg for instance, one gets a limiting angle which should be almost reached as the centre-of-mass energy gets beyond 20 GeV.

Jet fragmentation is interesting to study and particles near the limit of phase space should have quantum numbers strongly correlated with those of the primordial hadron constituent. In $e^+e^-$ annihilations, this should lead to important correlations among secondaries of high energy, taking one on each side so that they could be readily associated with the primordial quark and antiquark respectively. At present, available information refers to charge correlations only. Figure 6 shows SPEAR results with a particle with $x > 0.5$ (0.7) required in order to better define the jet. There is an important correlation on the same side. Two fast particles on the same side tend to balance out their charge. The expected quark-antiquark correlation on the away side appears only in the latter case or only when the two particles are required to take a very large fraction of the available energy.

While quark jets should be the common feature, search for gluon jets is of great importance. An a priori interesting hunting ground is offered by the narrow vector resonances since, in the framework of QCD, they should decay into 3 primordial gluon jets. The mass of the $T$ could be enough for the corresponding structure to emerge with, in particular, a coplanar but not colinear structure. An obvious consequence is that the mean observed sphericity should increase. This is indeed the case as shown in Figure 7. The sphericity is much higher on the $T$ peak than just outside the peak and it takes a value in agreement with QCD expectations. On the $T$ peak the mean observed sphericity is found to be $0.38 \pm 0.02$ for charged particles (Figure 7 from PLUTO results), and $0.37 \pm 0.02$ for neutrals (DESY-Hamburg-Heidelberg-Munich). Off resonance the corresponding values are $0.27 \pm 0.015$ and $0.19 \pm 0.02$, respectively.

The mean multiplicity also increases on the resonance, as "naively" expected from gluon jets. It rises from $4.9 \pm 0.1$ off resonance to $5.9 \pm 0.1$ on resonance.

Further studies are much needed on the $T$ peak. Another a priori interesting hunting ground for gluon jets is opened by $\gamma$ triggers. In such a case the high energy photon should be produced together with two primordial gluon jets rather than with a quark-antiquark pair in a positive $C$ configuration. The latter
Figure 6: Charge correlations among jet fragments
Figure 7: Increase in sphericity on the T resonance
configuration should rather correspond to soft photons. A particular case of special interest is \( \chi \) state production, the \( \chi \) recoiling from the \( \gamma' \) (\( \gamma'' \)) with \( \gamma \) emission. The angular distribution of the jet axis with respect to the photon direction should then change from \( 1 + \cos^2\theta \) to \( 1 - \frac{3}{2} \cos^2\theta \), depending on whether one considers a two-gluon system or a quark-antiquark pair. The angular distribution is then defined with respect to the photon direction, in the \( \chi \) rest frame.

**Search for Heavy Leptons**

Charged heavy leptons can be produced through the one-photon annihilation process. The production cross-section is then given by

\[
\sigma_{L^+L^-} = \sigma_{\mu^+\mu^-} \beta \left(1 + \frac{1 - \beta^2}{2}\right)
\]

where \( \beta = E/P \). When \( \sqrt{s} \gg 2 M_L \) the cross-section corresponds to the point-like value and contributes for one unit of \( R \). The decay depends on the nature of \( L \).

If the \( L \) lepton corresponds to the sequential series or is of the ortho type its decay modes will be of the type \( L^- \to \nu_L + (e\nu, \mu\nu, \tau\nu \text{ or } q\bar{q}) \).

The expected branching ratios (as estimated by Y. Tsai) depend on the mass. Keeping the \( \tau \) as a reference (\( M_\tau = 1.8 \) GeV), one can list for instance what is then expected for \( M_L = 6 \) GeV. It is as follows:

<table>
<thead>
<tr>
<th>Mode</th>
<th>( B (1.8) )</th>
<th>( B (6) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( e\bar{\nu} )</td>
<td>0.18</td>
<td>0.23</td>
</tr>
<tr>
<td>( \mu\bar{\nu} )</td>
<td>0.18</td>
<td>0.23</td>
</tr>
<tr>
<td>( \tau\bar{\nu} )</td>
<td>--</td>
<td>0.20</td>
</tr>
<tr>
<td>( \pi\nu )</td>
<td>0.10</td>
<td>0.01</td>
</tr>
<tr>
<td>( \rho\nu )</td>
<td>0.22</td>
<td>0.05</td>
</tr>
<tr>
<td>( A_\mu \nu )</td>
<td>0.1</td>
<td>0.02</td>
</tr>
<tr>
<td>Hadrons, ( \nu )</td>
<td>0.08</td>
<td>0.26</td>
</tr>
</tbody>
</table>

Other types of leptons can a priori exist. One may consider leptons with charges normally associated with anti-particles, or (and) with no specific neutrino of their own. This leads to decay modes which may indicate some apparent violation of the \( \mu e \) universality.

There may also be heavy neutral leptons, as those (right handed) which can be advocated in order to suppress parity violation effects in atomic experiments. In this case \( E \) and \( M \) neutral leptons could be the partners of the \( e \) and \( \mu \) respectively, in right-handed doublets. The mass eigenstates could be linear combinations of \( E \) and \( M \). Such leptons could be produced through \( W \) exchange in the \( t \) channel, hence together with a neutrino \( e^+e^- \to \nu_L \), or in pairs through \( Z^0 \) (s channel) or \( W \) (t channel) exchange. Production rates can then be estimated to be at the level of about \( 10^{-34} \) and \( 10^{-35} \) cm\(^2\) respectively, provided that they
exist at all, and, assuming a mass of the order of 3 GeV say. The production rate is large enough for experimentation, with 100 to 10 events per day at 15 GeV beam energy.

Once produced, such leptons should decay leptonically (eeν, eμν, μνν) about 50% of the time, the remaining decay modes associating charged leptons with hadrons.

Study of 2γ Processes

This can be well studied at PETRA and the PLUTO detector in particular could soon contribute important results. A detailed discussion of the 2γ process is to be found elsewhere in the proceedings of the Summer Study, in LEP Summer Study/1-13 and this will therefore not be reviewed here.

Weak Contributions

Weak effects should manifest themselves as the energy increases. At present energies the electromagnetic cross-section (one-photon exchange) decreases as α²/s whereas the weak cross-section increases as G²s (point-like four fermion coupling). At higher energies extrapolation of such behaviour gives an increasing role to weak interactions, with eventually s² > α²/G². Nevertheless, what is expected is rather some damping of the weak interaction effect, the point-like interaction being only a low energy approximation to Z⁰ exchange. This may however occur with both couplings playing a competing role, as predicted in gauge theories.

In the PETRA energy range, the emergence of weak contributions should lead to:
(i) A change in the behaviour of the cross-section.
(ii) Forward-backward asymmetry.
(iii) Changes in the differential cross-section according to beam polarization.

The weak coupling of the Z⁰ to a quark-antiquark pair or lepton pair ff is written

\[ G^\pm f_{\nu \mu} (\nu_\tau - a_\tau \gamma_5) \]

With a forward-backward asymmetry for quark

\[ A = \frac{F - B}{F + B} = \frac{3}{2} g \frac{a_\tau}{g_{\mu}} \frac{s m_\mu^2}{s - m_\mu^2} \]

The ratio \( \frac{a_\tau}{g_{\mu}} \) is 3/2 for the u, c and t quarks and 3 for the d, s and b quarks.
It is 1 for a muon pair.

One may thus expect asymmetries at the level of 18% at 30 GeV and 36% at 40 GeV, when studying jet production. We have used \( g = \frac{G}{8\sqrt{2} \pi a} \approx 4.4 \times 10^{-5} \text{ GeV}^{-2} \).
For lepton pair production, the cross-section divided by the one-photon exchange contribution reads
\[ 1 - v_e v_\mu g \frac{s}{(s - m_Z^2) + \frac{1}{s - m_Z^2}} \]
with \( v_e = v_\mu = -1 + 4 \sin^2 \theta W = 0 \)
\[ a_e = a_\mu = -1 \]
in the standard model of Weinberg-Salam.

The forward-backward asymmetry is then given by
\[ \frac{F - B}{F + B} \approx -\frac{3}{8} a_e a_\mu \frac{s m_Z^2}{s - m_Z^2} \]
which, with \( m_Z \) beyond the PETRA energy range, can be approximated as
\[ \frac{F - B}{F + B} = 7 \times 10^{-5} s \text{ with } s \text{ in } \text{GeV}^2. \]
This gives 6% at \( \sqrt{s} = 30 \text{ GeV} \) and 12% at \( \sqrt{s} = 40 \text{ GeV} \).

Figure 8.a) gives the ratio between the full cross-section for muon pair production and the cross-section calculated from one-photon exchange, as obtained for different values of \( a \) and \( v \). Figure 8.b) shows the related variation of the forward-backward asymmetry. Although some specific effects should occur over the PETRA energy range in its first stage (up to 30 GeV), the expected effects remain small and cannot provide any definite test of the Weinberg-Salam model used for calculating these effects.

1.3 The PETRA Detectors

At present there are 5 main detectors, all resulting from large collaborations. They are respectively:

- CELLO (DESY-Karlsruhe-Munich-Orsay-Paris-Saclay)
- JADE (DESY, Hamburg-Heidelberg-Lancaster-Manchester-Tokyo)
- MARK J (Aachen-DESY-MT-NIKHEF, Amsterdam)
- PLUTO (Aachen-DESY, Hamburg-Bergen-Maryland-Siegen-Wuppertal)

Four of them use a solenoid field configuration. The respective diameters \( D \), lengths \( L \) and Field strength \( B \) are (in metres and Tesla):

- CELLO \( D = 1.5 \), \( L = 3.5 \), \( B = 1.5 \)
- JADE \( 2 \), \( 3.6 \), \( 0.5 \)
- PLUTO \( 1.4 \), \( 1 \), \( 2 \)
- TASSO \( 2.7 \), \( 4.5 \), \( 0.5 \)
Figure 8.a) : The expected cross-section divided by the one-photon exchange cross-section
Figure 8.b): The forward-backward asymmetry
MARK J has a toroidal field configuration. It uses shower counters and magnetized iron. Figure 9 shows each of them. PLUTO, which recently moved away from DORIS, should eventually be replaced by CELLO in the interaction area where it is installed at present. One may rightfully be impressed by the sophistication of all these detectors and perhaps also surprized by some redundancy among them. Nevertheless, this simply reflects a physics where each event is a priori valuable and where there is only one most interesting process, namely the one-photon annihilation. This is very different from hadron physics where detectors usually try to focus on special types of processes, selecting one among many a priori interesting ones. As already mentioned, no detector is yet dedicated to the study of 2γ processes, though PLUTO can probably explore them rather far already.

CELLO (Figure 9.a)) has a 4π coverage for charged particles and neutral detection. It can separate photons from electrons with a high precision and can well distinguish between electrons, muons and hadrons. JADE (Figure 9.b)) has drift chambers and track sampling. Electrons, muons, pions, protons and kaons are separated over the 3 to 15 GeV/c range. The PLUTO detector (Figure 9.c)) has 4π coverage for charged particles and photon detection. It separates electrons, muons and hadrons. The TASSO detector (Figure 9.d)) which uses aerogel and Cerenkov detectors has full particle identification. The MARK J detector (Figure 9.e)) is specialized for muon and lepton pair detection (μ, μ⁺μ⁻, eμ, e⁺e⁻).

Apart from these large detectors, there is a monopole search experiment installed in the present PLUTO area. It uses capton foils inside the beam pipe and is based on a dE/dx measurement. It should be sensitive to a production cross-section two orders of magnitude lower than the point-like (muon pair) cross-section.

This concludes the survey of the PETRA programme.

2. The PEP Programme

2.1 The Machine

The layout of the machine is sketched in Figure 10. The length of the circumference is 2.2 Km. The beam energy can vary between 4 and 18 GeV (with foreseen extension up to 29 GeV). The designed luminosity varies as L = 10^{32} (E/15)^2 cm^{-2} sec^{-1} below 15 GeV. It then decreases and equals 10^{31} at 18 GeV.

The machine should be ready for first beams in October 1979 and the experimental areas will be ready for occupancy in the spring of '79. It is hoped that area 6 will be completed with building and crane by beam turn on time.
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CELLO (Figure 9.a)) has a $4\pi$ coverage for charged particles and neutral detection. It can separate photons from electrons with a high precision and can well distinguish between electrons, muons and hadrons. JADE (Figure 9.b)) has drift chambers and track sampling. Electrons, muons, pions, protons and kaons are separated over the 3 to 15 GeV/c range. The PLUTO detector (Figure 9.c)) has $4\pi$ coverage for charged particles and photon detection. It separates electrons, muons and hadrons. The TASSO detector (Figure 9.d)) which uses aerogel and Cerenkov detectors has full particle identification. The MARK J detector (Figure 9.e)) is specialized for muon and lepton pair detection ($\mu^+\mu^-$, $e^+e^-$).

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The machine should be ready for first beams in October 1979 and the experimental areas will be ready for occupancy in the spring of '79. It is hoped that area 6 will be completed with building and crane by beam turn on time.
Figure 9.1: The Detector CELLO

1. Zentrale Drift- u. Proportionalkammern
2. Zylindrische Schauerzähler (Flüssig Argon)
3. Supraleitende Spule des Detektors
4. Vakuum - Strahlrohr
5. Proportionalkammern für Myon- Nachweis
6. Eisenjoch
7. Helium-Verflüssiger
8. Argon - Kryogenerator
9. Versorgungsleitungen für flüssiges Helium
10. Fahrwerke
11. Elektronik - Schränke

Beteiligte Institute: DESY, Hamburg
DE, Karlsruhe
MP, München
LAL, Orsay
Universität (VI) Paris
CEN, Saclay

DETEKTOR CELLO
Gewicht: 1400 t
Magnetfeld: 15 kT

- 411 -
Figure 9.b) : The JADE Detector
Figure 9.c) : The PLUTO Detector
Figure 9.d) : The TASSO Detector
2.2 The Approved Experiments

No specific request for time or energy have been considered as yet. During a "first round" approvals for several "facilities" have been given. During a "second round" smaller experiments were also included in the programme. A third round just considered DELCO and discussed the programme status.

At present 7 experiments have been approved. They are as follows:

<table>
<thead>
<tr>
<th>Code</th>
<th>Experiment</th>
<th>Collaborators</th>
</tr>
</thead>
<tbody>
<tr>
<td>PEP 5</td>
<td>Mark II detector</td>
<td>SLAC/LBL</td>
</tr>
<tr>
<td>PEP 4</td>
<td>Using TPC chambers</td>
<td>LBL, UCLA, UCR, Johns Hopkins, Yale</td>
</tr>
<tr>
<td>PEP 9</td>
<td>Study of 2γ processes</td>
<td>UCSD, UCSB, UCD, Amsterdam</td>
</tr>
<tr>
<td>PEP 6</td>
<td>MAC detector</td>
<td>Stanford, Wisconsin, Northeastern, Utah, Colorado</td>
</tr>
<tr>
<td>PEP 12</td>
<td>High resolution spectrometer</td>
<td>Argonne, Indiana, Michigan, Purdue</td>
</tr>
<tr>
<td>PEP 14</td>
<td>Search for free quarks</td>
<td>Stanford, Northwestern, Hawaii, LBL, Frascati, Berkeley, SLAC</td>
</tr>
<tr>
<td>PEP 2</td>
<td>Monopole search</td>
<td>Berkeley, SLAC</td>
</tr>
</tbody>
</table>

It is also expected that one experiment will use the DELCO detector (Stanford, Caltech, SLAC).
Figure 11.a) shows a general view of the Mark II detector. It is a detector weighing 1500 tons. The magnet has an inner radius of 1.59 m and a length of 4.18 m. The field strength is 0.5 T and the thickness corresponds to 1.2 radiation lengths. The drift chambers cover 90% of the full solid angle with 3204 cells and 16 layers. The shower counters cover 90% of the full solid angle. There are 8 barrel counters. There are 18 layers with a thickness of 14 radiation lengths. The expected resolution is 0.1/ E. There is a stored read-out for position resolution.

The muon detector system covers 60% of the full solid angle. It is a 4 layer system. At average incidence their iron thickness corresponds to 47, 70, 101 and 132 cm respectively. The detector proper has 3808 triangular proportional tubes. The wire spacing is 2.5 cm.

The time of flight system covers 75% of the full solid angle with 48 double-ended counters. The expected time resolution is 250 psec. There is also a small angle tagging system.

Figure 11.b) shows a layout of the detector of Experiment PEP 4 with TPC's, while Figure 11.c) shows a schematic diagram of the TPC proper. In this set-up the TPC has a length of about 1 m with an inner and outer radius of 0.1 and 1 m respectively. The gas mixture at 10 atmospheres contains 80% Argon and 20% Methane. The drift voltage is 200 KV/m and the wire gain is $10^3$. There are 1200 wires per end cap and altogether $10^4$ "pads". The digital system uses an analogue CCP storage at 50 psec per bucket. The expected dE/dx resolution is less than 3% and the expected spatial resolution is 100 - 150 microns.

The magnet (Figure 11.b) has a radius of 1.1 m and a length of 3.8 m. The field strength is 1.5 T and the thickness corresponds to 0.6 radiation lengths. This includes the vessel of the TPC.

Figure 11.d) gives a layout of experiment PEP 9 with emphasis on 2γ processes.

Figures 11.e) and 11.f) show the MAC detector of Experiment PEP 6. The magnet has an inner radius of 0.5 m and a length of 2 m. The field strength varies from 0.5 to 1 T. The thickness corresponds to one radiation length. The drift chambers have 1,200 channels with a double sense wire system, with 10 layers. The shower counters are set up in a hexagonal array with 13,000 wires and 32 layers. There are 1200 read-out channels.

The hadron calorimeter has 30 layers of steel and proportional tubes with a total thickness of about 1 m. It includes approximately 50,000 wires with read-out in 3,600 channels. The total weight is 550 tons. It is toroidally magnetized for $\mu^+\mu^-$ determination. The time-of-flight system uses counters just
Figure 11.a) : The Mark II detector
Figure 11.b) : Layout of Experiment PEP 4
Figure 11.c): Schematic view of the Time Projection Chamber (TPC)
Figure 11.d) : The $2\gamma$ detector of experiment PEP 9
Figure 11.e) : Layout of the MAC detector of experiment PEP 6
Figure 11.f) : The MAC detector. Blown-up view
the steel with a conventional system. There are three sets of drift chambers. The first one is just outside the shower counters. The second and third ones are outside the steel and determine the muon direction. The total number of channels is 4,800. The overall expected resolution, using tracking, shower counters and calorimeter, is the order of $0.5\sqrt{E}$.

Figure 11.g) gives the layout of the high resolution spectrometer of experiment PEP 12. The magnet has an inner radius of 2.3 m and a length of 4 m. The field strength is 1.7 T. The thickness is very large. There are two sets (inner and outer) of drift chambers. The inner set is similar to the one used in Mark II. The outer layer consists of a double set of cylindrical tube drift chambers.

The shower (and trigger) counters are set as a band with two layers. They are of the lead scintillator sandwich type.

A second phase development should include additional muon counters outside the magnet using the magnet steel as absorber. It should also include photoionization Cerenkov counters between the inner and outer drift chambers.

Figure 11.h) shows the layout of the free quark detector of experiment PEP 14. In this case, the dE/dx and time-of-flight counters are set in 8 layers. They altogether use 400 phototubes. 32 lucite Cerenkov counters are set behind the time-of-flight counters. Finally, 14 multiwire proportional chambers are used to track charge 1/3 particles.

2.3 Progress of Detectors

At beam turn on, in October, 1979, Mark II (PEP 5) should be running and occupy one of the interaction areas (Area 12 on Figure 10). The area next to it (anti-clockwise) will be used for machine study, while accommodating the monopole detector of experiment PEP 2. Either of the two following areas could accommodate the DELCO detector together with either the high resolution spectrometer or the free quark experiments which will be in 6 and 8 respectively.

The next area around (Area 4) will be used for the MAC detector of experiment PEP 6. The last intersection area will house the TPC detector of experiment PEP 4 together with the $2\gamma$ detector of experiment PEP 9.

The present status of Mark II is such that runs should take place from mid-October 1978 to June 1979, for 25 weeks altogether. The detector will then be moved to PEP in June 1979 to start up in October 1979.

Figures 12 and 13 illustrate partially the performance of the detector with recent physics results and Figure 14 gives the efficiency for neutral detection.
Figure 11.g) : Layout of the high resolution spectrometer of experiment PEP 12
Figure 11.h): Free quark search experiment - experiment PEP 14
Figure 12.a) : 5-prong hadron
Figure 12.b): 7-prong hadron
Preliminary Mk II

$K_S^0 \rightarrow \pi^+ \pi^-$ Invariant Mass

$E_{c.m.} = 4.16$ GeV

Figure 13.a)
Figure 13.b)

Preliminary MK II
$K^-\pi^+ + K^+\pi^-$ Invariant Mass
$E_{c.m.} = 4.16 \text{ GeV}$
$D^0$ Recoil Mass
$E_{c.m.} = 4.16$ GeV

Figure 13.c)
Figure 13.d)
Figure 14: a) photon detection efficiency
b) $\pi^0\eta^0$ detection efficiency
Figure 15: Second Moment vs. Mean, for the truncated dE/dx energy loss distribution in the prototype TPC chamber for particles at 800 MeV/c momentum.
Figure 16: PEP Luminosity vs. Beam Energy

<table>
<thead>
<tr>
<th>Curve</th>
<th>$P_{RF}$ (MW)</th>
<th>$L_{RF}$ (m)</th>
<th>Quad/Sext</th>
<th>Cavities</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>6</td>
<td>50</td>
<td>Conven.</td>
<td>Conven.</td>
</tr>
<tr>
<td>B</td>
<td>15</td>
<td>200</td>
<td>Conven.</td>
<td>Conven.</td>
</tr>
<tr>
<td>C</td>
<td>15</td>
<td>200</td>
<td>Supercon.</td>
<td>Conven.</td>
</tr>
<tr>
<td>D</td>
<td>5</td>
<td>200</td>
<td>Supercon.</td>
<td>Supercon.</td>
</tr>
</tbody>
</table>

SINGLE BEAM ENERGY (GeV)

LUMINOSITY ($10^{31}$ cm$^{-2}$ s$^{-1}$)
Studies with TPC are progressing. A prototype chamber with 200 dE/dx samples and 8 "pads" is being tested. At present the precision on the measurement of dE/dx is at the 2.7% level. The precision on the "pads" is 140 μ. The CCD tests have been successful. Figure 15 shows the separation between pions, electrons and protons thus obtained.

A prototype magnet 0.7 m long is being tested. Results are satisfactory.

At present the coil of the TPC detector is under construction. It should be completed in January 1979. Design for the iron is completed and magnetic measurements should begin in August 1979. The construction of the TPC is beginning. The construction of the cylindrical calorimeter is delayed due to funding reasons. The detector should be sufficiently advanced for check-up in the beam in the spring of 1980.

The MAC detector is well into production for all components. It should be ready by October 1979. The high resolution spectrometer should be ready before the spring of 1980. At present, the iron is starting to arrive from Argonne while the coil is being modified. Work is beginning on the other components.

Equipment for the free quark search experiment should be ready by October 1979.

Delays with the 2γ experiment are mainly due to the NaI crystal part of the detector. 60 elements should be available by mid 1979. The remaining 60 should be ready early in 1980 only.

2.4 Future Possibilities

They are summarized in Figure 16 which gives the expected luminosity as a function of energy for different possible stages of development. They correspond to an increasing amount of radio frequency power and to the possible use of superconducting cavities.

This concludes the survey of the PEP programme.
LEP SUMMER STUDY
Organized under the Joint Sponsorship of
ECFA and CERN

Les Houches and CERN
10 to 22 September 1978

EXPERIMENTATION AT LEP*
by
B.H. Wiik
DESY

* Copies available upon request from Ch. Redman, CERN/ISR,
LEP Summer Study Secretariat
1. **Introduction**

In this talk, which was given at the beginning of the LEP Summer Study at Les Houches, some of the more basic processes in $e^+e^-$ annihilation were discussed and the rates estimated. The hope was that this might serve as an introduction and be the starting point for the work in the various working groups. The following topics were treated:

1. Estimate of $e^+e^- \rightarrow$ hadrons
2. Strong Interaction Issues
3. Weak Interaction Issues
4. The Higgs Particle(s)

An excellent review of all the topics discussed here can be found in the report written by J. Ellis and M.K. Gaillard published in CERN Yellow Report 76-18. A complete set of references can be found in this report.

1.1 **Estimate of $e^+e^- \rightarrow$ hadrons**

The total number of events observed with an ideal 4π detector in one day is given by $N = \sigma \cdot L \cdot \epsilon$. Here $\sigma$ is the total $e^+e^-$ annihilation cross-section and $L$ the predicted peak luminosity integrated over one day. $\epsilon$ is an efficiency factor which includes the solid angle coverage and the trigger efficiency of the detector, the difference between peak luminosity assumed and average luminosity obtained and it also includes the data collection efficiency. Based on the experience with SPEAR and DORIS we expect this factor to be of the order of 1/4. This reduction factor should be kept in mind since all the rates listed are evaluated assuming $\epsilon = 1$.

The luminosity is taken from the Blue Book and it is plotted in Fig. 1. Shown is the luminosity curve corresponding to a standard RF cavity (solid line) and the one which would result from the use of superconducting cavities (dashed lines). In the neighbourhood of 40 GeV, where LEP should "take over" from PETRA, one unit of $R$ corresponds to some 40 events/day. At the upper limit of the range allowed by the use of superconducting cavities (100 GeV of beam energy) one unit of $R$ yields about 8 events/day.

It is generally assumed, and this is supported by all the experimental information available, that the time-like current – electromagnetic or weak – couples directly to pointlike fermions as depicted in Fig. 2.

*Figure 2: $e^+e^- \rightarrow f\bar{f}$*
Figure 1: The luminosity as a function of beam energy in the present Design Study.

Luminosity: $L \left(10^{32} \text{cm}^{-2} \text{sec}^{-1}\right)$

- PETRA
- Standard RF Cavities
- With Superconducting RF Cavities
The corresponding contribution to $R$, $R_f$, is written as,

$$R_f = \frac{e_f^2}{2 \cdot s \cdot e_f \cdot v \cdot v_f \cdot g}{(s/m_z^2 - 1) + \frac{\Gamma_f^2}{s - m_z^2} + \frac{s^2 \cdot g^2 (v^2 + a^2)}{(s/m_z^2 - 1)^2 + \Gamma_f^2/m_z^2}} \tag{1}$$

with $s = (2E)^2$ and $g = \frac{G_F}{8 \cdot \sqrt{2} \cdot \pi \cdot \alpha} = 4.4 \times 10^{-5}$ GeV$^{-2}$.

The couplings of the neutral weak current are given by

$$a_f = 2(1^L - 1^R), \quad v_f = 2(1^L + 1^R) - 4 \cdot e_f \cdot \sin^2 \theta \tag{2}$$

We estimate all rates using the standard Salam-Weinberg model, which has only left-handed doublets of leptons and quarks. Indeed, at present, all the experimental data agree with the predictions from the standard model with $\sin^2 \theta = 0.25$. With this particular value for $\sin^2 \theta$ we find

$$v = 0, \quad a = -1 \quad (v = 1, \quad a_v = 1)$$

$$v_u = v_c = v_t = 1/3, \quad v_d = v_s = v_b = -2/3$$

$$a_u = a_c = a_t = 1, \quad a_d = a_s = a_b = -1. \tag{3}$$

The mass of the $Z^0$ and the $W^\pm$ are then respectively:

$$M_{Z^0} = \frac{37.4}{\sin \theta \cdot \cos \theta} = 86.4 \text{ GeV}$$

$$M_{W^\pm} = \frac{37.4}{\sin \theta} = 74.8 \text{ GeV} \tag{4}$$

The $Z^0$ will show up as a resonance in $e^+e^-$ annihilation with an intrinsic width given by:

$$\Gamma = \frac{G_F \cdot m_z^3}{24 \sqrt{2} \cdot \pi} \quad \sum_f (a_f^2 + v_f^2) \tag{5}$$

As explicitly written in (4), all "up" quarks and all "down" quarks in each of the left-handed doublets contribute in the same way. One might then rewrite the sum in (5) as a sum over the number of fermions $N_f$, appearing in 4 different kinds, namely

$$\sum_f (a_f^2 + v_f^2) = (1 + (1 - 4\sin^2 \theta)) \cdot N_L + 2 N_v$$

$$+ 3(1 + (1 - 8/3 \sin \theta)^2) \cdot N_{2/3}$$

$$+ 3(1 + (1 - 4/3 \sin \theta)^2) \cdot N_{-1/3} \tag{6}$$

$$\leq N_L + 2 N_v + 3.33 N_{2/3} = 4.33 N_{-1/3}$$
With the presently "known" number of flavours (including the yet to be found top quark) this gives

\[ \Gamma_{Z^0} \leq 2.1 \text{ GeV}. \] (7)

This width is much smaller than the natural energy spread in LEP 70 (dE/E = 1.23 x 10^{-3} with Wiggler magnets) and hence can be determined directly from a measurement of the total annihilation cross-section.

Note that a precise measurement of the width limits the number of fundamental fermions with \( m < m_{Z^0}/2 \).

The size and the energy dependence of \( R \), derived using the equations above, are shown in Fig. 3. (Note the logarithmic scale.) The value of \( R \) based on one photon exchange alone, is shown as a dotted line. The rates, obtained using the luminosity given in Fig. 1, are also shown.

The most prominent feature in the cross-section is the \( Z^0 \) peak and it will take a very large number of fermions to wash it out. Note that whereas the rates in the vicinity of the \( Z^0 \) peak are very large they are not overwhelming outside of the peak; i.e., one really needs all the anticipated luminosity and \( \epsilon \) must be kept as large as possible.

1.2 Strong Interaction Issues

Here we want to discuss three topics:

a) new flavours,
b) free quarks and gluons,
c) QCD.

New flavours are produced in e^+e^- annihilation with a "known" cross-section and they have well-defined signatures:

1. The occurrence of narrow states (onia) both with and without the quantum numbers of a photon;

2. A step in the total cross-section at the threshold for production of hadrons with open flavour.

We will first estimate the signal and the signal to background ratio for new 1^-- states. The cross-section for producing a vector meson of mass \( M \) integrated over the resonance is given by:

\[ \int \sigma \, dE = \frac{6\pi^2}{M^2} \frac{\Gamma_e \Gamma_h}{\Gamma} \] (8)

where \( \Gamma_e \), the width of the 1^-- to decay into lepton pairs, are given by
Figure 3: The parameter R as a function of centre-of-mass energy. Expected counting rates at different energies (number of events per day) are also indicated.
Present data are consistent with $|\psi(0)|^2 - M^2$. Plugging this into (8) yields

$$\int \sigma \ dE \sim \frac{e_f^2}{M^2} \quad (9)$$

The actual numerical value is obtained by normalizing to the $J/\psi$ and this leads to the following estimate:

$$\int \sigma \ dE = 10^4 \left( \frac{3}{N} \right)^2 \left( \frac{3 e_f^2}{2} \right)^2 \text{nb} \cdot \text{MeV}$$

$\Delta R$, the increment in cross-section, is given by

$$\Delta R = \frac{\int_{0}^{dE} \delta E}{(dE)^B}$$

where $\delta E_B$ is the energy spread in the beams.

With Wiggler magnets $\delta E/E = 1.23 \times 10^{-3}$ and without Wiggler magnets

$\delta E/E = 1.23 \times 10^{-3} (E/70 \text{ GeV})$.

The results are listed in Table 1.

Table 1
Production of New Flavours with $e_f = 2/3$

<table>
<thead>
<tr>
<th>$2 \cdot m_f$</th>
<th>$\Delta R(1^-)/R$</th>
<th>$\Delta R(1^-)/R$</th>
<th>Rate</th>
<th>$\Delta R(\bar{f}f)/R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>4.5</td>
<td>0.6</td>
<td>170</td>
<td>0.22</td>
</tr>
<tr>
<td>100</td>
<td>2.8</td>
<td>0.06</td>
<td>105</td>
<td>0.10</td>
</tr>
<tr>
<td>140</td>
<td>2</td>
<td>0.17</td>
<td>7.5</td>
<td>0.18</td>
</tr>
<tr>
<td>180</td>
<td>1.2</td>
<td>0.13</td>
<td>1.7</td>
<td>0.19</td>
</tr>
</tbody>
</table>

It might be possible to find new onia in this way, but the task is certainly not going to be easy.

The step in $R$, due to a new flavour, is coming from two terms, one associated with the point-like cross-section and the other one with $Z^0$-exchange. This gives

$$\Delta R = 4/3 + 0.104 \cdot R_Z \quad (e_f = 2/3)$$

$$\Delta R = 1/3 + 0.13 \cdot R_Z \quad (e_f = -1/3)$$

The relative changes in $R$ due to the production of new flavours are also listed in Table 1, last column.

This also looks rather marginal, however, if our present ideas are correct, then selecting events with large sphericity should lead to a much improved signal to background ratio without reducing the rate substantially.
The $Z^0$ is a very prolific source of new flavours with $m_f \leq M_\pi/2$. The decay $Z^0 \rightarrow f \bar{f}$ with $e_f = 2/3 (1/3)$ proceed with a rate of about 20000/day (27000/day). However in order to find the new flavour states among the debris of the other decays, one must be able to identify both charged and neutral particles and measure their momentum.

The search for quarks should be an important task at any new accelerator. $e^+e^-$ collisions are particularly well suited since the current couples directly to the quarks in a well-defined manner yielding an initial state where the $q\bar{q}$ pair has all the energy of the $e^+e^-$ pair. The signature for a free quark is less well defined. If we assume that a free quark behaves like a bound one (all other colour charges are shielded) then we have to search for particles with charge 2/3 and 1/3. Most detectors proposed for PETRA and PEP should be able to identify at least the 2/3 variety by measuring $dE/dx$ and it is fairly straight-forward to design a detector to measure particles with 1/3 charge. However, it has recently been suggested that a free quark might behave very differently from a bound quark. Such quarks which might have an extremely short range or even be non-ionizing and could easily escape detection in a general purpose detector.

Deep inelastic experiments with electrons or neutrinos have revealed the existence of yet another particle. These particles, christened gluons, only participate in the strong interaction. The gluons can therefore not be produced directly in $e^+e^-$ annihilation but they might be radiated off a struck quark. Onia states with $C = +1$ are expected to decay into two such gluons. It is conceivable that quarks are confined but gluons not. The signature for a gluon is even less clear than the signature for a quark.

Next I will discuss various tests of QCD. To first order, $e^+e^-$ annihilation leads to two well-defined, back-to-back jets of hadrons in the final state. This simple picture is modified (to first order in $\alpha_s$) in any sort of field theory of the strong interaction as shown in Fig. 4.

![Figure 4: Quark-antiquark production in electron-positron annihilation](image-url)
The hadrons in the jet will thus acquire larger transverse momenta with respect to the jet axis than naively expected, in some rare case one might even observe three jet events. However, note that such a feature is not unique to QCD but will arise in any field theory of the strong interactions.

To test QCD we have to answer the following questions:

1. Are the gluons vector particles, i.e. $J^{PC} = 1^{--}$?
2. Are the gluons flavour neutral?
3. Are they non-abelian, i.e. does the gluon-gluon coupling exist?

It might be that the cleanest information on these points will come from a study of onia decay, provided sufficiently heavy onia exist. Also, if the characteristic mass scale of the strong interaction is around 0.5 GeV, as indicated by present experiments, then PETRA and PEP might well be suited for the task. If one tentatively assigns 30 GeV to the toponium (with $e_T = 2/3$) one expects – using the predicted peak luminosity for PETRA – 30,000 events per day for the production of the lowest state and 10,000 events per day for the first excited vector state.

The predicted branching ratio for $T(1^{--}) \rightarrow 3$ gluons + hadrons is about 0.4 – and a small fraction of these events will presumably lead to three well separated jets. The background due to

$$T(1^{--}) \rightarrow L \bar{L} \rightarrow \text{hadrons and } T(1^{--}) \rightarrow 1\gamma \rightarrow \text{hadrons}$$

can be subtracted by considering events at adjacent energies.

Even cleaner gluon jets can be obtained by studying the decay of the $C$-even $P$ states ($P_{C/\chi}$ for charmonium) $T(1^{--}) \rightarrow \gamma + P(0^{++}, 2^{++}) \rightarrow \gamma + \text{gg} \rightarrow \gamma + \text{hadrons}$. The flavour neutrality of the gluons can be established by searching for long range quantum number correlations in the back to back gluon jets. The angular momentum of the gluon can also be uniquely determined from these decays. It is a more difficult task to establish the non-abelian nature of the gluons. However, given sufficient statistic the decay $2^{++}(0^{++}) \rightarrow 3\gamma$ where a gluon is radiated by one of the original gluons can be used. Also, the rate for the decay $T(1^{--}) \rightarrow \gamma \text{gg}$ might be sufficiently modified due to gluon-gluon interactions to serve as a test.

Although QCD will presumably have been thoroughly tested at PEP and PETRA before LEP starts operation, it clearly remains an important task for LEP to extend the tests into a new energy region.
1.3 Weak Interaction Issues

Here we want to discuss three topics:

a) Determination of the neutral weak coupling constants for quarks
b) New leptons,
b) $W^\pm$ production

One can determine, using unpolarized beams, $v \cdot v_f$ from a measurement of the event rate and $a \cdot a_f$ from a measurement of the angular asymmetry. Both $v$ and $a$ can be independently determined from the purely leptonic reactions $e^+e^- \rightarrow e^+e^-$ and $e^+e^- \rightarrow \mu^+\mu^-$ (Yellow Repor) and such a measurement of hadronic production can therefore be used to find the values for $v_f$ and $a_f$. Since the weak couplings to the valence quarks $u$ and $d$ can be determined from $v$ induced neutral current events we only have to worry about the heavier quarks. Let us consider $e^+e^- \rightarrow ss$. The angular distributions expected in the standard model is plotted in Fig. 5 for $\sqrt{s} = 60$ GeV and $\sqrt{s} = 120$ GeV. The predicted forward backward asymmetry is rather pronounced but the rates are marginal away from the $Z^0$ peak. We expect 25 ($\sqrt{s} = 60$ GeV) and 70 ($\sqrt{s} = 120$ GeV) $s\bar{s}$ events/day. However, in order to carry out such a measurement we have to determine the production angle of the quark and the quark flavour. Present wisdom says that the production angle is given by the direction of the hadron jet and hence "easy" to determine experimentally. It is more difficult to uniquely identify the flavour - in the case of $s\bar{s}$ production one might hope to find back to back jets - one with positive the other with negative strangeness. Therefore, in order to carry out such experiments one must be able to identify particles which travel close together within the jet, such a task might be possible by a combination of $dE/dx$ measurements and Cerenkov analysis. This procedure can easily be generalized to heavier quarks.

For example $e^+e^- \rightarrow c\bar{c}$ might be identified by observing leading $D^+$ respectively $D^-$ in the two back to back jets. Indeed, this identification might improve with increasing quark mass since it is presumably fairly improbable to create heavy quarks by the colour potential created by the original quark. Experiments at PETRA and PEP will be able to test these ideas and hopefully to extract values for the neutral weak couplings to the $s, c, b$ and maybe the $t$-quark.

We next consider the production of charged leptons. The rates are calculated in terms of $\gamma$ and $Z^0$ exchange according to equation (1). The expected rate rises from 40 events/day for a lepton of mass 30 GeV to 7300/day at the $Z^0$ peak. It then falls to 50 events/day for a mass of 70 GeV and 9 events/day only for a mass of 100 GeV. The corresponding steps in $R$ are only a few per cent at the $Z^0$ mass, and of the order of 10% at high energy. The classic signature for charged leptons is to search for final states $e^+\mu^\pm (4\nu)$ resulting from purely
Figure 5: Angular distribution for a pair of strange jets
leptonic decays. However the leptonic branching ratio $B_L$ is to first approximation given by

$$B_L = \frac{1}{2 \cdot N_L + 3 \cdot N_f},$$

where $N_L$ is the number of leptons and $N_f$ the number of flavour doublets; i.e. we expect $B_L \approx 0.085$. Therefore, only about 1.5% of all produced lepton pairs will yield an $e^\pm \mu^\pm$ (4\nu) final state or 0.62 events/day for $m_L = 30$ GeV, 106 events/day at the $Z^0$ peak, 0.7 events/day for $m_L = 70$ GeV and 0.1 event/day at $m_L = 100$ GeV. These rates are very low and it may seem preferable to try to identify one of the leptons through its hadronic decay. The signature would be rather spectacular - well above threshold we expect fast single leptons on one side and a jet of hadrons with comparably low multiplicity on the other side. The expected rate for this final state is nearly an order of magnitude higher than the rate for the purely leptonic final state.

The charged leptons discussed above are of a very orthodox type. Although the motivation for a neutral heavy lepton to partner the electron in a right-handed doublet has evaporated, a search for such unusual leptons, produced in association with a neutrino (W-exchange) or in pairs (W and Z-exchange) should be carried out at high energies. The production in association with a neutrino leads to a spectacular final state: a large transverse momentum imbalance caused by the neutrino and a single jet consisting of leptons and/or mixed lepton-hadrons. Unlike the production of charged leptons, the cross-section for neutral leptons is model dependent. A typical cross-section production for a neutral lepton of mass 3 GeV is indicated in Fig. 6 and compared to the rate for $e^+e^- \rightarrow \mu^+\mu^-$. 

We next turn to $W^\pm$ production. Single $W^\pm$ production is possible, the leading graphs are shown in Fig. 7.

The cross-section however is rather small and increases only logarithmically with energy. At $\sqrt{s} = 140$ GeV the cross-section ($M_W = 74.8$ GeV) is about $2 \times 10^{-37}$ cm$^{-2}$ sec$^{-1}$ or about 1.8 events a day assuming the peak luminosity. The signature will be rather clean; a large momentum imbalance and either a single lepton or a jet of hadrons at large angles resulting from the $W^-$ decay. The positron will in general be at small angles with respect to the beam axis. However, keep in mind that this was estimated assuming $\epsilon = 1$. Realistic estimates including all the cuts will reduce this rate to below 1 event/day.

It is much more attractive to pair-produce the $W$ via the diagrams depicted in Fig. 8.
Figure 6: Production cross-section for a neutral heavy lepton
Figure 7: Single $W^-$ production

Figure 8: $W$ production to lowest order in Gauge theories
In Fig. 8, the first diagram is a standard charged current interaction; the second is the pair production diagram via a virtual timelike photon, and the last diagram which proceeds via a timelike $Z^0$ is very interesting since the $Z^0W^\pm$ coupling is a unique feature of Gauge theory and is completely determined in the standard model. In this model there are characteristic large cancellation effects among the various diagrams.

The resulting cross-section is plotted in Fig. 9. The cross-section rises sharply above threshold. Such a Born term contribution would rise indefinitely if the cancellation among the different terms were not working. If they work, as expected, the production cross-section should have a characteristic rise and fall behaviour, as shown in Fig. 9. This should be enough to ascertain $W$ pair production with a peak value of the order of $1.5 \times 10^{-35}$ cm$^2$, which corresponds now to 90 $W$ pairs per day or 1.2 event per day with a clear $e^+\mu^- (2\nu)$ final state signature. Fig. 9 is drawn for $m_W = 74.8$ GeV. A machine with $\sqrt{s} \geq 200$ GeV would have a peak energy capable of matching the maximum of the production cross-section.

Detection could use the leptonic mode but also the more frequent jet modes. $W$ pair production should also change dramatically with beam polarization, since the first graph corresponds to a particular polarization state only. This has been studied in detail by K. Gaemers (see LEP Summer Study/1-6).

1.4 The Higgs Particle(s)

So far it has been impossible to construct a spontaneously broken gauge theory without introducing Higgs particles. The standard Weinberg-Salam model has one scalar Higgs particle, other gauge models have more, both charged and neutral. Since these particles are a special feature of the gauge models it is obviously very crucial to search for them, and high energy $e^+e^-$ collisions seem to offer the best possibilities. Unfortunately there are no stringent limits on the mass, although most theoreticians feel that the scalar Higgs in the standard model is above 7.0 GeV. However, note that the experimental bound only excludes Higgs particles with a mass less than 15 to 20 MeV and studies of $K$ decays give a bound on Higgs particles in the mass range between 140 MeV and $2 m_H$. These limits will be greatly extended by experiments at PETRA and PEP. The search for the Higgs mesons makes use of the fact that the Higgs likes to couple to heavy particles, with couplings to fermions proportional to $m_f$ and to vector mesons proportional to $m_V$. Higgs bosons are therefore expected to be produced in conjunction with heavy particles and to decay preferentially into the heaviest particles which are kinematically allowed.
Figure 9: W pair production cross-section
At PETRA and PEP energies the best way to produce a Higgs boson seems to be via heavy $1^{--}$onium states, $Q\bar{Q}(1^{--}) \to H + \gamma$. The branching ratio between this decay mode and the lepton pair decay mode is approximately given by:

$$\frac{B(1^{--} \to H + \gamma)}{B(1^{--} \to \mu^+\mu^-)} = \frac{G_F \cdot m_Q^2}{2 \pi \cdot d} \left(1 - \frac{m_H^2}{m_Q^2}\right)^{1/2} = 3.5 \times 10^{-4} \times \frac{m_Q^2}{m_V^2} \left(1 - \frac{m_H^2}{m_V^2}\right)^{1/2}$$

To translate this into a rate, one might assume a toponium with $m_V = 30$ GeV and $e_t = 2/3$ which is produced with a global rate of about 30,000/day at PETRA (assuming the design peak luminosity). With $B(1^{--} \to \mu^+\mu^-) \approx 5 \times 10^{-2}$, one expects that Higgs bosons with a mass of 10, 20 and 25 GeV will appear 110, 90 and 60 times per day respectively. The signature will be a monochromatic photon of respectively 13.3, 8.3 and 4.6 GeV recoiling against a pair of heavy particles. This might look very encouraging, but one should not forget the important background resulting from $1^{--} \to \gamma \cdot g \cdot g$ which, for the parameters above, occurs with a rate of 2,500 events per day. A prerequisite for such a search is therefore a detector with a good photon resolution in order to extract the monochromatic signal and with the ability to reconstruct the final state to ascertain that heavy states indeed were involved in the decay.

At LEP one may envisage higher $1^{--}$onium states. The rate for producing high mass $1^{--}$ states and the branching ratio into Higgs, normalized to the branching ratio into $\mu^+\mu^-$, are given below in Table 2.

### Table 2

<table>
<thead>
<tr>
<th>$m_V$ (GeV)</th>
<th>Onium Rate events/day</th>
<th>$1^{--} \to H\gamma$/$1^{--} \to \mu^+\mu^-$</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>260</td>
<td>0.14</td>
</tr>
<tr>
<td>60</td>
<td>170</td>
<td>0.32</td>
</tr>
<tr>
<td>80</td>
<td>140</td>
<td>0.56</td>
</tr>
<tr>
<td>120</td>
<td>90</td>
<td>1.26</td>
</tr>
</tbody>
</table>

Assuming a leptonic branching ratio which varies between 3% at 40 GeV and 0.5% at 120 GeV and neglecting the phase space factor $(1 - m_H/m_V)^{1/2}$ one expects about one Higgs event every day. Such a low rate might be acceptable since the signature is rather clean.

However, a more attractive way to search for Higgs bosons at LEP is exploiting its relatively strong coupling to the gauge boson. A particularly favourable final state is $e^+e^- \to Z^* \to Z^0H$. One would identify the $Z^0$ through its lepton pair decay mode. From the $Z^0$ one can compute the effective mass of the
system recoiling against the $Z^0$ and thus the $H$ should be visible as a peak in this recoil spectrum. That this peak is indeed associated with the Higgs can be verified by measuring its decay products which should contain heavy fragments. The maximum cross-section occurs at an energy $\sqrt{s} = m_Z + 2 \cdot \sqrt{2} \cdot m_H$ and is given by:

$$\sigma(Z^0H) = \sigma({\text{pointlike}}) \cdot \frac{m_Z}{38} \cdot \frac{m_Z}{2 \sqrt{2} m_H} (1 + v^2/a^2)$$

with $m_Z$ in GeV. The expected rates are given below in Table 3.

<table>
<thead>
<tr>
<th>$m_H$ (GeV)</th>
<th>$\sqrt{s}$ (GeV)</th>
<th>$\sigma$ (ZH) $\sigma$ (pointlike)</th>
<th>$HZ^0$ events/day</th>
<th>$\mu^+\mu^-H$ events/day</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>100</td>
<td>3.8</td>
<td>163</td>
<td>4.9</td>
</tr>
<tr>
<td>30</td>
<td>130</td>
<td>1.3</td>
<td>56</td>
<td>1.7</td>
</tr>
<tr>
<td>50</td>
<td>160</td>
<td>0.77</td>
<td>23</td>
<td>0.65</td>
</tr>
<tr>
<td>60</td>
<td>172</td>
<td>0.64</td>
<td>13</td>
<td>0.34</td>
</tr>
</tbody>
</table>

Thus, in order to search for Higgs bosons up to the mass predicted for the $W^\pm$ and the $Z$, we will need cms energies in the order of 200 GeV or above and a luminosity of the order of $10^{32}$ cm$^{-2}$ sec$^{-1}$.

Conclusions

LEP is a unique tool for the investigation of the strong, the electromagnetic and the weak interaction in a very clean environment. The upper energy should allow us to pair produce $W^+W^-$ with some margin and to search for Higgs mesons up to masses comparable to the anticipated mass of the $Z^0$. This can be achieved with 2 x 100 GeV and a luminosity of $10^{32}$ cm$^{-2}$ sec$^{-1}$. In order to exploit the unique feature of LEP we need much more sophisticated detectors than the ones available today, in particular we must be able to identify and measure momentum of particles inside a jet of particles.

It is a pleasure to thank Maurice Jacob and Christine Redman for organizing an enjoyable meeting and for providing the ambiance which made it into such a great success. I'm less grateful to Maurice Jacob for forcing me to write my talk.
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WEAK–ELECTROMAGNETIC INTERFERENCE

by

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LEP Summer Study Secretariat
1. Introduction

An \(e^+e^-\) storage ring such as LEP provides an opportunity to perform some exceedingly interesting experiments. If the present ideas on the Weak interaction are correct, the currently accepted value of \(\sin^2 \theta_W = 0.23\), fixes the mass of the \(Z^0\) at 88.9 GeV/c\(^2\), well within the range of LEP. The leptonic decays of the \(Z^0\) are very clean channels through which a study of weak-electromagnetic interference effects is made possible. The measurements around the \(Z^0\) pole of total cross-sections, angular asymmetries and final state lepton polarizations are reviewed.

The expected mass of the \(W^\pm\) of 78 GeV/c\(^2\) makes the production of \(W^\pm\) pairs possible if the beam energy can be extended to about 90 GeV. This channel, and the effect of beam polarization on its study, are also briefly discussed. Finally, possible detectors for leptonic final states are reviewed.

2. Experiments near the \(Z^0\) pole.

The reactions that are relevant here are:

\[e^+e^- \rightarrow e^+e^-\]
\[e^+e^- \rightarrow \mu^+\mu^-\]
\[e^+e^- \rightarrow \tau^+\tau^-\]

The diagrams involved are shown in Fig. 1.

2.1 Measurement of the total cross-section

In the absence of beam polarization and assuming the final state helicities are not measured, the differential cross section for \(\mu^+\mu^-\) and \(\tau^+\tau^-\) production is given by \(^1\)

\[
\frac{4\pi}{s} \frac{d\sigma}{d\Omega} = F_1(s) (1 + \cos^2 \theta) + 2 F_3(s) \cos \theta
\]

where \(s = 2 E_{\text{beam}}\)
\(\theta = \) angle between the incoming \(e^+\) and the outgoing \(\mu^+\).
\(F_1, F_3 = \) functions of the width \(\Gamma\) and mass \(M\) of the \(Z^0\), and of \(g_A, g_V\), the axial vector and vector coupling constants.
In principle the coupling constants entering (1) could be different for e, \( \mu \), \( \tau \). They would only be the same for e-\( \mu \)-\( \tau \) universality. The measurement of the \( \mu^+\mu^- \) and \( \tau^+\tau^- \) cross sections provides a test of universality. The \( e^+e^- \) cross section is expected to be different because of the exchange diagrams of Fig. 1.

Integrating (1) over an apparatus that is forward-backward symmetric in \( \theta \), yields a total counting rate that is proportional to \( F_\gamma \) only. The total cross section of lepton pairs as a function of \( s \) yields the mass and width of the \( Z^0 \). In the Weinberg Salam model 2) a measurement of the mass fixes \( \sin^2 \theta_W \) and hence the coupling constants \( g_A^2, g_V^2 \). In principle \( g_A^2 \) and \( g_V^2 \) can also be obtained from a measurement of the depth of the interference minimum preceding the \( Z^0 \) pole. However, given \( \sin^2 \theta_W = 0.23 \), the current best value, this minimum is too shallow to be measured accurately. The counting rate for \( e^+e^- \rightarrow \mu^+\mu^- \) (or \( \tau^+\tau^- \)) is shown in Fig. 2. It was computed assuming

i) The Weinberg-Salam model with \( \sin^2 \theta_W = 0.23 \).

ii) Two values of \( \Gamma \), the \( Z^0 \) width, \( \Gamma = 1 \) and \( \Gamma = 3 \) GeV/c\(^2\).

iii) An apparatus with an angular acceptance \( 30^0 < \theta < 150^0 \) and \( 0^0 < \phi < 360^0 \).

iv) A variation of the luminosity with the beam energy given by \( 3) \)

\[
\mathcal{L} = \left( \frac{E_{\text{beam}}}{70} \right)^2 \times 10^{32} \text{ cm}^{-2} \text{ sec}^{-1}.
\]

In addition to the solid curves, 20 data points, corresponding to 100 hours running time each, are shown, together with statistical error bars. It is clear that statistics will not greatly affect the measurement of the width that is expected to be about 2.5 GeV/c\(^2\). The beams energy spread of 100 MeV will also not be a significant effect. The more important effect is likely to be relative point to point normalization errors. A measurement of the width is of interest as it is a measure of the number of neutrinos with \( m_\nu < m_{Z^0}/2 \).

2.2 Forward Backward Asymmetry

The asymmetry, \( A \), is defined as

\[
A = \frac{N_F - N_B}{N_F + N_B}
\]
Figure 1: Lowest order contributions to the reactions

\[ e^+e^- \rightarrow e^+e^- \]
\[ \mu^+\mu^- \]
\[ \tau^+\tau^- \]
\[ \sin^2 \theta_W = 0.23 \]
\[ \mathcal{L} = \left( \frac{\sqrt{s}}{140} \right)^2 \times 10^{32} \text{ cm}^{-2} \text{ sec}^{-1} \]

**ACCEPTANCE**

30° < θ < 150°

FULL \( \phi \)

\[ e^+e^- \rightarrow \mu^+\mu^- \]
\[ e^+e^- \rightarrow \tau^+\tau^- \]

\( \Gamma = 1 \text{ GeV} \)

\( \Gamma = 3 \text{ GeV} \)

\( M_{Z^0} = 88.884 \)

Figure 2: Counting rate as a function of \( \sqrt{s} \) for the reactions \( e^+e^- \rightarrow \mu^+\mu^- \) and \( e^+e^- \rightarrow \tau^+\tau^- \) near the \( Z^0 \) pole for the assumptions described in the text and for two values of the \( Z^0 \) width, \( \Gamma = 1 \) and \( \Gamma = 3 \text{ GeV/c}^2 \).
where \( N_F \) and \( N_B \) are the number of events with a positive lepton in the forward and backward hemisphere respectively.

The \( e^+e^- , \mu^+\mu^- \) and \( \tau^+\tau^- \) final states can be used. In the \( \tau^+\tau^- \) reactions the \( \tau \rightarrow (\tilde{\tau}^0)\, \nu \nu \) decays can be used as the \( \mu \) or \( e \) emerge close to the original \( \tau \) direction thus preserving the original asymmetry. For the \( \mu^+\mu^- \) and \( \tau^+\tau^- \) reactions and \( 4\pi \) coverage

\[
A(s) = \frac{3}{4} \frac{F_3(s)}{F_1(s)}
\]

whereas for

\[
30^\circ < \theta < 150^\circ
A(s) = \frac{9.42}{13.60} \frac{F_3(s)}{F_1(s)}
\]

The asymmetry again determines \( g_A^2 \) and \( g_W^2 \) and hence \( \sin^2 \theta_W \). This value of \( \sin^2 \theta_W \) must agree with the one obtained from the mass of the \( Z^0 \) for the Weinberg-Salam model to hold. The relative sign of \( g_A \) and \( g_W \) is however not determined by the asymmetry. The asymmetry also provides an opportunity to test \( \mu - \tau \) universality.

The asymmetry in the \( \mu^+\mu^- \) final state is plotted in Fig. 3. It was calculated in the context of the Weinberg-Salam model for

\[
\sin^2 \theta_W = 0.23 \text{ and } 0.22 \text{ and for } \Gamma = 3 \text{ GeV/c}^2.
\]

A change of \( \sin^2 \theta_W \) of 0.01 is certainly measurable. The difference in the two curves arises mostly from the different values of \( M_{Z^0} \) derived from the two values of \( \sin^2 \theta_W \).

The reaction \( e^+e^- \rightarrow e^+e^- \) can also be used to measure the asymmetry. In this case however, the asymmetry is large and positive because of the exchange diagram of Fig. 1, except near the \( Z^0 \) pole where the asymmetry drops towards zero. The difference between the \( e^+e^- \) and \( \mu^+\mu^- \) asymmetries is illustrated in Fig. 4.

The angular asymmetry can also be used to estimate the mass of a possible second \( Z^0 \) lying outside the LEP energy range 4). It would influence the asymmetry near the top energy of the machine, (Fig. 5). For instance, for 200 hours running time at \( E_{\text{beam}} = 70 \text{ GeV} \), a second \( Z^0 \) with a mass of 230 GeV/c\(^2\) would affect the asymmetry at the two standard deviation level (Fig. 6).

A word of caution: radiative corrections will greatly affect the asymmetry even well above the \( Z^0 \) pole. They will have to be accounted
Figure 3: The angular asymmetry $A_{\mu\mu}$ calculated in the context of the Weinberg-Salam model and plotted as a function of $\sqrt{s}$ for two values of $\sin^2 \theta_W$, 0.23 and 0.22.
Figure 4: The difference in the angular asymmetry between the $e^+e^-$ and the $\mu^+\mu^-$ (or $\tau^+\tau^-$) final states for $\sin^2\theta_W = 0.25$. 
Figure 5: The angular asymmetry in the $\mu^+\mu^-$ channel plotted as a function of $\sqrt{s}$ for two hypotheses:

a) the Weinberg-Salam model with a single $Z^0$ at 79 GeV/c$^2$

b) the model of De Rujula, Georgi and Glashow with a second $Z^0$ at 214 GeV/c$^2$ (Ref. 4).
Figure 6: The number of running hours at $E_{\text{beam}} = 70$ GeV necessary to distinguish, at the two standard deviation levels, a two $Z^0$ hypothesis from a standard Weinberg-Salam model with a single $Z^0$ at 79 GeV/c$^2$ plotted as a function of the mass of the second $Z^0$. 
for before extracting a second $Z^0$ contribution. In the case of $A_{\mu\mu}$ and
$A_{ee}$ restricting the final state lepton to have an energy equal to the beam
energy within the measurement errors will reduce the effect of radiative
processes. Hence the importance of good energy or momentum resolution
in the detectors.

2.3 Polarization of outgoing lepton

The polarization of outgoing $\mu$ or $\tau$, $P_L (\mu, \tau)$, is given by

$$P_L (\mu, \tau) = \frac{-F_4 (1+\cos^2 \theta)^2}{F_1 (1+\cos^2 \theta) + 2F_3 \cos \theta}$$

where

$$F_4 = 2g_A g_V \frac{s(s-M_Z^2)}{e^2((s-M_Z^2)^2+\Gamma^2 M_Z^2)} + \frac{(g_V^2 + g_A^2)s^2}{4((s-M_Z^2)^2+\Gamma^2 M_Z^2)}$$

$F_4$ and $P_L$ are proportional to the product of $g_A$ and $g_V$. A measurement of
$P_L$ therefore determines the relative sign of $g_A$ and $g_V$. Both $\mu'$s and $\tau'$s
can be used to determine the polarization. Here again $\mu-\tau$ universality
can be checked. The experiment is to be done at the $Z^0$ pole.

i) $e^+e^- \rightarrow \mu^+\mu^-$

The method and apparatus to determine the polarization of out-
going muons was described in reference 5. Briefly it involves stopping
muons in about 33m of iron and observing their polarization by measuring
the forward/backward asymmetry in the decay electrons ($\mu\mu$ later).
Typically the polarization can be measured to $\sim 10\%$ in 100 hours.

ii) $e^+e^- \rightarrow \tau^+\tau^-$

$e^+e^- \rightarrow \mu^+\nu\bar{\nu}$

$e^+e^- \rightarrow (\mu^+\nu\bar{\nu})$

$e^+e^- \rightarrow (\mu^+\nu\bar{\nu})$

$\tau^+\tau^-$

$\tau^+\tau^-$

Here the $\mu\nu\nu$ and $e\nu\nu$ decays of the $\tau$ are used\textsuperscript{6}. They each
have a branching ratio of 17%. The signature is an electron and a muon
only in the final state. Most of the events have a collinearity of less
than $10^\circ$ (Fig. 7a). The average momentum of the $\mu$ and $e$ is 15 GeV/c for
$\tau$ pairs at the $Z^0$ pole. The fraction of the total energy going into
neutrinos is shown in Fig. 7b). The origin of the events can be checked by studying their angular asymmetry. It should be the same as the asymmetry for $\mu^+\mu^-$ events assuming $\mu$-$\tau$ universality has been found to hold.

Since the momentum spectrum of the $\mu$ or $e$ can be computed given the $\tau$ polarization the method is simply based on measuring this spectrum and deducing the polarization.

If $p_1 = \text{muon or electron momentum}$

$$x = 2p_1/\sqrt{s}$$

$$f(x, P_L) = \frac{dn}{dx}$$

Then

$$\frac{df}{dP_L} = 0 \text{ at } x = x_c = 0.4215$$

i.e. the spectrum is independent of polarization at that point. This provides a polarization estimator $R$ defined as

$$R = \frac{n_H}{n_L}, \text{ where } n_H = \int_{x_c}^1 f(x) \, dx$$

$$n_L = \int_{x_0}^{x_c} f(x) \, dx$$

where $x_0 = \text{minimum x (depending on apparatus and background)}$.

The variation of the polarization and of $R$ with $\sin^2 \theta_W$ is shown in Fig. 8, whereas the number of running hours required to achieve a precision of 10% and 30% is plotted in Fig. 9 again as a function of $\sin^2 \theta_W$. The detector acceptance has been assumed to be the same as described in the previous section.

Radiative corrections for this process have been calculated and introduced in the estimator $^7$). They of course affect the $R \leftrightarrow P_L$ relationship. This is shown in Fig. 10, where the dashed curve is without radiative correction and the solid curve is with. However, once these corrections are introduced the method remains valid. In particular, the error in the estimator arising from uncertainties in the estimator due to the machine energy spread and due to the uncertainty in the $Z^0$ width is always less than 1%.
Figure 7: a) Co-linearity of the muon and electron at the $Z^0$ pole in the reaction $e^+e^- \rightarrow \tau^+\tau^-$. 

b) $x_F$, the fraction of the total energy going into neutrinos in the above reaction near the $Z^0$ pole.
Figure 8: The mean polarization, $P_L$, of $\nu'$s or $\tau'$s and the estimator, $R$, described in the text plotted as a function of $\sin^2 \theta_W$. The dotted line shows the fractional error of $R$ which can be obtained in 100 hours of running time.
Figure 9: Number of running hours required to achieve a precision of 10% and 30% in the measurement of the polarization of the $\tau$ via its $\mu\nu\nu$ and $e\nu\nu$ decays plotted as a function of $\sin^2\theta_W$. The detector acceptance has been taken as $30^\circ < \theta < 150^\circ$ and $0 < \phi < 360^\circ$. 

$L = 10^{32} (M_Z/140)^2 \text{cm}^{-2} \text{s}^{-1}$

$\pi/6 < \Theta < 5\pi/6$

$0 < \phi < 2\pi$

$\sqrt{s} = M_Z^0$
Figure 10: The estimator $R$ plotted as a function of $<P_T>$ without taking into account radiative corrections (dashed curve) and after introducing radiative corrections (solid curve). The horizontal scale also shows the correspondence between $<P_T>$ and $\sin^2 \theta_W$ in the Weinger-Salam model.
iii) \( e^+e^- \rightarrow \pi^+\pi^- \rightarrow \ell^+\nu\ell^-\bar{\nu} \)

The method uses the \( \pi\nu \) decay mode for one of the \( \tau \)'s (8.5% branching ratio). The process is tagged by the other \( \tau \) decaying into an electron or a muon. Again the momentum spectrum of the pion depends on the \( \tau \) polarization but the effect is more pronounced since it is a two-body decay (Fig. 11).

\[
\frac{dN}{dE_\pi} \propto 1 + \langle p_{\tau^-} \rangle \left( \frac{E_{\pi^-} - E_{\text{beam}}/2}{E_{\text{beam}}/2} \right)
\]

\[
\propto 1 + \frac{F_4}{F_1} \left( \frac{E_{\pi^-} - E_{\text{beam}}/2}{E_{\text{beam}}/2} \right)
\]

As described earlier \( F_4/F_1 \) depends on the relative sign of \( g_A \) and \( g_V \). It is estimated that the polarization can be measured to \( \pm 4\% \) in 100 hours.

2.4 Polarized beams

If \( h^+ \) and \( h^- \) are the helicities of the positron and electron respectively, then

\[
\frac{\sigma(h^+h^-)}{\sigma(0,0)} = 1 - h^+h^- + (h^-h^+) F(g_A^*,g_V^*,s)
\]

where \( \sigma \) is the cross-section integrated over all angles. It can be seen that this is identically zero when \( h^+ = h^- \). However, for opposite helicities

\[
\frac{\sigma(-+)}{\sigma(++)} = -F(g_A^*,g_V^*,s) = -\frac{2g_A g_V}{g_A^2 + g_V^2} \text{ at the } Z^0 \text{ pole.}
\]

So, here too the relative sign of \( g_A \) and \( g_V \) can be extracted. However, the exact same combination of \( g_A \) and \( g_V \) is extracted from a measurement of the mean polarization of \( \mu^- \)'s and \( \tau^- \)'s in the final state.
Figure 11: The energy spectrum of the $\pi^-$ in the reaction
\[ e^+e^- \rightarrow \tau^+\tau^- \rightarrow \pi^-\nu_{\tau} \]
for two values of $\sin^2 \theta_W$, 0.2 and 0.3 corresponding to a reversal of the $\tau$ polarization in the Weinberg-Salam model.
3. **W pair production**

The diagrams contributing to $e^+e^- \rightarrow W^+W^-$ are shown in Fig. 12. The mass of the $W$ is 78 GeV/c$^2$ in the Weinberg-Salam model with $\sin^2 \theta_W = 0.23$. The cross-section peaks 30 - 40 GeV above threshold \(^9\) (i.e. $\sqrt{s} = 190-200$ GeV) at a value of $\sim 2 \times 10^{-35}$ cm$^{-2}$. Because of the $\nu$-exchange diagram the cross section is peaked forward but since the $W$'s are slow being near threshold the $\mu$'s or $e$'s from their decay will be nearly isotropic. An apparatus with an acceptance $30^\circ < \theta < 150^\circ$ and $0 < \phi < 360^\circ$ detects 70% of

$$e^+e^- \rightarrow W^+W^- \xrightarrow{\nu_e}$$

With a luminosity variation beyond $E_{\text{beam}} = 80$ GeV given by \(^3\)

$$\mathcal{L} = \left(\frac{80}{E_{\text{beam}}}\right)^3 10^{32} \text{ cm}^{-2} \text{ sec}^{-1}.$$ 

6000 $W$ pairs are produced in the apparatus in 2000 hours running. Assuming a 20% branching ratio into an electron or a muon, 2400 events with an $e$ or a $\mu$ are seen and 240 with both an $e$ and a $\mu$.

3.1 **Polarized beams in $e^+e^- \rightarrow W^+W^-$**

The $\nu$ exchange diagram provides the major part of the cross section. However, it is the $Z^0$ exchange diagram with the information it provides on the $WWZ$ couplings that is of greater interest. But the $\nu$ exchange can be "switched off" by selecting left handed $e^+$'s and right handed $e^-$'s using polarized beams. Unfortunately the event rate drops by a factor of $\sim 20$ (Fig. 13). The yield is then 120 events with a single lepton in the final state and 12 events with 2 leptons in 2000 hours running.

The study of $W^+W^-$ pairs is obviously a reaction that needs a beam energy in excess of 70 GeV and with the highest possible luminosity.
Figure 12: Contributions to the reaction
\[ e^+ e^- \rightarrow W^+ W^- \]
Figure 13: Cross-section for the production of $W$ pairs
   a) With unpolarized beams
   b) With lefthanded $e^+$'s and righthanded $e^-$'s.

\[ \sigma_{\text{tot}}(e^+e^-\rightarrow W^+W^-) \]

Unpolarized Beams

Polarized Beams

\[ \tau = S/(2M_W)^2 \]
4. Detectors

Several types of detectors can be considered to study leptons in the final state.

a) Magnetised iron toroids to measure muon momenta, with an inner sleeve of shower counters to measure the electron energies. This type of detector has the advantage of being relatively inexpensive and can cover very large solid angles. Its disadvantages however are that it does not measure the electron charge (an important factor in asymmetry measurements) and that it does not measure pion momenta thus ruling out $\tau$ polarization measurements via its $\tau\nu$ decay.

b) A superconducting air core toroid with 8 discrete coils (Fig. 14). Drift chambers around the vacuum pipe and beyond the coils measure the trajectories. An electromagnetic calorimeter is located behind the drift chambers and is itself followed by an iron muon identifier. The advantage is that this magnet leaves the forward direction free of end caps such that, for instance, a two-photon physics detector can be added. However, the coils obscure 35% of the azimuth thus limiting event rates.

c) Superconducting solenoid (Fig. 15).

Parameters:
- Length: 3 m
- Radius: 1 m
- Field: 15 K gauss
- Coil: aluminium stabilized (0.5 - 1.0 radiation length thick)

The chambers would be of the type used by JADE at PETRA and R-807 at the ISR. They are drift chambers with current division to measure the longitudinal coordinates. A total of 50 measurements per track would be used.

The shower detector positioned just outside the coil would be 20 radiation length thick and would be segmented longitudinally into 2 sections (5 and 15 rad. lengths) to reject hadrons using the early shower development technique. Lead scintillator sandwiches would be adequate to identify the electrons and measure their energy.

The muon identifier (or hadron filter) would simply consist of the 2 m thick return yoke of the magnet followed by drift chambers and scintillators.
Figure 14: Superconducting air core toroid viewed along the beam. The figure shows an overall view of the magnet as an insert, and a detail of one segment equipped with detectors.
Figure 15: Superconducting solenoid equipped with
a) drift chambers in the field region
b) electromagnetic calorimeters outside the coil
c) a muon detector outside the return yoke.
The resolution of the magnet-drift chamber system is given by

$$\frac{\delta p_T}{p_T} = \frac{3.3 \times 10^{-2} \text{ (cm)}}{B(\text{K gauss}) L^2} p_T \sqrt{\varepsilon^2 \text{ An}}$$

Where:

- \( L = \) lever arm of measurement = 100 cm
- \( B = \) field 15 K gauss
- \( \varepsilon = \) measurement error per point = 0.02 cm

The lead scintillator energy resolution is given by

$$\Delta E/E = 12\%/\sqrt{E} \text{ or } \pm 1.9\% \text{ at 40 GeV}.$$  

The first step in rejecting hadronic events is to reject multi-particle events since all the final states of interest are 2 particle states:

- \( \mu\mu, \mu e, ee, e\pi, \mu\pi \)

(The only exception would be the \( e^+e^- \rightarrow W^+W^- \) reaction). To use this method thoroughly an end cap detector would be useful.

The \( \pi/\mu \) separation is given by the return yoke:

- Punch through (2 m of iron) < 3%
- \( \pi \)-decay < 0.3%
- \( k \)-decay < 2.5%

The \( \pi/e \) separation requires \( E \sim p \) to within the measurement errors and a large energy deposition in the first 5 radiation lengths of the e.m. calorimeter. The rejection obtained by this method depends on the final energy spectrum of hadronic events but it is estimated to be less than 1%.

Finally, cosmic rays which could contaminate the \( \mu^+\mu^- \) data are discriminated against using time of flight between the scintillators located outside the yoke (the time difference is \( \sim 26 \text{ ns} \)).
5. **Conclusion**

To conclude, a study of lepton final states at LEP will yield very valuable information on the weak interaction. Near the $Z^0$ pole, weak-electromagnetic interference can be studied through measurements of the total cross section and angular asymmetries. These measurements determine the magnitude of coupling constants $g_A$ and $g_V$ as well as the mass and width of the $Z^0$. A measurement of the polarization of $\mu$'s and $\tau$'s will in addition determine the relative sign of $g_A$ and $g_V$. These measurements provide an opportunity to test $e^-\mu^-\tau$ universality. The existence of a second $Z^0$ of mass less than 230 GeV/c$^2$ could be inferred from measurements of the angular asymmetry even if the maximum beam energy does not exceed 70 GeV. On the other hand if the maximum energy can be extended to 100 GeV the $W^+W^-$ threshold is likely to be reached thus opening up a whole new field of research. In particular the $WWZ$ vertex could then be studied if the luminosity of the storage ring can be kept high at these higher energies. Finally, the study of the $W^+W^-$ final state would greatly benefit from the use of polarized beams.
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STUDY OF HEAVY-LEPTON POLARIZATION
IN $e^+e^-$ ANNIHILATION AT THE $Z^0$ POLE

G. Goggi

1. Introduction

The annihilation process into lepton pairs $e^+e^- \rightarrow \ell^+\ell^-$ in the proximity of the $Z^0$ peak yields information on the relative strength of the vector and axial couplings $g_V$ and $g_A$ through the cross-section energy dependence and the angular asymmetry of the final state leptons.

The measurement of lepton polarization, besides providing a more sensitive test of the vector-axial mixture in the interaction, is the only way to determine the relative sign of $g_V$ and $g_A$, in the absence of polarized beams.

The production of a pair of heavy leptons and the study of their subsequent leptonic decays provides a practical and effective way to determine final state helicities at energies around the expected $Z^0$ mass. The reduction of the $\tau^+\tau^-$ rate by a factor $B^2_{\tau}$ with respect to $\mu^+\mu^-$ due to the branching ratio of the leptonic decays of the $\tau$ is not a serious limitation, given the large cross-sections expected at the peak $Z^0$ energy. This reduction factor is more than compensated by the exceptionally clean experimental signature of $\mu e$ events, as opposed to hadronic $\tau$ decays, and by the relative ease of detecting the decay leptons with a conventional apparatus. This is to be compared to the massive detector of a dedicated muon polarimeter\(^1\) which requires about 0.4 $M_Z$ (GeV) metres of Fe to stop muons from the decay $Z^0 \rightarrow \mu^+\mu^-$. The purpose of this note is to study the experimental feasibility of a measurement of longitudinal polarization on heavy leptons produced near the $Z^0$ pole.

2. Polarized heavy lepton production

In the following the same conventions as in reference \(^1\) are used, as well as the general approach. The formalism adopted is that of Budny\(^2\). The Weinberg-Salam model\(^3\) is used to obtain numerical results.
The cross-section for unpolarized beams and unobserved final state helicities for the reaction

\[
e^+ e^- \to \tau^+ \tau^- \quad \tau \to l\nu\nu
\]

\[l = e, \mu\]

is in lowest order in the weak and electromagnetic interactions

\[
\frac{d\sigma}{d\Omega} = \frac{3}{16\pi} \sigma_p \left\{ F_1(1 + \cos^2\theta) + 2F_3\cos\theta \right\} 2B_L^2
\]

where \(\sigma_p\) is the point-like cross-section and the set of functions \(F_i\) is given in the appendix.

The Lorentz boost in \(\tau\) decay confines the final state leptons in a narrow collinearity distribution, as shown in Fig. 1a for \(M_Z = 77.3\) GeV. This allows also the determination of the asymmetry arising from the second term in (2) (inset in Fig. 1a), which sets an additional constraint on the origin of the \(\mu e\) events. An additional experimental signature is represented by the large fraction of the total energy carried by the decay neutrinos, shown in Fig. 1b. For this case the average lepton momenta are about 13 GeV/c.

The longitudinal polarization \(P_L\) of the heavy leptons produced is given by

\[
P_L(\cos\theta) = \frac{-F_4(1 + \cos^2\theta)^2}{F_1(1 + \cos^2\theta) + 2F_3\cos\theta} \quad (\tau^+)\]

Averaging over the polar aperture of any forward-backward symmetric apparatus yields

\[
\langle P_L(\tau^+) \rangle = \tau \frac{2\xi}{1 + \xi^2}
\]

where \(\xi = g_A/g_V\) is the ratio of the axial and vector couplings. The function (4) is shown in Fig. 2a for \(\tau^+\); as an example the dependence of \(\xi\) on \(\sin^2\theta_W\) in the W-S model is given in Fig. 2b.

Fig. 2a shows how the polarization determines the sign of \(g_A/g_V\); although the polarization is maximal at \(|\xi| = 1\), it appears that even small values of \(g_V\), as obtained by the current values of \(\sin^2\theta_W\), yield sizeable polarization effects.
Figure 1.a): Collinearity distribution for $e$ events from pair production at $\sqrt{s} = 77.3$ GeV. The inset shows the angular distribution of the muon in the lab system.

b): Distribution of the normalized missing energy for the same reaction.
Figure 2.a: Dependence of the average longitudinal polarization on the ratio of the axial to the vector couplings $\xi$. The polarization of a $\tau^+$ is shown.

b: Variation of the parameter $\xi$ as a function of $\sin^2 \theta_W$ in the $W$-$S$ model. The left branch of the curve corresponds to negative $\xi$ values.
3. Decay of polarized heavy leptons

In the measurement of the longitudinal polarization of muons at rest the relevant information on the spin direction is contained in the angular asymmetry of the decay electrons. The rapid decay in flight of heavy leptons transfers this information to the momentum distribution of the decay leptons. It is the final state momentum of the detected lepton which is then correlated with the $\tau$ spin. Assuming for the heavy lepton weak current the same V-A structure as for the muon and neglecting final state masses ($m_Z/m_\tau \to 0$) the differential momentum spectrum of the decay lepton in the lab. system has the form

$$\frac{dn}{dX} = f(X, P_L) = a(X) + P_L b(X) \quad \text{(for } \tau^+)$$

where $X = P_L/p_{\tau}$ is the normalized lepton momentum; the functions $a$ and $b$ are given in the appendix. Fig. 3a shows the momentum spectrum expected for $\tau^+$ decay with $\sin^2 \theta_W = 0.1$ or $<P_L> = 0.88$.

The sensitivity of the momentum distribution to the polarization value is shown in Fig. 3b, where the quantity $r(X)$, the fractional difference of $f(X, P_L)$ for two values of $\sin^2 \theta_W$, is displayed:

$$r(X) = 1 - f(X)_{\sin^2 \theta_W = 0.5} f^{-1}(X)_{\sin^2 \theta_W = 0.1}$$

The function $df/dP_L$ changes sign when crossing the critical value $X_c = 0.4215$, at which point the momentum distribution does not depend on the polarization. It follows that, rather than fitting the entire distribution, which requires an absolute energy calibration, a simple estimator of the polarization can be the quantity

$$R = \frac{n_H}{n_L}$$

where

$$n_H = \int_{X_c}^1 f(X)dX \quad n_L = \int_{X_0}^{X_c} f(X)dX$$
Figure 3.a): Normalized momentum distribution of the lepton in $\tau^+$ decay at $\sin^2 \theta_W = 0.1$.

b): The fractional difference $r(X)$ of the distribution in a) for $\sin^2 \theta_W = 0.1$ and $\sin^2 \theta_W = 0.5$ (solid line). The dash-dotted line indicates the sensitivity of the spectrum in a) to the polarization. The critical value $X = X_c$ at which the derivative is zero is shown.
where the lower limit $X_\alpha$ on the momentum is set by considerations of detection efficiency and backgrounds. The variation of the average longitudinal polarization for a $\tau^+$ and of the estimator $R$ on $\sin^2\theta_W$ are displayed in Fig. 4. Two separate curves for $R$ correspond to values of the lower limit $X_\alpha = 0.05$ and $X_\alpha = 0.1$. It is clear that the method is rather insensitive to the experimental momentum cut, which in turn is also dependent on the absolute energy calibration.

4. A polarization experiment

As a realistic example let us consider an experiment in which $\tau$ pairs from $Z^0$ decay are detected through their $e\mu$ decays over a reasonable fraction of the solid angle. The leptonic branching ratio of the $\tau$ is taken to be $B_\tau \approx 0.2$, the width of the $Z^0$ $\Gamma_Z = 1$ GeV, and the luminosity $\mathcal{L} = (M^2_Z/140)^2 \cdot 10^3$ cm$^{-2}$ s$^{-1}$.

The apparatus is supposed to be rather conventional, having a polar acceptance between $30^\circ$ and $150^\circ$ to the beams and full azimuthal coverage. It must be capable of measuring electron and/or muon momenta up to the beam value; momentum resolution is not a critical parameter provided it is sufficient to minimize (calculable) smearing effects on $R$ at $X = X_c$, around 17 GeV/c for $M_Z = 78$ GeV.

With these assumptions, and considering the energy spectrum of only one detected lepton, the fractional uncertainty on $R (X_\alpha = 0.05)$ obtained in 100 hours of running time is shown in Fig. 4b. In the worst case $R$ is measured to a 3% precision in such a short time.

Considering now the running time needed to measure the polarization to a given fractional error $\sigma_R/P$, including in the calculation the sensitivity of $<P_L>$ on $R$, we obtain

$$t = 1.3\times 10^{-4} \cdot \frac{g[P_L]}{R[P_L]} \left[ 1 + R[P_L] \right] \cdot \frac{1}{P} \frac{dP}{d\Omega} \left( \frac{dR}{d\Omega} \right)^{-2} \left( \frac{d\theta}{d\Omega} \right)^{-1} \left( \int \frac{d\Omega}{d\Omega} d\Omega \right)^{-1}$$

where $t$ is the running time in hours, $R[P_L]$ and $g[P_L]$ are given by equations (A10) and (A11) and we consider the contributions of both leptons.

Numerical results from equation (8) are shown in Fig. 5 for the range of $\sin^2\theta_W$, which can be explored with a 70 GeV machine. As can be seen a running time of the order of $10^2$ hours yields a 10% measurement of the $\tau$ longitudinal polarization almost everywhere. Such a precision allows a valuable cross-check
Figure 4.a): The $\tau$ longitudinal polarization averaged over a symmetric polar interval as a function of $\sin^2 \Theta_W$.

b): Dependence of the polarization estimator $R$ on $\sin^2 \Theta_W$ for two values of the momentum cut $X_0$ (solid lines). The dotted line shows the fractional error on $R$ which can be obtained in 100 hours of running time (see text).
Figure 5: Running time as a function of $\sin^2 \theta_W$ for given fractional errors on the polarization. The vertical dashed lines delimit the interval of $\sin^2 \theta_W$ which can be explored with a 70 GeV machine.
of the $g_A/g_V$ ratio obtained by an asymmetry measurement. The curve labelled $\Delta P_L = P_L/3$ shows the minimal running time needed for a determination of the sign of $g_A/g_V$, which can be achieved in most cases in a few hours of running. The number of $e\mu$ events at the $Z^0$ peak corresponding to the case $\Delta P_L/P_L = 0.1$ is given in Fig. 6. Also shown is the statistics expected in 100 hours of running time.

The above values show that, besides the considerations of this note, the study of the weak properties of heavy leptons at LEP-70 has considerable intrinsic validity if such a copious source as the neutral weak boson is within reach of the machine.

5. Conclusions and remarks

It has been shown in the previous sections that heavy-lepton pair production at the neutral vector boson mass can be easily detected and measured through the leptonic decay mode. This reaction is particularly suited for a determination of the $\tau$ helicity; no specialized apparatus is needed for this measurement, which is compatible with the physics programme of a general detector. In addition the expected rate of $e\mu$ events is very high, yielding a minimum of $6 \cdot 10^3$ events in 100 hours of running time, which makes a detailed investigation of the $\tau$ properties a physically interesting programme in itself. The information on the $\tau$ spin alignment can be easily extracted from the lepton decay spectrum; good sensitivity can be achieved in the region $0.2 < \sin^2 \theta_W < 0.4$ where the physical value is most probably found. More generally the value of the longitudinal polarization allows the determination in a model-independent way of the relative sign of the axial and vector weak couplings, as well as their absolute ratio.

Although in this case the need to observe final state polarizations can be circumvented by the use of polarized beams, it is generally true that some of the information which can be obtained by observing final state helicities is not accessible by simply controlling the longitudinal beam polarization and vice versa. This point is relevant to the study of possible contributions from non-conserved currents.

Acknowledgements

I wish to thank G. Barbiellini, L. Camilleri and J.H. Field for very useful discussions.
Figure 6: Number of events at the $Z^0$ pole as a function of $\sin^2\theta_W$. The solid line corresponds to the case $\Delta P_L/P_L = 0.1$ of Figure 5. The dotted line shows the expected number of events in 100 hours of running time.
APPENDIX

For unpolarized beams the differential cross-section for $\tau^\pm$ production with $\tau^\pm$ helicity $h_\pm$ is:

$$\frac{128\pi}{3\sigma_p} \frac{d\sigma}{d\Omega} = 2F_1(1 + \cos^2\theta) + 4F_3\cos\theta - h_+h_-(1 - \cos\theta)^2(F_1 - F_3)$$

$$+ (h_+ - h_-)(1 + \cos\theta)^2 F_4$$

(A1)

where

$$F_1 = 1 + 2g_V^2 R_e(R) + (g_V^2 + g_A^2)^2 |R|^2$$

$$F_3 = 2g_A^2 \Re(R) + 4g_V^2g_A^2 |R|^2$$

$$F_4 = 2g_Vg_A \Re(R) + 2g_Vg_A(g_V^2 + g_A^2) |R|^2$$

(A2)

with

$$R = \frac{e^2(s-M_Z^2)}{s^2 - (\tau - \tau Z)^2 + i\tau z_f}$$

From (A1) and (A2), averaging over the helicity of one $\tau$, we obtain the longitudinal polarization of the other:

$$P_L(\tau^\pm) = \frac{i F_4(1+\cos\theta)^2}{F_1(1+\cos^2\theta) + 2F_3\cos\theta}$$

(A3)

Averaging over any interval $(\theta, \pi-\theta)$ corresponding to the angular acceptance of a forward-backward symmetric detector we obtain

$$<P_L(\tau^\pm)> = \frac{i F_4}{F_1}$$

(A4)

At the $Z^0$ pole we obtain from (A2):
and for a reasonably narrow width

$$P_L = \frac{2\xi}{1 + \xi^2} \xi = \frac{g_A}{g_V} \ (s >> \Gamma^2) \quad (A6)$$

The normalized lepton momentum distribution in the laboratory from $\tau$ decays in flight has the form

$$f(X) = a(X) \hat{P_L} b(X) \quad (\tau^+) \quad (A7)$$

where

$$a(X) = \frac{5}{3} - 3X^2 + \frac{4}{3}X^3\quad (A8)$$
$$b(X) = \frac{1}{3} - 3X^2 + \frac{8}{3}X^3\quad X = \frac{P_L}{P_\tau}$$
$$b(X) = 0 \text{ at } X = X_c$$

Defining:

$$R = \frac{n_H}{n_L} \quad n_H = \int_{X_c}^{1} f(X) dX \quad n_L = \int_{X_0}^{X_c} f(X) dX \quad (A9)$$

The dependence of $R$ on the longitudinal polarization is given by:

$$R(P_L) = \frac{A \hat{P_L}}{C + D P_L} \quad \text{(for } \tau^+\text{)} \quad (A10)$$

with $A = 0.36186$, $B = -0.8666$, $C = 0.55493$, $D = 0.07011$ for $X_0 = 0.05$.

If $n_D$ is the number of events used in the determination of $R$ and $N$ the number in the whole distribution:
\[ N_D = n_L + n_H \quad \text{for } 0 < x < 1 \]
\[ N = g(P_L)N_D \quad \text{for } 0 < x < 1 \]

where

\[ g(P_L) = (0.91679 \pm 0.01655 P_L)^{-1} \quad \text{for } t^\pm \]

The statistical error on \( R \) is given by:

\[ \sigma_R = \sqrt{\frac{R(1+R)}{n_L}} = \sqrt{\frac{R(1+R)^2}{N_D}} \]
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USE OF $\tau$ HADRONIC DECAY MODES TO STUDY POLARIZATION NEAR THE $Z^0$ POLE AT LEP

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I. INTRODUCTION

The aim of this note is to assess the possibilities offered by the two-body or quasi-two body semi leptonic decay modes of the $\tau$ in order to measure the polarization of the $\tau$ produced in $e^+e^-$ annihilation in the vicinity of the $Z^0$ pole.

These $\tau$ decay modes have been studied by Tsai$^1$ and also by Thacker and Sakurai$^2$. Recent estimates$^7$ are presented in table 1.

Some results on $\tau^+\tau^-$ production are recalled in section II.

In section III, the $\pi^-\nu$ decay mode is shown to have very visible effects both on the pion energy spectrum and on the pion angular distribution. Finally in section IV we give some remarks on the $\rho^-\nu$ decay mode.

II. $e^+e^- \rightarrow \tau^+\tau^-$ (s,s,s)

With the following assumptions:

1. One neglects effects of the $\tau$ mass, which are at most of order $\frac{2m_\tau}{\sqrt{s}}$ where $\sqrt{s} = 2E$ is the center of mass energy in the $e^+e^-$ collision.

2. One sums over the spin orientations of one of the final $\tau$ particles, let's say $\tau^+$ spins. Then the cross section depends only upon the scattering
TABLE 1. (from ref. 7)

**τ Branching ratios in %**

<table>
<thead>
<tr>
<th>Decay mode</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\nu_\tau + \nu_\mu + \nu_\mu^-$</td>
<td>15.97</td>
</tr>
<tr>
<td>$\nu_\tau + \pi^-$</td>
<td>9.8</td>
</tr>
<tr>
<td>$\nu_\tau + K^-$</td>
<td>0.62</td>
</tr>
<tr>
<td>$\nu_\tau + \rho^-$</td>
<td>23.01</td>
</tr>
<tr>
<td>$\nu_\tau + K^{*-}$</td>
<td>1.57</td>
</tr>
<tr>
<td>$\nu_\tau + A_1^-$</td>
<td>9.34</td>
</tr>
<tr>
<td>$\nu_\tau + Q^-$</td>
<td>0.41</td>
</tr>
<tr>
<td>$\nu_\tau + \bar{u}d &gt; 1.1$ GeV</td>
<td>21.27</td>
</tr>
<tr>
<td>$\nu_\tau + \bar{u}S &gt; 1.1$ GeV</td>
<td>1.54</td>
</tr>
</tbody>
</table>

Life time: $2.53 \times 10^{-13}$ sec.
angle $\theta$ and the $\tau^-$ helicity $h_-$

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{8\pi} (F_1 (1 + \cos^2 \theta) + 2 \cos \theta F_3 + h_-(1 + \cos \theta)^2 F_4)$$

with: $F_1 = 1 + 2 v^2 \Re \chi + (a^2 + v^2)^2 |\chi|^2$; $\sigma_{\text{TOT}} = F_1 \sigma_{\text{point}}$.  

$$F_3 = 2a^2 \Re \chi + 4a^2v^2 |\chi|^2$$

$$F_4 = 2av (\Re \chi + |\chi|^2 (a^2 + v^2))$$

with: $\Re \chi = \frac{g m^2}{(S-m^2)^2 + \Gamma^2 m^2}$, $m = m_{\gamma^*}$, $\Gamma = \Gamma_{\gamma^*}$.

$$|\chi|^2 = \frac{g^2 m^2}{(S-m^2)^2 + \Gamma^2 m^2}$$

In the Weinberg-Salam model:

$$\frac{g m^2}{16 \sin^2 \theta_w \cos^2 \theta_w} = \frac{37.3}{m_{\gamma^*}}$$

$$a = 1$$

$$v = 4 \sin^2 \theta_w - 1$$

$$\Gamma = \Gamma_{\gamma^*} = m_{\gamma^*} g m^2 16 (1 - 2 \sin^2 \theta_w + \frac{2}{3} \sin^2 \theta_w)$$

(if only six quarks and three lepton doublets are present).

For example: $\Gamma = 3 \text{ GeV}$ at $\sin^2 \theta_w = 0.2$

The mean polarizations are: $<P_-> = \frac{F_4}{F_1}$, $<P_+> = -\frac{F_4}{F_1}$.

At the $Z_0$ peak: $\frac{F_4}{F_1} = \frac{2av}{a^2 + v^2}$
III. DECAY $\tau^- \rightarrow \pi^- \nu$

The angular distribution for this decay with a helicity $h_-$ for the $\tau$ is given by Tsai$^1$:

$$d\omega^\pm = B_\pi (1 \mp h_\pm \cos \theta^*) \, d \cos \theta^*$$

One simple way to use this result is to look at the total pion spectrum in the final state.

Then:

$$E_\pi = \frac{E}{2} (1 + \epsilon) + \frac{E}{2} (1 - \epsilon) \beta_\tau \cos \theta^* \, \rho \, \frac{E}{2} (1 + \cos \theta^*) \left( \epsilon = \frac{m_\pi^2}{m_\tau^2} \right)$$

So that:

$$\frac{dN}{dE_{\pi^-}} = 1 + \langle P_- \rangle \left( \frac{E_\pi - E/2}{E/2} \right) = 1 + \frac{F_4}{F_1} \left( \frac{E_\pi - E/2}{E/2} \right)$$

and:

$$\frac{dN}{dE_{\pi^+}} = 1 - \langle P_+ \rangle \left( \frac{E_\pi - E/2}{E/2} \right) = 1 + \frac{F_4}{F_1} \left( \frac{E_\pi - E/2}{E/2} \right)$$

The slope of the pion spectrum, irrespective of the charge, will give a measure of the sign and value of the $\nu/a$ ratio of the $Z_0$ (fig. 1).

More information can be gained by looking at the bi-dimensional distribution of pions in energy and angle. One finds (Appendix 1) that the forward-backward asymmetry of the $\pi^-$ will be a varying function of the pion energy.

$$A(E_\pi) = \frac{3}{4} \cdot \frac{F_3 + C(E_\pi) F_4}{F_1 + C(E_\pi) F_4} \cos \theta$$

where $C(E_\pi)$ varies from $-1$ to $+1$ when $E_\pi$ goes from minimum to maximum, and $\cos \theta$ is equal to $+1$ at minimum and maximum of $E_\pi$, and is always near $1$ at energies around the $Z_0$ peak.

The figure 2 shows the angular distribution of pions for $E_\pi < \frac{E}{2}$ and $E_\pi > \frac{E}{2}$. These asymmetries give an independent measurement of the $\tau$ helicity.
Figure 1: Pion energy spectrum

\[ \sin^2 \theta_W = 0.2 \]

\[ \sin^2 \theta_W = 0.3 \]
Figure 2: Pion angular distribution
Another way to use these distributions could be to cut the two-dimensional plot $E_\pi$, $\cos \theta_\pi$ into four quadrants. As an example, for $\sin^2 \theta_w = .2$, one gets the ratios:

<table>
<thead>
<tr>
<th>$E$</th>
<th>$\cos \theta_\pi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>22%</td>
<td>19%</td>
</tr>
<tr>
<td>23%</td>
<td>36%</td>
</tr>
</tbody>
</table>

In terms of counting rates necessary for a meaningful measurement, the reaction $e^+e^- \rightarrow \tau^+\tau^-$ will be signed by a leptonic (e or $\mu$) decay mode of one of the $\tau$'s, with a branching ratio of 32%. The fraction of events with a $\pi^0\nu$ decay of the other $\tau$ is $2 \times 10^2 \times 32\% = 6.4\%$ of all produced $\tau$-pairs.

In the Weinberg-Salam model, with the realistic $Z^0$ width of 3 GeV, the number of $\pi^0\nu$ decays produced at the top of the $Z^0$ is $\sim 17$/hour at a luminosity of $5 \times 10^{32} \text{cm}^{-2}\text{sec}^{-1}$.

In a hundred hours of running time, an absolute precision of 4% is reached on the $\tau$ polarization, or a relative precision of 10% for $\sin^2 \theta_w = .2$.

More realistic experimental conditions are under study with the help of a Monte-Carlo simulator. Some results are shown on figs 5 and 6.

One sees that these $\tau \rightarrow \pi\nu$ measurements are one of the best candidates for measurement of the sign of the $\frac{\nu}{\bar{\nu}}$ ratio of the $Z^0$.

More over, if it exists other heavy leptons with a mass greater than 4 GeV, and lower than $\frac{m_{Z^0}}{2}$, the rapid disappearance of their $\pi^0\nu$ mode would leave this $\tau$ decay mode a unique tool to study these polarization phenomena.

**IV. THE $\tau \rightarrow \rho\nu$ DECAY MODE**

The interest in this mode is its 23% branching fraction. The angular distribution for this decay is complicated by the vector structure of the $\rho$. Two
Figure 3: Rho energy spectrum

\[ \sin^2 \theta_w = 0.2 \]

\[ \sin^2 \theta_w = 0.3 \]
Figure 4: Rho angular distribution
Figure 5: Energy spectrum of pions from $\tau^+$ or $\tau^-$ in the Weinberg-Salam theory for 2 values of $\sin^2 \theta_W$. The pion energy is measured in a calorimeter with a resolution $\Delta E \sim 0.5 \sqrt{E}$ (GeV)
Figure 6: Monte Carlo generation of the pion distribution for $|\cos \theta| < 0.9$
different amplitudes, with $\rho$'s of helicity 0 and -1 contribute. The $\rho$ angular
distribution, following Tsai\(^1\) result, is given in appendix 2. We have:

$$dW^\pm = B_\rho \ (1 + a \ h_\rho \ \cos^2 \theta) \ d \cos \theta$$

with $a = \frac{m_\tau - 2m_\rho}{m_\tau + 2m_\rho} \approx .046$.

In terms of $\rho^\pm$ energy and angle distributions, (Figs 3 and 4) the effects are
very similar to the $\pi^\pm$ ones, except that the polarization effects are decreased by the
coefficient $a$. This loss of information will be partially recovered by the higher
branching fraction of the $\rho$ mode, but the number of produced $\tau$'s necessary to
reach a given precision on the polarization using the $\rho$ mode will be of order

$$\frac{B_\pi}{B_\rho} \ a^2 = 2$$

times greater than using the $\pi$ mode: it will be only a useful complement of infor-
mation. For the $A_1$ mode, $a \approx .14$ and this mode is useless. The $\pi - \nu$ mode thus
remains the most important mode to measure $\tau$ polarization.

A useful conversation with A. Courau is acknowledged.
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APPENDIX 1

e^+e^+ \rightarrow \tau^+\tau^-

The angular distribution is:

\[ d\sigma = \frac{2\pi \alpha^2 B_\pi}{4S} \left[ F_1 (1 + \cos^2 \theta) + 2 F_3 \cos \theta + F_4 (1 + \cos \theta)^2 \cos^2 \phi \right] d\cos \theta d\cos \phi d\phi \]

By a change of variables to the pion energy \( E_\pi \), the pion angle \( \cos \theta_\pi \), and the azimuthal angle of the \( \tau \) around the pion, and integrating over this last angle one gets:

\[ \frac{d\sigma}{d\cos \theta_\pi} \propto G_1 + 2 G_2 \cos \theta_\pi + G_3 \sin^2 \theta_\pi \]

with:

- \( G_1 = (F_1 + C(E_\pi) F_4) (1 + \cos^2 \theta) \)
- \( G_2 = (F_3 + C(E_\pi) F_4) \cos \theta \)
- \( G_3 = -\frac{1}{2} (F_1 + C(E_\pi) F_4) (3 \cos^2 \theta - 1) \)

where:

\[ C(E_\pi) = \frac{2 E_\pi - E (1 + \varepsilon)}{P_\tau (1-\varepsilon)} \frac{E_\pi - E/2}{E/2} \]

\[ \cos \theta = \frac{2 E E_\pi - m^2_\tau - m^2_\pi}{2 P_\tau P_\pi} \gamma \]

Integration over \( \cos \theta_\pi \) yields the energy spectrum of \( \pi^1 \).

The angular distribution yields a forward-backward asymmetry:

\[ A(E_\pi) = \frac{\int_0^1 d\sigma d\cos \theta_\pi - \int_1^0 d\sigma d\cos \theta_\pi}{\int_0^1 d\sigma d\cos \theta_\pi} = \frac{6G_2}{6G_1 + 4G_3} = \frac{3(F_2 + C(E_\pi) F_4) \cos \theta}{4 (F_1 + C(E_\pi) F_4)} \]

A\((E_\pi)\) depends upon the pion energy:

- at \( E_\pi = E_\pi \text{ min} \): \( A_{\text{min}} = \frac{3}{4} \left( \frac{F_3 - F_4}{F_1 - F_4} \right) \)
- at \( E_\pi = E_\pi \text{ max} \): \( A_{\text{max}} = \frac{3}{4} \left( \frac{F_3 + F_4}{F_1 + F_4} \right) \)
At the top of the $Z_0$: $A_{\text{min}} = \frac{-2av}{a^2 + v^2}$, $A_{\text{max}} = \frac{2av}{a^2 + v^2}$

$$\cos u = \frac{\vec{P}_\pi \cdot \vec{P}_\tau}{|\vec{P}_\pi||\vec{P}_\tau|}$$ is always positive if $E > 11.3$ GeV, and at $2 E \sim M_{Z^*}$, one gets $\cos u > .83$.

Such dependance of the asymmetry in the angular distribution as a function of the particle energy should also be useful for purely leptonic decay modes of the $\tau$. 
APPENDIX 2: $\tau^- + \rho^- \rightarrow \nu_\tau \pi^- \pi^0$

The squared matrix element is given by Tsai\(^1\) in term of dot products:

$$|M|^2 \propto \left[ 2(p_1 \cdot Q) (p_2 \cdot Q) - (p_1 \cdot p_2) Q^2 - m_\tau \right]^{-1} \left( W, Q \right) (p_2 \cdot Q) - (W, p_2) Q^2$$

where \(P_1, P_2, q_1, q_2\) are the four momenta of the \(\tau^-, \nu_\tau, \pi^-, \pi^0\) respectively, \(Q = q_1 - q_2\) and \(W\) is the four vector which reduces to the three dimensional polarization vector \(\vec{W}\) in the rest frame of \(\tau^-\).

If \(\alpha^*\) and \(\phi^*\) are the \(\rho\) decay angles in its rest frame and \(\theta^0\) the \(\rho\) production angle in the \(\tau\) frame we have:

$$P_1 \cdot Q = P_2 \cdot Q = \frac{m_\tau^2}{2} (1 - \eta) \beta^* \cos \alpha^*$$

$$P_1 \cdot P_2 = \frac{m_\tau^2}{2} (1 - \eta)$$

$$m_\tau (W, P_2) = \frac{m_\tau^2}{2} (1 - \eta) W \cos \theta^\rho$$

$$m_\tau (W, Q) = -\frac{m_\tau^2}{2} (1 + \eta) \beta^* W \cos \theta^\rho \cos \alpha^* - m_\tau \beta^* W \sin \theta^\rho \sin \alpha^* \cos \phi^*$$

and \(Q^2 = -m_\rho^2 \beta^{*2}\)

where \(\beta^* = \left(1 - \frac{4m_\tau^2}{m_\rho^2}\right)^{1/2}\), \(\eta = \frac{m_\rho^2}{m_\tau^2}\) and \(\vec{W}\) is along the \(z\) axis.

so that

$$|M|^2 \propto \cos^2 \alpha^* (1 + W \cos \theta^\rho) + \frac{m_\rho^2}{m_\tau^2} \sin^2 \alpha^* (1 - W \cos \theta^\rho)$$

$$-2 \frac{m_\rho^2}{m_\tau^2} W \sin \theta^\rho \sin \alpha^* \cos \alpha^* \cos \phi^* \quad (A2.1)$$

It is worth noticing that this can be expressed, for \(W = 1\), as

$$|M|^2 \propto \left| \frac{1}{8} \left( \frac{\theta^\rho}{d_1} Y_1^0 (\alpha^*) + \frac{\sqrt{2} m_\rho}{d_1} \frac{1}{\sqrt{2}} \left( \frac{\theta^\rho}{d_1} Y_1^1 (\alpha^*, \phi^*) \right) \right) \right|^2$$

where the two helicity states of the \(\rho\) are evident.
The formula A2.1 has been used for M.C. generation of events. When integrated on the $\rho$ decay angles, it gives the $\rho$ production angular distribution used in section 4:

$$\frac{dW}{d\cos\theta_\rho} \propto \left( 1 + \frac{2m_\rho^2}{m_t^2} \right) + \left( 1 - \frac{2m_\rho^2}{m_t^2} \right) W \cos\theta_\rho$$

It may be useful to note here that the $\pi^-$ energy angle spectrum in the $\tau^-$ rest frame given in ref 1 has a term missing which changes its shape: the formula 2 - 20 of this paper should have its last line changed to:

$$\pm \frac{(\mathbf{\tilde{q}} \cdot \mathbf{q_1}) \omega_1}{q_1^2} \left[ 16M_L^2 \left( \omega_1 - \frac{M_L^2 + M_D^2}{4M_L} \right)^2 + M_\rho^2 \left( 1 - \frac{4M_D^2}{M_\rho^2} \right) \left( 3M_L^2 - M_\rho^2 - \frac{M_D^2}{\omega_1} \right) \right]$$

so that the coefficient in the square bracket is not definite positive, and in fact changes sign in the $\omega_1$ range. Note also that the $\rho^-$ and $\Lambda^-_1$ decay modes are incorrect in ref 6.
THE REACTION \( e^+e^- \rightarrow \mu^+\mu^- \) (OR \( \tau^+\tau^- \))
WITH TWO NEUTRAL WEAK BOSONS
AT LEP ENERGIES

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At the present time, the standard Salam-Weinberg model of weak and electromagnetic interactions is in agreement with almost all the experimental data. The rates of neutral current-induced \( \nu \) or \( \bar{\nu} \) reactions on nucleons are now well accounted for with a Weinberg angle \( \Theta_W \) such that \( \sin^2 \Theta_W = 0.24 \pm 0.02 \)\(^{[1]} \). Recently, the discovery of parity violation in reactions of polarized electrons on nucleons at SLAC\(^{[2]} \) has provided a remarkable confirmation of the standard model with \( \sin^2 \Theta_W = 0.20 \pm 0.03 \). In the reactions \( \nu_\mu (\bar{\nu}_\mu) + e^- \) there is still some controversy in the data\(^{[1]} \) but all experiments suffer from low statistics.

The standard model is also the "minimal" model, and its success at center-of-mass energies much smaller than the mass of the weak neutral boson is understandable, even if the structure of neutral current is actually more complicated. However, since we are concerned by the energy range which should be covered by the LEP machine, it is particularly interesting to review the predictions of models involving larger gauge groups, thus more than one neutral weak boson \( Z \).
In Section 1, the model independent formulae giving the cross-section, the charge asymmetry and the longitudinal polarization of the final leptons are given for \( e^+ e^- \rightarrow \mu^+ \mu^- (or \tau^+ \tau^-) \) reaction. In Section 2, the general features of models based on the \((SU_2)_L \times (SU_2)_R \times U_1\) group are summarized. The predictions of two of them are presented in Section 3 for the reaction \( e^+ e^- \rightarrow \mu^+ \mu^- (or \tau^+ \tau^-) \).

1 - GENERAL MODEL-INDEPENDENT FORMULAE

The general formulae for the reaction \( e^+ e^- \rightarrow \mu^+ \mu^- \) are given by a straightforward extension of the calculations of R. Budny \([3]\), involving three annihilation graphs, (via \( \gamma, Z_1 \) and \( Z_2 \)). If only vector and axial couplings are present, the differential cross-section and the lepton longitudinal polarization are given by \([4]\):

\[
\frac{d\sigma}{dt} = \frac{\alpha^2}{4S} \left[ F_1(S) \left( 1 + \cos^2 \theta \right) + 2 \cos \theta \cdot F_3(S) \right]
\]

\[
P(\mu^+ or \tau^+) = -\frac{F_4(S) \left( 1 + \cos \theta \right)^2}{F_1(S) \left( 1 + \cos^2 \theta \right) + 2 \cos \theta \cdot F_3(S)}
\]

in which \( F_1, F_3 \) and \( F_4 \) are functions of the center of mass energy squared \( S \), and of the boson masses, widths and coupling constants.

In this report, we define the coupling constants \( g_{V1}, g_{A1} \) (for \( Z_1 \)) and \( g_{V2}, g_{A2} \) (for \( Z_2 \)) in such a way that the interaction lagrangian be written as:

\[
e^2 \frac{\partial}{\partial x} \cdot \bar{\epsilon}(g_{V1} \gamma \mu + g_{A1} \gamma \mu \gamma_5) e; \quad (i = 1, 2).
\]

(Note that in Ref. \([3]\), \( e^2 \) is included in the coupling constants)
If \((M_k, \Gamma_k, k = 1,2)\) are respectively the boson masses and widths, we define:

\[
R_k = \frac{S}{S - (M_k - \Gamma_k/2)^2} \quad (k = 1,2).
\]

Then, \(F_1, F_3\) are given by:

\[
(3) \quad F_1 = 1 + \sum_i (|R_i|^2 (g_{V1}^2 + g_{A1}^2) + 2(Re R_i) q_{V1}^2) + 2(Re R_i R_2^\dagger) (q_{V1} q_{V2} + g_{A1} g_{A2})^2
\]

\[
(4) \quad F_3 = \sum_i (4|R_i|^2 (g_{V1}^2 g_{A1}^2 + 2(Re R_i) q_{V1}^2 q_{V2} + 2(Re R_i R_2^\dagger) (q_{V1} q_{A2}^2 + g_{A1} q_{V2})^2
\]

and \(F_4\) is similarly given by:

\[
(5) \quad F_4 = \sum_i 2|R_i|^2 (q_{V1} g_{A1}^2 + g_{A1} g_{A1}^2) + 2(Re R_i) q_{V1} g_{A1}
\]

\[
+ 2(Re R_i R_2^\dagger) (q_{V1} q_{V2} + g_{A1} g_{A2}) (q_{V1} g_{A2} + g_{A1} q_{V2})
\]

The terms in \(|R_i|^2\) clearly result from the square of the \(Z_i\) amplitudes, the terms in \((Re R_i)\) from the \(\gamma - Z_i\) interference, and finally the term in \(2(Re R_i R_2^\dagger)\) from the \(Z_1 - Z_2\) interference.

Formulae (1) to (5) allow to calculate event rates, charge asymmetry and polarization \([4]\) as functions of \(S\). In the following, the detector is assumed to be Solenoidal, with full azimuthal acceptance, and \(30^\circ < \Theta < 150^\circ\), \(\Theta\) being the angle with respect to the beam axis; the luminosity of the machine is assumed to be:

\[
\mathcal{L} = 10^{32} \left(\frac{E}{70 \text{ GeV}}\right)^2 \text{cm}^{-2} \text{s}^{-1}.
\]

The model dependent inputs are thus the coupling constants, masses and widths of the bosons \(Z_1\) and \(Z_2\).
The extension of the usual SU$_2 \times$ U$_1$ gauge group had been initially motivated by experimental results, namely: anomalous trimuon production in $\nu$ reactions and absence of parity violation in bismuth atoms. At the present time, all experimental data agree on trimuon rates compatible with conventional processes $[6]$ and parity violation in e $N$ reactions is clearly proved $[2]$. 

However, the models based on the (SU$_2$)$_L \times$ (SU$_2$)$_R \times$ U$_1$ group have an interesting feature, namely the basic left-right symmetry, parity being spontaneously broken via the Higgs mechanism. A complete review of such models can be found in Ref. $[5]$. Their most important characteristics are summarized here:

(a) The (SU$_2$)$_L$ and (SU$_2$)$_R$ groups have the same coupling constant:
\[
g = \frac{e}{\sin \theta_W}. \quad \text{The } U_1 \text{ coupling constant is: } g' = \frac{e}{\sqrt{\cos 2\theta_W}}.
\]

(b) The generators of (SU$_2$)$_L$, (SU$_2$)$_R$ and U$_1$ being denoted by $T^a_L$, $T^a_R$ and $Y$ respectively, the electric charge is given by:
\[
Q = T^3_L + T^3_R + \frac{Y}{2}
\]
Left-handed leptons are classified in (SU$_2$)$_L$ doublets and (SU$_2$)$_R$ singlets, whereas right-handed leptons are classified in (SU$_2$)$_L$ singlets and (SU$_2$)$_R$ doublets. Both have $Y = -1$.

(c) There are 4 charged boson states $W^+_L$ and $W^+_R$ and 3 neutral boson states: $W^0_{3L}$, $W^0_{3R}$ and $B$, (B corresponding to U$_1$). The physical bosons are linear combinations of such states. The Higgs fields are chosen in such a way to give mass to charged bosons and to two neutral bosons, (the third one being the photon).
(d) Higgs bosons $\chi_L(T_L = \frac{1}{2}, T_R = 0, Y = -1)$ and $\chi_R(T_L = 0, T_R = \frac{1}{2}, Y = -1)$ are introduced with vacuum expectation values $\lambda_L$ and $\lambda_R$. They contribute to both neutral and charged boson masses. Neutral current violate parity only if $\lambda_R \neq \lambda_L$.

(e) Higgs bosons $\delta_L(T_L = 1, T_R = 0, Y = 0)$ and $\delta_R(T_L = 0, T_R = 1, Y = 0)$ are introduced with vacuum expectation values $b_L$ and $b_R$, contributing only to charged boson masses.

(f) An additional Higgs boson $\phi(T_L = T_R = \frac{1}{2}, Y = 0)$ mixes $W_L^\pm$ and $W_R^\pm$. It also affects the masses of the neutral bosons, and has no effect on parity violation in neutral currents.

In order to suppress right-handed charged currents at low energies, a very large mass has to be given to $W_R^\pm$. This can be achieved either by taking $b_R \gg b_L$, or by taking $\lambda_R \gg \lambda_L$ or both. We shall only consider here, as illustrations, the two following extreme cases:

1) **Fritzsch - Minkowski - Mohapatra - Sidhu model (FMMS)**

   In this model, $\lambda_L$ and $\lambda_R$ are equal and $b_L$ set equal to 0, whereas $b_R$ is very large. There is no parity violation in neutral currents since the lighter neutral boson ($Z_A$) is coupled to a purely axial current and the heavier one ($Z_V$) to a purely vector current. The predictions of this model for neutrino reactions coincide with those of the standard model with the same value of $\sin^2 \vartheta_W$. However, the recent SLAC experiment now excludes this model.

2) **De Rujula - Georgi - Glashow model (DGG)**

   In this model, $\lambda_L$ is set equal to 0 and only $\lambda_R$ is used to give mass to $W_R^\pm$. Parity is thus violated in neutral current processes. The two physical neutral bosons are now linear combinations of $Z_A$ (purely axial) and $Z_V$ (purely vector) states. The model parameters can be fitted to account for neutrino data and two solutions (denoted by DGG - and DGG+ respectively) are found. However, the DGG+ solution would imply a reduction of parityvio-
lating effects in eN reactions relatively to the standard model and is excluded by the recent SLAC results.[2].

Table 1 shows the range allowed for $\sin^2 \theta_W$ and for the neutral boson masses from the fits to neutrino data performed in Ref.[5], respectively for FMMS, DGG± models and for the standard Salam-Weinberg model. In the first three models, the mass of the first boson is expected to be slightly smaller than in the standard model. The mass of the heavier boson, however, is not likely in the energy range of LEP 70 if FMMS and DGG± models are discarded on the basis of SLAC results[2], but the present low energy data are still compatible with a mass of about 200 GeV resulting in observable effects at 70 GeV, especially in charge asymmetry.

In all models, the neutral boson widths have been calculated on the basis of 3 quark and 3 lepton doublets [9]. In the DGG± models, additional parameters have been fixed as explained in Appendix 1.

Figures 1 to 4 compare the FMMS model to the standard model for event rates and charge asymmetry. The effect of the heavier boson in charge asymmetry is the presence of a "dip" whose width strongly depends
Figure 1: Event rate per hour versus beam energy ($\sin^2 \theta_w = 0.20$)

a) FMMS model (-----)

b) Standard model (-------)
Figure 2: Event rate per hour versus beam energy \((\sin^2 \theta_W = 0.30)\)

a) FMNS model (---)

b) Standard Model (--------)
Figure 3: Charge asymmetry versus beam energy ($\sin^2\theta_w = 0.20$)

a) FMMS model with boson widths calculated
   ($M(Z_2) = 107.8$ GeV/c$^2$, $\Gamma(Z_2) = 2.7$ GeV/c$^2$) (———)

b) FMMS model with $\Gamma(Z_2)$ fixed at 1 GeV/c$^2$ (--- --- --- ---)

c) Standard model (--------)
Figure 4: Charge asymmetry versus beam energy ($\sin^2 \theta_W = 0.30$)

a) FMMS model with boson width calculated
   \[ M(Z_2) = 107.8 \text{ GeV/c}^2, \Gamma(Z_2) = 1.8 \text{ GeV/c}^2 \] (-----)

b) FMMS model with $\Gamma(Z_2)$ fixed at 1 GeV/c$^2$ (--.--.--.--.)

c) Standard model (--------)
on the boson width. Of course, the FMEIS model predicts no final lepton polarization, since parity is conserved; \( q_{V1} = 0 \) and \( q_{A2} = 0 \); (see formula (5)).

Figures 5 to 7 compare DGG* models to the standard model for event rates, charge asymmetry and lepton polarization. In DGG* models the dip effect in charge asymmetry is not concentrated at the heavy boson mass as in the FMEIS model; significant effects can be seen below the resonance energy; they are not sensitive to the width of the heavier boson. As far as polarization is concerned, the predictions depend on the deviation from \( \frac{1}{4} \) of \( \sin^2 \theta_W \); (for \( \sin^2 \theta_W = \frac{1}{4} \), all the preceding models predict no effect). Here also, significant effects of the heavier boson can be seen below the resonance energy.

The preceding features of the DGG* solution allow for significant deviations from the standard model at a beam energy of 70 GeV, even if the heavier boson is out of the range of LEP 70.

As far as the cross-section is concerned, the main effect is the shift of the lighter boson peak relatively to the prediction of the standard model with \( \sin^2 \theta_W = 0.24 \pm 0.02 \) namely \( M(Z_1) = 87 \pm 3 \text{ GeV/c}^2 \). However, \( \sin^2 \theta_W \) being essentially known from \( \nu + \text{nucleon reactions}, \) its present value may be affected by QCD effects so that the discovery of the \( Z_1 \) peak at e.g. \( M(Z_1) = 79 \text{ GeV/c}^2 \) would not be a decisive argument against the Salam-Weinberg theory. However, once the \( Z_1 \) mass is known, it is possible to compare the standard model and a two-boson-model, both accounting for the same measured mass. As an example, we assume \( M(Z_1) = 79 \text{ GeV/c}^2 \) and compare the DGG* models(*) and the standard model for effects in the charge asymmetry \( A \). The variation of \( A \) with the beam energy is shown in figure 8, both for \( M(Z_2) = 214 \text{ GeV/c}^2 \) and for an infinite value of \( M(Z_2) \) (standard model). On the basis of the statistical error only \( \Delta A = \sqrt{\frac{1-A^2}{N}} \), \( N \) being the number of observed events at 70 GeV, the number of hours of a 70 GeV run necessa-

(*) The DGG models have 3 parameters, namely \( \sin^2 \theta_W \) and the angles \( \alpha \) and \( \beta \) defined in Ref.[10]. Here, \( \beta \) has been fixed to 0 in order to agree with SLAC data on polarized electron scattering. Only \( \alpha \) and \( \theta_W \) are varied in order to keep \( M(Z_1) \) equal to 79 GeV/c².
Figure 5: Event rate per hour versus beam energy

a) DGG - model (---------)
b) DGG + model (-.-.-.-.)
c) Standard model ($\sin^2 \theta_W = 0.20$) (--------)

For the parameters of DGG $\pm$ models, see appendix 1.
Figure 6: Charge asymmetry versus beam energy
a) DGG - model (-----)
b) DGG + model (-.-.-.-)
c) Standard model \( \sin^2 \theta_W = 0.20 \) (------)

For the parameters of DGG ± models, see appendix 1.
Figure 7: Longitudinal polarization of the final lepton versus beam energy
a) DGG - model (-----)
b) DGG + model (-.-.-.-)
c) Standard model ($\sin^2 \theta = 0.20$) (--------)
d) Standard model ($\sin^2 \theta = 0.30$) (.......)

For the parameters of DGG - models, see appendix 1.
Figure 8: Charge asymmetry versus beam energy, for $M(Z_1) = 79$ GeV/c$^2$

a) DGG model with $M(Z_2) = 214$ GeV/c$^2$ (---)

b) Standard model (--------)
ry to obtain a 2 s.d. discrepancy with the standard model is plotted in figure 9 as a function of $M(Z_2)$. Assuming that the LEP machine can be operated at 100 GeV with a luminosity $4 \times 10^{31}$ cm$^{-2}$ s$^{-1}$, the number of hours of a 100 GeV run necessary to observe the same deviation is also indicated in figure 9. Similarly, the 90% confidence intervals for $A$ obtained at 70 GeV are shown in figure 10 as functions of $M(Z_2)$ both for a 200 h run and for a 1000 h run.

It can be concluded that indirect effects of a second boson can be detected by LEP 70, only if $M(Z_2) \lesssim 260$ GeV/c$^2$.

CONCLUSION.

The present data on $\nu +$ nucleon and $e^- +$ nucleon scattering (found to be in good agreement with the Salam-Weinberg model) impose severe constraints to $(SU_2)_L \times (SU_2)_R \times U_1$ models, so that no deviation from the Salam-Weinberg model is predicted in $e^+e^- \rightarrow \mu^+\mu^-$ reactions for beam energies lower than about 50 GeV. However, if there exists a second neutral boson $Z_2$, its effects in charge asymmetry can be detected by LEP 70 provided its mass is lower than $\sim 260$ GeV/c$^2$. 
Figure 9: Number of hours of run necessary to obtain a 2 s.d. discrepancy with the standard model, versus the mass of the heavier boson (DGG - model with $M(Z_1) = 79$ GeV/c$^2$)

a) Run at 70 GeV with a luminosity of $10^{32}$ cm$^{-2}$ s$^{-1}$ (-----)
b) Run at 100 GeV with a luminosity of $0.4 \times 10^{32}$ cm$^{-2}$ s$^{-1}$ (-------)
Figure 10: 90% confidence interval in charge asymmetry, versus the mass of the heavier boson.

(DGG-model with $M(Z_1) = 79$ GeV/c^2)

a) 1000 h run at 70 GeV (-----)
b) 200 h run at 70 GeV (--------)

The dotted line corresponds to the prediction of the DGG-model.
APPENDIX:

a) FMW model: Apart from $\sin^2 \theta_W$, this model uses a parameter $\varepsilon$ defined in Ref. [8]. This parameter (which has no effect on parity conservation in the model) has been taken equal to 0.

b) DGG model: Apart from $\sin^2 \theta_W$, this model uses two angles $\alpha$ and $\beta$ defined in Ref. [10]. The curves shown in figures 5 to 7 have been calculated using $\sin^2 \theta_W = 0.30$ and $\sin^2 \alpha = 0.16$, $\sin^2 \beta = 0$ for solution DGG-; $\sin^2 \alpha = 0.40$, $\sin^2 \beta = 0.23$ for solution DGG+

Those values are close to the ones fitted in Ref. [5], and thus agree with neutrino data.

The curves shown in figures 9 and 10 have been calculated with $\beta = 0$, and by varying $\alpha$ and $\theta_W$ simultaneously in order to keep $M(Z_1) = 79 \text{ GeV/c}^2$. The following table shows their corresponding variations together with those of $M(Z_2)$, of event rates per hour and of charge asymmetry at a beam energy of 70 GeV.

<table>
<thead>
<tr>
<th>$\sin^2 \alpha$</th>
<th>$\sin^2 \theta_W$</th>
<th>$M(Z_2)$ GeV/c$^2$</th>
<th>Rate/h</th>
<th>Charge Asymmetry</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.16</td>
<td>0.300</td>
<td>214</td>
<td>1.44</td>
<td>0.34</td>
</tr>
<tr>
<td>0.12</td>
<td>0.315</td>
<td>251</td>
<td>1.53</td>
<td>0.42</td>
</tr>
<tr>
<td>0.08</td>
<td>0.326</td>
<td>313</td>
<td>1.60</td>
<td>0.45</td>
</tr>
<tr>
<td>0.04</td>
<td>0.336</td>
<td>451</td>
<td>1.66</td>
<td>0.46</td>
</tr>
<tr>
<td>0.00</td>
<td>0.343</td>
<td>$\infty$</td>
<td>1.72</td>
<td>0.47</td>
</tr>
</tbody>
</table>

(standard model)


[10] A. DE RUJULA, H. GEORGI and S.L. GLASHOW,

Following ref. (1) and (2), we describe two methods to reach the weak coupling of the quarks by measuring the forward-backward asymmetries with jets produced in $e^+e^-$ annihilation.

We can define an orientation on a jet either by using the leading particles or by measuring the charges of the two jets. By doing so, we are unable to separate $u\bar{u}$ from $d\bar{d}$ jets: both $u$ and $d$ can give a leading $n^+$ and have very similar charge distributions. Fig. 1 - 2 show the charge distribution expected from the Feynman-Field\(^4\,5\) model for quark fragmentation. Fortunately, these two types of jets do not occur with the same frequency and have different asymmetries (fig. 3) so that in a wide range of energies we can measure a net effect which can be related to the weak interaction asymmetry. In the case of $s\bar{s}$ jets, the situation is less ambiguous and we will indicate a method which allows for an efficient separation from $c\bar{c}$, $u\bar{u}$ and $d\bar{d}$ jets provided one can afford a low efficiency.

I. Average charge method.

Given two back to back jets, one can define a class of events where the charge in one hemisphere is greater or equal to 1. We assume that all the particles are seen, so that no independent requirement on the opposite jet can be made.
Figure 3: Weinberg Salam predictions for the leptons and quark asymmetries with $\sin^2 \theta_W = .2$

- $\cdots$ d quarks
- $\cdots\cdots$ u quarks
- $\mu$
One defines reliability factors for each type of jet with the definition:

\[
R = \frac{\text{True} - \text{False}}{\text{True} + \text{False}}
\]

where 'True' refers to the number of times the criterion succeeded in selecting the correct flavor and 'False' to the number of times the criterion failed.

The observed asymmetry:

\[
A_Q = \frac{N_{\text{FORWARD}}(Q > 1) - N_{\text{BACKWARD}}(Q > 1)}{N_{\text{FORWARD}}(Q > 1) + N_{\text{BACKWARD}}(Q > 1)}
\]

can be expressed in terms of the quark asymmetries\(^{(3)}\) \(A_u\) and \(A_d\) as follows:

\[
(i) \quad A_Q = R_u A_u \frac{N_u}{N_u + N_d} - R_d A_d \frac{N_d}{N_u + N_d}
\]

where \(N_u\) and \(N_d\) are the number of \(u\bar{u}\) and \(d\bar{d}\) type jets selected with the charge criterion.

Table I gives \(R\) and efficiencies for both types of jets in the Feynman and Field model\(^{(4, 5)}\) referred from now on as the F.F. model.

Fig. 4 shows the expected variation of \(A\) with energy assuming that three isodoublets are produced at LEP.

Remarks:

1) The \(A_Q\) measurement provides a test of universality of the Weinberg-Salam theory since as, already mentioned, all isodoublets contribute. However, charge distributions may change with the type of flavor so that both reliability and efficiency vary from quark to quark. For our calculation, we have assumed universal behaviour of same charge quarks.

2) One could alternatively measure weighted charges\(^{(4)}\) or perform longitudinal momentum\(^{(3)}\) cuts. These variations are probably useful to check that the F.F. picture is correct but the size of the effect should remain the same.
Table I.

Average charge \((Q \geq 1)\) method parameters (F.F. predictions)

<table>
<thead>
<tr>
<th>Type</th>
<th>Efficiency</th>
<th>Reliability</th>
</tr>
</thead>
<tbody>
<tr>
<td>uıntı</td>
<td>.63</td>
<td>.78</td>
</tr>
<tr>
<td>dđ</td>
<td>.52</td>
<td>.65</td>
</tr>
</tbody>
</table>
Figure 4: Charge asymmetry expected requiring that the total charge in one hemisphere be greater or equal to 1. We take $\sin^2 \theta_W = .2$ and assume three isodoublets. We indicate statistical errors with 100 hours per point with $L = 10^{32} \left( \frac{E}{70} \right)^2 \text{cm}^{-2}\text{s}^{-1}$. 
II. Leading particle signature.

An orientation is given by selecting jets with a fast - about one half of the maximum momentum - particle with characteristic quantum numbers: $\pi^\pm, K^\pm$. Fig. 5 shows the ideal behaviour expected assuming that a leading $\pi^+$ means a u or $\bar{d}$ quark and a $K^+$ means a $\bar{s}$ or u quark.

Three effects will tend to dilute this ideal effect:
- Secondary dressings can often produce a leading particle with no relation with the leading quark.
- Primary dressings into resonances, e.g. $\rho^0$, can also lose the information.
- The majority of the flavours s, c, b, t, etc... will give no asymmetry through leading $\pi^\pm$ though still providing such $\pi$. Except for the s, we know no reliable way of estimating such contributions.

a) $\pi^\pm$.

We can write for the asymmetry parameter an expression similar to formula (i), although the probability to get a fast $\pi^\pm$ clearly changes when going to heavy isodoublets. Defining as $F_i$, the fraction of cases in which an isodoublet $i$ gives a fast charged $n^+$ ($n^-$) along the $u$ ($\bar{u}$) direction. We assume that all the asymmetry comes only from $u\bar{u}$ and $d\bar{d}$.

Table II gives a set of probabilities(5, 4) allowing an explicit calculation of $F_i$ for $u\bar{u}$, $d\bar{d}$ and $s\bar{s}$.

For numerical application we make the simplifying hypothesis that we have three isodoublets $(u, d)$, $(c, s)$ and $(t, b)$ and that $F_s = F_c = F_b = F_t$.

Formula (ii) reduces to:

$$A_{\pi} = \frac{K_u F_u A_u N_u - K_d F_d A_d N_d}{\varepsilon F_i N_i}$$
Figure 5: Ideal $\pi^+$ and $K^+$ asymmetries expected with reliability equal 1 and $\sin^2 \theta_W = .2$

\[ \text{--- $\mu$} \]
\[ \text{----- $\pi^+$} \]
\[ \text{..... $K^+$} \]
Table II.

Leading particle probabilities in F.F. in %.

a) $z_{\text{min}} > .5$

<table>
<thead>
<tr>
<th>quark</th>
<th>$\pi^+$</th>
<th>$\pi^-$</th>
<th>$K^+$</th>
<th>$K^-$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u$</td>
<td>10.6</td>
<td>3.2</td>
<td>3.6</td>
<td>1.1</td>
</tr>
<tr>
<td>$\bar{u}$</td>
<td>3.2</td>
<td>10.6</td>
<td>1.1</td>
<td>3.6</td>
</tr>
<tr>
<td>$d$</td>
<td>3.2</td>
<td>10.6</td>
<td>1.1</td>
<td>1.9</td>
</tr>
<tr>
<td>$\bar{d}$</td>
<td>10.6</td>
<td>3.2</td>
<td>1.9</td>
<td>1.1</td>
</tr>
<tr>
<td>$s$</td>
<td>2.9</td>
<td>2.9</td>
<td>.8</td>
<td>8.7</td>
</tr>
<tr>
<td>$\bar{s}$</td>
<td>2.9</td>
<td>2.9</td>
<td>8.7</td>
<td>.8</td>
</tr>
</tbody>
</table>

b) $z_{\text{min}} > .4$

<table>
<thead>
<tr>
<th>quark</th>
<th>$\pi^+$</th>
<th>$\pi^-$</th>
<th>$K^+$</th>
<th>$K^-$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u$</td>
<td>16.8</td>
<td>5.9</td>
<td>6.0</td>
<td>1.8</td>
</tr>
<tr>
<td>$\bar{u}$</td>
<td>5.9</td>
<td>16.8</td>
<td>1.8</td>
<td>6.0</td>
</tr>
<tr>
<td>$d$</td>
<td>5.9</td>
<td>16.8</td>
<td>1.9</td>
<td>3.3</td>
</tr>
<tr>
<td>$\bar{d}$</td>
<td>16.8</td>
<td>5.9</td>
<td>3.3</td>
<td>1.9</td>
</tr>
<tr>
<td>$s$</td>
<td>5.5</td>
<td>6.1</td>
<td>1.8</td>
<td>12.4</td>
</tr>
<tr>
<td>$\bar{s}$</td>
<td>6.1</td>
<td>5.5</td>
<td>12.4</td>
<td>1.8</td>
</tr>
</tbody>
</table>
In table III the efficiency and reliability parameters are summarized. Fig. 6 shows the expected variation of $A_{\mu}$ with energy. From this curve, we see that we obtain a rather poor sensitivity to $\sin^2\theta_W$ measurement with $u\bar{u}$ and $d\bar{d}$ jets. It does not matter so much since this parameter is already available from neutrino reactions.

b) $K^\pm$.

Using charge kaons asymmetries in $e^+e^-$ annihilations is the only way to reach weak coupling constants for strange quarks.

A similar procedure can be developed which will lead to a relation between $A_{K^\pm}$ and $A_{\mu}$ and $A_S$. However, in this case, we can think of a better method. We separate $s\bar{s}$ from $u\bar{u}$ by requiring a leading $K^-$ ($K^+$) in the jet opposite to the leading $K^+$ ($K^-$). In addition, a veto can be set on jets with two charged $K$ to depress $u\bar{u}$ and $d\bar{d}$ jet type events where $K$ are produced in pairs.

Charm is also a contribution although it will not produce so easily a leading kaon. Since $K^+$ come from the $\bar{c}$ quark jet with charge $-2/3$ we can use method I to separate them from $K^+$ originating from $\bar{s}$ jets with charge $+1/3$.

Table IV summarizes the useful estimates for this method. We give only an estimate for the upper limit for the charm contribution.

With these small contaminations, one is almost able to reach directly $A_S$. $A_d$ has the same behaviour as $A_S$ so the $d\bar{d}$ contamination simply adds up.

The "up" contribution is small and could even be reduced since the charm term tends to cancel the up asymmetry.

Fig. 7 shows the dependence of the asymmetry with $\sin^2\theta_W$ at the $Z^0$ pole. With 200 hours spent at the $Z^0$ pole, one will collect $10^5$ events giving about 2000 fast $K^+K^-$ pairs.

Conclusion.

From this study, we conclude that the average charge method gives the most efficient way of measuring weak interactions with quark jets. The leading particle method has nevertheless its own advantages:

- It does not require an exact counting of charges which looks hard experimentally.
- It relies less on the details of F.F. (especially on any cut off on the soft particles).
- It allows a separate measurement of $A_S$. 
Table III.

Leading $\pi^+$ method parameters in F.F.

a) $z > 0.5$

<table>
<thead>
<tr>
<th>jet</th>
<th>Efficiency</th>
<th>Reliability</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u\bar{u}$</td>
<td>0.18</td>
<td>0.40</td>
</tr>
<tr>
<td>$d\bar{d}$</td>
<td>0.18</td>
<td>0.40</td>
</tr>
</tbody>
</table>

b) $z > 0.4$

<table>
<thead>
<tr>
<th>jet</th>
<th>Efficiency</th>
<th>Reliability</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u\bar{u}$</td>
<td>0.30</td>
<td>0.37</td>
</tr>
<tr>
<td>$d\bar{d}$</td>
<td>0.30</td>
<td>0.37</td>
</tr>
</tbody>
</table>
Figure 6: Predicted asymmetry behaviour using leading $\pi^\pm$ with $E > .4$. Errors statistical with hypotheses of Figure 4.

- Curve (a) $\sin^2\theta_W = .2$
- Curve (b) $\sin^2\theta_W = .25$
Table IV.

Leading $K^+$ and $K^-$ probabilities.

$\bar{z} \min > .4$

<table>
<thead>
<tr>
<th>jet</th>
<th>$K^+K^-$ Efficiency</th>
<th>with charge cut Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u\bar{u}$</td>
<td>.36</td>
<td>.23</td>
</tr>
<tr>
<td>$d\bar{d}$</td>
<td>.1</td>
<td>.05</td>
</tr>
<tr>
<td>$s\bar{s}$</td>
<td>1.5</td>
<td>.75</td>
</tr>
<tr>
<td>$c\bar{c}$</td>
<td>&lt; 1.5</td>
<td>&lt; .08</td>
</tr>
</tbody>
</table>
Figure 7: Asymmetry behaviour with leading ($Z > 0.4$) $K^+$ and $K^-$ detected in each hemisphere versus $\sin^2 \theta_W$ at the $\pm 0^\circ$ pole. Statistical errors assume 2000 hours spent at the $\pm 0^\circ$ pole.
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LEP SUMMER STUDY

Organized under the Joint Sponsorship of
ECFA and CERN
Les Houches and CERN
10 to 22 September, 1978

$\gamma\gamma$ PHYSICS AT LEP

P.V. Landshoff
DAMTP, Cambridge
and
J.H. Field
DESY, Hamburg

Summary Reports from the $2\gamma$ Session at Les Houches
Copies available upon request from Ch. Redman, CERN/ISR
LEP Summer Study Secretariat
Foreword

It is well known that while the point-like cross-section falls as $E^{-2}$, the total cross-section associated with the $2\gamma$ process slowly rises with energy. At LEP energies, the latter is already overwhelming as compared to the former. However, while a large fraction of the available energy is often concentrated on particles produced at wide angles in the former case, most of the energy is usually found with the two quasi-ellostically scattered electron and positron in the latter case. The two main questions which then arise are:

(i) How to escape from the very large background associated with $2\gamma$ processes in order to study the a priori far more interesting annihilation processes;

(ii) How to properly trigger on $2\gamma$ processes in order to study their interesting and specific features.

Present expectations about $2\gamma$ processes and ways in which they could be studied at LEP are reviewed in this report. At Les Houches, it was deemed appropriate to approach questions from different sides: theoretical, experimental and machine design. This is reflected in this paper which puts together the contributions of P.V. Landshoff and of J. Field, which cover the theoretical and experimental aspects of $2\gamma$ processes.

Present understanding and expectations concerning $2\gamma$ processes were closely related to some of the conclusions reached when discussing experimental zones in the interaction areas. This is discussed by P. Strolin in his report LEP Summer Study/1-7.
1. Introduction

In the early 1970's, there were a very large number of theoretical papers on \( \gamma \gamma \) collisions. These are very well reviewed in references 1 to 5. Reference 5 lists 244 papers, of which I suppose about half directly concern \( \gamma \gamma \) reactions. Since 1975, there have appeared only a few relevant papers; I have listed these in references 6 to 13, though I may have missed some.

The \( \gamma \gamma \) contribution to \( e^+e^- \) scattering is shown in Figure 1. Because of the poles in the photon propagators, small values of \( q^2 \) and \( q^2 \) dominate, and result in large cross-sections. Some particular examples, to be compared with the "pointlike" cross-section \( \sigma(e^+e^- + \mu^+\mu^-) = 4 \times 10^{-36} \) cm\(^2\) at 70 + 70 GeV, are:

\[
\begin{align*}
\sigma(e^+e^- \rightarrow e^+e^-e^+e^-) &= 2 \times 10^{-26} \\
\sigma(e^+e^- \rightarrow e^+e^-e^+e^-) &= 2 \times 10^{-30} \\
\sigma(e^+e^- \rightarrow e^+e^-\mu^+\mu^-) &= 3 \times 10^{-31}.
\end{align*}
\]

Figure 1

While the point-like cross-section falls rapidly, as \( E \rightarrow 0 \), with increasing beam energy \( E \), the \( \gamma \gamma \) processes vary only slowly with \( E \).

Mostly, the particles produced in the \( \gamma \gamma \) processes go down the beam pipes. Nevertheless, with such relatively huge cross-sections, the small fraction of events where they do not can be a troublesome background to more interesting \( e^+e^- \) physics. This is considered by Field later in this report. I shall be more concerned with the question whether \( \gamma \gamma \) processes are interesting in their own right. My conclusion will be that they are not sufficiently interesting to form a significant part of the case for building LEP. However, once LEP exists, the processes will certainly be worth studying.

2. Lepton Pair Production

The Feynman graph for the processes \( e^+e^- \rightarrow e^+e^-\mu^+\mu^- \) is shown in Figure 2. With untagged e\(^\pm\), it leads to an asymptotic cross-section

\[
\sigma = \frac{28}{27} \frac{\alpha^4}{\pi m_e^2} \left( \log \frac{4E^2}{m_e^2} \right)^2 \log \frac{4E^2}{m_e^2} \tag{1}
\]

\[
\text{Figure 2}
\]
Particular examples are

\[
\begin{align*}
\mu^+\mu^- : & \ 3 \times 10^{-31} \\
\tau^+\tau^- : & \ 5 \times 10^{-34} \\
\text{mass 25 GeV} : & \ 10^{-36}
\end{align*}
\]

Figure 2

This assumes that the lepton has spin \(\frac{1}{2}\). For larger spin, the cross-section could be greater, and it might grow with increasing beam energy \(E\), perhaps like a power.

These cross-section values should again be compared with that for producing the \(\ell^+\ell^-\) pair via the single-\(\gamma\) process. The latter is independent of the mass \(m_{\ell}\) (provided that \(m_{\ell}\) is rather less than \(E\)), and at \(E = 70\) it is \(4 \times 10^{-36}\). So at \(E = 70\) and for \(m_{\ell} \leq 24\) the \(\gamma\gamma\) process gives a larger cross-section.

However, in the \(\gamma\gamma\) process the \(\ell^+\ell^-\) pair is almost always produced with an invariant mass that is nearly as small as it can be: unlike in the single-\(\gamma\) process, the pair is almost at rest in its centre of mass. Also, this centre of mass tends to move fast in one or other of the two longitudinal directions, so that the leptons emerge close to the beam direction. But they do sometimes have large \(p_t\); I return to this later.

3. Equivalent Photon Approximation

This approximation ignores the facts that the photons are not quite on shell and not completely transverse. The literature contains extensive discussions\(^5\) as to how accurate this approximation is, but it is certainly good enough for estimating expected counting rates. In terms of the energies \(\omega\) and fluxes \(N\) of the two photons, it gives

\[
d\sigma = \sigma_{\gamma\gamma} N_1 N_2 \frac{d\omega_1}{\omega_1} \frac{d\omega_2}{\omega_2}
\]

Here, the photon-photon cross-section \(\sigma_{\gamma\gamma}\) is a function of the invariant energy \(\sqrt{s} - (4\omega_1\omega_2)^{1/2}\) of the produced system. The photon flux \(N\) is a function of the photon energy \(\omega\) and of the scattering angle \(\theta\) of its parent electron or positron. For the case of tagging between angles \(\theta_{\min}\) and \(\theta_{\max}\) the angular dependence of \(N\) is roughly as

\[
\int_{\theta_{\min}}^{\theta_{\max}} \frac{d\theta^2}{\theta^2 + m^2/e^2}
\]
So about half the cross-section comes from angles less than $\sqrt{m_e/E}$, which is 2.5 mrad for $E = 70$, and tagging at realistically accessible angles reduces the cross-section by an order of magnitude in the case of single tagging ($e^+$ or $e^-$ only), and by two orders of magnitude for double tagging. The effect of double tagging on the formula (1) is to replace the factor $(\log 4E^2/m_e^2)^2$ by $(\log \sigma_{\text{max}}/\sigma_{\text{min}})^2$.

To calculate $d\sigma/ds$, one has to integrate (2) over $\omega_1$ and $\omega_2$, subject to the constraint $\omega_1\omega_2 = \frac{4}{3}s$. The result is that values of $\omega_1-\omega_2$ far from 0 dominate; hence the produced system usually has large longitudinal momentum.

4. Hadron Production

Integration of (2) gives, when there is no tagging,

$$\sigma = \frac{1}{4} \left( \frac{a}{\pi} \log \frac{4E^2}{m_e^2} \right)^2 \int_{\sigma_{\text{min}}}^{4E^2} \frac{ds}{s} f \left( \frac{\sqrt{s}}{2E} \right) \sigma_{YY}(s)$$

(3)

$$f(z) = (2 + z^2)^2 \log \frac{1}{z} - (1 - z^2) (3 + z^2)$$

Because of the factor $s^{-1}$ under the integral, low values of $s$ are relatively important. It is usual to estimate $\sigma_{YY}$ in the low energy region by assuming that, as in purely hadronic reactions, the resonances are dual to Regge exchange. At large $s$, the cross-section is assumed to be purely diffractive and to obey the usual pomeron factorisation

$$\sigma_{YY} \sim \frac{\sigma_{\gamma p}}{\sigma_{pp}} = 2.5 \times 10^{-31}$$

(An interesting question to explore experimentally is whether $\sigma_{YY}$, like $\sigma_{\gamma p}$ and $\sigma_{pp}$, rises slowly with energy). These assumptions give

$$\sigma(e^+e^- \rightarrow e^+e^- + \text{hadrons}) \sim 10^{-31}$$

Again, this should be compared with the single-$\gamma$ cross-section, which at $E = 70$ is

$$\sigma(e^+e^- \rightarrow \text{hadrons}) = (4 \times 10^{-36}) \, \text{R}$$

with, perhaps, $\text{R} \cong 10$.

It is worth remarking that the duality assumption is open to question, because amplitudes that describe the scattering of photons have fixed poles in the Regge plane. It has been suggested that in fact, there is a substantial additional contribution to $\sigma_{YY}$ in the resonance region, resulting in the conventional estimate of $\sigma(e^+e^- \rightarrow e^+e^- + \text{hadrons})$ being too small by perhaps a factor of 3.
5. Large Transverse Momentum

In their classic 1971 paper on large transverse momentum\(^{14}\) BBK pointed out that \(\gamma\gamma\) processes would contribute significantly to the production of large transverse momentum jets of hadrons. The mechanism is that shown in Figure 3, which is similar to Figure 2, but with the lepton line replaced by a quark. The emerging quark and antiquark fragment into a pair of jets; these are precisely similar to the single jets already seen at SPEAR and DORIS. However, the \(\gamma\gamma\) events should be readily distinguishable from the single-\(\gamma\) events, even without \(e^\pm\) tagging, because in the \(\gamma\gamma\) case a large fraction of the energy is usually taken by the final-state \(e^\pm\) and usually the jets do not come out back-to-back.

![Figure 3](image)

Because of the similarity of Figures 2 and 3,

\[
\sigma(e^+e^- \to e^+e^-q\bar{q}) = R_{\gamma\gamma} \sigma(e^+e^- \to e^+e^-k^+k^-)
\]

\[
R_{\gamma\gamma} = 3 \sum_{\text{quark flavours}} e_i^u
\]

\[
= \frac{34}{27} \text{ for } u, d, s, c
\]

Notice that the relation (4) applies only to the case where the quarks are produced with large transverse momentum; at small \(p_t\) there is no reason to believe that the simple diagram of Figure 3 will dominate over more complicated ones. It is interesting to note that the value given for \(R_{\gamma\gamma}\) applies only to fractionally-charged Gell-Mann/Zweig quarks; for Han/Nambu quarks below the colour threshold the corresponding value is \(R_{\gamma\gamma} = \frac{10}{3}\). In this respect the \(\gamma\gamma\) processes differ from the single-\(\gamma\) ones; for the latter \(R\) takes the same value in the Han/Nambu case below the colour threshold as in the Gell-Mann/Zweig case.

The cross-section that Figure 2 gives for the production of a large-\(p_t\) lepton pair is\(^{\text{11}}\)

\[
\sigma(p_t > p_t^{\text{min}}) = \frac{16\pi}{3} \left(\frac{a^2}{\pi} \log \frac{4E^2}{m^2 e}\right)^2 \frac{\log E/p_t^{\text{min}}}{p_t^{\text{min}}^2}
\]
This asymptotic form applies provided that $p_t^\text{min}$ is rather greater than the mass of the lepton. With $p_t^\text{min} = 10$, it provides about 4 units of $R$ at $E = 70$ for each type of lepton.

According to (4), the production of a pair of hadronic jets having $p_t^{\text{jet}} > p_t^\text{min}$ occurs at approximately the same rate. The original calculation\textsuperscript{14)\textsuperscript{11}} of Figure 3 was within the framework of the scaling parton model. It is now known that the corrections to Figure 3, where gluons are added as internal lines or are emitted as part of the quark jets, just cancel in leading order. However, QCD provides\textsuperscript{11-13)} also additional diagrams of importance. Two of these

![Diagram](image)

are shown in Figure 4. Their contributions to the inclusive production of transverse jets is\textsuperscript{11-13)} only a little smaller than that of Figure 3, but they are experimentally distinguishable from Figure 3 through the presence of longitudinal hadronic jets in one or both of the beam directions. In the second diagram of Figure 4, one of the transverse hadronic jets is a fragmenting gluon; this could be one of the cleanest ways to study gluon jets.

Each of the diagrams of Figures 3 and 4 gives a pair of transverse jets, having approximately equal and opposite total rapidity, and the directions of the two jets are only weakly correlated. As is seen\textsuperscript{11)} in Figure 5, the jets nearly always take much less than the maximum available energy, so there is no risk of confusing them with the jets from single-$\gamma$ events. Because jets are reluctant to give a significant fraction of their total energy to any one particle this means also that the $\gamma\gamma$ jets do not contribute significantly to the production of high-$p_t$ particles; when $E = 70$ for $\theta = 90^\circ$ the contribution to $\sigma(e^+e^- \to \pi + \text{anything})$ from $\gamma\gamma$ is\textsuperscript{11)} already an order of magnitude below that from single-$\gamma$ at $p_t = 6$ GeV/c.
6. Deep Inelastic Processes

So far I have considered events where both photons are near their mass shells, \( q_1^2 = q_2^2 = 0 \). If now one photon is far off shell, one effectively has deep inelastic scattering on an on-shell photon target. Two contributions to this are expected. The first is rather similar to that expected from a vector meson target; the corresponding structure function \( F_2(x) \) will nearly scale and the longitudinal structure function \( F_L(x) \) will be small (the Callan-Cross relation). The other contribution occurs because of the pointlike nature of the target photon. It arises again from Figure 3, and it gives an \( F_2(x) \) that grows logarithmically with the \( q^2 \) of the off-shell photon, with an \( F_L(x) \) that is not small but rather scales:

\[
F_2(x) \sim f_2(x) \log q^2/m_0^2
\]

\[
F_L(x) \sim f_L(x)
\]

(6)

The value of the scale parameter \( m_0 \) is not known, but it seems likely to be of the order of 1 GeV.

In the simple parton model, \( f_2(x) \) and \( f_L(x) \) are very simple functions, for example \( f_L \) is proportional to \( x(1-x) \). Witten\(^8\,13\) has derived the remarkable result that QCD correlations retain the general forms (6), but they change the
functions $f_2(x)$ and $f_L(x)$: see Figure 6. At small $x$, the vector-meson-dominated part of $F_2(x)$ will probably swamp the pointlike contribution; as for a nucleon target, it will be non-zero at $x = 0$.

Notice that, even with unpolarized $e^\pm$, there is a third structure function. This is because each of the two photons tends to be polarised in the plane of the emitting $e^\pm$. Thus there is a term in the cross-section proportional to $F_3(x, q^2) \cos 2\phi$, where $\phi$ is the angle between the scattering planes of the $e^+$ and the $e^-$. The contribution from Figure 3 gives an $F_3$ that scales.

7. Doubly Deep Inelastic Events

Finally, I come to those rare events where both photons are far off shell. There have been a very large number of papers on this subject; a good summary has been given by Walsh\(^2\). When both $q_1^2$ and $q_2^2$ are large, one might consider a variety of limits; in most of them the invariant mass $\sqrt{s}$ of the produced hadronic system will also be large. It is uncertain how large the variables must be in order that theoretical predictions, which are always asymptotic predictions, shall be realised experimentally, and the expected counting rates do not seem to have been calculated fully, though partial estimates are given in Ref. 15.

a) $s >> q_1^2 q_2^2$

b) $s, q_1^2, q_2^2 \rightarrow \infty$ with $s/(q_1^2 q_2^2)$ fixed

c) $s, q_1^2, q_2^2 \rightarrow \infty$ with $s/q_1^2$ and $s/q_2^2$ fixed

d) $q_1^2, q_2^2 \rightarrow \infty$ with $q_1^2/q_2^2$ fixed, then $s \rightarrow \infty$

e) $q_1^2, q_2^2 \rightarrow \infty$ with $q_1^2/q_2^2$ and $s$ fixed.

The case a) is just the Regge limit, while d) and e) are associated with light-cone commutators of currents. In the cases b), c) and d) the diagram of Figure 3
will again dominate asymptotically and its contribution is completely calculable but in limit e) the quark and the antiquark suffer an ordinary final-state strong interaction and the contribution is not calculable. For b), c), d) and e) there are scaling predictions: the amplitudes are functions of the variables that are kept finite. For the case $q_1^2 = q_2^2$ the various limits are as depicted in Figure 7.

Figure 7

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COLLISIONS: EXPERIMENTAL ASPECTS

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1. Introduction and Summary

This review is divided into 4 sections. In section 1 the problem of separating 1γ and 2γ processes is rediscussed in the light of new theoretical expectations of high \( p_T \) hadron production from jets in 2γ processes\(^1\)\(^-\)\(^4\)). Here there is some overlap with P. Landshoff's review, but as a rather complete monte carlo study of single particle inclusive production in both 1γ and 2γ processes has now been done\(^5\)) some quite firm conclusions can be reached. In spite of the 20 times larger ratio of 2γ/1γ cross-sections at LEP, as compared to PEP or PETRA, no problem is expected in separating processes with hadronic final states. Heavy lepton production is also considered in section 1 and here the conclusions are not so optimistic, particularly if several heavy leptons exist within the energy range of the machine. More work is needed here. Section 2 considers 3 potentially interesting fields of 2γ physics: (i) jet production, (ii) deep inelastic γγ scattering, (iii) production of C = +1 resonances. Experimental signatures are discussed and rates are given. In section 3 tagging is discussed. The main points here are: a) Tagging efficiency, in particular the effect of vector meson propagators\(^6\)), which may suppress the tagging efficiency for some 2γ processes by an order of magnitude or more as compared with previous assumptions. b) Backgrounds. These include the "intrinsic" background resulting from \( \gamma/e \) misidentification, as well as various external backgrounds resulting in production of electrons at small angles. By far the most serious of the latter is beam-gas bremsstrahlung, which imposes quite severe constraints on the vacuum in the LEP straight sections, if tagging is to be a viable proposition. Finally, in section 4, single particle inclusive production of hadrons in various 1γ and 2γ processes are shown for different angular acceptance regions of a practical detector, and some features of a possible 2γ detector for LEP are summarized. More detailed discussions of the problems of 2γ detector design are presented elsewhere\(^7\),\(^8\)).

2. Separation of 1γ and 2γ Processes

2.1 Hadronic Final States

A number of different possible processes resulting in jets in the final state are shown in Fig. 1a)-g). Of these, the dominant contribution at high \( p_T \) is expected to come from the QED graph, Fig. 1a). The cross-section for this graph, when the quarks have large \( p_T \), is expected\(^1\),\(^3\)) to be related to 2γ production of \( \mu \) pairs:
Figure 1: Jet Physics in $\gamma \gamma$ Collisions.

- **2 JET**
  
  - a) 
    - Q.E.D.
    - $\sigma \sim \frac{(1-x_R)}{p_\perp^4}$
  
  - b) 
    - C.I.M.
    - $\frac{(1-x_R)}{p_\perp^6}$
  
  - c) 
    - C.I.M.
    - $\frac{(1-x_R)^3}{p_\perp^8}$

- **3 JET**
  
  - d) 
  
  - e) 
    - C.I.M.
    - $\frac{(1-x_R)^2}{p_\perp^6}$

- **4 JET**
  
  - f) 
    - Q.C.D.
    - $\frac{(1-x_R)^3}{p_\perp^4}$
  
  - g) 
    - Q.C.D. Quark Annihila.
\[ \text{do}(e^+e^- \rightarrow e^+e^-q\bar{q}) = R_{YY} \text{do}(e^+e^- \rightarrow e^+e^-u^+u^-) \]

where

\[ R_{YY} = 3 \sum_i q_i^4 = 34/27 \]

\[ q_i \text{ quark charge.} \]

Defining \( x_R = E^\text{quark}/E = E^\text{JET}/E \) (\( E = \) beam energy) and \( \theta = \theta^\text{quark} = \theta^\text{JET} = \) polar angle to the beams, the differential cross-section for \( x_R \sim 1 \) is given by:

\[
\frac{d^2\sigma}{d\Omega dx_R} = \frac{4 R_{YY}}{s} \left( \frac{\alpha^2}{\pi} \ln \frac{E}{m_e} \right)^2 \left( 1 - x_R \right) \frac{(1 + \cos^2\theta)}{\sin^4\theta} 
\]

where \( \alpha = \) fine structure constant

\[ s = 4E^2 \]

Eq. (1) may be compared with the corresponding differential cross-section for the \( 1\gamma \) process:

\[ e^+e^- \rightarrow q\bar{q} \rightarrow 2 \text{jets} \]

which is:

\[
\frac{d\sigma}{d\Omega} = \frac{R_\gamma \alpha^2}{4s} (1 + \cos^2\theta) 
\]

where \( R_\gamma = 3Eq_i^2 = 10/3 \)

Separation of the \( 1\gamma \) and \( 2\gamma \) processes will be most difficult for large values of \( x_R \). Integrating Eq. (1) over the range \( 0.8 < x_R < 1.0 \) and taking the ratio to Eq. (2) gives:

\[ r = \frac{d\sigma^{2\gamma}/d\Omega}{d\sigma^{1\gamma}/d\Omega} \bigg|_{x_R > 0.8} = 1.89 \times 10^{-2} \left( \frac{\alpha \ln \frac{E}{m_e}}{\sin^4\theta} \right)^2 \frac{1}{\sin^4\theta} \]

\( r \) is plotted as a function of \( \theta^\text{JET} \) in Figure 2.

At beam energies of 15, 70 GeV, \( r = 1 \) at angles of 102, 109 mrad so the "cross over" of the \( 1\gamma \) and \( 2\gamma \) processes occurs at \( \theta^\text{JET} \approx 6^\circ \) almost independantly of the beam energy. For \( E = 70 \text{ GeV} \) this corresponds to a \( p_t \) of the jet of \( \sim 6 \text{ GeV} \). With \( \theta^\text{JET} > 20^\circ \) the \( 2\gamma \) cross-section is only 1% of the \( 1\gamma \). It is interesting to note that the curve in Fig. 2 is independent of the number of quarks, provide these always occur in doublets of charge \( 2/3, -1/3 \) and both members of each doublet are either excited, or above threshold. In this case the ratio \( R_{YY}/R_\gamma \) has the universal value 17/45.

One may conclude from the above analysis that, providing jets can be identified in the final state, the \( 2\gamma \) background will become negligible for \( \theta^\text{JET} > 20^\circ \) i.e. within the normal acceptance of a central solenoidal detector. Since however the experimental definition of a "jet" is rather more fuzzy than the theoretical one it is of interest to ask what separation of \( 1\gamma \) and \( 2\gamma \) processes
\[ E = 700 \text{ GeV} \]
\[ x_R = \frac{E_{\text{jet}}}{E} \]
\[ r = \frac{\frac{\frac{d\sigma}{d\Omega}}{x_R > 0.8}}{\frac{d\sigma}{d\Omega}} \]
\[ \frac{1.6 \times 10^{-4}}{\sin^4 \theta_{\text{jet}}} \]
\[ R_y = \frac{10}{3} \]
\[ R_{\gamma\gamma} = \frac{34}{27} \]
can be obtained by use of more straightforward kinematical cuts. Two variables which may be expected to give good discrimination between $1\gamma$ and $2\gamma$ hadronic events are:

(i) **The total observed energy** $E_{\text{vis}}$

For $1\gamma$ processes this should peak at $2E$ with a width given by the detector resolution, and a tail extending to lower energies, due to unobserved final state particles. For $2\gamma$ processes this variable peaks at low values due to the luminosity function of the $\gamma\gamma$ collisions which is roughly $\frac{1}{E_{\gamma 1}} \times \frac{1}{E_{\gamma 2}}$ where $E_{\gamma 1}, E_{\gamma 2}$ are the lab. energies of the colliding photons.

(ii) **The polar angle** $\theta$ of produced hadrons

For $1\gamma$ processes, this is expected to result from the fragmentation of quarks produced with a $1 + \cos^2 \theta$ distribution at the quark level, and so to be almost isotropic. In the $2\gamma$ process the largest contribution is expected on the basis of VDM to result from quasi-diffractive $pp$ scattering, and so to have the most energetic particles at small angles.

In Figures 3 and 4 $E_{\text{vis}}$ is plotted for respectively, diffractive and high $p_t$ (two jets as in Fig. 1.a) $2\gamma$ processes\(^9\). In both cases a cut $\theta > 10^\circ$ is made on the produced hadrons. In Fig. 4 the expected $1\gamma$ signal, assuming a resolution of $0.5 \sqrt{E_{\text{GeV}}}$ for $E_{\text{vis}}$ is also shown. It is clear that a cut $E_{\text{vis}} > 100$ GeV will reduce the background even from the high $p_t$ $2\gamma$ process to negligible levels, while retaining all but a few % at the $1\gamma$ signal. More details of the Monte Carlo simulation used for these plots are given in Ref. 9.

### 2.2 Heavy Lepton Production

Here the process of interest is supposed to be $1\gamma$ production of a new heavy lepton $L$ of mass greater than the $\tau$. As for the $\tau$, the cleanest experimental signature is expected to be in the purely leptonic decay channels, in particular the $e\mu$ channel i.e.

$$e^+e^- \rightarrow L^+L^- \rightarrow e\mu + 4\nu$$

There are a number of different $2\gamma$ processes contributing background:

- $e^+e^- \rightarrow e^+e^-\mu^+\mu^-$
- $e^+e^- \rightarrow e^+e^-\tau^+\tau^-$
- $e^+e^- \rightarrow e^+e^- e\mu + 4\nu$
- or $e^+e^- \mu\mu + 4\nu$
- $e^+e^- \rightarrow e^+e^- \ell^+\ell^- \rightarrow e^+e^- \mu\mu + 4\nu$

where $\ell$ is a heavy lepton with $m_{\tau} < m_{\ell} < m_L$.

Backgrounds also arise from $1\gamma$ production of lighter heavy leptons:

- $e^+e^- \rightarrow \tau^+\tau^- \rightarrow e\mu + 4\nu$
- $e^+e^- \rightarrow \ell^+\ell^- \rightarrow e\mu + 4\nu$
Figure 3

$ee \rightarrow (ee) \times$

Low $p_T$, $\gamma \gamma \rightarrow X$

$V_S = 140 \text{ GeV}$

$W > 4 \text{ GeV}$

$\theta > 10^\circ$

$\int L dt = 10^{38} \text{ cm}^{-2}$

$E_{\text{vis}} \text{ (GeV)}$
Figure 4

\[ \text{No of events/10 GeV} \quad \int L \, dt = 10^{28} \text{cm}^{-2} \]

\[ \text{ee} \rightarrow \text{ee } q\bar{q} \]

\[ \theta > 10^\circ \]

\[ \text{ee} \rightarrow q\bar{q} \]

\[ E_{\text{vis}} \text{ (GeV)} \]

\[ 50 \quad 100 \quad 150 \]
Because of the large energy carried away by neutrinos, $E_{\text{vis}}$ is no longer a useful parameter in separating background. The problem is illustrated in Fig. 5 where the $\ell \gamma$ production of a 40 GeV heavy lepton is compared to the $2\gamma$ background from $e^+e^- \rightarrow e^+\mu^-\mu^+$ \cite{9}. The $\mu\mu$ signature is used and $E_{\text{vis}} = E_{\mu^+} + E_{\mu^-}$. Cuts $\theta_{\mu^+}, \theta_{\mu^-} > 10^6$ are used to suppress the $2\gamma$ background, but even so, the heavy lepton signal is buried by some two orders of magnitude under the background.

This problem has been studied in some detail at PEP/PETRA energies by Vermaseren\cite{10}. To separate the $e^+e^- \rightarrow e^+e^-\mu^+\mu^-$ background, where an $e$ and a $\mu$ are unobserved, use can be made of strong peaking of the unobserved electron in the beam direction. Suppose that the $e^+$ and $\mu^-$ are observed, the $e^-$ and $\mu^+$ being considered "missing" whether or not they are within the acceptance of the apparatus. The polar angle of the missing momentum vector can be calculated from the kinematical variables of the observed particles:

$$\cos^2 \theta_{\text{miss}} = \frac{\text{p}_{\text{miss}}}{E_{\text{miss}}} = -\frac{(\text{p}_e + \cos \theta_{e^+} + \text{p}_{\mu^-} - \cos \theta_{\mu^-})}{2E - E_{e^+} - E_{\mu^-}}$$

This variable is plotted for $E = 15$ GeV, $M_L = 5$ GeV, in Fig. 6\cite{10}. Making a cut $|\cos^2 \theta_{\text{miss}}| < 0.3$ retains all the heavy lepton signal. In most cases when $|\cos^2 \theta_{\text{miss}}| < 0.3$ either the $e^-$ and/or the $\mu^+$ will also be seen in the detector so the background level will in fact be even lower than shown in Fig. 6. However, scaling to LEP energies, the $2\gamma$ signal will be relatively $\sim 20$ times higher, corresponding to a signal/background $\sim 1/1$ on the peak of the heavy lepton signal. As noted above the background level will certainly be suppressed further by observation of the $e^-$ or $\mu^+$. The background level can also be estimated by taking different charge combinations. With $e^+\mu^+$, $e^-\mu^-$ identical distributions to $e^+e^-$ should be obtained for the background contribution.

To separate the channels:

$$e^+e^- \rightarrow L^+L^- \rightarrow e\mu + 4\nu$$
$$\rightarrow \tau^+\tau^- \rightarrow e\mu + 4\nu$$
$$\rightarrow e^+e^-\tau^+\tau^- \rightarrow e\mu + (e\nu)_{\text{unseen}} + 4\nu$$

Vermaseren\cite{10} suggests use of the variable $p_t^\ell$. If $p_t^e$ ($p_t^\mu$) is the transverse momentum of the $e$($\mu$), with respect to the $\mu$($e$) direction, $p_t$ is the minimum of $p_t^e$, $p_t^\mu$. This variable measures the transverse momentum in the heavy lepton decay, and has a kinematic limit determined by the heavy lepton mass. In Fig. 7 $\frac{d\sigma}{dp_t}$ is shown for $E = 15$ GeV and heavy lepton masses of 1.8, 5, 10, 14 GeV. Other cuts are detailed in Ref. 10. Also shown is the contribution from the $e^+e^-\tau^+\tau^-$ final state. Remembering that this signal will be some 20 times higher at LEP energies one cannot be too optimistic as to the possibilities of making a clean separation. $2\gamma$ production of intermediate mass heavy leptons $\ell$ will further complicate the situation.
Figure 5

$\text{Nb of events/10 GeV}$

$ee \rightarrow (ee) \mu^+ \mu^-$

$\sqrt{s} = 140 \text{ GeV}$

$W > 4 \text{ GeV}$

$\theta_{\mu^+}, \theta_{\mu^-} > 10^\circ$

$\int L \, dt = 10^{38} \text{ cm}^2$

$e^+e^- \rightarrow \mu^+ \mu^-$

$P_T > 7 \text{ GeV}$

$e^+e^- \rightarrow L^+L^-$

$\mu^+\mu^-4\gamma$

$E_{\text{vis}} \text{ (GeV)}$
\[ \frac{d\sigma}{d\cos \theta_{\text{miss}}} \] (\text{pb})

\[ e^+e^- \rightarrow e^+e^- \mu^+\mu^- \]

\[ e^+e^- \rightarrow \nu \bar{\nu} \]

Cut

\[ \cos \theta_{\text{miss}} \]

Figure 6
Figure 7
In conclusion more work must be done and more ideas are needed before a clear separation of heavy lepton events from various backgrounds can be expected at LEP energies. The existence of such leptons could no doubt be established by looking for thresholds in the energy dependence of the $e\mu$ signal. To produce clean event samples for more detailed studies seems more difficult.

3. 2$\gamma$ Physics

3.1 Jet Production

A large number of different processes leading to jets in the final state are expected in $2\gamma$ collisions. Some of these processes are shown in Fig. 1\textsuperscript{1}). The characteristic $x_R$ and $p_T$ behaviour of the produced jets is indicated. All reactions except that in Fig. 1.c) which corresponds to diffractive $pp$ scattering in the VDM model, have two jets at high $p_T$, accompanied by 0, 1 or 2 jets in the beam directions. The $p_T$ behaviour of some of the processes is shown in Figs 8, 9 for $E = 15, 70$ GeV\textsuperscript{,2)}. $E_{p_T} \frac{d^2\sigma}{d^2p_T}$ is plotted versus $p_T$ for $\theta_{jet} = 90^\circ$. The already mentioned leading behaviour of the QED graph of Fig. 1.c) is evident. Of particular interest is the 1st order QCD graph of Fig. 1.d) which leads to a 3-jet event. The contribution is comparable to that of the QED graph, and dominates the competing C.I.M. (Constituent Interchange Model) 3-jet process, for $p_T^{jet} > 10$ GeV, $E = 70$ GeV (Fig. 9). The interest of this process is that a single gluon jet should be produced, clearly separated from the quark jets, so that the properties of the gluon fragmentation function may be directly studied. Another interesting point is the absolute cross-section of the QED graph. As pointed out in P. Landshoff's review, this is 2.5 times larger in the integer charge (Han Nambu) model than in the fractionally charged (Gell-Mann Zweig) model and should allow an easy experimental discrimination between these two models.

All the processes in Fig. 1 have a common experimental signature, two jets should be observed, co-planar with the beams, but in general non-collinear. In addition there may be further jets along the beam pipes. Because of the $\gamma\gamma$ luminosity function, generally $E_{vis} \ll 2E$, making the experimental identification of the jets more difficult than in $1\gamma$ reactions. To disentangle the different topologies good acceptance near the beam pipes, and double tagging to give kinematically constrained events will be needed. The jets themselves, however, particularly from the leading QED graph, should be evident even without tagging. In fact a clear separation of the 2-jet process, Fig. 1.a), from the diffractive process, Fig. 1.c), can already be seen at the level of the single hadron inclusive distributions, in the Monte Carlo studies of Ref. 5, when suitable cuts are made. This is shown in Fig. 10, where the number of charged tracks is plotted versus their momentum for the QED ($q\bar{q}$) and diffractive ($pp$) processes.
Figure 8: Jets in $\gamma\gamma$ collisions. Brodsky, Gunion, DeGrand, Weis
P.R.L. 41 (1978) 572
Figure 9: T. DeGrand. ECPA/LEP 37
Figure 10: Separation of high $p_T$ and low $p_T$ $2\gamma$ processes from single particle inclusive distributions. M. DAvier, ECFA/LEP 26
3.2 Deep Inelastic $\gamma\gamma$ Scattering

The physical interest of this process, as a particularly clean test of quark parton and QCD ideas, has been stressed in Landshoff's review. The numbers of events which may be expected above the kinematical region accessible to PEP and PETRA, for an integrated luminosity of $L = 10^{38}$ cm$^{-2}$ and $E = 70$ GeV are:

<table>
<thead>
<tr>
<th>$Q^2$(GeV/c)$^2$</th>
<th>$W_{\gamma\gamma}$ = 10 - 20 GeV</th>
<th>20 - 50 GeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 - 25</td>
<td>265</td>
<td>850</td>
</tr>
<tr>
<td>25 - 100</td>
<td>15</td>
<td>7</td>
</tr>
<tr>
<td>&gt; 100</td>
<td>5</td>
<td>10</td>
</tr>
</tbody>
</table>

These figures assume a 5% double tagging efficiency and equal contributions from the box diagram discussed by P. Landshoff, and a VDM contribution which is expected to dominate when $x = \frac{Q^2}{Q^2 + W_{\gamma\gamma}^2} \approx 0$. Over $10^3$ events are expected, which should be sufficient to test the two most interesting theoretical predictions:

- the shape of $F_2(x)$
- the rise $\propto \ln Q^2$ in $F_2(x)$ near $x = 1$.

3.3 Production of $C = +1$ Mesons

$\gamma\gamma$ collisions give a unique opportunity to study the direct production of $C = +1$ states, via the process

$$e^+e^- \rightarrow e^+e^-\gamma\gamma \rightarrow e^+e^-X$$

where $X$ is a $C = +1$ meson, e.g. $n^0$, $\eta^0$, $\eta'$, $\eta_c$, $\chi$, $\eta_b$, $\eta_t$, ... This method has the advantage, over scanning for new states with $J^{PC} = 1^{--}$, in the annihilation channel, that a single run at the maximum beam energy makes available the entire spectrum of $C = +1$ states to which the machine is sensitive. No time-consuming and rather hazardous (if the states are very narrow) energy scanning is needed. However, if the states are narrow, the available $\gamma\gamma$ luminosity at any given mass is rather low. Consider for example $\eta_c$ and $\eta_b$ states with parameters:

- $\eta_c$; $M = 2.8$ GeV, $\Gamma_{\gamma\gamma} = 10$ keV
- $\eta_b$; $M = 9.2$ GeV, $\Gamma_{\gamma\gamma} = 20$ keV

The total cross-sections for production of these states using beam energies of 15, 70 GeV are, in pb:

<table>
<thead>
<tr>
<th>$E$(GeV)</th>
<th>$\eta_c$</th>
<th>$\eta_b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>104 (116)</td>
<td>2.0 (2.0)</td>
</tr>
<tr>
<td>70</td>
<td>265</td>
<td>9.4</td>
</tr>
</tbody>
</table>

The cross-sections were calculated in the DEPA (Double Equivalent Photon Approximation)$^{11}$. The bracketed numbers are the result of an exact Feynman diagram calculation$^{12}$. On the assumption of an integrated luminosity of $L = 10^{38}$ cm$^{-2}$
and 5% double tagging efficiency, the following number of events are obtained:

\[
\begin{align*}
\eta_c & = 1330, \\
\eta_b & = 50
\end{align*}
\]

Whether these represent observable signals depends on the level of background underneath the resonance peaks. The widths of the latter are determined by the experimental resolution in the \( \gamma \gamma \) effective mass, \( \sigma_{\gamma \gamma} \). This is given, for production of the resonance at rest in the lab. system, by:

\[
\sigma_{\gamma \gamma} = 2E \sqrt{\frac{1}{2} \left( \frac{\sigma E}{E} \right)^2 + \frac{1}{2} \left( \frac{\sigma E'}{E'} \right)^2}
\]

(3)

The dependence of \( \sigma_{\gamma \gamma} \) on the rapidity of the produced state is weak, so only a small error is made by using Eq. (3). \( \sigma E/E \) is determined by the beam energy spread in the machine, and is typically \( 10^{-3} \) (13). The best value that can be expected for \( \sigma E'/E' \) is that obtained by using NaI detectors for the scattered electrons (14):

\[
\frac{\sigma E'}{E'} = 0.02 \left( \frac{E'}{E} \right)^4 \approx 7.0 \times 10^{-3} \quad (E' \approx 70 \text{ GeV})
\]

so the error on \( E' \) is the dominant one and

\[
\sigma_{\gamma \gamma} \approx \sqrt{2} E \frac{\sigma E'}{E'} \approx 900 \text{ MeV} (E = 70 \text{ GeV})
\]

Defining the resonance peak by a region \( \pm 2\sigma_{\gamma \gamma} \), centered on the maximum, the expected number of background events is (11):

\[
N_b = e_{DT} \sigma_{\gamma \gamma} \sigma_{ee}^{\text{tot}} \left( \frac{2a}{\pi} \right)^2 \left( \ln \left( \frac{E}{m_e} \right) \right)^2 \frac{1}{z} \left( 4 \ln \frac{1}{z} - 3 \right) \frac{2\sigma_{\gamma \gamma}}{E}
\]

(4)

where: \( z = \frac{M_X}{2E} \ll 1 \), \( e_{DT} = \text{double tagging efficiency}, L_{ee} = \text{luminosity} \)

\[
L_{ee} = 10^{38} \text{ cm}^{-2} \text{ s}^{-1}
\]

Taking \( \sigma_{\gamma \gamma}^{\text{tot}} = 250 \text{ nb} \) leads to the following number of background events, and statistical significance for the signals:

\[
\begin{align*}
\eta_c & = 1330, \\
\eta_b & = 50
\end{align*}
\]

Background \( 6.9 \times 10^4 \)

Statistical Significance \( 5\sigma \)

\[
0.6\sigma
\]

The situation for the \( \eta_b \) may not be quite so pessimistic as these numbers suggest. As will be discussed in the following section, arguments can be given why the tagging efficiency for background events may be considerably less than the simple expression:

\[
e_{DT} \approx \ln^2(\theta_{\text{max}}/\theta_{\text{min}}) \left( \ln \left( \frac{E}{m_e} \right) \right)^2,
\]
though the signal should not be so suppressed. The signal/noise ratio can also be improved by making cuts on the $p_T$ of the produced hadrons. Those resulting from the resonance decay, coming, for example, from the fragmentation of two wide angle gluons, should extend to higher $p_T$ values than the background which is expected to be predominantly diffractive.

4. Tagging of Scattered Electrons

4.1 Tagging Efficiency

Except in the region very close to $x_e = E'_e/E = 1$ the single tagging efficiency is almost independent of the scattered electron energy $E'_e$, and is given approximately (within 10%) by the expression

$$
\varepsilon_{ST} = \ln \frac{\theta_{\text{max}}}{\theta_{\text{min}}} / \ln \left( \frac{E}{m_e} \right)
$$

(5)

The tagging efficiency in the region near $x = 1$, (actually where $\frac{1 - x}{x} < \theta^2$) is given to within ~ 1% by replacing $\ln \left( \theta_{\text{max}}/\theta_{\text{min}} \right)$ by the expression:

$$
\ln \left( \frac{\theta_{\text{max}}}{\theta_{\text{min}}} \right) \left[ \frac{(1 - x_e)^2 + x_e \theta_{\text{max}}^2}{(1 - x_e)^2 + x_e \theta_{\text{min}}^2} \right]^{\frac{1}{2}}
$$

(6)

The definition of tagging efficiency in Eq. (5) is the ratio of the flux of virtual photons at a given value of $x_e$ in the angular region $0 < \theta < \theta_{\text{max}}$ to the flux in the full angular range $0 < \theta < \pi$. It has been pointed out by M. Davier$^6$ that in processes where the virtual photon couples to the produced hadronic system via the propagator of a light vector meson ($\rho, \omega, \phi$) the tagging efficiency will be considerably suppressed compared to the value given by Eqs. (5) and (6). In the case when the vector meson propagator is given by $\frac{1}{(1 - q^2/m^2)^2}$ and the scattered electron angular distribution in the absence of the propagator is $d\theta^2/\theta^2$, the suppression factor may be calculated analytically with the result:

$$
S(x_e, \theta_{\text{max}}, \theta_{\text{min}}) = \frac{1}{\ln \left( \frac{\theta_{\text{max}}}{\theta_{\text{min}}} \right) \left[ \frac{(1 - x_e)^2 + x_e \theta_{\text{max}}^2}{(1 - x_e)^2 + x_e \theta_{\text{min}}^2} \right]^{\frac{1}{2}}} \left( \frac{\ln \theta_{\text{max}}}{\theta_{\text{max}}} \right) \left( \frac{P(x_e, \theta_{\text{min}})}{P(x_e, \theta_{\text{max}})} \right) + \frac{1}{\ln \left( \frac{\theta_{\text{max}}}{\theta_{\text{min}}} \right) \left[ \frac{(1 - x_e)^2 + x_e \theta_{\text{max}}^2}{(1 - x_e)^2 + x_e \theta_{\text{min}}^2} \right]^{\frac{1}{2}}} \left( \frac{\ln \theta_{\text{min}}}{\theta_{\text{min}}} \right) \left( \frac{P(x_e, \theta_{\text{max}})}{P(x_e, \theta_{\text{min}})} \right)
$$

where $P(x, \theta) = 1 + \frac{x e^2 \theta^2}{m^2}$. Figure 11 shows $S$ for $E = 70$ GeV, $\theta_{\text{min}} = 10$ mr, $\theta_{\text{max}} = 100$ mr and $m_\rho = 0.773$ GeV. In the region near $x_e = 1$, $S$ is $\approx 0.05$.

It should be pointed out however, that by no means all hadronic final states are expected to be produced by VDM like coupling to virtual photons. Some exceptions are:

- production of high $p_T$ jets (point-like coupling of both photons)
- deep inelastic photon coupling (point-like coupling at high $Q^2$ photon)
- heavy $C = +1$ resonance production. If $\eta_C$, for example, is produced via a VDM-type diagram the propagators might be expected to have a mass $M_{J/\psi}^2$ rather than $M_C^2$ and so have a much flatter $Q^2$ dependence.

For these processes one might hope that the propagator effects would improve the signal/background ratio by suppressing uninteresting diffractive background. However, it should be stressed that it is quite unknown how much of the total $\gamma\gamma$ cross-section is VDM-like and how much point-like, so the curve of Fig. 11 should be treated as a lower limit. It is also interesting to note that the suppression is least important in the region of small $x$, corresponding to large effective masses of the produced $\gamma\gamma$ system. This is the kinematic region where it is important to have samples of tagged $2\gamma$ events to estimate background levels to $1\gamma$ processes. Clearly, one of the most interesting quantities to measure in a $2\gamma$ experiment, at a very early stage, will be the $Q^2$ dependence of the total hadronic cross-section at relatively low values of $Q^2 \leq 1 \text{ GeV/c}^2$ so as to shed light on the VDM versus point-like nature of the coupling of photons to hadrons in $2\gamma$ collisions.

Figure 11: $S(x) = \text{suppression factor of single tagging rate due to } \rho \text{ propagator } 1/(1 - q^2/m_C^2)$. $E = 70 \text{ GeV}, 10 \text{ ms} < \theta < 100 \text{ ms}, m_\rho = 0.773 \text{ GeV}$
4.2 Backgrounds in Tagging

Two different types of background are considered here. The first is an "intrinsic" background resulting from misidentification of forward produced hadrons as electrons. The second results from various external processes that produce electrons at small angles.

In Fig. 12 the $x_e$ distribution of scattered electrons in a typical range of tagging angles $10 \text{ mrad} < \theta < 100 \text{ mrad}$ is shown with 3 different assumptions:

\begin{align*}
\text{Curve B: } & \frac{dN}{dx_e} = \frac{1 + x_e^2}{1 - x_e} \ln \frac{\theta_{\text{max}}}{\theta_{\text{min}}} \\
\text{Curve C: } & \frac{dN}{dx_e} \text{ given by exact EPA expression}^{11)} \\
\text{Curve D: } & \frac{dN}{dx_e} = \frac{1 + x_e^2}{1 - x_e} \ln \frac{\theta_{\text{max}}}{\theta_{\text{min}}} \times S(X_e, \theta_{\text{max}}, \theta_{\text{min}}) \\
\end{align*}

i.e. $\rho$ propagator effect from Eq. (7) included.

Also shown in Fig. 12, with the correct relative normalization, is the expected distribution of charged hadrons in the same angular region, from diffractive type 2$\gamma$ events\(^5\) where, in almost all cases, the corresponding scattered electrons are in the beam pipe and unobserved (Curve A). It can be seen for small values of $X_e, X_h$ the flux of hadrons is some 2 orders of magnitude higher than the scattered electrons. This can also be seen in Fig. 13 where the ratio of Curve A to Curve D is shown. To reduce the number of false tags to acceptable levels a hadron/electron discrimination factor in the tagging system of the order of $10^3$ is needed.

If the total energy of the produced hadrons $E_{\text{vis}}$ is measured with good efficiency, this background should be largely removed by accepting only events where this directly measured energy agrees with the value $E_{\text{tag}} = 2E - E'_1 - E'_2$ calculated from the energies $E'_1, E'_2$ of the scattered electrons. For the background events, coming predominantly from low energy misidentified hadrons, it is expected that $E_{\text{tag}} >> E_{\text{vis}}$.

Some order of magnitude estimates of backgrounds due to various sources of small angle electrons are presented in Table 1. The angular range is $15 \text{ mrad} < \theta < 150 \text{ mrad}, E = 70 \text{ GeV}$ and $L = 10^{32} \text{ cm}^{-2} \text{ s}^{-1}$. The entries are the single tagging rate and the double tagging rate, resulting, in all cases except DBBB, from accidental coincidences. These latter have a rate:

\begin{equation}
\frac{f_{\text{DT}}}{f_{\text{ST}}} = \frac{(f_{\text{ST}})^2}{f_{\text{B}}} e^{-f_{\text{ST}}/f_{\text{B}}}
\end{equation}

where $f_{\text{ST}}$ is the single tagging frequency and $f_{\text{B}}$ is the bunch crossing frequency $= 54 \text{ kHz}$ in LEP 70 with 4 bunches in each beam. Also indicated in Table 1 are
Figure 12: Comparison of fluxes of charged hadrons and scattered electrons in
the tagging region 10 mrad < θ < 100 mrad
E = 70 GeV, W > 4 GeV
A: hadrons
B: electrons, tagging eff: \( \frac{\theta_{\text{max}}}{\theta_{\text{min}}} \) / ln E/m_e
C: electrons, tagging eff: complete EPA formula\(^{11}\)
D: electrons, as B but p propagators effect included
Figure 13: Ratio of charged hadrons to scattered electrons in the tagging region $10 \text{ mrad} < \theta < 100 \text{ mrad}$. $E = 70 \text{ GeV}, W > 4 \text{ GeV}$
the main characteristics of the energy spectra of the electrons from the various sources. The bremsstrahlung rates BBB and DBBB, as well as the pair production and Compton rates were taken from formulae and plots given in Ref. 15. The BGB rates were taken from the LEP-70 study\textsuperscript{16} and refer to a pressure of $10^{-10}$ torr.

It can be seen that the most serious background because of its high rate, and because the electrons are quite hard and so cannot be significantly reduced by energy thresholds, is that due to beam-gas bremsstrahlung (BGB). This background consists of electrons, which lose energy in collisions with residual gas in the long straight sections, but remain trapped in the machine until they encounter the strong field gradients of the low-$\beta$ quadrupoles just before the intersection region, which deflect them into the experimental detectors. The bracketed BGB rates in Table 1 refer to a vacuum of $5 \times 10^{-9}$ which is typically what is aimed for at PEP and PETRA. Such a vacuum gives a single tag rate of $-1.4 \times 10^5$ Hz, which is $10^6$ times larger than the rate from $2\gamma \rightarrow$ hadrons, and corresponding to more than two background hits per beam crossing. If $2\gamma$ physics is to be possible, or more generally, if any type of tagging is contemplated, the vacuum in the straight sections is of crucial importance. This must be at the $10^{-10}$ torr level if the BGB rates are to be $-a few\%$ per beam crossing. Other methods of reducing this background are:

- High $\beta$ shielding in the vacuum pipe to absorb electrons not passing close to the interaction point;
- A requirement in the fast trigger, by the use of coincidence matrices, that accepts only electrons pointing from near the interaction point.

5. Detector Design

As discussed in more detail in Refs. 7 and 8, technical limitations impose on the design of $2\gamma$ detectors typically four different regions of polar angle $\theta$, relative to the beams, of produced particles. These regions with typical values of $\theta$ are:

- **Beam pipe**
  
  $0 < \theta < 10$ mrad

- **Tagging**
  
  $10 < \theta < 100$ mrad

- **Forward Detector**
  
  $100 < \theta < 300$ mrad

- **Central Detector**
  
  $\theta > 300$ mrad

Particles can be detected only in b), c) and d) and there are often dead areas between these regions, due again to various technical constraints.

In Figures 14 - 16 are shown inclusive hadron momentum spectra for the 4 regions a) - d) for the following three processes\textsuperscript{5})

(i) $e^+e^- \rightarrow e^+e^- + \text{hadrons (low-}p_T, \ pp \text{ scattering})$
Figure 14: Inclusive hadron spectra (M. Davier ECFA/LEP 26)
e^+e^- \rightarrow e^+e^-X \text{ (low } p_t\text{)}
charged + neutral, E = 70 \text{ GeV, } W_{YY} > 4 \text{ GeV}
Figure 15: Inclusive hadron spectra (M. Davier ECFA/LEP 26)
$e^+ e^- \rightarrow e^+ e^- q\bar{q}$ (high $p_T$)
charged + neutral, $E = 70$ GeV, $10 < W_{YY} < 20$ GeV
Figure 16: Inclusive hadron spectra (M. Davier ECFA/LEP 26)
e^+e^- \rightarrow qq \text{ (annihilation)}
Charged + neutral, E = 70 GeV, \Sigma \xi = 10^{38} \text{ cm}^{-2}
Figure 17: LEP y detector (plan view, one quadrant)

Central Detector (Solenoid)

D1-D7 Drift Chambers
P1-P3 Prop Chambers
T1-T3
I1-I2 e/hadron disc. detectors.
X^* rod^*, or dE/dx

Shower Counters

Superconducting Dipole B ~ 1T

~ 5 m space for particle identification dE/dx or e Counters

Interaction Point

Compensating Solenoid

Superconducting flux excluding pipe ~ 4 cm thick
(ii) \( e^+e^- \rightarrow e^+e^- + q\bar{q} \) (high \( p_T \), as Fig. 1.a)
\[ \rightarrow \text{hadrons} \]

(iii) \( e^+e^- \rightarrow q\bar{q} \) (annihilation)
\[ \rightarrow \text{hadrons} \]

Figures 15 and 16 have the same relative normalization. These plots show two general features of the hadrons from \( 2\gamma \) events:
- the spectra are soft, as compared with those from annihilation events. Even at small angles, in the tagging region, the spectrum peaks around 2 GeV for 70 GeV beam energy;
- large numbers of very soft (< 1 GeV) particles are produced in the central detector region.

These features imply that in the design of \( 2\gamma \) detectors, at least for analyzing the final state hadrons, quite modest magnetic fields are adequate. Another consequence is that particle identification techniques should be aimed at rather low momentum particles. This has not always been the case in previous conceptual designs of \( 2\gamma \) detectors. Thus relativistic ionization rise and time of flight are expected to be important techniques for charged-particle identification.

A possible dedicated detector for \( 2\gamma \) physics, based on the design study in Ref. 17 is shown in Fig. 17. The main features of the detector are summarized in Table 2. Special emphasis is placed on the following points in the design:
- large tagging efficiency
- good \( e/\text{hadron} \) discrimination in the tagging region (see section 4.2)
- high magnetic field to match electron momentum measurements from bend to NaI energy resolution
- the provision of space for good particle identification \( (dE/dx, C, \text{T.o.F.}) \) in the forward direction
- full coverage for neutral detection outside the beam pipe.

Finally, it may be remarked that other design philosophies may have advantages for studying specific physics topics. To improve acceptance, and provide magnetic analysis of momentum down to much lower angles than the detector shown in Fig. 17 one possibility is to use lower magnetic fields, with unshielded beams. These possibilities are discussed at some length in Ref. 8.

Acknowledgements

The work described in this talk owes much to discussions with and contributions from the other members of the \( 2\gamma \) ECFA/LEP subgroup, in particular G. Barbiellini, M. Davier and F. Erne. I am also indebted to K. Kajantie, P. Landshoff and C.H. Llewellyn-Smith for their enlightening remarks on theoretical questions.
<table>
<thead>
<tr>
<th>Process</th>
<th>Single Tag Rate Hz</th>
<th>Double Tag Rate Hz</th>
<th>Electron Energy Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Signal: $e^+e^- \rightarrow e^+e^- + \text{hadrons}$ (no propagator suppression, see section 4.1)</td>
<td>0.11</td>
<td>0.015</td>
<td>$4 &lt; E_e^l &lt; 67 \text{ GeV (bremss. spectrum)}$</td>
</tr>
<tr>
<td>BBB: $e^+e^- \rightarrow e^+e^- \gamma$</td>
<td>11.6</td>
<td>$1.2 \times 10^{-3}$</td>
<td>&quot; &quot; &quot; &quot; &quot; &quot;</td>
</tr>
<tr>
<td>DBBB: $e^+e^- \rightarrow e^+e^- \gamma\gamma$</td>
<td>0.01</td>
<td>$&lt; 0.002$</td>
<td>&quot; &quot; &quot; &quot; &quot; &quot;</td>
</tr>
<tr>
<td>BGB: e.g. eco + ecoγ</td>
<td>$2.7 \times 10^3$</td>
<td>64</td>
<td>Flat 0 - 50 GeV</td>
</tr>
<tr>
<td></td>
<td>$(1.4 \times 10^5)$</td>
<td>$(1.4 \times 10^4)$</td>
<td></td>
</tr>
<tr>
<td>Pair production on synchrotron radiation:</td>
<td>~ $10^3 - 10^4$</td>
<td>~ $10 - 10^3$</td>
<td>$1 &lt; E_e^l &lt; 100 \text{ MeV (peaked low)}$</td>
</tr>
<tr>
<td>$e\gamma \rightarrow ee^+e^-\gamma$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Compton scattering on synchrotron radiation</td>
<td>~ 200</td>
<td>~ 0.4</td>
<td>$1 &lt; E_e^l &lt; 1000 \text{ MeV (peaked low)}$</td>
</tr>
<tr>
<td>$e\gamma \rightarrow e\gamma$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

For BGB: $p = 10^{-10} \text{ torr}$, in ( ) $p = 5 \times 10^{-9} \text{ torr}$
N.B. bunch crossing frequency = 54 kHz
15 mrad < $\theta$ < 150 mrad, $E = 70 \text{ GeV}$, $L = 10^{32} \text{ cm}^{-2} \text{ sec}^{-1}$
<table>
<thead>
<tr>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
<th>VI</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 - 12.5 mr</td>
<td>12.5 - 62.5 mr</td>
<td>62.5 - 75.8 mr</td>
<td>75.8 - 150 mr</td>
<td>150 - 367 mr</td>
<td>&gt; 367 mr</td>
</tr>
</tbody>
</table>

**Beam-pipe**

- Tagging, B = 0
- NaI
- \( \sigma_p/E = 0.02/E^4 \)
- Prop\(^n\) chambers

- \( \pi/e \) disc:
  - 3 layers of \( X^n \) rad\(^n\) detectors\(\dagger\)
  - Length 60 cm
  - \( \pi/e \) rej:
  - \( 10^3 \) (2 GeV)
  - e detection eff: \( (0.97)^3 = 0.91 \) (2 GeV)
  - No \( \mu/\pi \) disc

**Super-conducting pipe**

- Tagging, horiz.
- dipole field B ~ 1T
- \( B_1 = 3 \) Tm
- \( \sigma_p/p = 2 \times 10^{-4} \) p

**Dead**

- Drift chambers:
  - \( \sigma \sim 200 \) \( \mu \)
  - NaI, Prop\(^n\) ch. as II
  - \( \pi/e \) disc. as II
  - \( \pi/K \) T.o.F.
  - \( p < 1.5 \) GeV
  - K/p T.o.F.
  - \( p < 2.5 \) GeV
  - \( \sigma_t = 0.2 \) ns, 30 sep
  - \( \mu/\pi \) disc
  - \( 1 \) m Fe
  - few % rej.

- Solenoid
- Modest Resolution
- \( \sigma_p/p = 0.01 - 0.03 \times p \sin \theta \)

- e.g.
- CELLO
- JADE
- TASSO

\(\dagger\) See ref. 18.
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SEARCH FOR NEW PARTICLES*

M. Banner
Saclay
In this review, we consider only these particles which, according to present theoretical prejudices, will or might exist. We examine the feasibility of observing such particles at large $e^+e^-$ energies.

The standard model of Weinberg-Salam assumes the existence of a triplet of heavy bosons, that is $Z^0$ and the two charged $W^\pm$s. If the $Z^0$ exists with a mass of $M_{Z^0} = 3.4$ TeV, its mass and natural width can in principle be easily measured. It will not be discussed further.

It is assumed in what follows that the detector measures the direction and the momentum of all charged particles, and of photons.

**DETECTION OF THE W**

If the total energy $\sqrt{s}$ is less than twice the $W$ mass it can only be produced singly. The feasibility of an experiment which detects the single $W$ has been investigated by W.D. Schlatter (ECFA/LEP/10, 1978). The cross section is

$$\sigma(e^+e^- \rightarrow svW) = \frac{\alpha\bar{\sigma}}{2\pi\sqrt{s}} \log\left(\frac{s}{M_{e^+e^-}^2}\right) F(a)$$

where $a = m^2_W/s$. The function $F(a)$ is shown in fig.2. For $m_W = 76$ GeV and $\sqrt{s} = 140$ GeV, $\sigma_{\text{total}}(Wv) = 2 \times 10^{-37}$ cm$^2$.

In this reaction the electron distribution is peaked in the forward direction. The $W$ is supposed to decay into two jets. The 2-photon annihilation is the main background contribution. The transverse momentum distributions of the 2 hadronic jets differ, for the $W$ signal, and the 2-photon backgrounds. By requiring $p_T > 10$ GeV/c, the background is reduced by a factor 10, with a loss in the signal of only 10%. Figs.3 and 4 summarise the situation. A resolution $\sigma_{\text{E}}/E = 0.3/\sqrt{E}$ of the hadronic energy is assumed in Fig.3. Fig.4 assumes $\sigma_{\text{E}}/E = 0.6/\sqrt{E}$. The superimposed curve (A) is the $W$ signal. Curve (B) is the calculated background for $p_T > 4$ GeV and curve (C) for $p_T > 10$ GeV. It should be stressed that the background calculation is not very reliable; the background could be larger.
Figure 1: Diagram for the reaction $e^+e^- \rightarrow W^- e^+ \nu$

Figure 2: Plot of the function $F(a) = \frac{m_W^2}{s}$
Figure 3: Relative counting rate as a function of the dijet mass
(a) $W$ production
(b) for $\gamma\gamma$ jets with $p_T \geq 4$ GeV/c
(c) for $\gamma\gamma$ jets with $p_T \geq 10$ GeV/c
Calorimeter resolution $\sigma_E = 0.3/\sqrt{E}$
Figure 4: Same as Fig. 3 but with $\sigma_E = 0.6/\sqrt{E}$
The efficiency for detection of the W with $p_T > 10$ GeV/c and an electron tagging between $1^\circ$ to $15^\circ$, is 0.65. If the electron tagging is between $3^\circ$ and $15^\circ$, which seem easier to achieve, then the efficiency becomes 0.34. In these conditions, at a luminosity of $10^{32}$ cm$^{-2}$/s, the rate is 0.5 events per day. Assuming that the average luminosity is one quarter of the peak luminosity, 200 days are necessary to collect 25 events. The experimental situation is difficult. If the W is seen, its mass will not be measured better than several GeV/c$^2$.

If the energy of the machine is large enough, it is possible to detect the W via the reaction $e^+e^- \rightarrow W^+W^-$. Fig.5 shows the cross section for this process as given in the Yellow Book (CLEO 76-10). The dominant W decay mode is hadronic, into 2 jets. The possibility of detecting the W in these conditions has been investigated by De Brion (ECFA/LEP/14). Again, we will give only the main features here. It is assumed that the hadron-jet energy is measured with a resolution of $\sigma/E = 0.6/\sqrt{E}$.

In the detector, 4 jets will be observed and the problem is to correctly associate each jet with its partner. Two jets originating from a W decay should have a total energy equal to the beam energy. The decay of two W's into 4 jets has been simulated by a Monte Carlo method. The energy distribution of all 2-jet combinations is given in Fig.6. A peak at the beam energy is seen and therefore the pairing can be selected by this method. When the pairing is determined, the W mass can be reconstructed. The result is shown in Fig.7. A clean W signal is measured. The background in Fig.7 is due to the few cases where the pairing is incorrect. The W width is given by the energy resolution. If one is worried about the loss of events due to angle of the jets with respect to beam direction, or the angle between them, Fig. 8 and 9 show the two angular distributions. These distributions are broad and a maximum of 10% of the detected events are lost due to overlap between 2 jets. In principle, $e^+e^- \rightarrow$ hadron should show a typical two jet structure and should not produce any background to the four jet events. About 50% of the $W^+W^-$ pairs produced decay into four hadron jets. At full luminosity one expects at the energy for maximum production cross section assuming $(\sin^2 \theta_W = 0.25)$, 70 events per day. Again, assuming the average luminosity to be one quarter of the peak...
Figure 5: W pairs cross-section as a function of the total energy $\sqrt{s}$
Figure 6: Total energy distribution of any two pairs of jets from W decays in e+e- → WW.
Figure: 7: Reconstructed invariant mass of two jets selected on the basis of the result of Fig. 6

\[
W^+ \text{ mass}
\]

\[
\text{background}
\]
Figure 8: Distribution of the closest jet to the beam axis (two $W$ production)
Figure 9: Distribution of the cosinus of the angle between any two jets
luminosity it would take 20 days to accumulate 300 events. With this method the W mass can be determined to within few GeV/c². But, an inspection of Fig. 5 shows that the rise of the cross section is very steep and therefore the measurement of few values of the excitation curve can give a very precise determination of the W mass.

A large fraction of the time (between 20 and 40% depending on the number of quarks) one W decays into a hadron jet, and the other into a charged lepton plus neutrino. The event pattern is relatively simple; 2 hadron jets with an invariant mass equal to the W mass, and 1 lepton plus missing energy. The branching ratio of the W into lepton plus neutrino is measured as well as the angular distribution of the leptonic decay in the W rest frame.

Also, the number of events is sufficient to perform an analysis of the W angular distribution, and to obtain information on the W couplings (Darriulat-Gaillard, ECFA/LEP/1).

It is worth while to add few comments concerning the effect of polarized beams... A detailed theoretical study on this subject exists (K.J.F. Gaemersand, G.J. Gounaris, TH.2548 CERN). Here, we would like to remark the following simple fact.

\[ \sigma_{\text{unpolarized}} = \frac{1}{4} \left[ \sigma^{LR} + \sigma^{RL} + \sigma^{LL} + \sigma^{RR} \right] \]

L and R stand for the beam helicities. The three last terms are small and hence, if the beams were 100% polarized, the rate of \( W^+ W^- \) events would be increased by a factor of four. If \( a = \frac{\sigma^{RL}}{\sigma^{LR}} \ll 1 \), and \( P \) is the beam polarization, then

\[ R = \frac{\text{Rate RL}}{\text{Rate LR}} = \frac{(1-P^2) + 4 \, P^2 a}{4 \, P^2 + (1-P^2)} \]

\( a \) is small ( \( \ll 1/20 \)) and in practice the ratio \( R \) will depend only on the beam polarization. The measurement of \( a = \frac{\sigma^{RL}}{\sigma^{LR}} \) therefore requires a precise determination of the beam polarization. The observation of these polarization effects would be a remarkable confirmation that only left handed electron interact weakly.
SEARCH FOR HEAVY LEPTONS

If the heavy lepton mass is less then half of the Z° mass, the cleanest way to detect their existence is to measure those electron-muon coincidences having a large fraction of the missing energy. The branching ratio $Z^0 \rightarrow L^+L^-$ is of the order of $3 \times 10^{-2}$. The branching ratio $L^+L^- \rightarrow e\mu$ is of the order of $2 \times 10^{-2}$. At LEP, at the energy of the $Z^0$ pole it is reasonable to assume a $Z^0$ production rate of 2000/hour. This leads to 1.2 $e\mu$ coincidence per hour, or 30 per day. The dominant background could come from the reaction

$$e^+e^- \rightarrow e^+e^- \mu^+\mu^-$$

but the transverse momentum of the electrons is in this case much less than for the decay electron of the heavy lepton.

The distribution of the angle $\theta$ between the electron and the muon, has been calculated by Fujikawa et al (Phys. Rev. D13, 2534) and is shown in Fig.10. Because this angular distribution depends on the heavy lepton velocity, it will be possible, if the number of heavy leptons between the $Z$ mass and $m_{Z^0}/2$ is not too large, to have an approximate determination of its mass. A precise measurement of its mass would probably require a scan in energy and because of the low rate outside the $Z^0$ pole, it would be very difficult to perform.

For the same reason, a heavy lepton of 80 GeV/c² mass will also be difficult to detect. With a total energy $\sqrt{s} = 200$ GeV, an $L^+$ mass of 80 GeV/c², and a 10% branching ratio into electron or muons, one expects a rate of $e\mu$ of 14 per 1000 hours. The situation is better if the detector permits the measurement of 1 lepton + 2 hadron jets + missing energy. 70 events of this type could be detected in 1000 hours. A scan in $\sqrt{s}$ is not feasible.

SEARCH OF HIGGS

If the mass of the Higgs boson is much smaller than the $Z^0$ mass, the cross section for the reaction $e^+e^- \rightarrow Z^0 + H$ has been given in the Yellow Report (CERN 76.18). The best signature for the $Z^0$ is its decay in $e^+e^-$ or $\mu^+\mu^-$. The Higgs is identified by the missing mass to the $Z^0$. 
Figure 10: Collinearity angle distribution between the decay electron and the decay muon in the case of heavy lepton pair production.
For a 40 GeV Higgs mass and a total energy of $\sqrt{s} = 146$ GeV the production cross section is $2.8 \times 10^{-36}$ cm$^2$ and in 1000 hours a total number of 100 Higgs could be detected. On the other hand, if it is possible to identify the $Z^0$ via its two jet decay, then the rate will be 10 times larger.

**QUARK SEARCH**

According to current theoretical ideas, the virtual photon of $e^+ e^-$ annihilation couples to a quark-antiquark pair. The quark itself then fragments to produce a jet of particles. If at sufficiently high energy the quark becomes free, a measurement of the quark charge is possible. A clean signature can be obtained from ionisation in scintillation counters or in gaseous detectors.

If the quark is liberated in a jet, ionisation measurements might be difficult to perform. Some people have proposed the use of the "superenergy" method (ECFA/LEP41). It is based on the fact that if the quark charge is $\frac{1}{3}$, then its apparent momentum is larger than its energy. Table 1 (taken from ECFA/LEP 41) summarizes the situation.

**NEUTRAL HEAVY LEPTON**

A neutral heavy lepton can decay into a lepton pair plus neutrinos.

$$L^0 \rightarrow e^+ \mu^- + \nu_e + \nu_\mu + \bar{\nu}_L$$

Neutral heavy leptons are produced in pairs and therefore a clean signature would be events with 2 electrons, 2 muons and the rest of the energy missing.

**CONCLUSION**

A good detector (Fig.11) for the detection of new particles must have good electron and muon identification. It must also have a good hadron calorimetry system. A solenoid of 1m radius, followed be electromagnetic and hadron calorimeter is probably a good detector. The hadron calorimeter can also be used as a muon identifier, if incorporated as part of the solenoid return yoke.
Table 1

Reconstructed final state energy $E_f$ for $E_{in} = 200$ GeV and different reactions using the LEP jet detector.

<table>
<thead>
<tr>
<th>Reaction</th>
<th>$E_f$</th>
<th>rms spread</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e^+ e^- (\text{hadrons})<em>{\text{jet 1}} = (\text{hadrons})</em>{\text{jet 2}}$</td>
<td>200.8 GeV</td>
<td>1.8 GeV</td>
</tr>
<tr>
<td>$e^+ e^- (u+\text{hadrons})<em>{\text{jet 1}} + (u+\text{hadrons})</em>{\text{jet 2}}$</td>
<td>234 GeV</td>
<td>8.0 GeV</td>
</tr>
<tr>
<td>$e^+ e^- (d+\text{hadrons})<em>{\text{jet 1}} + (d+\text{hadrons})</em>{\text{jet 2}}$</td>
<td>340 GeV</td>
<td>32.0 GeV</td>
</tr>
<tr>
<td>$e^+ e^- (\text{hadrons})<em>{\text{jet 1}} + (u\bar{u}+\text{hadrons})</em>{\text{jet 2}}$</td>
<td>227.4 GeV</td>
<td>5.2 GeV</td>
</tr>
<tr>
<td>$e^+ e^- (\text{hadrons})<em>{\text{jet 1}} + (d\bar{d}+\text{hadrons})</em>{\text{jet 2}}$</td>
<td>309.6 GeV</td>
<td>21.2 GeV</td>
</tr>
</tbody>
</table>
Figure 11: Possible detector for the search for new particles
LEP SUMMER STUDY
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ECFA and CERN
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10 to 22 September 1978

e+e− PHYSICS AT LEP ENERGIES

Zedology J. Ellis*
Weak Interactions Beyond the Z Pole(s) M.K. Gaillard**
Testing Strong Interaction Theories J. Ellis*

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Copies available upon request from Ch. Redman, CERN/ISR,
LEP Summer Study Secretariat
The key motivation for LEP is the unique possibility which it would offer to study the merging of weak and electromagnetic interactions, as expected in the framework of present gauge theories, and as supported by a whole array of recent and spectacular experimental results.

Of particular importance is the expected presence of the $Z^0$, which should lead to impressive effects over a good fraction of the LEP energy range. $Z^0$ formation and decay, and related theoretical issues, are covered by J. Ellis in Section 1 of this report. There are however very important weak interaction issues for the study of which energies still higher than the $Z^0$ mass are required. Production of $W$'s, $W$ pairs and perhaps Higgs mesons is in particular of great interest. This question is surveyed by M.K. Gaillard in Section 2 of this report. The importance of the $e^+e^-$ annihilation cross-section over the $Z^0$ peak should rapidly lead to registering tens of millions of events at very large $Q^2$. Hadron production, with perhaps the occurrence of new quarks, should then provide very interesting clues testing strong interaction theories. This question is covered by J. Ellis in the third section of this paper.

This paper combines three of the plenary theoretical talks presented at Les Houches. Weak interaction issues, with emphasis on very high energy questions, were also covered by M. Veltman, but this will not appear as a special write-up in the proceedings. While the present paper mainly describes what is now referred to as the standard approach, other scenarios may be entertained. This was discussed at Les Houches by S.L. Glashow and LEP Summer Study/1-9 actually covers both his CERN and Les Houches talks. The theoretical review of the $2\gamma$ process given by P.V. Landshoff is included in LEP Summer Study/1-13.

These 6 talks at Les Houches contributed to the discussion of theoretical questions at LEP energies, with conclusions summarized by Ch. Llewellyn Smith in LEP Summer Study/1-2.
1. Introduction

It is clear that the Z^0 peak will be a very important landmark in e^+e^- collisions\cite{1,2,3,4,5}. It will be a rich cornucopia of all conceivable particles with masses \( \leq m_Z \) or \( m_Z/2 \). If its properties resemble at all the predictions of simpler gauge models such as the Weinberg-Salam model\cite{6}, it will produce about an event per second for a LEP luminosity of the order of \( 10^{32} \text{ cm}^{-2} \text{ sec}^{-1} \). As far as the weak interactions are concerned, Z^0 decays present the opportunity to measure in detail the neutral weak couplings of all lepton and quark flavours with masses \( \leq m_Z/2 \). As for the strong interactions, one can imagine experiments with \( 0(10^6 \text{ or } 10^7) \) hadronic events at \( Q^2 = 0(10^4) \text{ GeV}^2 \), for more than are conceivable in any experiments at space-like \( Q^2 \)\cite{7}. These events will provide a unique window into the dynamics of quarks and gluons at short distances. Strong interaction studies with LEP are discussed later in this report\cite{8}: the rest of this section will concentrate on weak interaction studies at the Z^0 pole\cite{9}. Two main aspects will be emphasized: the fact that high statistics enable precision measurements which bear on the fundamental structure of the theory, and the possibility to search for very rare decay modes involving exotic particles.

2. General Features of the Z^0

We will be mainly concerned with the couplings of the Z^0 to fundamental fermions (quarks, leptons) which we parametrize\cite{2,5} in the following form:

\[
\mathcal{L}_f = -m_Z \left( \frac{c_P}{\sqrt{2}} \right)^{\frac{1}{2}} \gamma^\mu \left( \frac{v_f - a_f \gamma_5}{\sqrt{2}} \right) f Z^\mu \tag{1}
\]

Constant factors have been removed to the front of equation (1) so that the reduced couplings \( v_f \) and \( a_f \) are expected to be of order unity in any unified gauge model for which the coupling constant would be of order \( e \). As an example, we can consider the standard SU(2) x U(1) Weinberg-Salam model\cite{6} in which

\[
\begin{align*}
    v_e &= v_\mu &= v_\tau &= -1 + \frac{4}{3} \sin^2 \theta_W, \\
    v_d &= v_s &= v_b &= -1 + \frac{4}{3} \sin^2 \theta_W, \\
    v_u &= v_c &= v_t &= 1 - \frac{8}{3} \sin^2 \theta_W, \\
    a_e &= a_\mu &= a_\tau &= -1 \\
    a_d &= a_s &= a_b &= -1 \\
    a_u &= a_c &= a_t &= 1
\end{align*} \tag{2}
\]

Before discussing the size of the Z^0 peak implied by the couplings (1) and (2) we need a standard reference cross-section, which we take to be:

\[
\sigma_{\text{pt}} = \sigma(e^+e^- \rightarrow \gamma^* + \mu^+\mu^-) = \frac{4\pi}{3} \frac{a^2}{Q^2} = \frac{87}{Q^2 \text{ (GeV)}^2} \text{ nb} \tag{3}
\]
Relative to this cross-section we define, for any $f\bar{f}$ final state,

$$R_f = \frac{\sigma(e^+e^- + f\bar{f})}{\sigma_{pt}}$$

(4)

Also used often will be the integrated forward-backward asymmetries

$$A_f = \frac{\int_{-1}^{0} d(cos\theta) \frac{d\sigma}{d(cos\theta)}(e^+e^- + f\bar{f}) - \int_{0}^{1} d(cos\theta) \frac{d\sigma}{d(cos\theta)}(e^+e^- + f\bar{f})}{\int_{-1}^{0} d(cos\theta) \frac{d\sigma}{d(cos\theta)}(e^+e^- + f\bar{f}) + \int_{0}^{1} d(cos\theta) \frac{d\sigma}{d(cos\theta)}(e^+e^- + f\bar{f})}$$

(5)

Gauge theories generally expect that the mass of the $Z^0$ will be

$$m_{Z^0} = 0(e/G_F) = O(100) \text{ GeV}$$

(6)

The simplest Weinberg-Salam model\textsuperscript{6}) in fact predicts:

$$m_Z = \frac{\pi\alpha}{\sqrt{2}G_F} / \sin^2\theta_W \cos\theta_W = \frac{37.4 \text{ GeV}}{\sin^2\theta_W \cos\theta_W}$$

(7)

Taking $\sin^2\theta_W \approx 0.20$ consistent with the latest neutral current experiments\textsuperscript{10}), we find $m_Z < 94 \text{ GeV}$. In this energy region, equation (3) tells us that

$$\sigma_{pt} \sim 10^{-35} \text{ cm}^2, \text{ corresponding to 3.6 events/hour at a luminosity of 10}^{32} \text{ cm}^{-2} \text{ sec}^{-1}.$$  

Elementary considerations tell us that, neglecting radiative corrections, the cross-sections at the peak of the resonance are given by:

$$\frac{\sigma(e^+e^- + Z^0 + X)}{\sigma_{pt}} = \frac{9}{\alpha^2} B(Z^0 \rightarrow e^+e^-) B(Z^0 \rightarrow X)$$

(8)

Taking three generations of each type of fermion ($\nu$, $k^-$, charge $-1/3$ quark, charge $2/3$ quark) we are led to expect:

$$B(Z^0 \rightarrow e^+e^-) = 0(\frac{1}{20} \text{ to } \frac{1}{30})$$

(9)

Inserted into equation (8) this branching ratio suggests that:

$$\frac{\sigma(e^+e^- + Z^0 + all)}{\sigma_{pt}} \sim \text{ few thousand}$$

at the resonance peak, corresponding to the order of ten thousand events/hour.

In fact, in the simplest Weinberg-Salam model\textsuperscript{6}) with $\sin^2\theta_W = 0.20$, we find:

$$\frac{\sigma(e^+e^- + Z^0 + all)}{\sigma_{pt}} \bigg|_{\text{WS}} \leq 5100$$

(11)

if there are just three generations, corresponding to about 5 events/second.

Before continuing, a word should be said about the reliability of estimates of the $Z^0$ mass. The prediction\textsuperscript{7}) of the Weinberg-Salam model depends on the SU(2) symmetry being spontaneously broken by an isodoublet of Higgs fields\textsuperscript{11}). More complicated Higgs sectors could alter the prediction\textsuperscript{7}), but they are already severely constrained by neutral current data. Since the couplings\textsuperscript{1,2}) are fixed
in the Weinberg-Salam model, we see that the neutral to charged current cross-section ratio

\[
\frac{\sigma(\text{NC})}{\sigma(\text{CC})} = \frac{1}{m_Z^2}
\]  

(12)

The present agreement of neutral current cross-sections with Weinberg-Salam constrains the \(Z\) mass within the \(SU(2)_L \times U(1)\) framework:

\[
\frac{m_Z^2}{m_Z^2 (\text{WS})} = 1.02 \pm 0.05
\]  

(13)

If one goes beyond the \(SU(2) \times U(1)\) weak electromagnetic gauge model the restriction (13) is of course greatly relaxed. However, it can be argued that in a wide class of models with more than one \(Z^0\) boson, at least one of them must have a mass smaller than that predicted by Weinberg and Salam\(^1\)).

For an arbitrary \(Z^0\), the formulae (1) and (2) correspond to decay widths

\[
\Gamma(Z^0 \rightarrow f\bar{f}) = \frac{G_F m_f^2}{24\sqrt{2}\pi} (v_f^2 + a_f^2)
\]  

(14)

for \(m_f << m_Z/2\). For the favoured range of values of \(m_Z\) and \(v_f\), \(a_f\) of order unity, equation (14) implies that \(\Gamma(Z^0 \rightarrow f\bar{f}) = O(100)\) MeV. Including 3 generations of fermions one would therefore expect a total \(Z^0\) decay width

\[
\Gamma(Z^0 \rightarrow \text{all}) = O(2 \text{ to } 3) \text{ GeV}
\]  

(15)

which is much wider than the expected machine energy resolution \(O(10^{-3})m_Z = O(100)\) MeV. In the simplest Weinberg-Salam model (since one expects decays into fermion-antifermion pairs to dominate) one finds\(^2,5\) from equation (2) that

\[
\Gamma(Z^0 \rightarrow \text{all}) = \frac{G_F m_Z^2}{24\sqrt{2}\pi} \left[ 2N_v + \left( 1 + \frac{8}{3} \sin^2 \theta_W \right)^2 N_{\ell^-} + 3\left( 1 + \frac{8}{3} \sin^2 \theta_W \right)^2 N_{2/3} + 3\left( 1 + \frac{4}{3} \sin^2 \theta_W \right)^2 N_{-1/3} \right]
\]  

(16)

If we take \(\sin^2 \theta_W = 0.20\) we find that

\[
\Gamma(Z^0 \rightarrow \nu\bar{\nu}) : \Gamma(Z^0 \rightarrow e^+e^-) : \Gamma(Z^0 \rightarrow \mu^+\mu^-) : \Gamma(Z^0 \rightarrow u\bar{u}) : \Gamma(Z^0 \rightarrow d\bar{d}) = 2 : 1.04 : 3.63 : 4.67
\]  

(17)

with the decay rate

\[
\Gamma(Z^0 \rightarrow e^+e^-) = 90 \text{ MeV}
\]  

(18)
Combining the results (17) and (18) we see that if there are $N_G$ generations of fundamental fermions

$$\Gamma(Z^0 \to \text{all}) \sim 1.0 N_G \text{ GeV}$$

(19)

and

$$B(Z^0 \to e^+e^-) \sim 1/11 N_G$$

(20)

resulting in $\Gamma(Z^0 \to \text{all}) \sim 3 \text{ GeV}$, $B(Z^0 \to e^+e^-) \sim 3\%$ for the minimal case of 3 generations.

3. Determining the Fermion Spectrum

The above results are encouraging, in the sense that the $Z^0$ peak is large and dramatic, as long as there are not too many generations of fermions. Is it conceivable that there might be so many fermions as to wash out the $Z^0$ peak? The "established" fermions are the three generations:

$$\begin{pmatrix} \nu_e \\ e^- \\ \nu_u \\ u^- \\ \nu_e \\ e^- \\ \nu_c \\ c^- \\ \nu_t \\ t^- \end{pmatrix}$$

(21)

and the question arises whether we have any constraints on the total number of other as yet undiscovered fermions. Neutrinos are a particular headache because they seem to have negligibly small masses, while charged fermion masses increase sufficiently rapidly that not too many of them can be reasonably expected to have masses $< m_Z/2$.

What limits do we have on the number of unobserved neutrinos? The best limit from high energy physics at the present time may come from the upper limit on the decay $K \to \pi \nu\bar{\nu}$:

$$B(K \to \pi \nu\bar{\nu}) < 6 \times 10^{-7}$$

(22)

which when compared with the theoretical branching ratio

$$B(K \to \pi \nu\bar{\nu}) \sim 0(10^{-10}) N_\nu$$

(23)

suggests that $N_\nu < 6000$ - not a very stringent limit! It has been proposed that one might establish a good limit on $N_\nu$ from decays of heavy quark–onia into neutrinos. One finds that

$$\frac{\Gamma(V \to Z^0 \to \nu\bar{\nu})}{\Gamma(V \to \gamma^* + e^+e^-)} \geq \frac{G_F^2}{64 \pi^2} \frac{m_\nu^4}{e_q^2} (1 - 4 |e_q^2 | \sin^2 \theta_W)^2$$

(24)

$$= 0.2 \times 10^{-8} m_\nu^4 N_\nu \text{ for } e_q = \frac{2}{3}$$

(25)

Putting in $V = J/\psi : m_\psi \sim 3 \text{ GeV}$, and assuming an upper limit $\Gamma(J/\psi \to \nu\bar{\nu})/\Gamma(J/\psi \to e^+e^-) \leq 1$, one finds $N_\nu < 5 \times 10^6$, an even less stringent limit! However, if there happens to be toponium with a mass of 30 GeV, the strong
mass dependence in (25) would enable a much more stringent limit to be set on \( N_v \).

The decay \( V \rightarrow \nu \bar{\nu} \) could be looked for by looking for events of the type \( e^+e^- \rightarrow V' \rightarrow V + \pi\pi, \ V \rightarrow \text{nothing visible}. \)

There are some limits on neutrinos and other neutral, heavy leptons which come from cosmology. The standard big-bang cosmology is only consistent with the present astrophysical density of Helium if there are at most 3 or 4 "light" neutrinos with masses \( \leq m_\nu^{14} \). For heavier neutral leptons, there are no very strong constraints on unstable species, but stable neutral leptons are constrained\(^{14}\) by the large scale dynamics of the universe and of galaxies to have masses \( \geq 10 \text{ GeV} \).

To limit the number of unstable massive neutral leptons or neutrinos, we fall back on the observation that generally \( m_{\nu c} \ll m_c \), for any given \( \nu_c \) doublet. From the results of PLUTO and SPEAR, we believe that any new heavy lepton must have a mass \( \geq 5 \text{ GeV} \). However, the number of such heavy leptons is theoretically constrained. In the simplest Weinberg-Salam model the result \( m_Z = m_w / \cos \theta_w \) which underlies equation (7) is subject\(^{15}\) to radiative corrections from all doublets containing massive fermions:

\[
\frac{M_W^2}{m_Z^2} = \cos^2 \theta_w \left[ 1 + \left( \frac{1}{3} \text{ for leptons} \right) \frac{G_F}{8\sqrt{2}\pi^2} \left( \frac{2m_w^2}{m_1^2} \right) \left( \frac{\beta_n}{m_n^2} \right) \left( \frac{m_2^2 + (m_1^2 + m_2^2)}{m_1^2} \right) \right]
\]

The experimental constraint (13) already means that for heavy leptons \( L \) with masses \( \gg \) their associated neutrinos

\[
\Sigma m_L^2 < 0(500 \text{ GeV})^2
\]

Since present experimental limits tell us that any such heavy lepton has \( m_L > 5 \text{ GeV} \), the restriction (27) means there are less than \( 0(10^4) \) such heavy leptons, and so \( N_L < 0(10^4) \). PETRA can soon improve the lower limit on \( m_L \) to \( \sim 15 \text{ GeV} \), in which case \( N_L \) would be \( < 10^3 \). LEP could eventually improve the lower limit on \( m_L \) to about 100 GeV, corresponding to \( N_L < 0(25) \). It is clear from equation (8) that \( 0(10^4) \) neutrinos would be required if the \( Z^0 \) were washed out to the extent that \( \sigma(e^+e^- \rightarrow Z^0 + X)/\sigma_{Pe} = 0(10) \). Therefore either PETRA and LEP find vast numbers of heavy leptons, or the \( Z^0 \) will be a large peak\(^{16}\).

Assuming that the \( Z^0 \) peak is indeed big and not very wide, one can then imagine a precision determination of the \( Z^0 \) mass. A precision measurement of the \( WW \) threshold to get the \( W \) mass, and detailed neutral current measurements (see part 4 of this report) would then enable the radiative corrections in equation (26) to be severely restricted, so that the bound (27) could be improved. In this way, one could perhaps exclude the possible existence of any heavy lepton with
mass > 100 GeV (and hence outside the mass range accessible to LEP), and of course
detect any heavy lepton with mass < 100 GeV. A check that the studies of fundamental
ermion spectroscopy were indeed completed by LEP would be furnished by measuring
\( \Gamma(Z^0 \rightarrow \sum \nu \bar{\nu}) \). Several ways of doing this come to mind. One possibility is looking
for the decay chain \( e^+e^- \rightarrow \nu' \rightarrow \nu \pi \pi, \nu + \nu \bar{\nu} \) mentioned earlier\(^5,\)\(^13\). Another is a
precision measurement of the \( Z^0 \) width, which increases by \( 0(5 \text{ to } 10)\% \) for each
neutrino in addition to the canonical three. Another possibility is to look for
the reactions \( e^+e^- \rightarrow \nu \bar{\nu} + \gamma \), where the only particle visible in the final state
would be an energetic large angle \( \gamma \)\(^17\). The rate for this seems prohibitively
small at PETRA energies, but the experiment may be feasible\(^18\) above the \( Z^0 \) mass,
where the dominant contribution to the cross-section is the radiative correction
reaction \( e^+e^- \rightarrow Z^0 + \gamma, Z^0 \rightarrow \nu \nu \). Depending on the total number of neu-
trinos, this process may have\(^18\) a cross-section of the same order as \( \sigma_{pt} \) for
centre-of-mass energies between 120 and 200 GeV.

4. Detailed Measurements Near the \( Z^0 \) Peak

We will now survey the different neutral current measurements that can
be made by observations at and near the \( Z^0 \) peak. Radiative corrections will not
be taken into account, because a complete calculation of these is only just be-
coming available\(^19\), but we do not think they will make qualitative changes in
the classes and qualities of measurements that can be made.

The Shape of the Total Cross-Section

If we consider an arbitrary fermion-antifermion pair (with the exception
of \( e^+e^- \)) then

\[
R_f \equiv \frac{\sigma(e^+e^- \rightarrow \gamma, Z \rightarrow \ell \ell \gamma)}{\sigma_{pt}}
\]

\[
= \frac{Q_f^2}{2s} - \frac{2s \cdot 0_f V_{\ell f} V_{\ell f}}{s} + \frac{s^2 a^2 (v^2 + a^2)}{s} + \frac{(s^2 m_Z^2 - 1)^2 + r^2 Z^0 / m_Z^2}{2}
\]

\( (28) \)

where \( \rho \equiv (G_F/\sqrt{2})\alpha \).

We notice that the total cross-section shape is sensitive to the products
of vector (or of axial) weak couplings. If we specialize to \( \mu^+\mu^- \), assume \( \mu-e \) uni-
versality: \( v_\mu = v_\ell \equiv v \)
\( a_\mu = a_\ell \equiv a \)

\( (30) \)

and neglect \( \Gamma_Z \), we find

\[
R_\mu = 1 + 2v_\mu^2 \chi + (v^2 + a^2) \chi^2
\]

\( (31) \)

where \( \chi \equiv m_Z^2 \rho \left( \frac{s}{s - m_Z^2} \right) \)

\[
= 0.39 \left( \frac{s}{s - m_Z^2} \right)
\]

if \( m_Z = 94 \text{ GeV} \)

\( (32) \)
In general, \( R_\mu \) exhibits a minimum at

\[
\frac{s}{m_Z^2} = \frac{\delta}{1 + \delta} = \frac{\left( \frac{1}{m_\mu^2} \right)}{(v^2 + a^2)^2}
\]

(33)

corresponding to \( \sqrt{s} = 29 \) GeV if we take the Weinberg-Salam model with \( \sin^2 \theta_w = 0.20 \). The value of \( R_\mu \) at its minimum is

\[
R_{\mu}^{\text{min}} = 1 - \frac{v^4}{(v^2 + a^2)^2}
\]

(34)

which is not very exciting if \( \sin^2 \theta_w = 0.20 \):

\[
R_{\mu}^{\text{min}} = 0.9985
\]

(35)

The general shapes of \( R_\mu \) for different choices of \( v \) and \( a \), and a \( Z^0 \) mass of 83 GeV, are shown in Fig. 1. It is clear that if \( a = 0 \), which is not expected in the Weinberg-Salam model and is indeed disfavoured by experiments finding parity violation in deep inelastic electron scattering and atoms, then \( R_{\mu}^{\text{min}} = 0 \) at \( \sqrt{s} \approx 0.85 m_Z \) - rather dramatic!

Forward-Backward Asymmetry

The angular distributions for the processes \( e^+e^- \to f\bar{f} \) (\( f\bar{f} \neq e^+e^- \)) have the following form near the \( Z^0 \) peak:

\[
\frac{d\sigma}{d\cos \theta} (e^+e^- \to f\bar{f}) = \frac{\pi q^2}{2s} \left\{ Q_f^2 (1 + \cos^2 \theta) - 2Q_f \chi (v_e v_f (1 + \cos^2 \theta) + 2a_e a_f \cos \theta) + \chi^2 \left[ (v_e^2 + a_e^2) (v_f^2 + a_f^2) (1 + \cos^2 \theta) + 8v_e v_f a_e a_f \cos \theta \right] \right\}
\]

(36)

if we neglect the decay width \( \Gamma_Z \) compared with \( m_Z \). If we define the integrated forward-backward asymmetry

\[
A_f = \frac{\int_0^1 d\sigma}{\int_0^1 d\cos \theta} \frac{d\sigma}{d\cos \theta} - \frac{\int_1^0 d\sigma}{\int_0^1 d\cos \theta} d\cos \theta - \int_0^1 d\sigma \frac{d\sigma}{d\cos \theta} (\cos \theta)
\]

(37)

it is found from the angular distribution (36) to be

\[
A_f = \frac{3}{2} \chi \left( \frac{-Q_f a_e a_f + 2v_e v_f a_e a_f \chi}{Q_f^2 - 2Q_f v_e v_f + \chi^2 (v_e^2 + a_e^2)(v_f^2 + a_f^2)} \right)
\]

(38)

Since the angular distribution is only quadratic in \( \cos \theta \), there is a trivial bound \( |A_f| \leq 3/4 \).
Figure 1: The ratio \( R_{\mu} \) of \( (e^+e^- \to \mu^+\mu^-) \) relative to \( \sigma_{pt}(3) \), plotted for different values of the vector and axial couplings of the e and \( \mu \).
We see from equations (36) and (38) that the forward-backward asymmetry is non-zero if \( a_e \) and \( a_f \) \( \neq 0 \), as expected in the Weinberg-Salam model. Notice however that it does not permit a determination of the relative signs of \( v \) and a couplings (a limitation shared of course by the total cross-section formula (28)). Even at low (PETRA-PEP) energies, the asymmetry (38) can become quite large.

When \( \sqrt{s} \ll m_Z \),

\[
a_f = \frac{-3}{2} \chi \frac{a_e a_f}{Q_f}
\]

which at \( \sqrt{s} = 40 \) GeV is already

- 10\% for \( \mu^+ \mu^- , \tau^+ \tau^- \)
- 14\% for \( \mu^+ \mu^- , \tau^+ \tau^- \)
- 28\% for \( d\bar{d} , s\bar{s} , b\bar{b} \)

In the particular case of \( e^+ e^- \rightarrow \mu^+ \mu^- \) or \( \tau^+ \tau^- \), and assuming charged lepton universality \( a_e = a_\mu = a_\tau \equiv a \), \( v_e = v_\mu = v_\tau \equiv v \), we find

\[
A = \frac{\frac{3}{2} \chi \left( a^2 + 2v^2 a^2 \chi \right)}{1 + 2v^2 + \chi^2 (v^2 + a^2)^2}
\]

which goes through a minimum at

\[
\frac{s}{m_Z^2} = \frac{1}{1 + (m_{Z'}^2) (a^2 + 3v^2)}
\]

(\( \sqrt{s} = 78 \) GeV for \( \sin^2 \theta_W = 0.20 \)). At this point \( A \) takes the value

\[
A_{\text{min}} = \frac{-3}{4} \frac{1}{\left( 1 + \frac{2v^2}{a^2} \right)}
\]

and in fact attains the kinematic bound of \(-0.75\) when \( v = 0 \) corresponding to \( \sin^2 \theta_W = 0.25 \), while \( \sin^2 \theta_W = 20 \) would yield \( A = -0.69 \). The asymmetry also goes through a maximum at

\[
\frac{s}{m_Z^2} = \frac{1}{1 - (\rho m_Z^2) (a^2 - v^2)}
\]

(\( \sqrt{s} = 118 \) GeV for \( \sin^2 \theta_W = 0.20 \)) at which point it takes the limiting value \( A_{\text{max}} = 0.75 \). At the peak of the resonance, \( \sin^2 \theta_W = 0.20 \) would imply an asymmetry \( A \approx +0.11 \) at \( \sqrt{s} = 94 \) GeV. General forms of the asymmetry for muons are shown in Fig. 2, corresponding to different choices of the values of \( v \) and \( a \).

The above analysis does not apply to \( e^+ e^- \rightarrow e^+ e^- \) because there are also crossed-channel \( \gamma \) and \( Z^0 \) exchange diagrams. The cross-section formulae therefore become more complicated\(^{20}\), and will not be reproduced here. We will just make the qualitative observation that the behaviour of the forward-backward asymmetry
Figure 2: The forward-backward asymmetry $A$ (37) for $e^+e^- \rightarrow \mu^+\mu^-$, plotted for different values of the vector and axial couplings of the $e$ and $\mu$. 
parameter is very different in this case from the reaction \( e^+e^- \rightarrow \mu^+\mu^- \). In the case of \( e^+e^- \rightarrow e^+e^- \) there is a large positive asymmetry, of the order of \( 3/4 \) off resonance, which may be drastically reduced on resonance – see the asymmetry in a restricted angular range \( 30^\circ < \theta < 150^\circ \) plotted in Fig. 3. For \( \theta \) sufficiently small the well understood crossed channel \( \gamma \) exchange will always be at low enough \( Q^2 \) to dominate the cross-section and provide a reliable luminosity monitor even near the top of the \( Z^0 \) peak.

**Helicity Measurements**

Another observable which is potentially interesting in the reaction \( e^+e^- \rightarrow f\bar{f} \) is the helicity of the outgoing fermion. If we specialize for the moment to the case of \( e^+e^- \rightarrow \mu^+\mu^- \) or \( \tau^+\tau^- \), the helicity is maximal in the forward direction

\[
H^Z_\tau(s, \cos \theta = +1) = \frac{-4\chi a\nu (1 + \chi(a^2 + \nu^2))}{1 + 2\chi(\nu^2 + a^2) + \chi^2 [(a^2 + \nu^2)^2 + 4a^2\nu^2]} \tag{45}
\]

so that on the resonance peak itself

\[
H^Z_\tau(m_Z^2, \cos \theta = +1) = \frac{-4a\nu (a^2 + \nu^2)}{(a + \nu) + 4a\nu} \tag{46}
\]

If \( \sin^2 \theta_w = 0.20 \), this value on the peak is \( +0.13 \). The variation with \( s \) if \( \sin^2 \theta_w = 0.35 \) is shown in Fig. 4. We notice in (45) (46) that the helicity is sensitive to the product of \( a \) and \( \nu \), enabling their relative sign to be measured, which was not possible with the cross-section and angular asymmetry measurements discussed earlier.

So helicities are interesting and non-zero in general. Can they be measured? An early suggestion was to stop muons produced on resonance in a polarimeter, and determine their polarization from observations of the decay electrons. Such an experiment would be very cumbersome and difficult\(^{20} \), and a better idea may be to use the decays of the \( \tau \rightarrow e\nu\nu \) or \( \pi\nu \) as convenient polarization analyzers. Either of these seems possible, with \( \tau \rightarrow \nu \nu \) measurements perhaps more sensitive to \( H^\tau \)\(^{21} \). Could one perform polarization measurements on quarks? In the absence of e-fermion universality, the numerator in (45) becomes

\[
4a\nu(a^2 + \nu^2) + \nu \frac{a}{f} \frac{a}{f} \frac{a}{e} \frac{a}{e} (a^2 + \nu^2) (1 + \cos^2 \theta) + 2a \frac{e}{e} \frac{e}{e} \frac{e}{f} \frac{e}{f} (a^2 + \nu^2) \cos \theta \tag{47}
\]

and we have sensitivity to \( a f v_f \) as well as \( a v_e \). The only problem is to find a quark helicity analyzer. It has been suggested\(^{22} \) to look at correlations of the type \( p_{jet} \cdot (p_1 \times p_2) \), where \( p_1 \) and \( p_2 \) are the momenta of the two fastest particles in a quark jet. Unfortunately, neutrino data suggest\(^{23} \) that any such correlations are in fact washed out. A related suggestion is to look at the polarization of
Figure 3: The corresponding asymmetry for $e^+e^- \rightarrow e^+e^-$ in the Weinberg-Salam model with different values of $\sin^2 \theta_W$. 
Figure 4: The helicity of a muon or $\tau$, for different values of the vector and axial couplings.
final state hadrons with non-zero spin, such as $\rho$, $K^*$ and $\Lambda^{2+}$). A guaranteed quark polarimeter has yet to emerge, but we note that in a sense none is needed. Cross-section and angular asymmetry measurements enable the relative signs of all the $v_f$ to be determined, and also those of all the $a_f$. The $\tau$ helicity experiment would then determine the signs of the $a_f$ relative to the $v_f$, and any further information would in principle be redundant.

Polarized Beams

These would enable the extraction of physics similar to that obtainable from helicity measurements. If the incoming $e^+$ and $e^-$ have helicities $h^+$, $h^-$ respectively, then5)

\[
\frac{d\sigma^{\pm}}{dcos\theta} (h^+, h^-) = \frac{1}{2} (1 - h^+h^-) \frac{d\sigma}{dcos\theta} \text{ unpolarized}
\]

\[\text{plus} \left( h^- - h^+ \right) \frac{\pi a^2}{2s} \chi \left\{ Q_f \left[ \frac{V_{e_f} a_f}{2} (1 + \cos^2\theta) + 2a_v \frac{v_f}{v_e} \cos\theta \right] \right. \]

\[\left. - \chi \left[ \frac{v_f a_f}{2} \left( a^2_e + v^2 \right) \left( 1 + \cos^2\theta \right) + 2a_v \frac{v_f}{v_e} \left( a^2_f + v^2_f \right) \cos\theta \right] \right\} \tag{48}
\]

Equation (48) again exhibits sensitivity to the relative signs of $v$ and $a$ couplings. If polarized beams were freely available they would probably be more powerful probes of these signs than the helicity measurements. Certainly, the prospect of being able to turn on, or off, an $e^+e^- \rightarrow f\bar{f}$ cross-section by adjusting the beam polarizations seems very attractive. On the other hand, it should be emphasized that from a logical point of view, within the standard gauge theoretical point of view no new information is gained thereby. It is not clear how seriously this should be taken into account when considering the cost of developing polarized beams. They would certainly be invaluable analyzers if the standard gauge picture were wrong.

$Z^0 \rightarrow \text{Heavy } f\bar{f}$

The $Z^0$ decays democratically into all fermions with essentially equal rates (if $m_f < m_Z$). In fact the event rates may be larger than those close to the associated thresholds. If $m_q << m_Z$, then for equal luminosities one finds

\[
\frac{\text{Rate } (Z^0 \rightarrow f\bar{f})}{\text{Rate } (\text{threshold } \rightarrow f\bar{f})} = 1000 \left( \frac{m_q}{m_Z} \right)^2 \tag{49}
\]

which is promising for studies of $t\bar{t}$ and $b\bar{b}$ final states. Measurements of these final states may be the only way to determine heavy quark neutral current couplings. At present we know in principle the neutral current couplings of $u,d,e,v_\mu$ and to some extent $v_e$. Those for $s,c,t,b,\tau$ and even $u$ are still essentially unknown.
The problem resides in finding ways to detect heavy quark decays of the \( Z^0 \). One might look for

- events with many final state leptons\(^{25}\) (> 3 e\(^\pm\) and \( \mu^\pm \) + hadrons, or events with 2 identically charged leptons in the same jet);
- fat jets\(^{2,5}\), because one expects\(^{25,26}\) a heavy quark to decay into three light quarks and antiquarks: \( Q \rightarrow qq\bar{q} \) as in Fig. 5. The transverse momenta of the associated hadrons would then probably be quite large;
- long-lived heavy mesons? In the conventional six-quark extension of the Weinberg-Salam model, the decay rates of heavy quarks into light quarks are suppressed\(^{25}\). Careful analysis\(^{27}\) suggests bounds on the lifetimes of bottom mesons: \( B^0 \equiv b\bar{d}, \ B^- \equiv b\bar{u} \):

\[
\sum_{\text{hadrons}} |p_t| = \sum_{\text{quarks}} |p_t| - \frac{2}{3} m_Q \quad \text{(50)}
\]

If the lifetime is in the upper half of this range, it might be observable in \( e^+e^- \) collisions at high enough energies to give a useful relativistic dilation of the decay track length. One might therefore look for "staggered" events of the type shown in Fig. 6, where there is a set of particles produced in the initial \( e^+e^- \) annihilation, and two other sets of tracks converging on separate \( B \) and \( \bar{B} \) decay points.

In this section we have chiefly discussed relatively common \( Z^0 \) decays with branching ratios \( > 10^{-2} \). The large event rates available at the \( Z^0 \) peak should enable detailed studies of these channels. It may be worth emphasizing again the interest of such studies. For example, there are theories which purport to calculate \( \sin^2 \theta_W \) with a precision better than 0.01\(^{28}\). It would be nice to know if these theories were correct. It would also be nice to reduce the errors on the combination \( \frac{m_W^2}{m_Z^2 \cos^2 \theta} \) to see how close it is to 1, and thereby (recall\(^{15}\) equation (26)) get a useful constraint on the fermion mass spectrum. For this purpose, detailed measurements at the \( Z^0 \) peak must be combined with precise measurements of the \( e^+e^- \rightarrow W^+W^- \) threshold.

5. Decays Involving Higgs Bosons

The previous section dealt with the relatively common decays of the \( Z^0 \) into fermion pairs. What other important decays of the \( Z^0 \) are expected? Other members of the elementary particle zoo include the \( W^\pm \) and the Higgs particles. Decays into the \( W^\pm \) are expected to be very rare: in the Weinberg-Salam model with \( \sin^2 \theta_W = 0.35 \) so that \( m_W = 62 \text{ GeV}, m_Z = 80 \text{ GeV} \), it was found\(^{29,2}\) that

\[
\Gamma(Z^0 \rightarrow W^+e^-\nu) \sim 3 \times 10^{-7} \text{ GeV} \quad \text{(52)}
\]
Figure 5: The expected dominant decay mode $Q \to q\bar{q}q$ of a heavy quark.

Figure 6: The possible "staggered" event structure of a decay $Z^0 \to Q\bar{Q}$ into a heavy quark-antiquark pair with lengthy lifetimes.

Figure 7: The dominant diagram for $Z^0 \to H^0 l^+ l^-$. 
and the presently preferred value of $\sin^2\theta_w = 0.20$ would imply a still smaller decay rate. So we turn to the detection of Higgs bosons. Their importance has been adequately stressed in the literature. Higgs bosons play the essential rôle in generating the spontaneous symmetry breaking necessary to realistic renormalizable weak interaction models. Verification of their existence is therefore a crucial test of the entire gauge theory approach to weak interactions. Other promising ways of looking for Higgs bosons have been proposed which involve either lower $e^+e^-$ centre-of-mass energies (e.g. the decay of a vector meson $V \rightarrow H + \gamma$) or higher centre-of-mass energies (e.g. the reaction $e^+e^- \rightarrow Z^0 + H^0$). What are the possibilities in $Z^0$ decays?

$Z^0 \rightarrow H^0 + \mu^+\mu^-$ or $e^+e^-$

This decay would proceed via the diagram shown in Fig. 7. In terms of the variable $x = 2E_{\text{Higgs}}/m_Z$ the decay spectrum has been computed to be:

$$\frac{1}{\Gamma(Z \rightarrow e^+e^-)} \frac{d\Gamma(Z^0 \rightarrow H^0 e^+e^-)}{dx} = \frac{\alpha}{4\pi \sin^2\theta_w \cos^2\theta_w} \left[ 1 + \frac{x^2}{12} + \frac{2}{3} \frac{m_H^2}{m_Z^2} \left( x^2 - \frac{4m_H^2}{m_Z^2} \right)^2 \right] \left( \frac{x - m_H^2}{m_Z^2} \right)^2$$

The resulting total branching ratio is plotted in Fig. 8. We see that for $m_H < 40$ GeV, the branching ratio $B(Z^0 \rightarrow H^0 e^+e^-)$ is $> 3 \times 10^{-6}$. This may give an acceptable rate if one can indeed do experiments with tens of millions of $Z^0$ decays, though more thought about backgrounds is required. The signature for Higgs decays is its propensity for decaying into the heaviest fermions available: $H \rightarrow Q\overline{Q}$, which should mean that its final states will contain an unusually high fraction of prompt decay leptons, and tend to have fatter jets on average than in the $e^+e^-$ continuum.

$Z^0 \rightarrow H^0 + \gamma$

The branching ratio for this process has recently been calculated. It was found that

$$\frac{\Gamma(Z^0 \rightarrow H^0 + \gamma)}{\Gamma(Z^0 \rightarrow \mu^+\mu^-)} \approx 8 \times 10^{-5} \left( 1 - \frac{m_H^2}{m_Z^2} \right)^3 \left( 1 + 0.17 \frac{m_H^2}{m_Z^2} \right)$$

for $\sin^2\theta_w = 0.20$. We therefore see that $B(Z^0 \rightarrow H + \gamma) \leq B(Z^0 \rightarrow H^0 e^+e^-)$ for $m_H < 0.6 m_Z$ (see Fig. 8), with a total branching ratio $B(Z^0 \rightarrow H + \gamma) \sim (1$ to 2 $) \times 10^{-6}$. The final state may be cleaner than in the $H^0 e^+e^-$ case, which could be polluted by decays involving heavy quarks and their subsequent semileptonic decays. On the other hand, the $H^0\gamma$ final state may be confused with the radiative reaction
Figure 8: The ratios $\Gamma(Z^0 \rightarrow H^0 \mu^+ \mu^-)/\Gamma(Z^0 \rightarrow \mu^+ \mu^-)$ and $\Gamma(Z^0 \rightarrow H^0 \gamma)/\Gamma(Z \rightarrow \mu^+ \mu^-)$ as functions of $m_H/m_Z$ for $\sin^2 \theta_W = 0.20^{+0.04}_{-0.03}$.
\( Z^0 \rightarrow q\bar{q} \gamma \) which occurs in lower order in \( a \) than \( Z^0 \rightarrow q\bar{q} e^+ e^- \). This background can be reliably computed in QCD\(^3\) and should enable a reliable assessment of the gravity of this potential background.

\( Z^0 \rightarrow H^+ H^- \)

The previous two reactions involved the single neutral Higgs boson found in the minimal Weinberg-Salam model where symmetry breaking is obtained from just one Higgs multiplet. If there are more than one Higgs multiplet, there will be additional physical charged Higgs bosons, as well as extra neutral ones. The decay rate

\[
\Gamma(Z^0 \rightarrow H^+ H^-) = \frac{G_F m_Z^3}{96 \sqrt{2} \pi}
\]

for charged Higgs particles with \( m_H \ll m_{Z/2} \), corresponding to a branching ratio at the percent level. One might guess that each charged Higgs particle may like to decay into pairs of heavy quarks: \( H^+ \rightarrow Q \bar{Q}' \) resulting in distinctive final states which should be detectable if \( H^+ \) exist.

\( Z^0 \rightarrow H^0 H_2^0 \)

If there is only one Higgs multiplet and hence only one neutral Higgs boson, the decay \( Z^0 \rightarrow H^0 H^0 \) is forbidden by Bose symmetry. On the other hand, if there is more than one Higgs multiplet and hence more than one neutral Higgs boson, decays like \( Z^0 \rightarrow H^0 H_2^0 \) become possible, and might have branching ratios up to the percent level. As in the case of \( Z^0 \rightarrow H^+ H^- \), decays into heavy quarks might provide a useful signature for such final states.

6. **Summary**

Theoretical studies of the \( Z^0 \) peak in \( e^+ e^- \) annihilation suggest the following conclusions:

a) Large event rates can be expected near the \( Z^0 \) peak, which should enable precision measurements of important fundamental parameters like \( m_Z, \Gamma_Z, \) the neutral current couplings of fermions, \( \sin^2 \theta_W \), etc.

b) One should also be able to search for rare decay modes of great interest, such as the Higgs boson processes \( Z^0 \rightarrow H^0 t^+ t^- \), \( Z^0 \rightarrow H^0 \gamma \).

c) Combining \( Z^0 \) width and mass measurements one should be able to determine the complete fermion spectrum: the number of neutrinos, the possible existence of massive fermions, copious decays into fermions with masses \( < m_{Z/2} \).

d) A topic not emphasized here, but discussed elsewhere\(^8\), is the possibility of detailed strong interaction studies with tens of millions of events at \( Q^2 \sim 10^4 \) GeV\(^2 \) - a cornucopia not available in any other way.
Of course we hope for and expect surprises, but even the above minimal shopping list of predictable physics suggests that the "Z^0 factory" aspect of LEP should be a copious source of new physics.

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10. A recent review was compiled by C. Baltay for the 1978 Tokyo International Conference on High Energy Physics.
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19. M. Veltman. Talk presented at this meeting and private communications.
20. See Section 3 of this report, reference 2, and references therein.
23. ABCLOS Collaboration, private communication.


Most theorists now believe that weak, electromagnetic and strong interactions are described by gauge theories: the leading candidate for the strong interactions is quantum chromodynamics (QCD), unbroken colour SU(3); for the electroweak interactions the minimal model is that of Weinberg-Salam. More generally I shall refer to the electroweak gauge theory as quantum asthenodynamics (QAD: the Greek word 'asthenos' means "weak, without strength").

So far we have acquired a certain amount of indirect evidence that gauge theories are relevant to nature: the anomalous dimensions of the deep inelastic structure functions measured in BEBC agree with those predicted in QCD; QAD is supported experimentally by the verification of both the Weinberg-Salam and GLM predictions. The anticipated discoveries of the W ± Z in forthcoming facilities will provide even more compelling evidence. However, at least in the case of QAD, these data provide evidence only for couplings of vector bosons to fermions which obey certain symmetry requirements. The property specific to a locally gauge invariant theory is the self interactions of vector bosons: trilinear and quadrilinear vector couplings characterized by the same strength as the vector-fermion couplings. Isolation of these couplings would provide direct evidence for gauge theories.

For QCD, isolation of tri- and quadrilinear gluon vertices is in principle possible in, for example, high $p_T$ jet production in hadronic collisions or in $\Upsilon$ decay. However interpretation of such data will be difficult to say the least. Here I shall be concerned with the self interactions of the heavy vector bosons of QAD as well as a second signature, unique to spontaneously broken gauge theories: The Higgs boson.

1. The Trilinear Boson Couplings of QAD

The obvious way to measure the $ZW^\pm$ and $\gamma W^\mp$ vertices in $e^+e^-$ collisions is via the annihilation diagram of Figure 1. As this requires a machine energy above the $W$ threshold, one may ask if there are means of studying these vertices at lower energy. Candidate processes for probing the $\gamma W$ vertex are depicted in Figure 2. The total cross-section for the process

$$e^+e^- \rightarrow e^\pm + W^\mp + \nu,$$  \hspace{1cm} (1)

including the contribution of Figure 2.a) has been calculated and found to have a very small cross-section for energies below threshold.
Figure 1: Gauge theory annihilation diagram for $W$-pair production.

Figure 2: Processes which might probe the $\gamma WW$ vertex below the $W$-pair threshold.

Figure 3: $Z$ decay channel which might probe the $ZZW$ vertex.
For example, in the Weinberg-Salam model with $\sin^2 \theta_W = 0.20$, $m_W = 84$ GeV, the cross-section at $\sqrt{s} = 150$ GeV, i.e. nearly at $2W$ threshold, is only

$$\Gamma(e^+e^- \rightarrow W\nu) = 8 \times 10^{-38} \text{ cm}^2$$

considerably lower than the $W$-pair production cross-section just above threshold, and falls rapidly at lower energies. The contribution of Figure 2.b) to the process

$$e^+e^- \rightarrow \gamma W^+W^-$$

has not been calculated, but is expected to be similar to (1) in order of magnitude. Therefore, unless there are very large deviations from the QAD couplings, these processes do not seem to provide useful probes.

A possible probe of the $ZWW$ vertex is through the decay of the $Z$ into one real and one virtual $W$

$$Z \rightarrow W + "W" \rightarrow f\bar{f}$$

shown in Figure 3. The branching ratio for this process in the Weinberg-Salam model with currently accepted parameters has been found to be

$$\Gamma(Z \rightarrow W + X)/\Gamma_Z \rightarrow 10^{-7}$$

giving a cross-section at the $Z$ peak

$$\sigma_p(e^+e^- \rightarrow W + X) \sim 10^{-38} \text{ cm}$$

which is at the limit of detectable rates.

We are therefore led to consider $W$ pair production

$$e^+e^- \rightarrow W^+W^-$$

as the only sensitive probe of trilinear vector boson couplings. The total cross-section, calculated in the Weinberg-Salam model for several values of $\sin^2 \theta_W$, is shown as a function of energy in Figure 4. For $\sin^2 \theta_W = 0.20$ the threshold will be higher ($\sqrt{s} = 170$ GeV), but the cross-section will peak (at about $\sqrt{s} = 200$ GeV) at a higher value. In addition to the "interesting" annihilation processes of Figure 1, there is a contribution from the "uninteresting" neutrino exchange diagram of Figure 5.a) - uninteresting in the sense that the couplings involved are already known from low energy data. Unfortunately, this gives the dominant contribution for energies not far above threshold as seen in Figure 6, where different contributions to the cross-section are plotted separately. However, Figure 6 also illustrates the sensitivity of the total cross-section to the delicate cancellations required by a gauge theory: each of the interference terms gives a negative contribution comparable in magnitude to the square of each contribution.
Figure 4: Cross-section\textsuperscript{9)} for $e^+e^- \rightarrow W^+W^-$ in the Weinberg-Salam model.
Figure 5: Conventional (a) and exotic (b) exchange diagrams for $e^+e^- \rightarrow W^+W^-$. 

Figure 6: Separate contributions $^9$ to the cross-section for $e^+e^- \rightarrow W^+W^-$. 

$\sin^2 \theta_W = 3/8$
Karel Gaemmers has explained in detail how the use of longitudinally polarized beams together with measurements of W polarizations through the angular distributions of their decay products allows the experimental separation of different contributions to the cross-section\textsuperscript{10}). I shall discuss instead what information can be extracted without the benefit of polarization. Discarding the possibility of the exchange of a doubly charged fermion as in Figure 5.b), there are only two independent amplitudes at fixed energy in any gauge model. This is because the form of the trilinear vector boson coupling is uniquely determined by the requirement of renormalizability. The annihilation amplitude, Figure 1, can differ from one gauge model to another only by the coupling strengths and the number and masses of the exchanged neutral bosons. The exchange annihilation and interference contributions to the total cross-section can be written, respectively, in the form:

\[
\frac{d\sigma_{\text{ex}}}{d\cos \theta} (s, \theta) = M_{\text{ex}}(s) F_1(s, \theta) \\
\frac{d\sigma_{\text{an}}}{d\cos \theta} (s, \theta) = M_{\text{an}}(s) F_2(s, \theta) \\
\frac{d\sigma_{\text{int}}}{d\cos \theta} (s, \theta) = M_{\text{int}}(s) F_3(s, \theta)
\]

(7)

where the angular functions\textsuperscript{9)} \(F_i(s, \theta)\) are model independent; the model dependence, including boson propagators is contained in the amplitudes \(M(s)\), which can in principle be extracted from experiment by weighting the data with appropriate angular projection operators\textsuperscript{11)} \(G_i(s, \theta)\):

\[
\int_{-1}^{1} d\cos \theta \ G_i(s, \theta) \ F_j(s, \theta) = \delta_{ij}.
\]

(8)

The angular dependence of the \(F_i\) and \(G_i\) is plotted in Figure 7 for different values of \(r = s/4 M_W^2\). As an exercise to test the sensitivity of a prototype experiment, a gedanken experiment\textsuperscript{11)} was done at 10 values of \(r\) assuming the true model to be Weinberg-Salam with \(\sin^2 \theta_W = 3/8\) and 250 running hours, or 100 events, per \(r\)-value. The resultant error bars are shown in Figure 8, along with theoretical values of the amplitudes \(M\) for a variety of models and two values of \(\sin^2 \theta_W\). Most of these models are by now experimentally disfavoured, but Fig. 8 serves to illustrate a few points.

(a) Unless there is a \(Z^0\) with mass above or just below the WW threshold (e.g. model c with \(\sin^2 \theta = 3/8\)) the energy dependence of the amplitudes is very insensitive to the \(Z\) propagators. (On the other hand it is sensitive to substantial deviations from the gauge theory three-vector vertex). It would therefore seem more profitable to accumulate data at the maximum cross-section energy which is about 200 GeV in the centre-of-mass for a W mass of 85 GeV.
Figure 7: Angular dependence of the functions $F_i(s, \theta)$ and the projection functions $G_i(s, \theta)$ for different values of $r = s/s_{thr}$. 
A right-handed $e\nu W$ coupling ($M_\nu << M_W$) doubles the exchange cross-section and is therefore clearly discernable. Different models for the annihilation channel will be harder to disentangle, and the annihilation exchange interference amplitude appears to provide the best probe of the $Z^0$ couplings.

The curves in Figure 8 were plotted assuming a branching ratio of 20\% for the trigger channel. The properties of the $W$ are of course model-dependent; we shall recall its properties in the Weinberg-Salam model. The $W$ mass is given by:

$$ M_W = \left( \frac{\alpha}{\sqrt{2}G_F} \right)^{1/2} \frac{1}{\sin^2 \theta_W} \approx 84 \text{ GeV, for } \sin^2 \theta_W = 0.2 $$

(8)

The partial decay width for $W \to e\nu_e$ is

$$ \Gamma(W \to e^-\bar{\nu}_e) = \frac{G_F M_W^3}{\sqrt{2}6 \pi} \approx 230 \text{ MeV, for } \sin^2 \theta_W = 0.2 $$

(9)

giving a total width of

$$ \Gamma_W = 4N \Gamma(W \to e\nu_e) = 900 \text{ MeV x N, for } \sin^2 \theta_W = 0.2 $$

(10)

going more than the number of lepton doublets and the number of coloured quark doublets. We now know that if this model is correct we must have at least $N = 3$, giving

$$ \Gamma_W \sim 2.7 \text{ GeV} $$

(11)

The leptonic branching ratio is

$$ B(e\nu) = B(\mu\nu) = B(\tau\nu) = \frac{1}{4N} = \frac{1}{12} \,, \text{ for } N = 3 $$

(12)

(provided $m_\nu << m_W$; in this model $1/12$ is therefore the upper limit for the leptonic branching ratio.

If the trigger for $W$-pair production is either one $e\nu$ or one $\mu\nu$ decay, with one hadronic decay required for mass reconstruction, the effective branching ratio is $1/8 - or 1/4 if the experiment can trigger on both muons and electrons.

A much higher counting rate will be obtained if 4-jet events can be used as a trigger. Our present ideas\(^1\) about perturbative QCD, as well as the jets observed at DESY at a c.m. energy of about 10 GeV, suggest that a purely hadronic trigger should indeed be feasible.

2. The Quadrilinear Boson Coupling

The quadrilinear boson coupling (Figure 9) can in principle be probed by the processes (Figure 10)

$$ e^+e^- \to W^+W^- \gamma $$

(13)

and
Figure 8: Predicted energy dependence$^{11)}$ of the amplitudes $M(s)$ defined in eq. (7) for several models: (a) photon and $\nu$ exchange only; (b) Weinberg-Salam; (c) $\text{SU}(2)_L \times \text{SU}(2)_R \times \text{U}(1)$; (d) $\text{SU}(2) \times \text{U}(1)$ with an $(e_R, \nu_R)$ weak doublet; solid lines: $\sin^2 \theta_W = 3/8$; broken lines: $\sin^2 \theta_W = 1/4$. 


which have been studied by Renard\textsuperscript{14}). The contribution of the diagram of Figure 10(a) to the cross-section for the process (13) is shown in Figure 11. It peaks at a value

\[ \sigma_{\text{max}} \approx 2 \times 10^{37} \text{ cm}^2 \]  

at c.m. energy

\[ \sqrt{s} = 4M_W - \frac{M_Z^2}{4M_W} = M_W(4 - \frac{1}{4\cos^2\theta_W}) = 310 \text{ GeV} \]

in the Weinberg-Salam model with \( \sin^2\theta_W = 0.2 \).

For \( \sqrt{s} \lesssim 200 \text{ GeV} \), the cross-section is only a few \( 10^{-38} \text{ cm}^2 \) and probably not observable, but such a study might become feasible if the energy is pushed slightly higher. Other contributions to the process (13) are shown in Figure 11. The (for present purposes uninteresting!) trilinear vertex contributions from diagrams like Figure 12(a) could be eliminated if the beams were longitudinally polarized, and the electron bremsstrahlung contribution of Figure 12(b) can be suppressed by excluding small angle photons.

The two-photon process of Eq. (14) and Figure 10(b) does not appear promising. The cross-section for \( \gamma\gamma \rightarrow W^+W^- \) is less than \( 10^{-35} \text{ cm}^2 \), giving a cross-section for (14) less than \( 10^{-38} \) without electron tagging.

3. What is the Higgs Particle?

The second characteristic of a spontaneously broken gauge theory is the Higgs particle, and I will briefly recall its properties in the "minimal" Weinberg-Salam model with just one physical Higgs boson. One starts with a gauge theory of massless fermions and vector bosons: the vector bosons are massless in order to preserve local gauge invariance, i.e. group transformations which differ from one space-time point to another; the fermions are massless because the gauge group is chiral: left and right helicity components transform differently and must therefore be decoupled. This theory is renormalizable and wrong: it does not describe the real world. To cure this one adds scalar particles with gauge invariant couplings to fermions, vector bosons and each other. The minimal scalar content which can account for the observed mass spectrum is a complex scalar doublet:

\[ \phi = \left( \begin{array}{c} \phi^+ \\ \phi^0 \end{array} \right) \]

Its self-couplings specify the "Higgs potential":

\[ v(\phi) = -\mu^2 |\phi|^2 + \lambda |\phi|^4 \]
Figure 9: Quadrilinear vector boson vertex in lowest order.

\[ \chi = \chi + \chi + \chi \]

Figure 10: Processes which could probe the quadrilinear vector boson vertex

Figure 11: Contribution of the diagram of Figure 10(a) to the cross-section for \( e^+e^- \rightarrow w^+w^-\gamma \).

Figure 12: Trilinear vector coupling contributions to the process \( e^+e^- \rightarrow w^+w^-\gamma \).
which, for $\mu^2 > 0$, has an absolute minimum at non-zero field strength:

$$|\phi|_{\text{min}}^2 = \frac{\mu^2}{2\lambda} \equiv \frac{v^2}{2} \neq 0.$$  \hspace{1cm} (19)

In order to apply conventional perturbation theory we must redefine the Higgs field so that the vacuum (minimum potential) corresponds to zero field strength:

$$\phi \rightarrow \phi' \equiv \phi - \phi_{\text{min}}' \hspace{1cm} \begin{vmatrix} V(\phi) \rightarrow V'(\phi') \end{vmatrix} : \begin{vmatrix} \frac{3v'}{3\phi'} \phi' = 0 \end{vmatrix}$$  \hspace{1cm} (20)

Now, we started with a complex doublet field which contains four real fields. After symmetry breaking the $w^+$, $w^-$ and $Z$ will acquire a mass, and therefore a longitudinal component of polarization. These additional degrees of freedom are provided by three of the scalar fields:

$$\phi^+, \phi^- \equiv \tilde{\phi}^+, \chi \equiv (\phi^0 - \bar{\phi}^0)/\sqrt{2}$$  \hspace{1cm} (21)

which disappear from the theory as physical particles. There remains one scalar degree of freedom which must necessarily appear as a physical particle. We write

$$\frac{\phi_0 + \bar{\phi}_0}{\sqrt{2}} = H + v$$  \hspace{1cm} (22)

where $H$ is the physical Higgs field and $v$ is the vacuum expectation value of the field $\phi$ which can always be chosen real:

$$\phi_{\text{min}} = \frac{v}{\sqrt{2}} = \phi'_{\text{min}}.$$  \hspace{1cm} (23)

The Higgs potential (18) depends only on $|\phi|^2$; the direction in SU(2) space chosen by $\phi_{\text{min}}$ is what explicitly breaks the symmetry. Since we know that electric charge remains conserved in nature, $\phi_{\text{min}}$ must be of the form (23).

To see how the fermions and vector mesons of the theory acquire masses, we look at their primary couplings to the scalar doublet $\phi$, or more particularly its neutral component. For fermions there are Yukawa couplings of the form:

$$L_Y = \frac{g_{\bar{f}f}}{\sqrt{2}} \bar{f}f \phi + \text{h.c.},$$  \hspace{1cm} (24)

which, after the redefinition (22) becomes

$$L_Y = g_{\bar{f}f}(H + v) \equiv g_{\bar{f}f} \bar{f}fH + m_f \bar{f}f$$  \hspace{1cm} (25)

A fermion mass term has appeared, related to the Yukawa coupling of the physical scalar $H$ by

$$g_{\bar{f}f} = m_f/v = g_{\bar{f}f}.$$  \hspace{1cm} (26)
Vector boson masses arise from the gauge couplings

\[ |(\tilde{w}_\mu + ig (\sigma^i/2) \tilde{w}_\mu^i + i \frac{g}{2} B_\mu) \phi|^2 \tag{27} \]

where \( \tilde{w}_\mu \), and \( B_\mu \) are the triplet and singlet respectively, of SU(2) \(_L\) \( \times U(1) \). For example there is a quartic coupling term in (27):

\[ \frac{g^2}{2} w^\mu w^-|\phi^0|^2 = \frac{g^2}{4} w^\mu w^- (H^2 + 2vH + v^2) \]

\[ \equiv \frac{g^2}{4} H^2 w^\mu w^- + g_{WH} H w^\mu w^- + M^2 w^\mu w^- \tag{28} \]

giving the relation between mass and coupling constant:

\[ g_{WH} = 2M_w^2/v = g^2 v/2. \tag{29} \]

Since we know the Fermi constant from experiment

\[ GF/\sqrt{2} = \frac{g^2}{8M_w^2} = \frac{10^{-5}}{\sqrt{2} M_P^2} \tag{30} \]

we have a determination of the vacuum expectation value \( v \)

\[ v^2 = 4M_w^2/g^2 = (\sqrt{2}G_F)^{-1} \tag{31} \]

which in turn specifies the coupling to any matter or gauge particle:

\[ g_{fH} = \frac{M_f}{v} = 3.8 \times 10^{-3} \frac{M_f}{M_P} \tag{32(a)} \]

for any fermion \( f \), and

\[ g_{VH} = 2M_V^2/v = 3.8 \times 10^{-3} \times 2M_V^2/M_P \tag{32(b)} \]

for any (complex) vector particle \( V \) (for real vector fields the coupling is half of (32(b)).

Equations (32) represent the physics of the Higgs particle\(^{15}\). Their immediate implications are

a) the Higgs particle will be most copiously produced in association with heavy particles, and

b) the Higgs particle will decay preferentially into the heaviest kinematically available particle).

If light enough, the Higgs particle can be produced in the decays of heavy particles; examples are Z-decay

\[ Z \rightarrow H + \bar{e}^+ e^- \tag{33(a)} \]

and the decay of heavy onia

\[ V(Q\bar{Q}) \rightarrow H + \gamma \tag{33(b)} \]
as discussed in the review by J. Ellis, later in this paper. If it is not light enough to appear as a decay product, one will have to look for Higgs bremsstrahlung\cite{15}) in heavy particle production as shown in Figure 13.

For Higgs masses pertinent to LEP (i.e. which forbid production via decays as in (33)), the cross-section\cite{16}) for heavy fermion pair production with Higgs bremsstrahlung (Figure 13.b)) appears to be too small to be useful ($< 10^{-38}$ cm$^2$), and the most promising production mode for a Higgs of mass greater than 40 GeV or so is via Bremsstrahlung from a virtual $Z^0$, Figure 13.a). Cross-sections\cite{17}) for different masses and energies are shown in Figure 14, calculated in the Weinberg-Salam model with $\sin^2 \theta_w = .20 - .27$. An advantage of this mechanism is that it provides a very clear signal via the leptonic decays $Z^0 \rightarrow e^+ e^-$ or $\mu^+ \mu^-$; the Higgs particle can be detected by looking for a peak in the recoil mass. However as the leptonic branching ratio is expected to be rather small

$$B(\mu \mu) = B(ee) \approx 3\%$$

if there are no more than three lepton and quark doublets with mass less than $M_{Z/2}$, it may be necessary to try to select the $Z$ via its two-jet hadronic decay in order to gain in rate if $M_H \geq 30$ GeV. The Higgs decays will be messy; they will probably contain a relatively high percentage of leptons from heavy quark and/or lepton cascade decays and may have a two-jet structure, depending on the relative mass of the Higgs meson and its primary decay product.

4. What is the Mass of the Higgs Particle?

Making the substitution (22) in the Higgs potential (18) we get

$$V(\phi) = V(H + v) = -\frac{u^2}{2}(H + v)^2 + \frac{\lambda}{4}(H + v)^4$$

$$= u^2(H^2 + \frac{1}{v}H^3 + \frac{1}{4v^2} H^4)$$

(34)

where we have used the relation (19). The potential (34) determines the Higgs self couplings as well as its mass

$$M^2_H = 2u^2 = 2\lambda v^2$$

(35)

in terms of an arbitrary parameter of the model: $u^2$ or $\lambda$. In fact, the self coupling constant $\lambda$ is not completely arbitrary, since radiative corrections induce effective self couplings (Figure 15) which are calculable, and it has been shown\cite{18}) that the stability of the vacuum, i.e. the requirement that $|\phi| = v^2/2$ be an absolute minimum:

$$V(v^2/2) - V(0) < 0$$

(36)

imposes a lower bound on the mass of the physical Higgs particle

$$M_H \geq 9 \text{ GeV}$$

(37)

in the Weinberg-Salam model with $\sin^2 \theta_w = 0.2$. The bound (37) is not iron-clad.
Figure 13: Higgs production via Bremsstrahlung from a heavy vector meson (a) or a fermion (b).

Figure 14: The cross-section\(^{17}\) for \(e^+e^- \rightarrow Z + H\) relative to the QED cross-section or \(e^+e^- \rightarrow \mu^+\mu^-\) as a function of the Higgs mass and for several values of the c.m. energy. The error bars include the range of values \(0.22 \leq \sin^2\theta_W \leq 0.29\).
Figure 15: Radiative corrections to the quartic coupling in the Higgs potential.

Figure 16: The $J = 0$ partial wave amplitude for $W_L^+ W_L^- \to W_L^+ W_L^-$ elastic scattering in the tree approximation.
It could be violated if:

(a) the physical vacuum is not an absolute minimum of the potential. In this case vacuum tunneling could occur, and the age of the universe bounds the tunneling amplitude; if the potential difference (36) is positive, it cannot be too large\(^1\). This gives\(^2\)

\[
M_H \geq 300 \text{ MeV.} \tag{38}
\]

(b) there is a fermion with mass comparable to the W and Z masses; the fermion loop of Figure 15.b) contributes with opposite sign to the vector boson loop of Figure 15.a) and could cancel the effect (recall: coupling \( \propto \) physical mass).

(c) the Higgs sector is more complicated. If one adds more Higgs doublets to the Weinberg-Salam model, the bound (37) applies only to the heaviest physical state.

It would be more useful to LEP studies if we could bound the Higgs mass from above rather than from below. However, the only upper bound at present is a philosophical one. From Eq. (35) we see that if we let the Higgs mass become arbitrarily large, the coupling constant

\[
\lambda = \frac{M_H^2}{2v} \tag{39}
\]

also becomes large, inducing arbitrarily large couplings of the Higgs meson to itself and to longitudinally polarized vector mesons. While the theory remains technically renormalizable, it becomes practically uncalculable. If we simply demand that the perturbation expansion converge,

\[
\frac{\lambda^2}{4\pi^2} \ll 1 \tag{40}
\]

we obtain a bound

\[
M_H \ll \sqrt{\frac{\lambda}{\pi}} v = 1.2 \text{ TeV.} \tag{41}
\]

A more meaningful question is: at what energy must we see either the Higgs particle or the effects of a large self coupling? As stressed by Veltman, if \(M_H \geq 300 \text{ GeV,} \) renormalization effects due to Higgs exchange may become observable (~ 10\%) at a centre of mass energy \(\sqrt{s} = 300 \text{ GeV.} \) In an alternative approach, other authors\(^{21}\) have considered the scattering of longitudinally polarized W's. The \(J = 0\) partial wave amplitude for \(W^+_L W^-_L\) elastic scattering in the tree approximation is shown in Figure 16 for \(M_H = 300 \text{ GeV and 1.2 TeV.} \) For the lower mass, the amplitude remains well below the unitarity bound \(t^0 \leq 1\) except at \(s = M_H^2\) when the finite decay width will regularize it. However for \(M_H \geq 1.2 \text{ TeV,}\) partial wave elastic unitarity is violated in the tree approximation. This means that the tree approximation fails and one encounters a strong interaction problem in the \(H, W^+_L, Z^-_L\) sector and one may anticipate bound states and Regge trajectories as in low energy hadronic physics. Whether any of these objects will fall in an energy
range accessible at LEP is under study\textsuperscript{22}). A more modest question is whether final state $W^+W^-$ interactions due to Higgs exchange might be important (say $\geq 10\%$), providing a probe of Higgs masses above threshold. Unfortunately, the $J = 0$ channel, where this can indeed be the case, is not accessible in $e^+e^- \rightarrow W^+W^-$ because of helicity conservation for the quasi-massless electrons (related to the effective decoupling of the Higgs from the electron in the minimal model). A study of the full radiative corrections to $e^+e^- \rightarrow W^+W^-$ which might disclose some sensitivity to high mass Higgs exchange is under way\textsuperscript{23}).

5. Complex Higgs Sectors

A priori there is a complete arbitrariness in the specialization of the Higgs sector. However, experiment already tells us that the relation of the Weinberg angle (which specifies the admixture of the V-A weak isospin and the vector electromagnetic currents in the weakly coupled neutral current) to the vector boson masses (which specify the magnitude of the effective fermi couplings) agrees with the prediction of the minimal model

$$\frac{M_Z^2 \cos^2 \theta}{M_W^2} = 1$$

(42)

to within about five per cent. In a general model with Higgs mesons if weak isospin ($I_1$, $I_3$) and vacuum expectation values $v_i$, one has

$$\frac{M_Z^2 \cos^2 \theta}{M_W^2} = \frac{2 \sum I_3^2 v_i^2}{\Sigma I(I+1) - I_3^2 v_i^2}$$

(43)

The right hand side of (43) is unity for $I = \frac{1}{2}$; the next lowest multiplet\textsuperscript{24}) which gives unity is $I = 3$, $I_3 = \pm 2$. Since the member with $v_i \neq 0$ is necessarily neutral, the latter case would involve a typical Higgs boson with charge $|Q| = 5$. The next lowest multiplet after $I = 3$ is $I = 25/2$, $I_3 = \pm 15/2$, which would require a physical particle with $|Q| = 20$! While such objects, if not too heavy, would imply spectacular physics at LEP, I shall dismiss them as implausible and consider only Higgs multiplets with $I = \frac{1}{2}$. Then the only open question is the number of such multiplets.

If there are more than one Higgs doublet, their couplings are not uniquely specified. Instead of Eqs. (28) and (29) one gets

$$M_W^2 = \frac{g_e^2}{4} \sum v_i^2; \quad |g_{WH_i}| = |v_i \frac{g_e^2}{2} | < gM_W$$

(44)

which means that the coupling of each Higgs meson to the $W$ is smaller than in the minimal model. For fermions, one gets instead of (25) and (26)
Since the signs of the $v_i$ and the $f^{\phi_i}_{f_i}$ are arbitrary, there are no inequalities bounding the Yukawa couplings which are a priori arbitrarily large. In addition, as mentioned above, in a model with more than one Higgs doublet, the lower bound on the Higgs mass applies only to the heaviest state. Therefore, in a more general model one can have arbitrarily light Higgs mesons with arbitrarily strong couplings to fermions.

However, the Yukawa coupling constant is really a coupling matrix:

$$g_{fi} = \Sigma f^{\phi \alpha}_{\beta} g_{\phi_i}$$

which must be diagonalized to define the strong interaction eigenstates. The transformation which diagonalizes the matrix (47) will not in general diagonalize the separate coupling matrices $g_{Hi}$ and one expects flavour changing, hadronic charge conserving, transitions. It was precisely to prevent such transitions that charm was conceived, and to remain consistent with the observed suppression of $\Delta S \neq 0, \Delta Q = 0$, one must impose

$$M_{Hi}^2 \geq G_F^{-1} \approx \left(300 \text{ GeV}\right)^2$$

which is edging into the realm of a strongly interacting Higgs sector. This problem can be evaded by imposing an extra symmetry which inhibits flavour mixing in the Higgs sector; one such example is the axion\(^25\).

Generally, if there are $N$ weak doublets of Higgs particles in the Weinberg-Salam model, there will be $N - 1$ physical states with $Q = +1$ and with $Q = -1$ (one of each having become longitudinal components of $W^\pm$), and $2N - 1$ neutral states (one having been similarly eaten by the $Z^0$). Predictions of their specific properties are highly model dependent, but one would expect their couplings to violate $e, \mu, \tau, \ldots$ universality. Charged Higgs production could occur in $e^+e^-$ annihilation via the usual one-photon exchange process (Figure 17.a)) and would be characterized by a change in $R$ of one quarter unit and a $p$-wave threshold behaviour.

If the coupling to electrons were anomalously larger for some neutral Higgs boson it might be visible as a direct channel resonance (Figure 17.b)). If longitudinally polarized beams are feasible, a sensitive probe for such objects would be scattering in the 'wrong' helicity channel,

$$e^+e^- \text{ or } e^+_L e^-_R \rightarrow \text{ anything}$$

(49)
Figure 17: Mechanisms for production of charged (a) and neutral (b) Higgs mesons in models with more than one Higgs doublet.

Figure 18: Non-conventional t-channel exchange process.
which excludes conventional channels except for photon-photon scattering processes and elastic electron scattering. A signal for the process (49) with no final state electrons and with relative cross-section $\gg 0(m_e^2/s)$ would imply either a non-conventional Higgs sector or a non-conventional t-channel exchange process as depicted in Figure 18. The latter processes are in themselves interesting to look for, and they are asymptotically ($s \gg m_X^2$) favoured relative to s-channel exchange processes:

$$\frac{\sigma_L}{\sigma_S} \rightarrow \frac{3s}{s} \frac{m^2}{m_e^2} \text{ coupling ratio} \quad (50)$$

In the conventional theory, the only allowed final states in t-channel exchange processes are $(L_1, L_2) = (e^+, e^-)$ or $(\bar{\nu}_e, \nu_e)$. The moral is then that, whatever the true theory, scattering in the "wrong" polarization channel would be a sensitive probe of new phenomena if the two-photon scattering background can be eliminated.

6. New Things

What new phenomena might we hope for beyond the verification of strong and electro-weak gauge theories? We have at present virtually no understanding of the fermion mass spectrum. If new lepton and quark states were discovered, there might emerge some discernable pattern of masses and mixing angles which would give us a clue to this problem. It is also widely speculated that a larger, "grand unified", gauge group underlies the presently "observed" SU(3)$_{\text{colour}}$ $\otimes$ SU(2)$_L$ $\otimes$ U(1) gauge group. It might be hoped that higher energies will begin to probe these additional couplings.

The most pessimistic picture for LEP is that the minimal "grand unified" model$^{26}$ is the correct one. This model has met with some phenomenological success in that it predicts$^{27}$

$$\sin^2\theta_W = 0.20 \pm 0.01$$

$$m_S \simeq 500 \text{ MeV}$$

$$m_b \simeq (5 - 6) \text{ GeV} \quad (51)$$

when the mass predictions refer to "constituent" s and b quarks and if there are only six quark flavours. The "unification mass", i.e. the energy at which new interactions become comparable to the usual electro-weak ones, is of the order of $10^{15}$ GeV. This picture then predicts that absolutely nothing will happen between present energies and $10^{15}$ GeV except for the discoveries of the t-quark, the intermediate bosons, $W^\pm$, Z, of the Weinberg-Salam model, and possibly one or more Higgs bosons. The only open questions, experimentally, are how many Higgs bosons are there, and what are their masses. (In addition one might observe proton decay with sufficient effort but this is not an experiment for LEP!)
Figure 19: Pair production and annihilation of (real or virtual)
(a) electrically charged colour SU(3) monopoles, and
(b) magnetic monopoles for $\sqrt{s} - 2 m_H$. 
However, nature may not choose to be so boring, and one can imagine more colourful possibilities. For example, the grand unified model proposed by Pati and Salam\(^{28}\) is SU(4)\(^4\) with a multitude of observable phenomena: liberated quarks and gluons, more W's and Z's (but with masses \(\approx 300\) GeV from low energy phenomenology and probably \(\geq 1\) TeV if arguments from astrophysics are taken into account). The "unification mass" in this model is of the order of \(10^4\) GeV, much better than \(10^{15}\) for experimentalists (but still with a tiny effective Fermi coupling constant at LEP). One amusing possibility of this model\(^{30}\) is the potential existence of SU(3)\(_{\text{colour}}\) monopoles with mass

\[
m_M \sim \frac{m}{\alpha_s} \approx 100 \text{ GeV}
\]

if the gluon mass is \(m_g \approx 10\) GeV as has been conjectured. The monopole would couple to gluons with coupling strength \(0(1/\alpha_s)\) so one could expect a splash of gluons (hadrons) near threshold for \(e^+e^- \rightarrow 2\) monopoles via the electric charge coupling of the monopoles (Figure 19.a) (whether such objects really exist in the theory depends on the details of the Higgs mechanism which gives the gluons their masses).

An even more spectacular phenomenon could be the production of "conventional" electromagnetic monopoles since they couple to photons with strength \(0(1/e)\) and would give a very large jump in \(R\) via, for example, the two-photon production process of Figure 19.b) followed by annihilation (near threshold) into a multi-photon final state. (However, there are cosmological arguments\(^{31}\) which tend to rub out electromagnetic monopoles of accessible masses.)

Another exotic possibility is that super-symmetric theories are related to the real world, in which case one expects exotic partners\(^{32}\) of all known particles. The most relevant such objects for LEP would be the scalar partners of leptons\(^{33}\) which could be pair-produced and would subsequently decay into other new objects

\[
e^+e^- \rightarrow \bar{s}_e + s_e \rightarrow e^+ + \text{new fermions}
\]

Such events would appear as heavy leptons in their signatures, but would be distinguishable via their kinematics.

It should be clear that whatever surprises may or may not await us, even the most "boring" physics beyond the Z('s) - isolation of the multiboson couplings and probes of the Higgs sector - is of prime importance.
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Testing Strong Interaction Theories

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1. Alternative Theories of the Strong Interactions

You sometimes get the impression these days that there is only one theory of the strong interactions, namely Quantum Chromodynamics (QCD). There is merit to this view, but conclusive evidence of its truth and applicability to the real world is not yet available, so we are forced to entertain possible alternative theories, at least for a while. To generate these possibilities, let us go through the process of conceptual logic which leads one to QCD, and ask what alternatives exist at each stage of the logical development.

Most theorists believe that the strong interactions are described by field theory. The reasons for this are, first that the other fundamental interactions are well described by field theories, and secondly that field theory is a tight framework with well-defined calculated rules enabling one to make hard and fast predictions, the essence of a scientific theory. The applicability of field theory to the strong interactions can of course be doubted. While QED is one of the most successful of physical theories, no conclusive evidence yet exists of the field-theoretical nature of the weak interactions, and our only knowledge of gravity is purely classical, with no evidence for quantum field theoretic aspects such as the graviton or a higher order quantum correction. Alternatives to field theory usually resemble the naïve parton model which had strong interactions cut off arbitrarily at small distances. Quite often strings (or bags which can be elongated to look like strings at high momentum transfers) are postulated, which join together quarks or partons.

Field theories contain gluons. Clearly we need some force to hold quarks together in hadrons. In order to be calculable, a field theory must be renormalizable, and the only known way to give quarks renormalizable interactions is to couple them to bosonic gluons (see Figure 1.a)). Furthermore, renormalizability stipulates that these bosons have either spin 0 or spin 1 - two viable alternatives, with QCD belonging to the second class.

Coupling constraints are scale-dependent. Suppose one considers a theory with a single coupling constant g. Higher order corrections (as in Figure 1.b)) will renormalize this coupling, and they will vary with the scale at which it is measured - say the momentum transfer Q^2:

\[ g \rightarrow g + A_3 g^3 \ln Q^2 + A_5 g^5 \ln^2 Q^2 + \ldots \] (1)
These corrections can be absorbed into the definition of a $Q^2$-dependent effective coupling constant $\bar{g}(Q^2)$. Since we will be interested at LEP in momenta which are large on a typical hadronic scale, we then ask how $\bar{g}(Q^2)$ varies as $Q^2 \to \infty$. Other possibilities exist, but let us assume that some limit $g^*$ exists. Then we have three alternatives. Perhaps $g^* = 0$: such a theory is called asymptotically free, and QCD is the only example among renormalizable field theories. Perhaps $g^* \neq 0$: such theories are known as fixed point theories, and examples are known which combine vector and scalar gluons. Perhaps $g^* = \infty$: no field theory is known to have this property, because no-one knows how to do reliable calculations in theories with large coupling constants.

These various alternatives are shown in Figure 2, with the various statements underlined above formulated as questions in a sort of logical flow chart. Alternative theories are generated by different answers to these various questions.

2. Crucial Experimental Tests

We are here concerned with $e^+e^-$ annihilation at high energies. In order to discuss tests of strong interaction theories in such reactions, it is convenient to divide the centre-of-mass energy range available into four parts as in Figure 3, and discuss in turn the strong interaction studies possible in each region. The four regions in Figure 3 are located relative to a new heavy $Q\bar{Q}$ threshold, and are the subthreshold region of new $Q\bar{Q}$ onia, the threshold region of quasi-elastic production of new $Q\bar{Q}$ mesons-antimeson pairs, a continuum region where theories may be applied to the total cross-section and to inclusive hadron cross-sections but the phase space is perhaps not sufficient for the expected jet structure to have emerged, and finally the high-energy region where jet structure may be clearly seen. We will now sketch briefly the predictions of different categories of strong interaction theories for these regions, which are all summarized in Figure 2.

2.1 Onia

The best-known predictions for these bound states are the charmonium predictions of QCD. One eventually expects quasi-Coulombic spectroscopy, the widths of these states should get narrower at higher masses (the Zweig rule improves), there are many specific predictions for decay and transition rates, and the final states should contain gluon jets. By contrast, naive quark-parton or string models, while perhaps expecting a steadily improving Zweig rule, would not expect gluon jets in onium final states, which generally might contain just a pair of normal light $q\bar{q}$ jets. Field theories with fixed point couplings presumably would not lead to the usual charmonium picture. With the coupling constant finite or infinite at large mass, there would be no justification for
Figure 1: a) Quarks coupled via bosonic gluons with a coupling which is b) renormalized by higher order graphs.

Figure 3: The different regions of $e^+e^-$ centre-of-mass energy $Q$ in which one may make different tests of strong interaction theories, as shown in Figure 2.

Figure 4: The fundamental process $e^+e^- \rightarrow q\bar{q}g$ giving a three-jet final state.
Figure 2: Different theories of the strong interactions, and crucial experimental tests to distinguish them
applying perturbation theory, so no motivation for quasi-Coulombic spectroscopy or a steadily improving Zweig rule, let alone the dominance of a small number of gluon jets in the final state. Heavy $Q\bar{Q}$ would be as complicated as the $\rho$ or $\omega$.

### 2.2 Threshold Region

Many properties of heavy $Q\bar{Q}$ mesons are calculable in QCD. One expects them to be produced in $e^+e^-$ collisions with $Z = \frac{2E_{\text{meson}}}{Q} = 1^3$, that their weak decay rates should be reliably calculable in terms of free quark diagrams, with $Q \rightarrow q\bar{q}$ and $q\bar{v}$ final states dominating with a calculable semileptonic branching ratio$^{10}$. If the quark $Q$ is heavy enough, the nonleptonic final states should be populated with 6 light quark jets and probably be essentially like phase space close to threshold$^{11}$, producing jumps in the average values of integrated quantities like sphericity or thrust$^{12}$. The lepton spectra in semileptonic heavy quark decays are also calculable from the free quark diagram with calculable strong radiative corrections$^{10}$. In many ways, the production and decays of new heavy quarks would resemble that of new heavy leptons. This intuitive picture, based on the smallness of the strong interaction coupling at large mass, would presumably be even more valid in a naïve parton or related model where the coupling is cut off at large $Q^2$ $^9$. By contrast, no such intuition would apply to theories with a non-zero fixed point, where the total decay rates, semileptonic branching ratios, leptonic spectra and final state jet structures would all be incalculable because of large strong radiative corrections. Heavy meson production and decay would be as complicated as the dynamics of $K$ mesons.

### 2.3 Continuum

QCD expects$^1$ that the hadron to muon total cross-section ratio

$$R \rightarrow \frac{3}{Q^2 \rightarrow \infty} \sum_{\text{quarks}} \frac{e^2}{q} \left( 1 + \frac{\alpha_s(Q^2)}{\pi} + \ldots \right)$$

where $\alpha_s(Q^2) = \frac{12\pi}{(33 - 2f)} \ln Q^2$

where $f$ is the number of quark flavours. The inclusive hadron cross-sections are expected to exhibit logarithmic scaling violations very similar to those in deep inelastic structure functions$^1$:

$$\begin{align*}
\int_{0}^{1} dZ \int_{0}^{Q^2} \frac{dz}{z^n} &= \int_{0}^{Q^2} \frac{dz}{z^n} \left( \frac{d\sigma}{dz} (e^+e^- \rightarrow \text{hadron} (Z) + X) \right) \\
&\sim (\ln Q^2)^{-\gamma_n} \\
&Q^2 \rightarrow \infty \\
&x \equiv Q^2/2q_p \text{target}
\end{align*}$$

\[ (x \equiv Q^2/2q_p \text{target}) \]
for flavour non-singlet combinations, where

\[ \gamma_n = \frac{4}{33 - 2f} \left[ 1 - \frac{2}{n(n+1)} + 4 \sum_{j=2}^{n} \frac{1}{j} \right] \]  \hspace{1cm} (4)

For comparison, the na"ive parton model\textsuperscript{3} predicts also that

\[ \frac{R}{Q^2} \rightarrow 3 \sum_{q} e_q^2 \]  \hspace{1cm} (5)

but with corrections generally suppressed by powers of $Q^2$. Also the model is explicitly constructed to give exact scaling in inclusive hadron distributions, as well as in the Bjorken limit of deep inelastic scattering. Finally, theories with non-zero fixed point couplings generally predict that $R \rightarrow \text{constant as } Q^2 \rightarrow \infty$, but see no reason why that constant should be $3 \sum q e_q^2$. They predict violations of scaling in inclusive hadron distributions and deep inelastic structure functions by powers of $Q^2$

\[ \int_0^1 dZ Z^n - 1 \frac{d\sigma}{dZ} (e^+e^- \rightarrow \text{hadron} (Z) + X) \]  \hspace{1cm} (Q^2)^{-dn} (6)

\[ \int_0^1 dx x^n - 2 f_2 (x, Q^2) \]

The anomalous dimensions $dn$ are not in general calculable. If however the fixed point coupling were small so that low order perturbation theory were still applicable, one would find

\[ \frac{dn}{n} \alpha \left\{ \begin{array}{ll}
1 - \frac{2}{n(n+1)} & \text{for vector gluons} \\
1 - \frac{2}{n(n+1)} & \text{for scalar gluons}
\end{array} \right. \]  \hspace{1cm} (7)

\subsection*{2.4 Jet Structure}

QCD predicts the dominance of 2-jet final states in $e^+e^-$ annihilation\textsuperscript{13,14}. However, the $p_T$ transverse to the jet axes should not be limited:

\[ \frac{\langle p_T \rangle}{Q} = 0 \left( \frac{\alpha_s}{\pi} \right) Q^2 \rightarrow \propto \ln Q^2 \]  \hspace{1cm} (8)

In sufficiently extreme cases, the large $p_T$ will be manifested in the form of three-jet events\textsuperscript{13} (see Figure 4), with a cross-section

\[ \frac{1}{\sigma_{\text{total}}} \frac{d^2\sigma}{dz_q dz_{\bar{q}}} \approx \left( \frac{2\alpha_s}{3\pi} \right) \left( \frac{Z_q^2 + Z_{\bar{q}}^2}{(1 - Z_q)(1 - Z_{\bar{q}})} \right) \]  \hspace{1cm} (9)

In general there is a sort of jet perturbation theory with

\[ \frac{\sigma(n_{\text{jet}})}{\sigma_{\text{total}}} \approx \left( \frac{\alpha_s}{\pi} \right)^{n-2} \]  \hspace{1cm} (10)
Naïve parton or string models would also expect the dominance of 2-jet final states\(^3,^4\), but corrections analogous to (8), (9) and (10) should rather be suppressed by powers of \(Q^2\) as in large \(p_T\) hadron-hadron collisions. By contrast, theories with a non-trivial fixed point would see no reason why 2-jet final states should dominate, instead
\[
\frac{\langle p_T \rangle}{Q} = O(1)
\]
and, assuming one would in fact have "jets", the multiple jet cross-sections would presumably all be \(O(1)\)\(^5\).

3. Present Knowledge

After the previous section's brief summary of the predictions of different strong interaction theories, we will now look at the present evidence bearing on these predictions, to see whether any theory emerges as the most likely to be correct.

3.1 Onia

It seems that heavy \(Q\bar{Q}\) spectroscopy does indeed become simpler than that of the bound states of light quarks. The \(J/\psi - \psi' - P_c - \chi\) and \(T - T' - T''\) systems begin to look qualitatively Coulombic, the Zweig rule is better for the \(J/\psi\) than it was for the \(\phi\), and non-relativistic quark models for radiative transitions seem to work well qualitatively in the charmonium system. It is known\(^16\) that \(T\) hadronic final states have higher sphericity and lower thrust than the adjacent \(e^+e^-\) continuum, and that events are consistent with the planar production of three gluons and their subsequent evolution into hadrons with finite \(p_T\). On the other hand, there are still problems with the quantitative analysis of charmonium spectroscopy and decay rates, and there is as yet no conclusive evidence of the "smoking gluon", in the form of gluon jets\(^4\) in \(T\) decays. Thus oniology qualitatively favours QCD over other strong interaction theories, but the evidence is still rather circumstantial.

3.2 Threshold Region

As far as \(D\)-meson production is concerned, there is some evidence from neutrino collisions that the \(c\) quark \(\rightarrow D\) meson fragmentation function is flatter than that for \(u\) or \(d\) quarks \(\rightarrow n\)\(^17\), but no such evidence from \(e^+e^-\) collisions. As for \(D\)-meson decays, there is evidence\(^18\) that the decay rate is within a factor 2 of the expectation of a suitably sophisticated free quark calculation\(^19\), and strong evidence that non-leptonic decays are not strongly enhanced relative to semi-leptonic decays, as expected\(^10\) in QCD or naïve parton calculations. Not much evidence here.
3.3 Continuum

It is found in $e^+e^-$ annihilation that the total cross-section is consistent with 10 to 20% with the predictions (2) and (5), both above and below the charm threshold - a weak point against theories which do not predict $R = 3\xi e^2$. There is no evidence from $e^+e^-$ annihilation concerning the predictions (3) and (6), but there is very strong (conclusive?) related evidence from neutrino collisions. First the ABCLOS collaboration verified the prediction (3) for the moments of $x F_3$, confirming both that the ratios of anomalous dimensions were as predicted by lowest order perturbation theory for vector gluons as in QCD, and that the $Q^2$ dependence of the moments was logarithmic (3) rather than like a power (6). These data included rather low $Q^2 = 0(1 - 5)$ GeV$^2$, which led to the reproach that quasi-elastic events were important, and that target mass effects as manifested in the choice of naïve (3) or sophisticated Nachtmann moments were also significant, so that the agreement of ABCLOS data with QCD should be regarded as fortuitous. However, their results have recently been confirmed in a higher range of $Q^2$ (5 to 75 GeV$^2$) by the CDHS collaboration, in a kinematic region with little sensitivity to elastic events or target mass effects. The two sets of results on ratios of anomalous dimensions are shown in the Table below. QCD seems to be the only available strong interaction theory consistent with the ABCLOS and CDHS data. In $e^+e^-$ annihilation we are concerned with predictions for inclusive final state hadron production. Related data in neutrino production are now being analyzed by the ABCLOS collaboration, and there are some preliminary indications of agreement with the QCD predictions (3) for the moments of inclusive hadron cross-sections.

<table>
<thead>
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<tr>
<th>CDHS Experiment</th>
<th>ABCLOS</th>
<th>Theory</th>
</tr>
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<tbody>
<tr>
<td>$\int_0^1 x^{-n-1} F_3(x, Q^2) dx$</td>
<td>Nachtmann Moments</td>
<td>Nachtmann Moments</td>
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<tr>
<td>-----------------</td>
<td>--------</td>
<td>--------</td>
</tr>
<tr>
<td>$\gamma_5/\gamma_5$</td>
<td>$1.58 \pm 0.12$</td>
<td>$1.34 \pm 0.12$</td>
</tr>
<tr>
<td>$\gamma_4/\gamma_6$</td>
<td>$1.34 \pm 0.07$</td>
<td>$1.18 \pm 0.09$</td>
</tr>
<tr>
<td>$\gamma_3/\gamma_6$</td>
<td>$1.76 \pm 0.15$</td>
<td>$1.38 \pm 0.15$</td>
</tr>
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</table>
3.4 Jet Structure

The dominance of 2-jet structures in $e^+e^-$ annihilation at $Q \gtrsim 6$ GeV is well-established\textsuperscript{2,3).} The agreement with the naïve parton\textsuperscript{3} (or QCD) predictions is striking, and qualitative evidence against fixed-point theories which see no reason for 2-jet final states to dominate\textsuperscript{1,5).} On the other hand, there is as yet no evidence at all for any of the QCD predictions of jet broadening (8), let alone for the rare 3-jet events\textsuperscript{13)(9)}\textsuperscript{). If these exist, they can presumably only be seen at higher centre-of-mass energies.

4. QCD Tests in $e^+e^-$ Collisions

The discussion of the previous section revealed some striking (section 3.3) and some qualitative (sections 3.1, 3.2 and 3.4) evidence in favour of QCD and against either naïve parton or fixed-point theories. We are therefore strongly tempted to conclude prematurely that QCD is correct and the alternative strong interaction theories wrong. However, this conclusion obviously needs much confirmation, and we will now see how this may be done in $e^+e^-$ collisions at high energy. Because the focus is now on QCD, most of the discussion will centre on the detailed predictions of this theory. Because we do not yet know how many (if any!) of these predictions will be confirmed at PETRA or PEP, we will include in our catalogue some topics which may already be checked by the time LEP comes into operation.

4.1 Heavy $Q\bar{Q}$ Mesons (Onia)

In QCD the effective coupling constant at some momentum scale $Q$ goes to zero\textsuperscript{1) at large momenta as

$$a_s(Q^2) \equiv g^2(Q^2) = \frac{12\pi}{(33 - 2f) \ln(Q^2/\Lambda^2)}$$

where $f$ is the number of quark flavours with masses $< Q$. The intuition behind the charmonium picture of QCD\textsuperscript{7)} is that for heavy quark-antiquark systems the relevant scale is often $Q^2 = 0(M_Q^2)$, so that many of their properties are calculable in a weak coupling regime where the theory may resemble QED, as manifested in the positronium system. In this picture, the quarks look point-like because of their high masses, and orbit each other with a Bohr momentum

$$k_B = O(a_s)M_Q$$

and are non-relativistic when $a_s \ll 1$. Radiative and other transitions between onium states are calculated by the methods of atomic and nuclear physics. Decays into conventional hadrons not containing heavy quarks are envisaged as involving the annihilation of the heavy $q\bar{q}$ pair at a point with a probability $\alpha/|\psi(0)|^2$.\textsuperscript{2}
(the wave function at the origin squared) into final states of gluons and quarks calculable in low order QCD perturbation theory. Leptonic decays are also governed by $|\psi(0)|^2$, which in the non-relativistic approximation can be factored out of decay rates as in Figure 5, to give predictions for ratios of decay rates such as

$$\frac{\Gamma(1^3S_1 \rightarrow \text{hadrons})}{\Gamma(1^3S_1 \rightarrow \gamma + e^+e^-)} = \frac{(\pi^2 - 9)}{18} \frac{5}{a_s^3}$$

(14)

for ortho-onium decay and

$$\frac{\Gamma(1^1S_0 \rightarrow \text{hadrons})}{\Gamma(1^1S_0 \rightarrow \gamma\gamma)} = \frac{9}{8} \frac{\alpha_s^2}{\alpha^2}$$

(15)

for para-onium decay.

Do this simple picture and predictions like (14) and (15) apply to present-day onium systems such as charmonium itself or the $T$ system? For $c\bar{c}$ states one would seek to use $\alpha_s(Q^2) = Q^2 - 3(M^2_c)$ to $10(M^2_\gamma/Q)$. In this range $\alpha_s$ is probably $O(0.5)$ or more\(^{20,21}\). But the measured value of the ratio (14) would correspond to $\alpha_s \approx 0.19$, indicating a failure of the charmonium formula (14) by an order of magnitude. There are also indications of similar problems with the prediction (15). Of course, if $\alpha_s=0(0.5)$ for the $c\bar{c}$ system, then the validity of the non-relativistic approximation is questionable. Even within this framework, it is known that the radiative corrections to the prediction (15) are very large\(^{24}\) and it is generally believed that they would be even larger in the ortho-onium formula (14). The situation will presumably be improved with the $T$ system where one would imagine $\alpha_s \approx 0.2$ to 0.3, but this is not yet known. It may well be that more quantitative tests of the onium picture of QCD will have to await higher mass $q\bar{q}$ bound states. Perhaps PETRA, CESR and PEP will find and study some, but this is as yet unknown. If not, it is quite possible that several fundamental questions about onium dynamics will not be answered before LEP.

- Can one really factorize onium decay rates into $|\text{wave function}|^2 \times |\text{matrix element}|^2$?

- Is asymptotic freedom applicable to the matrix element factor? Does the Zweig rule in fact improve as $m_q \rightarrow \infty$ as suggested by (14) and (15)? Can one measure the decrease of the strong coupling constant in this way?

- Does the hadronic wave function eventually become Coulombic as $m_q \rightarrow \infty$? The $q\bar{q}$ potential is a complicated mixture of neo-Coulombic at short distances and confining at large distances. If the wave function eventually becomes mainly Coulombic, the absolute decay rates also become predictable, e.g.
and the spectrum of ortho-onia becomes

\[ m(N^3S_1) = 2m_q - \left( \frac{4}{3} \alpha \right)^2 \frac{m_q}{4N^2} \]  

Realistic calculations suggest\(^{24}\) that these results would only be relevant for quarks beyond the reach of PETRA and PEP: \( m_q \gtrsim (30 \text{ to } 50) \text{ GeV} \).

The charmonium picture of course also makes many well-known predictions for the final states in onium decays, particularly for jet structures\(^4,12\). The leading order prediction for \( 3S_1 \) decay is for it to produce three gluons\(^7\) giving three final state hadronic jets with energy distributions \( x_i \equiv 2E_i/(m_{3S_1}^2) \)

\[ \frac{1}{\Gamma_{3g}} \frac{d^2\Gamma}{dx_1 dx_2} = \left( \frac{1}{\pi^2 - 9} \right) \left\{ \frac{(1 - x_1)^2}{x_2^2 x_3^2} + \frac{(1 - x_2)^2}{x_1^2 x_3^2} + \frac{(1 - x_3)^2}{x_1^2 x_2^2} \right\} \]  

Where and how would one test predictions like (18)? A start has been made\(^{16}\) in the T system, measuring global quantities like the average sphericity, spherocity, thrust, acoplanarity, etc. The results agree with a Monte Carlo based on the three-gluon decay distribution (18) and subsequent hadronization, but no meaningful three-jet structures or unambiguous evidence for the QCD prediction (18) has yet emerged. It seems that a higher mass onium state is needed - which may or may not be located in the PETRA-CESR-PEP energy range, and may or may not enable the detailed jet predictions (18) to be checked. How would one analyse for such 3-jet final states? A possible strategy\(^{12,29}\) is

1. For each event find the axis which maximizes the thrust:

\[ T \equiv \left( \frac{\sum_h |\mathbf{p}_h \cdot \mathbf{n}|}{\sum_h |\mathbf{p}_h|} \right) \]  

where the sum is over all hadrons \( h \), and \( \mathbf{n} \) is an arbitrary unit vector. The axis maximizing \( T \) (19) is called the thrust axis. In a three-jet picture it is parallel to the most energetic jet, and should have an angular distribution\(^{26}\)

\[ \frac{d\sigma}{d(cos\theta)} \propto (1 + 0.39 \cos^2\theta) \]  

according to QCD.

2. Check that events lie approximately in a plane, e.g. by checking that they have small acoplanarity.
\[ A \equiv 4 \min \left( \frac{\sum |F^h_\text{out}|}{\sum |F^h|} \right)^2 \]  

where the minimization is respect to the choice of plane perpendicular to which \( |F^h_\text{out}| \) is measured. (\( T \) decay events do indeed seem to have \( \langle A \rangle \) less than a phase space Monte Carlo would suggest.)

3. Align events in the event plane such that the angle \( \theta = 0 \) corresponds to the thrust axis. The forward-backward ambiguity is resolved by demanding that the hemisphere \( |\theta| < \frac{\pi}{2} \) has less \( \sum F^h \), while the right-left ambiguity is resolved by requiring that the hemisphere \( |\theta - \frac{\pi}{2}| < \frac{\pi}{2} \) has more \( \sum F^h \).

4. When this procedure is adopted, events with low thrust (\( T \leq 0.8 \)) should look like propellors in the \( \theta \) plane, while events with high thrust (\( T \geq 0.9 \) say) resemble two-jet events (see Figure 6). Unfortunately, simple kinematics will strongly favour this type of configuration when one studies relatively low-massonia with small hadronic multiplicities in the decays. To check the non-triviality of the QCD prediction (18) one can proceed by

5. Making a Jet Boost. Hadrons with \( |\theta| < \frac{\pi}{2} \) should have finite \( (p^h_T) \) in this picture - they make the most energetic jet. Boost the particles with \( \pi > |\theta| > \frac{\pi}{2} \) by an amount \( \zeta \):

\[ s\zeta = \frac{T}{2 \sqrt{1 - T}}, \quad c\zeta = \frac{2 - T}{2 \sqrt{1 - T}} \]  

In this frame, the remaining particles should ideally be in their centre-of-mass, and show up as two thin back-to-back jets, with a characteristic angular distribution:

\[ \frac{dN}{d(\cos \tilde{\theta})} = \frac{(6T^2 - 12T + 4) + T^2 \cos^2 \tilde{\theta}}{8(1 - T)^2 + (2 - T)^2} \cos^2 \tilde{\theta} \]

\[ = \frac{(1 - \frac{T^2}{(2 - T)^2} \cos^2 \tilde{\theta})^2}{(1 - \frac{T^2}{(2 - T)^2} \cos^2 \tilde{\theta})^2} \]

\[ = 1 - 0.1 \cos^2 \tilde{\theta} \quad \text{for} \quad T \sim 0.8 \text{ to } 0.9 \]

where \( \tilde{\theta} \) is the angle relative to the boost axis in the boosted reference frame (see Figure 7). We will see later that the distribution (23) is very different from that expected for three jet events in the \( e^+e^- \) continuum.
Figure 5: The charmonium model for ortho-onium decay into 3 gluons

Figure 6: Final state jet structure predictions\textsuperscript{12} for $T$ decays
Figure 7: The normalized angular distributions \( \frac{dN}{d(\cos \theta)} \) for
A: \( q\bar{q} + \) vector gluon,  
B: \( q\bar{q} + \) scalar gluon,  
C: \( ggg \) plotted for 
\( T = 0.8 \) and 0.9. The vertical lines are kinematic cutoffs\(^{25}\).
Other QCD tests in onium decays are of course possible but we will not
discuss them in detail here. The $^1S_0$, $^3P_0$ and $^3P_2$ states are expected to decay into two gluonic jets with angular distributions which should enable the spin 0 and spin 2 alternatives to be distinguished already in the $T$ system. The $^3P_1$ decay is interesting in that the two real gluon decay mode is forbidden. The dominant decay is expected to be via $g + (g^* + q\bar{q})$ where the real gluon is soft with energy the quark binding energy, so that the final state should contain two energetic $q\bar{q}$ jets and a possible low energy third jet as in Figure 8. Verification of this QCD prediction would be a decidedly non-trivial check on the non-relativistic binding dynamics. Seeing the third small jet may well require a quark mass beyond the reach of PETRA and PEP.

So far, in keeping with the avowed purpose of this talk, we have concentrated on strong interaction tests with onia. However, it is worth emphasizing that several interesting studies of the weak interactions may be possible with heavy onium systems.

**Charged Semi-weak Interactions?** If there is a charged Higgs boson $H^\pm$ then it can be produced in decays $q + q' \rightarrow H^\pm$ such as $t \rightarrow b + H^+$, $b \rightarrow c + H^-$, and these will go so fast as to dominate conventional charged current weak decays:

$$
\Gamma(q + q' \rightarrow H^\pm) = \frac{G_F m_q^3}{32\pi} \text{(angle factors)} \text{(phase space factor)}
$$

which could easily be $0(1)$ KeV for $b \rightarrow c + H^-$ if $m_{H^-} \leq 2$ GeV. In fact the decay rate (24) is so big it could even be a competitive decay mode of a heavy $t\bar{t}$ onium state:

$$
^3S_1(t\bar{t}) \rightarrow (bH^+)\bar{t} \rightarrow (b\bar{b}) (H^+H^-)
$$

which would require a decay rate $0(1)$ KeV, not negligible by comparison with

$$
\Gamma(3S_1(t\bar{t}) \rightarrow \gamma^* \rightarrow e^+e^-) \sim 5 \text{ KeV}.
$$

**Neutral Semi-weak Interactions?** If the neutral Higgs boson $H$ is sufficiently light, it can be produced by radiative decays of onia in

$$
\Gamma(3S_1 \rightarrow H + \gamma) \approx \left( \frac{G_F m_q^2}{\sqrt{2} \pi \alpha} \right) \left[ 1 - \frac{m_H^2}{m_{3S_1}^2} \right]
$$

Putting in numbers we find
\[
\frac{\Gamma(3S_1 + e^+e^-)}{\Gamma(3S_1 + \gamma^* + e^+e^-)} = \left( \frac{G_F^2}{128 \pi^2 a^2} \right) \frac{m_T^2}{m_z^2} \frac{1+(1-4\sin^2\theta_W)^2)(1-4|e_q|\sin^2\theta_W)^2}{e_q^2(m_z^2 - m_T^2)^2}
\]

which would be > 1 for a bound state of charge $-\frac{1}{3}$ quarks with $|m_{3S_1} - m_z| < 15$ GeV.

Thus the prospects look dim for Higgs hunting in $T$ decays—particularly since there is a (moderately reliable) lower bound

\[
m_H^2 \geq \frac{3\alpha^2 (2 + \sec^2\theta_W)}{16 \sqrt{2} G_F \sin^2\theta_W} = 7.1 \text{ GeV for } \sin^2\theta_W = 0.20
\]

It seems from equation (27) that the prospects are an order of magnitude better for a toponium system near the top of the PETRA-PEP energy range, but the sort of onium accessible to LEP is clearly a much more attractive hunting ground, from the point of view of the range of $m_H$ kinematically accessible, as well as the rate.

**Charged Weak Interactions?** If one naively calculates the decay rate for a heavy quark such as $t$:

\[
\Gamma(t \rightarrow q\bar{q}) = \frac{1}{192\pi^3} G_F^2 m_T^5 \times 0(8) \times (\text{angle factors}) \times (\text{phase space})
\]

where the numerical factor is a guess at the number of different fundamental fermion channels open for decays. The formula (29) yields $\Gamma \sim O(1)$ KeV for $m_T > 20$ GeV, suggesting another interesting non-strong decay mode for a heavy onium:

\[
^3S_1 (t\bar{t}) \rightarrow t(q\bar{q}) \rightarrow (q\bar{q})(q\bar{q})
\]

A 6-jet final state? Again large quark masses (LEP energies) are at a premium.

**Neutral Weak Interactions?** Direct observation of the axial neutral current in the process $e^+e^- \rightarrow Z^0 \rightarrow 3P_1$ is expected to be difficult because of the vanishingly small wave function at the origin expected for $P$-wave onium states. On the other hand, the neutral vector current decays of ortho-onia become large for sufficiently massive quarks: in the Weinberg-Salam model

\[
\frac{\Gamma(3S_1 \rightarrow 3P_1 \rightarrow e^+e^-)}{\Gamma(3S_1 \rightarrow \gamma^* \rightarrow e^+e^-)} = \left( \frac{G_F^2}{128 \pi^2 a^2} \right) \frac{m_T^2 m_Z^4}{m_{3S_1}^2} \frac{(1+(1-4\sin^2\theta_W)^2)(1-4|e_q|\sin^2\theta_W)^2}{e_q^2(m_z^2 - m_T^2)^2}
\]
In fact, if the difference $m_{3S_1} - m_{Z}$ is comparable with the $Z^0$ width (-2 to 3 GeV?) one can imagine very exotic interference and mixing effects which could look very different in different decay channels of the $Z^0$ and ortho-onium state\textsuperscript{29).}

Another amusing and important manifestation of the weak neutral current would be in the decay $3S_1 \rightarrow \nu \overline{\nu}$\textsuperscript{30,2}:

$$\frac{\Gamma(3S_1 \rightarrow Z^0 \rightarrow \nu \overline{\nu})}{\Gamma(3S_1 \rightarrow \gamma^* \rightarrow e^+e^-)} = N_M \left( \frac{G_F^2}{64 \pi^2 a^2} \right) \frac{m_{3S_1}^4}{a^2 \left( \frac{m_Z^2}{m_{3S_1}^2} - m_Z^2 \right)} \left( 1 - 4 |a_q| \sin^2 \theta \right)^2$$

(32)

This decay mode would still be relatively rare for PETRA-PEP onia, but would be very accessible for LEP onia. Probably the best way to look for the $\nu \overline{\nu}$ mode and count the total number of neutrinos would be via the decay chain

$$e^+e^- \rightarrow 2S_1 \rightarrow 3S_1 \rightarrow \nu \overline{\nu}$$

(33)

where the hadronic final state would just contain $2\pi$ and the specified missing mass of $m_{3S_1}$. A do-able experiment?

4.2 Heavy $Q\overline{Q}$ Mesons

As mentioned earlier, weakness of the strong interactions at large $Q^2$ motivates the idea that the weak decays of heavy mesons may proceed as if their constituent heavy quark decays almost freely, so that their decays would in some respects resemble those of heavy leptons, though with extra attendant hadronic junk. As the mass of a heavy quark is increased, the decay rate for $Q \rightarrow q\overline{q}q$ (see Figure 9.a)) increases as $m_Q^5$, whereas other mechanisms increase more slowly. For example, the annihilation mechanism $Q + \overline{q} \rightarrow q + \overline{Q}$ (see Figure 9.b)) should only increase as $m_Q$ for heavy quarks. Indeed, calculations suggest that probably already for charm and almost certainly\textsuperscript{31) for b quark mesons, the $Q \rightarrow q\overline{q}q$ mechanism should dominate. In QCD the rate for this decay is enhanced by a gluonic correction (see Figure 9.c)) factor which decreases to 1 as $m_Q$ becomes $\geq m_W^3$\textsuperscript{31)}

$$\left( \frac{\Gamma}{\Gamma_{\text{free}}} \right) = \frac{\frac{\alpha_s(m_Q^2)}{\alpha_s(m_W^2)}}{24 \frac{33 - 2f}{2f}}$$

(34)

where $f$ is the number of quarks with masses $\leq m_Q$. The fact that $Q^2 - m_Q^2$ is the relevant scale in (34) has been demonstrated by explicit calculations\textsuperscript{10}. Also, it is an experimental fact that the semileptonic branching ratio for charmed particles is much larger than that expected on the basis of the nonleptonic decay rate enhancement factors found for strange particles, and is closer to the naïve ratio.
Figure 8: Dominant decay mode of a $^3P_1$ onium state

![Diagram of $^3P_1$ onium state decay](image)

Figure 9: Decay modes for heavy mesons: (a) quasi-free decay $Q \rightarrow q\bar{q}q$, (b) annihilation $Q + \bar{q} \rightarrow q + q$, (c) gluonic corrections to $Q \rightarrow q\bar{q}q$

![Decay modes diagrams](image)

Figure 10: Final state from $e^+e^- \rightarrow Q\bar{q} \rightarrow (q\bar{q}) (q\bar{q})$

![Final state diagram](image)
expected on the basis of the decreasing non-leptonic enhancement (34). The $O(\alpha_s)$ corrections to the ratio (35) have been partly calculated. For heavy quarks one would expect that semileptonic branching ratios would generally fall in the range of (10 to 20)%. One would expect the relatively simple decay mode to be reflected in the hadronic final states, which should exhibit 3 jets ($q + q + \bar{q}$) for sufficiently heavy quarks. The spectra of the energies of these jets can be calculated from the classic $\mu \rightarrow e\nu \overline{\nu}$ decay spectrum. Strong radiative corrections to quark and also leptonic spectra in $Q \rightarrow qe\nu$ can also be calculated. One can define a Michel parameter $\rho$ and in principle calculate

$$\rho = 0.75 + O(\alpha_s(m_Q^2))$$

The production of heavy mesons in $e^+e^-$ annihilation is also expected to be relatively simple. It is expected that the heavy mesons will tend to have relatively large fractions

$$Z \equiv \frac{E_{\text{meson}}}{E_{e^\pm}} - 1$$

of the beam energies, because of the relatively low probability of gluon bremsstrahlung by high mass quarks. Figure 10 shows the result of a "back-of-the-calculator" estimate indicating that mesons made from 10 GeV quarks may carry $Z \sim 0.9$ at $E_{e^\pm} = 30$ GeV, and $Z \sim 0.8$ at $E_{e^\pm} = 100$ GeV. Combining our pictures for heavy meson production and decay, we are led to quite distinctive expectations for the hadronic final states in $e^+e^- \rightarrow$ heavy $QQ$. Close to threshold we expect

$$e^+e^- \rightarrow QQ \rightarrow (qq\bar{q})(\overline{q}\bar{q}q) + 6 \text{ jets} = \text{blob}$$

as in Figure 11, with the final states looking rather like phase space, with high sphericity, spherocity and acoplanarity, and low thrust. This should enable the extraction of a relatively refined $QQ$ sample just above threshold. Figure 12 shows some preliminary PLUTO data on the distributions of eigenvalues of the sphericity tensor for continuum events at $E_{e^\pm} = 4.7$ GeV, compared with those generated by a phase space Monte Carlo intended to simulate the production and decay of $b\bar{b}$ systems just above threshold. The proposed experimental cuts should enable the $b\bar{b}$ sample to be purified by a factor of 3 or 4. The same technique should work even better for heavier mesons, just because the jet-like continuum events will be that much jettier at high energies. Above threshold, the sphericity, thrust and acoplanarity of heavy quark events are all expected to fall as $(m_Q/E_{e^\pm})$ until they are comparable to ordinary light quark jets. Figure 13 shows an estimate of the likely change in $E_{1-T}$ near and above the $b\bar{b}$ threshold.
Figure 11: An approximate QCD estimate\textsuperscript{32} of the beam energy fraction $Z$ carried by heavy quark mesons.
Figure 12: Data (solid lines) and a phase space Monte Carlo (dashed lines) for different combinations of the sphericity tensor eigenvalues\(^2\). Indicated also are cuts proposed to separate heavy QQ production from continuum background.

Figure 13: The average \(<1-T\) expected\(^{1,2}\) to come from QCD perturbation theory, non-perturbative QCD effects, the T resonances and heavy quark pair production above threshold.
in $\text{e}^+\text{e}^-$ annihilation. Similar structures may be expected near heavier quark thresholds. Indeed, the best way of scanning for a very heavy quark threshold may be$^{11}$ to look for a threshold in high sphericity events, which should persist some way above threshold as shown in Figure 13, enabling a relatively coarse-grained scan to be made.

4.3 Scaling Violations in the Final State

It was emphasized in section 3.3 that while the best evidence to date for the validity of QCD may come from studies of scaling violations in deep inelastic lepton-hadron scattering, there is as yet no evidence for the predictions (3) in $\text{e}^+\text{e}^-$ annihilation. There, data$^{33}$ are more or less consistent with scaling at the present time. There are several reasons why measurements of hadronic scaling violations in $\text{e}^+\text{e}^-$ annihilation have been more difficult than in deep inelastic scattering. One is that the $Q^2$ lever arm has generally been smaller ($Q^2$ - 10 to 50 GeV$^2$ typically$^{33}$), whereas the ABCLOS analysis$^{20}$ used $Q^2$ - 1 to 100 GeV$^2$), another is the presence of new quark thresholds in $\text{e}^+\text{e}^-$ annihilation, which are not so bothersome in deep inelastic scattering. LEP will provide copious hadronic data at $Q^2$ - $10^4$ GeV$^2$, which enable the first of these problems to be avoided, but perhaps not the second. It may well be that even in this range of $Q^2$ the best probes of scaling violations may come from deep inelastic scattering with ep colliding ring machines$^{34}$). However, there is one interesting aspect of scaling violations which is difficult to probe in that way, which concerns gluons. The ABCLOS data$^{20}$ depend on the existence of the $q + g$ bremsstrahlung diagram and on the crossed diagram $g + q + \bar{q}$ of Figure 14.a). The CDHS data$^{35}$ provide some evidence for the existence of the $g + g + g$ vertex of Figure 14.b), but one would like something more conclusive. This may be provided by scaling violations in gluon jets, which should be much more rapid in $Q^2$ than those in quark jets, because of the larger colour charge of the gluon. Onia with different masses should provide gluon jets at $Q^2$ - $M^2$, and comparing the T and toponium hadronic decay spectra may enable this prediction to be tested$^{36})$. Will toponium be found at PETRA/PEP, or will it have to wait for LEP?

4.4 Jets, Broadening of $p_T$, Multi-jets, ...

As mentioned already in section 2.4, QCD predicts that most $\text{e}^+\text{e}^-$ annihilation events should have a 2-jet structure, and one may define

$$f \equiv \frac{\sigma(2\text{-jet})}{\sigma_{\text{total}}} = 1 - \frac{\alpha_s(Q^2)}{\pi} \tilde{f}$$

(38)

where $\tilde{f}$ is a function of how the jet is specified. To avoid infra-red problems, the natural specification is to use fixed cutoffs in energy fraction and angle: demand that a fraction $< \epsilon$ of the total centre-of-mass energy fall outside two
oppositely directed cones of opening angle $\delta$. For $e^+e^-$ annihilation when $\delta$ and $\epsilon$ are chosen small one finds\(^{14}1\)

$$\bar{f} = 4 \left( \ln 2\epsilon + 3 \right) \frac{4}{3} \ln \delta + \ldots$$ (39)

This formula has a limited range of applicability in $\epsilon$ and $\delta$, and more complete calculations of $\bar{f}$ exist\(^{37}\). Formula (39) applies to quark jets.

The corresponding formula for gluon jets is\(^ {38}\)

$$f_n = 1 - \frac{\alpha_s(Q^2)}{\pi} \left( 12 \ln 2\epsilon - (11 - \frac{2}{3}f) \right) \ln \delta + \ldots$$ (40)

Comparison with (39) shows that gluon jets should look wider than quark jets, in a region where perturbation theory is applicable. However, it seems likely that the jets observed in $e^+e^-$ collisions to date are probably predominantly non-perturbative, and it is not clear that the formulae (38), (39) and (40) are at all applicable. PLUTO T data\(^ {16}\) are even consistent with gluon jets having the same $<p_T>$ as quark jets.

A refinement of the lowest order perturbation predictions (38), (39) and (40) can be made\(^ {39}\) by summing all the leading $O(\alpha_s \ln \delta)^n$ terms in the perturbation expansion. This enables one to calculate the angular opening $\delta(Q^2)$ outside which a finite fraction $f$ of events deposit a fraction $\gg \epsilon$ of the total energy. As one might anticipate from equations (38), (39) and (41), $\delta$ is power-behaved with $Q^2$:

$$\delta(Q^2) \sim (Q^2)^{-d(f,\epsilon)}$$ (41)

and the table below lists $d(f,\epsilon)$ for both quark and gluon jets. Again one sees that perturbative gluon jets are much wider than perturbative quark jets, though
the relevance of this result to present-day data is still unclear.

\[
\begin{array}{|c|c|c|c|}
\hline
\varepsilon & 0.5 & 0.7 & 0.9 \\
\hline
0.1 & 0.28 & 0.19 & 0.08 \\
0.15 & 0.09 & 0.03 & quark jets \\
0.03 & & & gluon jets \\
\hline
\end{array}
\]

Table

If a new onium system is found with mass close to the upper end of the PETRA-PEP energy range, there should be little problem with extracting therefrom evidence for gluon jets. How about the prediction\textsuperscript{13)} (9) for three-jet \( q \bar{q} g \) final states in \( e^+e^- \) annihilation? The situation again largely depends on the location of the next heavy quark threshold. If one does not intervene, we expect\textsuperscript{12)} a perturbative tail in the thrust distribution \( \frac{1}{\sigma_{\text{total}}} \frac{d\sigma}{dT} \) coming from \( q \bar{q} g \) processes to be visible above the "nonperturbative" 2-jet background for \( T < 0.9 \) at \( Q \gtrsim 18 \) GeV. Analysis of such events should follow the same lines as suggested for onium \( \to 3 \)-jet events in section 4.1\textsuperscript{12,25).} Verify that low thrust events have low acoplanarity \( A \) (a cut in \( A \) may be necessary to remove \( e^+e^- \to \text{heavy } qq \) events). Make a jet boost \textsuperscript{(22)} to the centre-of-mass of the two less energetic jets, and study the angular distribution in the boosted reference frame, which should sharply distinguish between vector and scalar gluons:

\[
\begin{align*}
\frac{dN}{d(\cos \theta)} \text{(vector gluons)} & = \frac{1 + \frac{3(2-T)T^2 \cos^2 \theta}{4T^3 + (2-T)^3}}{1 - \cos^2 \theta} \approx 1 + 2 \cos^2 \theta \\
\frac{dN}{d(\cos \theta)} \text{(scalar gluons)} & = \frac{1 + \frac{T(3T-4) \cos^2 \theta}{4-3T^2}}{1 - \cos^2 \theta} \approx 1 + 0.2 \cos^2 \theta
\end{align*}
\]

(42)

where the approximations apply to events with \( T > 0.8 \) or 0.9 \textsuperscript{\textendash}). Comparing the predictions (23) and (43) shown in Figure 7 we see that the QCD predictions on and off resonance may be sharply distinguished, and the spin of the gluon determined in this way.
What of 4-jet events in $e^+e^-$ annihilation? These should have small cross-sections (10) and be difficult to see at PETRA and PEP$^{(40)}$, even in the absence of a new quark threshold. The large LEP event rates on the $Z^0$ peak with $Q^2 \sim 10^4$ may enable such events to be picked out, but their intrinsic interest is unclear. The one good thing about the smallness of the four-jet cross-section is that it should mean there will be negligible background to the search for 4-jet events due to $e^+e^- \rightarrow W^+W^-$ at centre-of-mass energies $O(200 \text{ GeV})$.

5. Conclusions

Although QCD is presumably the correct theory of the strong interactions, it must be confessed that this remains to be proved beyond the shadow of a doubt. Many tests of QCD can be made in $e^+e^-$ annihilations at higher energies than those available up to now. How many of these will be performed with the PETRA-CESR-PEP generation of machines remains to be seen. It largely depends on the location of the next heavy quark thresholds and the masses of the associated onia. QCD tests on onia are much cleaner for higher masses, where one would hope to see

- evidence for asymptotic freedom in decay matrix elements,
- the wave-functions becoming Coulombic,
- possible decays into Higgs particles ($H^\pm, H^0$),
- the decay into $\nu\bar{\nu}$ to count neutrino types.

Similarly, the higher the mass of a heavy quark $Q$, the more applicable QCD perturbation theory should be to the production, decay rates and final-state spectra for $Q\bar{Q}$ mesons.

Finally, one needs continuum data at high energies to see the scaling violations, jet broadening, etc., predicted by QCD. How high these energies should be depends on the locations of heavy quark thresholds. Perhaps PETRA and PEP will have high enough energies, perhaps not.

If we are not totally satisfied already, LEP will clearly be able to confirm to us the validity of QCD and rule out other strong interaction theories. In any case, LEP will be a great machine for strong interaction studies. The initial states are provided with great purity and at very high $Q^2$. The prospect of $O(10^7)$ hadronic events at $Q^2 \sim 10^4 \text{ GeV}^2$ sounds very enticing.
References


LEP SUMMER STUDY

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DETECTORS FOR LEP: METHODS AND TECHNIQUES

C. Fabjan
CERN, Geneva

Copies available upon request from Ch. Redman, CERN/ISR
LEP Summer Study Secretariat
1. Introduction

This note surveys detection methods and techniques of relevance for the LEP physics programme. The basic principles of the detector physics are sketched, as recent improvement in our understanding points towards improvements and also limitations in performance. Development and present status of large detector systems is presented and permits some conservative extrapolations. State-of-the-art techniques and technologies are presented and their potential use in the LEP physics programme assessed. Table 1 is a simplified guide through the detection requirements of some of the more frequently discussed LEP physics questions.

2. Momentum Measurement

Magnetic spectrometers for momentum analysis of charged particles will remain the "workhorses" of detector systems. Axial magnetic field geometries (Solenoid, Helmholtz coils, Open Axial Fields, etc.) will remain the favoured choice, but other topologies - such as dipoles and toroids - will again be considered for more specialized experiments. Progress in the technology and the evolution of geometries of magnets have recently been summarized1) and in this section therefore only developments on tracking devices will be discussed.

Achievable spatial resolution with different tracking techniques is summarized in Table 2 (Ref. 2). Among those listed, drift-chamber principles have been applied to several, ingenious detector geometries. Imaginative developments3) indicate that streamer chambers may offer yet the highest spatial resolution: conceivably, further advances on the initiation and the read-out of streamers could make this device a viable choice for specialized experiments requiring ultimate spatial measurement.

The limits on spatial resolution in drift-chamber geometries are rather well understood4). They are affected both by the correlation between the track and its associated ionization and by the techniques of its measurement.

The various energy loss mechanisms determine the track-ionization correlation. The dominating effect is emission of $\delta$-rays, which may have a range up to and beyond 100 $\mu$m (see Fig. 1). Their energy spectrum is shown in Fig. 2. In addition to $\delta$-emission a substantial fraction of the energy loss proceeds via atomic excitation. The measured ionization is produced either through photo- or Auger-electron emission. Relative strengths of these processes and the range of the resulting electrons are given in Fig. 3. This again indicates the extension of the ionization information to distances up to $\approx 100 \ \mu$m around the trajectory. Little can be gained to narrow this distribution by increasing the operating pressure, if measurements on track segments of a given length are considered. Scaling the pressure as $P' = \alpha P$, one estimates from the range-energy relation and the spectral
<table>
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<th>$\sigma_x$</th>
<th>$\Delta x$</th>
<th>$\sigma_E^{\pi}$</th>
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x ... good; xx ... excellent; xxx ... (beyond) state-of-the-art.
### Table 2

**Tracking Techniques: Spatial Resolution**

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<td>$\phi_{wire} = \phi/α$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>L' = L/α²</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Streamer</td>
<td>50</td>
<td>P' = aP</td>
<td>~ 10</td>
<td>.05</td>
<td>Sandweiss et al.</td>
</tr>
<tr>
<td>chamber</td>
<td></td>
<td>E' = aE</td>
<td></td>
<td></td>
<td>Laser firing?</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$t'_r = t_r/α$</td>
<td></td>
<td></td>
<td>Holographic R/O ?</td>
</tr>
<tr>
<td>Drift chamber</td>
<td>≥ 150</td>
<td></td>
<td>≥ 20</td>
<td>.5</td>
<td></td>
</tr>
<tr>
<td>Scintillator</td>
<td>≥ 300</td>
<td></td>
<td>~ 300 (?)</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Focusing ¥</td>
<td>~ 100</td>
<td></td>
<td>~ 100</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Transition</td>
<td>~ 200</td>
<td></td>
<td>~ 200</td>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

(P = pressure, $\phi$ = diameter, E = electric weld, $t_r$ = rise time, L = wire length, X = wire separation)
Figure 1: Number of $\delta$-electrons produced in Argon at STP, with range $R \geq R_o$ (Reference 4)

Figure 2: Number of $\delta$-electrons with energy $E \geq E_o$, produced by 1 GeV/c protons in 1 cm of Argon at STP conditions (Reference 4)
distribution of the $\delta$-rays, Fig. 1, that the number of electrons with range $r > r_0$ increases as $a^n$, $n = 0.3$ to 0.4. Only, of course, the range of excitation electrons is narrowed as $a^{-1}$. The net effect of this intrinsic width of the ionization distribution on the achievable localization accuracy depends critically on chosen "read-out" methods, and will be discussed later. Part of the process of "reading-out" the ionization track consists in drifting the charge segment to a reference anode, resulting in a further diffusion spreading of the form

$$\sigma = \sqrt{2D_1t},$$

where the index $i$ of the diffusion coefficient $D_1$ refers to the direction of diffusion relative to the drift direction (in general, longitudinal and transverse diffusion coefficients are different). In a "classical" drift chamber geometry, the longitudinal diffusion along the drift path will limit the position resolution for drift distances larger than ~1 cm. For a single electron drifting during a time $t$, the r.m.s. displacement $\sigma$ is

$$\sigma = \sqrt{2Dt} = \sqrt{\frac{2DX}{\mu E}} = P \left( \frac{E}{P} \right) \sqrt{\frac{X}{P}}.$$  

$P$ is the pressure, $\mu$ is the mobility and $E$ the electric field. For a given value of the reduced field $E/P$, $\sigma$ is proportional to $P^{-\frac{1}{2}}$, which is confirmed by measurements (Fig. 4). Conventionally, the drift time is determined by the arrival of the first electron of the ionization swarm consisting of $N$ ionization clusters and giving rise to an uncertainty of

$$\sigma = \frac{\sigma}{\sqrt{2 \ln N}} \sqrt{\frac{\tau^2}{6}}.$$  

Instrumental sensitivity to the full ionization distribution, which is achieved e.g. in "center-of-gravity" read-out methods, could result in an optimum resolution of $\sigma_N = \sigma/\sqrt{N}$. For typical operating conditions one may have: $N = 100$, $\sigma_1 = 0.426$ and $\sigma_N = 0.16$, demonstrating a potentially significant improvement.

The influence of magnetic fields on the diffusion coefficient can be estimated and has also been measured. Writing $D = \frac{v_e \lambda}{3}$, where $v_e$ denotes the velocity of electrons and $\lambda$ their mean free path, one may express the magnetic field effect as $D(B) = D(B = 0) \left( \frac{1}{1 + \omega^2 \tau^2} \right)$, with the usual symbols $\omega = eB/\mu c$, the cyclotron frequency and $\tau = \lambda/v_e$, the mean time between collisions. In the strong field limit, $\omega^2 \tau^2 >> 1$ the diffusion dispersion becomes

$$\sigma = \sqrt{\frac{2L}{3VP}} \cdot \frac{v^3}{\omega^2 \lambda},$$

which can therefore be reduced in gases with small electron-velocity and long mean free path. Figure 5 represents some measurements on transverse diffusion for several gas mixtures.

The conclusions on spatial resolution limitation may be summarized as follows:

i) the dispersion of the ionization around the trajectory extends up to 100 $\mu$m in typical detector geometries;
Figure 3: Range of photoelectrons ($E_1$) and Auger-electrons ($E_2$, with probability $p$ created by a photon with energy $E_x$, in Argon and Xenon at STP$^4$)

Figure 4: Influence of the gas pressure in the space resolution $\sigma$ as a function of drift distance$^5$
ii) for long drift geometries larger effects are superimposed due to diffusion;

iii) the measurement error depends critically on the read-out technique;
if information from all ionization clusters is used, the achievable intrinsic accuracy could be at the 10 μm level.

Besides better understanding of the operational principles of wire chambers, considerable advances in detector geometries and read-out techniques were necessary for the new generation of experiments at PETRA, ISR and PEP. Table 3 provides a summary of track chamber characteristics of these experiments in historical sequence and increasing complexity. Individual cylindrical chambers were used by the CCOR collaboration. Conventional drift time measurement of the first electron of the ionization swarm results in a spatial resolution approaching ~300 μm. The drift chamber operates in a 1.5 T magnetic field requiring a compensation by tilting the drift field. Unambiguous space points were obtained with a delay-line read-out capacitively coupled along the sense wires. A similar solution was adopted by the CELLO collaboration. The first step towards increased density of track points was taken by the MARK II group. The development of a new technique to localize the ionization avalanche along a sense wire through the measurement of the electric charge diffused to the two ends of the wire — "charge division" — and its first employment in an ISR experiment suggested a new class of track detectors. The track chamber readied for the AFS-installation allows up to 42 unambiguous space point determinations along a particle trajectory. Its layout has been optimized for high-multiplicity-"jet"-physics and is expected to operate at particle rates above 10⁷ s⁻¹. The possibility offered by the Charge-Division Technique to obtain multiple dE/dx-information has been maintained with the appropriate choice of relatively low gas gain (M ≤ 2 x 10⁴) to ensure true proportional operation. The JADE-chamber uses the same techniques in a more advanced form: it is designed to operate at 4 atm. pressure to improve the dE/dx-resolution; specially developed electronics, taking advantage of the bunched operation of PETRA, extends the multiple-hit capability also to the charge-division measurement.

Even more detailed information will be extracted from the "Time-Projection Chamber" (TPC) under preparation for a PEP experiment. The drift-geometry is schematically shown in Figure 6. The track-correlated ionization is projected onto the endplates resulting in up to 12 unambiguous two-dimensional coordinate
### Table 3
Read-out Schemes for Ionization

<table>
<thead>
<tr>
<th>Type</th>
<th>Experiment</th>
<th>$\Delta \phi \cdot r$ (mm) Method</th>
<th>$\Delta Z$ (mm) Method</th>
<th>Radial separation of coordinate meas. (mm)</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Layered Chambers</td>
<td>CCOR/ISR</td>
<td>$\geq 0.2 \tau_0$ of 1st electr.</td>
<td>5 Delay line</td>
<td>- 150 individual chambers</td>
<td>unambiguous space points</td>
</tr>
<tr>
<td></td>
<td>CELLO/PETRA</td>
<td></td>
<td>Cathode R/o</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total Volume</td>
<td>MARK II/PEP</td>
<td>- 0.2 &quot;</td>
<td>- 3) Stereo view</td>
<td>- 100 layered cells in common</td>
<td>some $dE/dx$ info designed for 4 atm.</td>
</tr>
<tr>
<td>Drift Cylinder</td>
<td>TASSO/PETRA</td>
<td>- 0.2 &quot;</td>
<td>- 3)</td>
<td>- 100 gas volume</td>
<td></td>
</tr>
<tr>
<td>Total Volume</td>
<td>AFS/ISR</td>
<td>- 0.2 &quot;</td>
<td>10) Charge</td>
<td>- 10 continuous</td>
<td>full identification capability to - 10 GeV/c</td>
</tr>
<tr>
<td>Drift + $dE/dx$</td>
<td>JADE/PETRA</td>
<td>- 0.1 (?) &quot;</td>
<td>20) Division</td>
<td>- 10 sensitivity</td>
<td></td>
</tr>
<tr>
<td>TPC Geometry</td>
<td>TPC/PEP</td>
<td>0.15 Center of gravity</td>
<td>1 Drift time</td>
<td>.5 for $dE/dx$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>p+p/SPS (UA 1)</td>
<td></td>
<td></td>
<td>- 80 for tracking</td>
<td></td>
</tr>
</tbody>
</table>
Figure 5: Measured transverse diffusion in 2 Tesla field for various A-X mixtures (X = CH₄ or CO₂) (Reference 7)
Figure 6: Schematic view of the TPC geometry. The track-correlated ionization is projected onto the endplates, resulting in a two-dimensional unambiguous coordinate readout with the drift time providing the third coordinate (Ref. 7)
readouts. As shown in Figure 7, the coordinate along these "spatial" wires is obtained by a centre-of-gravity readout of the charge induced on cathode pads, which results in a precision of approximately 150 μm. The third coordinate, the distance of the track from the endplates, is obtained via drift time by a centre-of-gravity readout of the ionization cloud: for this a novel electronics readout system had to be developed. It allows to continuously measure the charge arriving at the drift wires, binned in ~ 50 ns time intervals. In this way a spatial resolution of ~ 1 mm is accomplished despite the diffusion of the ionization cloud, which may reach values of up to ~ 1 cm. Operation of the chamber at 10 atm combined with the 192 dE/dx measurements should result in a dE/dx-resolution of σ = 2%, sufficient for π/K/p separation in the PEP energy range. A cut through the complete TPC detector is shown in Figure 8. It shows that the magnetic volume encloses in addition to the track chamber electromagnetic shower counters covering essentially 4π sr.

Readout techniques similar to the TPC methods are employed for a large 4π detector in preparation for the future p̅p-Collider in the CERN-SPS tunnel. The total track chamber volume of 6 m length and 2 m diameter is subdivided into six drift chamber modules, allowing two different drift directions for optimized curvature measurement (Figure 9). The evolution to date of these track chamber devices is documented further in Table 4. The trend is clear: increased energy of the machines results in increased complexity of events; ever-increasing granularity and improved spatial information is required. Improved and cheaper electronics allows for more readout channels and makes dE/dx measurements practical. Some possible developments are diagrammatically indicated in Table 5. They represent only narrow extrapolations on present state-of-the-art techniques and are likely to become useful concepts in the next generation of large tracking devices.

Further investigation on the principles of wire chamber operation, and in particular on gas mixtures optimized for the new class of track detectors, must continue. For TPC-like geometries (i.e. long drift distances) it would be advantageous to achieve low values of diffusion at relatively low values of E/P. This would permit the larger TPC geometries, which are of interest for LEP experimentation and would reduce the high-voltage problems. Geometries with \( E \times B \neq 0 \) may be unavoidable and their influence on drift properties could be further studied. The largest impact, however, is likely to come from electronics developments. We are expecting cheap preamplifiers with noise levels of \( 10^3 \) or less equivalent electrons. Further reduction in the cost and size of ADC's should make practical the extensive use of centre-of-gravity readout methods, both for drift-time and cathode-plane-type readouts. These ADC's might have a total digitization time as low as 20 ns, allowing continuous TPC-like readout and extremely fine granularity (~ 1 mm\(^3\)) with > \( 10^5 \) channels to become practical features of
Figure 7: Readout arrangement of the TPC endplates. Each of the wedges contains twelve "spatial" wires equipped with cathode pads to measure track curvature. Also, 192 "dE/dx" wires sample the ionization loss (Ref. 7).
Figure 8: Cut view through the TPC facility. The magnetic volume is filled with the track chamber and a 4π electromagnetic shower counter (Ref. 7)
Figure 9: Cutaway view of the central detector of the UA-1 facility for the CERN p\bar{p} collider, showing the B-field and drift directions (Ref. 17)
### Table 4

Some Statistics on Present and Future Track Devices

<table>
<thead>
<tr>
<th>Detector (year of 1st operat.)</th>
<th>Unit cell size (approx.)</th>
<th>Space points</th>
<th>Granulatiry</th>
<th>Number of Channels</th>
<th>Words/Event (Track ch.)</th>
<th>Evolution</th>
</tr>
</thead>
<tbody>
<tr>
<td>SFM (1973)</td>
<td>2 1000 1000</td>
<td>NO</td>
<td>2 1000 1000</td>
<td>~ 70'000</td>
<td>~ 10²</td>
<td></td>
</tr>
<tr>
<td>CCO (1976)</td>
<td>0.2 5 150</td>
<td>YES</td>
<td>2 10 150</td>
<td>~ 2'000</td>
<td>~ 10³</td>
<td></td>
</tr>
<tr>
<td>MARK II (1977)</td>
<td>0.2 5 100</td>
<td>NO</td>
<td>2 50 100</td>
<td>~ 3'000</td>
<td>~ 10³</td>
<td></td>
</tr>
<tr>
<td>JADE (1979)</td>
<td>0.1 20 20</td>
<td>YES</td>
<td>2 20 20</td>
<td>~ 4'000</td>
<td>&gt; 10³</td>
<td>higher density of channels</td>
</tr>
<tr>
<td>TPC-PEP (~1980)</td>
<td>.15 1 5</td>
<td>YES</td>
<td>.15 3 5</td>
<td>~10⁴ x 10²</td>
<td>~ 10⁴</td>
<td>- x 10 more pads for ~ 3 x higher momentum res.</td>
</tr>
</tbody>
</table>
Table 5

<table>
<thead>
<tr>
<th></th>
<th>Principles</th>
<th>Electronics</th>
<th>Mechanics</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Search for gases with smaller diffusion at lower E/P</td>
<td>Low noise ($&lt; 10^3$ e r.m.s.) → low gas gain</td>
<td>center of gravity $t_D$ ← parallel plate structures (no more wires?)</td>
</tr>
<tr>
<td></td>
<td>higher resolution, larger TPC's</td>
<td></td>
<td>highly integrated electronics ← multigap structures</td>
</tr>
<tr>
<td></td>
<td>investigate</td>
<td></td>
<td>TPC's with $\geq 10^5$ pads</td>
</tr>
<tr>
<td>$E \times B \neq 0$ geometries</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Anticipated Performance $\frac{\Delta p}{p^2} = \frac{0.002}{L^2 B (\text{m}^2)}$ ; $\sigma \left( \frac{dE}{dx} \right) - 2\%$ ; $\left[ \frac{dE}{dx} \right.$ (Cerenkov) $\left. ?? \right]$
such detectors.

The development of low-noise preamplifiers will be an essential element in the use of parallel plate proportional chambers, which may be operated in stable conditions if the gas gain is restricted to levels of \( \sim 10^3 \) \(^{19}\). Besides the obvious advantage of improved reliability of a detector system without delicate proportional wires, these low-gain chambers offer the novel feature of efficient single electron detection. This will allow the "imaging" of ionization tracks in ultimate detail. Its potential for particle identification will be discussed later. The resulting performance of these track chambers may be summarized as follows:

- **momentum resolution:**
  \[
  \frac{\Delta p}{p^2} \approx \frac{0.002}{L^2B (T/m^2)}
  \]

- **dE/dx resolution** \( \sigma \approx 2\% \).

Have we overlooked any other signatures of the charged particle traversing the gas volume? The highest-momentum particles, difficult to identify with the dE/dx-method, emit a considerable amount of Čerenkov light. Concepts are presently pursued to detect this light in gaseous detectors and it may perhaps be possible to extract from one integrated detector useful information on position, ionization and Čerenkov energy loss.

3. Particle Identification

In the present generation of PEP, PETRA and ISR detectors, increasing emphasis is placed on the possibility of particle identification. Its importance for the LEP physics programme has been repeatedly stressed and its detailed evaluation remains one of the important points for further study. Table 6 summarizes presently accessible techniques.

The range of conventional threshold Čerenkov counters has recently been increased through the development of a new radiator material, "aerogel", with refractive indices \( n = 1.01 \) to \( n \approx 1.05 \) \(^{20}\). It is therefore useful in a range, where pions and kaons have their cross-over point in ionization and for which only time-of-flight techniques (TOF) have been available. The TOF technique may be expected to see further applications with the development of new fast photon-detectors (such as channel plates) or other specialized devices with superior time resolution. Recently \(^{21}\) work has been reported on a new type of spark chamber with an intrinsic time resolution of \( \sigma < 100 \) ps.

Gaseous Čerenkov counters continue to be used in storage ring applications. As can be judged from Figure 10, the very large radial distances required exclude however this technique for general storage ring application.
<table>
<thead>
<tr>
<th>Technique</th>
<th>$\gamma$ Range</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aerogel</td>
<td>$\gamma \leq 6$</td>
<td>with $\gamma \approx 6$ can resolve $\pi/K$ ambiguity at cross-over of ionization</td>
</tr>
<tr>
<td>Time-of-flight</td>
<td></td>
<td>$\sigma \leq 100$ ps have been obtained with special spark chambers</td>
</tr>
<tr>
<td>Gas Threshold Čerenkov</td>
<td>$\gamma &gt; 10$</td>
<td>- Not suitable for Storage Ring (SR) applications where $2\pi$ coverage is required</td>
</tr>
<tr>
<td>Multiple Ionization</td>
<td>$2 \leq \gamma \leq 50$</td>
<td>Requires $\sigma(dE/dx) \sim 2-3%$; achieved so far in &quot;easy&quot; geometry only</td>
</tr>
<tr>
<td>measurement in homogeneous medium</td>
<td></td>
<td>Č-photons detected with UV-sensitized proportional chamber structures (PIC)</td>
</tr>
<tr>
<td>Imaging Čerenkovs</td>
<td></td>
<td>Useful as compact threshold detector for specialized applications</td>
</tr>
<tr>
<td>Transition Radiation</td>
<td>$\gamma \approx 1000$</td>
<td></td>
</tr>
</tbody>
</table>
Figure 10: A layout of the AFS Detector at the CERN ISR showing the configuration with the particle identification arm.
Particle identification through multiple dE/dx measurements has become feasible in recent years and to-date is recognized as the most practical method. It also meets a major criterion for storage ring detectors - compactness - because the dE/dx instrumentation can be integrated with the track detector. The γ range, over which identification can be accomplished, covers a major part of the LEP physics requirements.

Several advances on this technique have been reported in recent years. Very useful for the optimization of such detectors has been refined modeling of the energy loss process\(^\text{22}\)\). The reliability of these calculations was improved through more careful treatment of the atomic collision process, replacing the customary average value for the ionization potential value \(<I>\) with a more correct summing \(\sum f_i I_i\) over all atomic subshells. The agreement between these calculations and measured spectra can be evaluated from Figure 11. The same model, besides reproducing in detail the energy-loss distributions, can also account well for the observed "relativistic rise" (Figure 12). Figure 13 indicates the most probable energy loss of various particles in a 1 cm sample of Argon at 10 atm. and exhibits the two major difficulties of this technique: a measurement resolution of \(\sigma < 2\%\) is required for useful identification and saturation of the relativistic rise limits the γ range.

The relation between sample thickness, number of measurements and achievable resolution is given in Fig. 14\(^\text{23}\)\). Related curves are plotted in Fig. 15 to exhibit the relation between resolution and sampling frequency for a given detector length\(^\text{24}\)\). An upper limit to the improvement due to increasing sampling frequency may be estimated from the number of primary collisions in the detector. Monte Carlo calculations indicate\(^\text{25}\)\) that such counting of primary collisions may give a rather important improvement (up to a factor two) when compared to the more conventional integrated sampling over typically 1 cm track length.

The new chamber geometries, which were mentioned in Section 2 and which have single electron detection capability, conceptually allow for such detailed ionization sampling. The possible use for these applications should be investigated. This cluster counting technique and its effect on dE/dx-resolution was experimentally demonstrated recently\(^\text{26}\)\) where improvement factors of approximately 1.5 were found when compared to integral dE/dx sampling. To indicate the expected performance of such devices we give in Figure 16 the particle separation in units of standard deviation separation. In one case a filling of Argon/CH\(_4\) at 4 atm. with 1 cm thick samples was assumed\(^\text{27}\)\). As an indication that performance improvements are possible, curves are also shown for a Xe-CH\(_4\) filling. The pressure (1 atm) has been chosen to allow K/p separation up to \(-100\) GeV/c and a performance based on "cluster" counting was assumed.
Figure 11: Absolute calculation and measurement of the ionization loss for several particle momenta. Note that the calculations reproduce correctly the mean, the width and the Landau tail ("overflow") (Ref. 22)
Figure 12: Comparison of a Monte Carlo calculation of the relativistic rise of the energy loss with some measurements. Ref. a) is Ramana Murthy, Nucl. Instr. Methods 63 (1968) 77. Ref. b) is Harris et al., Nucl. Instr. Methods 107 (1973), 413. Ref. c) is Jeanne et al., Nucl. Instr. Methods 111 (1973) 287.
Most Probable Energy Loss ($E_{mp}$) for $e, \mu, \pi, K, p$ in 1 cm of Argon at 10 Atm.

Figure 13: Most probable energy loss for various particles in 1 cm of Argon at 10 atm. (Ref. 7)
Figure 14: Relation between detector thickness and number of dE/dx measurements for a given resolution and total detector length (Ref. 23)
Collision counting

Figure 15: Calculated particle separation (\(n_{\pi} - n_{K}\)) as a function of gas pressure and number of drift measurements for a fixed limit, based on counting of collisions, is also shown.

\[ \frac{E_{\pi} - E_{K}}{\sigma} \]

\(L = nt = 128.8\, \text{cm}\)
Figure 16: Expected particle separation (in units of standard deviations) in a detector of 170 cm depth, at 4 atm. and 140 samplings per track (Ref. 27). Also shown (dashed lines) is the performance of the same detector, filled with Xe/CH₄ at 1 atm. and assuming cluster counting (see text).
The relative merits of different gases for \( \text{d}E/\text{d}x \)-identification\(^{24}\) indicate that for the LEP energy range the heavy gases remain the preferred choice in a delicate balance between resolution and identification range. We should also be reminded that the saturation of the relativistic rise due to the density effect is intimately associated with extended homogeneous media. It has been pointed out theoretically\(^{28}\) and verified experimentally\(^{29}\) that this effect is reduced or absent in media where inhomogeneities (zones of energy loss) are separated by distances larger than the "formation" zone, the relaxation length characterizing the polarization effects and responsible for the density effect. While surely extensive development work must continue to evaluate the full potential of the \( \text{d}E/\text{d}x \) identification method, we anxiously await the first results from the \( \text{d}E/\text{d}x \) detectors under preparation at PEP, PETRA and ISR: so far the 2-3\% resolution has only been achieved in "easy" geometries, with collimated tracks in well-defined test set-ups. Absolute ionization measurements in storage ring detectors will be plagued by a number of systematic effects, depending on track inclination, gas gain and instrumental effects.

Ring focusing \( \Upsilon \) Čerenkov counters have recently been considered, following a suggestion by T. Ypsilantis to use UV-sensitive MWPC's as the photon detector\(^{30}\). The proposed detector geometry is given in Fig. 17, and the status and trends of development are tabulated in Table 7. The concept is attractive as it combines a geometry suitable for storage rings and high efficiency for the UV-Čerenkov photons requiring short radiators only. The resolution in the Lorentz factor \( \gamma \) is such that the complete \( \gamma \) range required for LEP physics could be covered with three radiators only. One of the difficulties of this technique for use in the LEP physics programme would appear to be the identification of all the particles belonging to a densely collimated jet: in the pattern recognition one will have to disentangle several (up to ~ 15) Čerenkov-ring images, mostly superimposed. Experimentally, the detection of Č-photons in UV-sensitive chambers has been accomplished for a threshold-Č-geometry and approximately half the expected number of photoionization electrons were measured\(^{31}\)One of the present points of activity concerns the readout problem. Here again, the combination of chambers with demonstrated single electron-detection efficiency\(^{19}\) and reduced electronics cost appear most promising. Research on new gases with lower photoionization threshold also continues. This might circumvent the requirement of special UV-transparent glasses, such as LiF, CaF\(_2\) or sapphire, a major practical obstacle.

To complete the discussion on identification techniques for LEP physics, we mention the possibility of Transition Radiation Detectors\(^{32}\). The theory of these detectors has been verified experimentally in detail\(^{33}\) and can be used for
<table>
<thead>
<tr>
<th>Concept</th>
<th>Status</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Present concept needs UV transmitting windows for cryogenic radiators for $Y \leq 10$</td>
<td>Various R/O schemes under preparation (Ypsilantis, Charpak, et al.)</td>
<td>R/O problem may become more manageable with new generation of electronics (Isabelle: 0.5 X 10$^{-4}$ detection channels)</td>
</tr>
<tr>
<td>R/O with UV sensitive gas and Prop. Ch.</td>
<td>Search for gases with lower ionization threshold (Ypsilantis)</td>
<td>Try different radiator-detector geometries (e.g., integration)</td>
</tr>
</tbody>
</table>

Table 7: Photoionization Imaging Čerenkov Counters
Figure 17: Schematic representation of a sequence of ring imaging Čerenkov counters, using UV-sensitive MWPC-like readout elements (See also Ref. 30)
the optimization of the detectors\textsuperscript{34}). These detectors are sensitive above thresholds $y > 500 - 1000$ and they might therefore be of value for special detection requirements. As an example, we estimate that a counter of total thickness $d \leq 20$ cm, optimized for electron detection, would yield a hadron-rejection of 30 to 40 and detect electrons with momenta $p \geq 5$ GeV/$c$ with an efficiency of $\geq 90\%$. This should be useful in a dedicated spectrometer set-up for $2\gamma$-physics, where electron tagging is important. It might also be used to identify electrons inside very collimated jets in sensitive searches of new thresholds.

4. Total Absorption Calorimetry

Absorptive spectroscopy has been increasingly used in recent years. Its consideration has been favoured by the increasing energy of the new accelerators and storage rings, where an energy resolution can be achieved which competes favourably with magnetic analysis. At LEP energies the momentum along a given axis and the energy of an event will be measured with a few percent accuracy and essentially independent of the final state. There, magnetic momentum analysis may only compete, if the total energy is shared in a multiparticle final state.

Electromagnetic Shower Detectors (ESD)

The physics of such detectors has been known for many years\textsuperscript{35}) and the required instrumental techniques are sufficiently straightforward to have permitted their use for a number of years. Recently, efforts were concentrated on developing readout methods suited to the requirements of storage ring detector geometries. Table 8 documents this activity\textsuperscript{36}).

New performance requirements will have to be imposed on ESD's to be used in the LEP physics programme, where it will be the most common and also most difficult task to estimate the fractional jet energy carried by photons. Due to the narrow collimation of LEP jets, this measurement has to be accomplished in the presence of partially developed hadronic cascades in the ESD. The influence of this hadronic shower background is summarized for various assumptions in Table 9\textsuperscript{37}). The evaluation is based on a 20 rad. lengths deep ESD, of which the first five and the subsequent 15 R.L. are read out individually. Therefore, should the intrinsic photon resolution be required, sufficiently fine transverse subdivision of the ESD is one of the necessary requirements.
<table>
<thead>
<tr>
<th>Technique for Electromagnetic Shower Detectors</th>
<th>Lead Glass</th>
<th>Lead Glass + RPC</th>
<th>RPC Sandwich</th>
<th>Scintillator Sandwich</th>
<th>Flashtube Sandwich</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta_0 = K/E$</td>
<td>&gt; 0.2</td>
<td>&gt; 0.2</td>
<td>&gt; 0.3</td>
<td>&gt; 0.2</td>
<td>&gt; 0.2</td>
</tr>
<tr>
<td>$\Delta_0 / E$</td>
<td>&gt; 0.2</td>
<td>&gt; 0.2</td>
<td>&gt; 0.3</td>
<td>&gt; 0.2</td>
<td>&gt; 0.2</td>
</tr>
<tr>
<td>$\Delta(E)$ (typically)</td>
<td>3 cm</td>
<td>0.3 cm</td>
<td>0.3 cm</td>
<td>0.3 cm</td>
<td>0.3 cm</td>
</tr>
<tr>
<td>Energy range</td>
<td>3 cm</td>
<td>3 cm</td>
<td>3 cm</td>
<td>3 cm</td>
<td>3 cm</td>
</tr>
<tr>
<td>$\Delta\tau$ (typ.)</td>
<td>100 ns</td>
<td>300 ns</td>
<td>100 ns</td>
<td>60 ns</td>
<td>60 ns</td>
</tr>
<tr>
<td>Multidet. Separ. (typ.)</td>
<td>2.4 cm</td>
<td>2.4 cm</td>
<td>2.4 cm</td>
<td>2.4 cm</td>
<td>2.4 cm</td>
</tr>
<tr>
<td>$\langle \sigma \rangle$ (g/cm$^2$)</td>
<td>2.4</td>
<td>3.7</td>
<td>2.4</td>
<td>3.7</td>
<td>2.4</td>
</tr>
<tr>
<td>Typ. Dim. (typ. $&lt; 15 X_0$)</td>
<td>60 cm</td>
<td>60 cm</td>
<td>60 cm</td>
<td>60 cm</td>
<td>60 cm</td>
</tr>
<tr>
<td>$\gamma$-influence</td>
<td>few Gauss</td>
<td>few Gauss</td>
<td>few Gauss</td>
<td>few Gauss</td>
<td>few Gauss</td>
</tr>
<tr>
<td>Hadron discrim.</td>
<td>1%</td>
<td>1%</td>
<td>1%</td>
<td>1%</td>
<td>1%</td>
</tr>
<tr>
<td>Calibration</td>
<td>1% - 2%</td>
<td>1% - 2%</td>
<td>1% - 2%</td>
<td>1% - 2%</td>
<td>1% - 2%</td>
</tr>
<tr>
<td>Price ($4m$ cover.)</td>
<td>4.5 MSF</td>
<td>4.5 MSF</td>
<td>4.5 MSF</td>
<td>4.5 MSF</td>
<td>4.5 MSF</td>
</tr>
<tr>
<td>$t$ ... sampling step in $X_0$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* *
Table 9
Calorimetric Measurement of Photons in a Jet

<table>
<thead>
<tr>
<th>Method</th>
<th>Photon Component</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>True Energy GeV</td>
</tr>
<tr>
<td>EDS measurement only; optimized estimates based on longitudinal development</td>
<td>20</td>
</tr>
<tr>
<td>EDS + momentum analysis of charged particles (assumed without error)</td>
<td>20</td>
</tr>
<tr>
<td>EDS + momentum measurement, transverse segmentation 5 x 5 cm²</td>
<td>20</td>
</tr>
</tbody>
</table>

Hadronic Shower Detectors

A LEP physics programme requiring a 4π Hadronic Shower Detector (HSD) has been discussed previously and some of its instrumental implications are summarized in Table 10. Further studies of such physics problems are required to assess the following questions:

- which instrumental performance of the HSD's is required?
- what sensitivity in the study of the various reactions can be achieved, given the limitation of the instrumental (e.g. tails in resolution) or physics "background" (flavour cascades, where a substantial fraction of the energy is expected to be carried away by neutrinos)?

In addition to the investigation of the above physics topics the value of a total energy measurement can surely be argued: it will be the only way to differentiate between events where neutral energy was carried by hadrons or by neutrinos. Figure 18 is shown to indicate that the interesting fraction of K0's and neutrons is expected to increase with p and should not be ignored.

Our understanding of the physics of hadron calorimeters has considerably progressed in recent years. Here we give a tabular summary of the inherent physics limitations of the energy resolution of such devices, followed by a discussion of instrumental effects (Tables 11 - 14).

Fluctuations in Nuclear Binding Energy losses dominate the energy resolution. These fluctuations can however be effectively compensated by correlated Fission Energy deposit, if Uranium 238 is used as the absorber, with a resulting energy resolution of σ(E)/E ≈ 0.3/√E(GeV). It is summarized graphically in Figure 19. Table 12 also shows that there are a number of small intrinsic effects
<table>
<thead>
<tr>
<th>Physics</th>
<th>Measurement</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jet studies</td>
<td>E, $\vec{p}$, invariant mass</td>
<td>Reasonable spatial resolution; control of tails of resolution function</td>
</tr>
<tr>
<td>$W \rightarrow 2$ jets</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Spherocity</td>
<td>$\vec{p}$</td>
<td>Very good spatial resolution; increases sensitivity to rare events with large spherocity</td>
</tr>
<tr>
<td>Reactions producing neutrinos</td>
<td>$\vec{p}$ missing</td>
<td>Very good energy and momentum resolution required; background from flavor cascades has to be evaluated.</td>
</tr>
<tr>
<td>Quark searches</td>
<td>E</td>
<td>&quot;Super energy&quot; Method$^{39}$</td>
</tr>
</tbody>
</table>
Table 11

Contribution to Energy Resolution (Absorber)

<table>
<thead>
<tr>
<th>Effect</th>
<th>Energy</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>visible</td>
<td></td>
</tr>
<tr>
<td>Decay in front of Absorber</td>
<td>x</td>
<td>For LEP jets and a decay length of 1.50 m, the average loss is ~ 1% (C.W. Fabjan and H. Grote, ECFA/LEP 7, 1978)</td>
</tr>
<tr>
<td>Back leakage</td>
<td>x</td>
<td>Determines depth of device; optimization for LEP jets indicates: 7 I.L. of Uranium (~ 44 cm); will give ~ 3% leakage for 20 GeV K^0. Non gaussian contribution.</td>
</tr>
<tr>
<td>Front leakage (Albedo)</td>
<td>x</td>
<td>~ 3 - 4% for 10 GeV particle, ~ 2% for 20 GeV particle (less for K^0); non gaussian; energy in form of low momentum p, n, γ. May be background for tracking device!</td>
</tr>
<tr>
<td>μ, ν Production</td>
<td>x</td>
<td></td>
</tr>
<tr>
<td>Nuclear Effects:</td>
<td></td>
<td>See Table</td>
</tr>
<tr>
<td>Excitation</td>
<td>x</td>
<td>Produces n, p, γ in MeV range; non gaussian.</td>
</tr>
<tr>
<td>Binding Energy</td>
<td>x</td>
<td>See Table</td>
</tr>
<tr>
<td>Fission compensation</td>
<td>x</td>
<td>Consequence: different response for π^0 and h; good yardstick of Nuclear effects is R = e/h = signal for e/signal for h at same energy. Fluctuations in nuclear effects limit the energy resolution.</td>
</tr>
</tbody>
</table>
Table 12
Some Contributions to "Invisible" Energy

Estimated Energy of Escaping Muons and Neutrinos

<table>
<thead>
<tr>
<th>( E_0 )</th>
<th>40 GeV</th>
<th>300 GeV</th>
<th>1000 GeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{E_\nu + E_\mu}{E_0} )</td>
<td>1.3%</td>
<td>0.4%</td>
<td>0.3%</td>
</tr>
</tbody>
</table>

Binding Energy Losses

<table>
<thead>
<tr>
<th>Particle/energy (GeV)</th>
<th>Material</th>
<th>Binding energy in %</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \pi^-/1 )</td>
<td>Organic scint.</td>
<td>16%</td>
<td>10%</td>
</tr>
<tr>
<td>( \pi^-/3 )</td>
<td>Organic scint.</td>
<td>13%</td>
<td>8%</td>
</tr>
<tr>
<td>( \pi^-/7 )</td>
<td>Fe/Scint.</td>
<td>20%</td>
<td>15%</td>
</tr>
<tr>
<td>( \pi^-/10 )</td>
<td>A.L. = 40 cm</td>
<td>24% ± 4%</td>
<td>17% ± 3%</td>
</tr>
<tr>
<td></td>
<td>Fe/L Ar</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>A.L. = 31 cm</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 13

Summary on Calorimetry

**Electromagnetic Shower Sampling Calorimeters**

Energy Resolution: $\sigma(E)/E = 0.05 \left( \frac{dE}{dx} / \text{MeV} \right)^{\frac{1}{4}} \cdot \frac{1}{\sqrt{E(\text{GeV}) \text{ particle}}}$

$dE/dx...$ ionization loss per sampling cell
verified for iron, lead, uranium

Position Resolution: $\sigma(x) = 2.77 \text{ mm measured with 5 mm strips}$

Hadron Rejection: factor $\sim 100$, based on two longitudinal divisions.

References: W. Willis, V. Radeka NIM 120, 221 (1974)
C. Cerri et al., NIM 141, 207 (1977)
pp Note 05.

**Hadronic Shower Sampling Calorimeters**

$$\sigma(E)/E = 0.55 \cdot \frac{1}{\sqrt{E(\text{GeV}) \text{ particle}}} \quad \text{(non U-absorber)}$$

Energy resolution: verified 1-10 GeV/c

$$\sigma(E)/E = 0.25 \cdot \frac{1}{\sqrt{E(\text{GeV}) \text{ particle}}} \quad \text{(U238 absorber)}$$

(Sampling step finely enough chosen not to influence resolution)

Shower dimensions: longitudinal $\sim 4$ absorption lengths
transverse $\sim 1.5$ absorption lengths for $> 80\%$ containment of showers
(measured in U/LA cal. at 10 GeV/c)


Position Resolution: $\sigma_x = 0.67 \frac{\text{Collision length (cm)}}{\sqrt{E \text{ (GeV)}}}$ (cm)
(extrapolated from WA 18 data)
Table 14
Contribution to Energy Resolution (Read out)

<table>
<thead>
<tr>
<th>Effect</th>
<th>Magnitude</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sampling</td>
<td>E.M.S. see Table</td>
<td>Approx. valid for [1 \geq \frac{\text{Energy loss (active)}}{\text{Energy loss (passive)}} \geq 0.1]</td>
</tr>
<tr>
<td></td>
<td>H.S.: approximately two times the value of E.M.S. in same detector</td>
<td>For hadron showers: sampling fluctuation does not limit resolution.</td>
</tr>
<tr>
<td>Saturation effects</td>
<td></td>
<td>Present in Organic Scint. and Liquid Argon, but found not to influence resolution.</td>
</tr>
<tr>
<td>Noise, calibration effects, etc.</td>
<td></td>
<td>Strongly dependent on Read out method. For example in Pb-glass array (\sigma \text{ (Calib)} \leq 3%)</td>
</tr>
</tbody>
</table>
Figure 18: Momentum spectra of $\pi^\pm$, $K^\pm$ and $\bar{p}$ (Reference 40)
Figure 19: Measured energy resolution for sampling HSD's (Reference 42)
Figure 20: Schematic layout of the AFS Calorimeter under construction for the CERN ISR. The total volume is subdivided into 800 "Towers" viewed by four photomultipliers each, and thus allowing two longitudinal readouts and shower localization.
which reduce the total visible energy in such calorimeters by a few percent. 
Fluctuations in this invisible energy will be one of the limitations to a Total 
Energy Measurement. Table 13 provides a summary of energy and position resolution.

The development of readout techniques carried out for ESD's clearly also 
benefits HSD-instrumentation. These are summarized in Table 14. Considerable 
experience is being accumulated with liquid Argon Counters at all the major labora-
tories\(^3\)). Soon it will be possible to compare the merits of this charge-collecting 
technique with the more recent Wavelength Shifter Readout of Scintillator Calori-
meters. A large 8 sr Uranium-Scintillator Calorimeter is under construction for 
the AFS at the ISR (Fig. 20).

5. Conclusions

The extensive development of large detector systems for storage ring 
physics during recent years justifies confidence that a substantial part of the 
physics programme planned for LEP can be carried out with presently available 
methods and techniques. Much recent refined understanding of the physics of det-
tectors combined with spectacular advances in electronics techniques will surely 
bring us closer to the "ultimate" detector in the next years. Many of the most 
interesting research topics on detectors would benefit from an interdisciplinary 
approach and could therefore be profitably pursued at University laboratories.

It was a pleasure to have been associated with this Summer Study. Its 
stimulating atmosphere owes much to its Organizer, M. Jacob.
References


2. W. Willis, unpublished notes.


5. W. Farr et al., Nucl. Instr. and Methods 154 (1978) 175


17. A. Astbury et al., A 4π Solid Angle Detector for the SPS used as a Proton-Antiproton Collider at a Centre of Mass Energy of 540 GeV, CERN/SPSC/P~92 1978.


25. I am indebted to J.H. Cobb, who carried out the Monte Carlo calculations.


DETECTION OF EVENTS WITH JET STRUCTURE

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CERN, Geneva

Copies available upon request from Ch. Redman, CERN/ISR LEP Summer Study Secretariat.
1. Introduction

This note summarizes the work on the design of a detector suited to investigate the jet structure of events produced in $e^+e^-$ collision at LEP energies.

It outlines the physics aims of the study, then reports on the generation of events with jet structure and the tracking of such events through a first approximation of detector. Performance of the detector, possible improvements and obvious shortcomings are discussed at the end.

2. Physics Aims

Jet physics is likely to represent an interesting topic at LEP energies. Predictions based on phenomenological extrapolations of present data and on QCD allow an oversimplified division of the field of interest into two extreme situations.

a) Study of jet kinematics in both cases where the jets originate from resonances, by for instance a three-gluon process, and when they originate from the continuum. Experimentally, kinematical quantities (sphericity etc.) related to the structure of the jet must be constructed from measured values of momentum and energy (with errors). In order to test the sensitivity of these quantities to different mechanisms of jet production and design feasible experiments one needs simulated exclusive particle distributions in the jets and a realistic guess of the errors in the detector.

b) Study of the particle composition of jets to reveal the underlying quark flavour structure and to distinguish genuine jets from extended cascade decays. Again one needs to produce a simulation of the particle composition of the jet and estimate the identification efficiency and errors.

3. Jet Models

The need for simulating exclusive events with a jet structure comes, on the one hand, from the stringent requirements that a tight bunch of particles, charged and neutral, put on a detection system. The evaluation of detector performance and its optimization for a given type of physics are crucially dependent on the structure of the event and can be studied only by realistic simulation and extensive tracking.

On the other hand, the choice of efficient estimators constructed from measured quantities depends on accuracy of measurements as well as on the structure and variety of events to be studied. The sensitivity of the estimators proposed (sphericity, spherocity, thrust, acoplanarity, triplicity etc.) has to be tried in comparing different jets in a realistic detector.
We have chosen a CM energy of $70\pm70$ GeV for all the jet simulation reported here [1].

3.1 Jet Production

i) Two-Jet Case ($q\bar{q}$)

The process is assumed to take place via one photon:

$$e^+ + e^- \rightarrow \gamma + q + \bar{q},$$

each quark giving a jet. The angular distribution for unpolarized beams is $dN = (1 + \cos^2\theta)\ d(\cos\theta)$ where $\theta$ is the polar angle with respect to the beam direction.

ii) Three-Jet Case - Gluon Bremsstrahlung ($q\bar{q}g$)

In the production of two jets a gluon is radiated from one of the quarks and makes its own jet. The 5-fold differential cross section of [2] has been reformulated for massive particles and used in the simulation.

iii) Three-Jet Case - Resonance Decay by Three Gluons ($ggg$)

This topology is thought to give three jets with a cleaner mutual separation than $q\bar{q}g$. Hence it has not been considered for this study.

3.2 Jet Fragmentation

i) Longitudinal Phase-Space

Our first generation of two-jet events was made by assuming a phase-space distribution with limited $p_\perp$ with respect to the jet axis. Energy-momentum conservation is correctly enforced. However, the nature of the hadrons produced and their multiplicity are not predicted. Therefore all particles are assumed to be pions and the average total multiplicity is fed into the calculation: $<n_{\pi}> = 25$.

The model is adequate for a variety of studies, primarily when jet kinematics or the properties of pions (photons) alone are concerned. See figs 1 and 2.

ii) Quark Jet Fragmentation

A more elaborate fragmentation of quark jets was generated according to Fields and Feynman [1,3]. It predicts completely the nature of the particles produced, by using as input the relative frequency of quark flavours and the known branching ratios for vector and pseudoscalar meson decay. Limitations of the model as it stands are non conservation of momentum per jet but only per event and non conservation of quantum number either per jet or per event. Baryons are ignored. Only $u$, $d$ and $s$ quarks are present. See table I and figs 3 to 7.
Figure 1: Display of a two-jet event. Magnetic field is along the beam axis at a strength of 1.5 T. Solid lines are charged pions, dotted lines neutral pions. The two projections shown are: a) in the plane of the beams and the jets and b) perpendicular to the beams.
Figure 2: Display of a two-jet event. Magnetic field is along the beam axis at a strength of 1.5 T. Solid lines are charged pions, dotted lines neutral pions. The two projections shown are: a) in the plane of the beams and the jets and b) perpendicular to the beams.
<table>
<thead>
<tr>
<th></th>
<th>Primaries</th>
<th>Stable Primaries</th>
<th>Stable Secondaries</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^0$</td>
<td>1.27</td>
<td>1.27</td>
<td>5.23</td>
</tr>
<tr>
<td>$\pi^-$</td>
<td>1.26</td>
<td>1.26</td>
<td>4.97</td>
</tr>
<tr>
<td>$K^-$</td>
<td>0.59</td>
<td>0.59</td>
<td>0.71</td>
</tr>
<tr>
<td>$\pi^+$</td>
<td>1.26</td>
<td>1.26</td>
<td>4.98</td>
</tr>
<tr>
<td>$\eta$</td>
<td>0.70</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\bar{K}^0$</td>
<td>0.60</td>
<td>0.60</td>
<td>0.72</td>
</tr>
<tr>
<td>$K^+$</td>
<td>0.69</td>
<td>0.69</td>
<td>0.67</td>
</tr>
<tr>
<td>$K^0$</td>
<td>0.61</td>
<td>0.61</td>
<td>0.66</td>
</tr>
<tr>
<td>$\eta'$</td>
<td>0.62</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho^0$</td>
<td>1.18</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho^-$</td>
<td>1.08</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$K^{*-}$</td>
<td>0.61</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho^+$</td>
<td>1.12</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\omega$</td>
<td>1.04</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\bar{K}^*0$</td>
<td>0.58</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$K^{*+}$</td>
<td>0.55</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$K^{*0}$</td>
<td>0.54</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\phi$</td>
<td>0.29</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma$</td>
<td></td>
<td></td>
<td>1.36</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>14.60</td>
<td>6.28</td>
<td>19.30</td>
</tr>
</tbody>
</table>
Figure 3: Average total multiplicity for: a) (q\bar{q}) two-jet events and b) (q\bar{q}g) three-jet events (multiplicity of the gluon jet alone shown by a dashed line)[1]
Figure 4: Momentum distribution of all particles for (q̅q) two-jet events, with $\pi^0$ stable [1]
Figure 5: Momentum distribution of leading particles for (q\bar{q}) two-jet events, with \(\pi^0\) stable, where leading is defined as the highest momentum particle in each jet [1].

Figure 6: Cumulative distribution of angular separation of any two charged particles in the same jet for (q\bar{q}) two-jet events [1].
Figure 7: Sphericity of $(q\bar{q})$ two-jet events (solid line). Sphericity of $(q\bar{q}g)$ three-jet events (dashed line) [1]
iii) Gluon Jet Fragmentation

A gluon goes into a (qq) pair and each quark is treated then as in the fragmentation of a quark jet [1] (figs 3 and 7).

4. Detector Design to First Order

We took as goals for a first attempt of a detector design to do the best conceivable job on:

a) Charged particle detection in a magnetic field for momentum measurement and particle identification by dE/dx in a gas.

b) Detection of electromagnetic showers from all γ in the jet and from e⁺ for normalization, inclusive and exclusive studies.

In carrying on the design we tried to keep open the possibility of complementing our basic goals by:

c) A hadron calorimeter to detect a fraction of the neutral energy escaping from the EM shower detector.

d) A muon identifier both for normalization and for inclusive muons.

e) A time-of-flight system.

f) A tagging system to discriminate between one and two photon processes.

We will skip all arguments in favour of a solenoid as magnetic field configuration, but we stress the choice of installing the electromagnetic shower detector inside the coil in order not to spoil the performance.

All considerations of detector design which follow are based upon tracking two-jet events.

4.1 Charged Particle Detector

A starting point for our early choice of charged particle detector came from jet separation studies during the 1977 ECFA Hamburg Study Week [4]. There it had become clear that a granularity of the detector of the order of 1 cm² in the plane perpendicular to the beams was necessary.

A second very stringent requirement came from our choice of using dE/dx measurements in a gas as the only way of achieving particle identifications in an almost 4π geometry. Since track length and momentum resolutions are needed simultaneously [5,6,7] we were lead to propose a relatively high magnetic field over a volume large at first sight but still insufficient, so that a pressurized detector should be envisaged.

The charged particle detector we propose is a large cylindrical gas volume (ID = 20 cm; OD = 360 cm; L = 600 cm) pressurized at 4 atm with sense wires parallel to the beams. It works as a drift chamber combining drift and pulse height measurements in order to give position (by time and charge division) and dE/dx (by summing pulse heights).
In discussing the granularity of such a detector we have to separate between its performance for position and for energy loss measurements, as they are understood today.

i) Position Measurements

Let us consider a track in a plane perpendicular to the beam axis travelling (almost) radially away from the intersect (fig. 8).

The track will be measured as many times as there are measuring wires crossing the plane. An increase of the number of measuring points \( N \), hence the number of measuring channels, improves the momentum measurement as \( \sim \sqrt{N} \). The minimum azimuthal separation of two tracks on the same wire is determined by the recovery time of the system after an avalanche has reached a wire (50 ns corresponding to 0.25 cm) as well as by the multihit capability of the electronics.

The measurement of the coordinate along the wire is done by charge division, where a large charge collection time is needed. No double hit resolution seems possible here, the first track drifting to the wire being recorded only.

Granularity in the radial direction means number of wires. In the azimuthal direction, means threshold of electronics and multihit capability. There is no granularity in the beam direction.

ii) Energy Loss Measurements

As pointed out in the previous paragraph, pulse height measurements require a long charge collection time and it seems hard to perform measurements on tracks drifting to the same wire when separated in time by \(< 200 \text{ ns} (\sim 1 \text{ cm})\). Moreover, immediately above this value of separation, precision may be insufficient for the second track.

Granularity in the radial direction means number of wires. Fine granularity in the azimuthal direction means number of wires; coarse granularity means fast ADC's and/or analog switches. There is no granularity in the direction of the beams.

We conclude that the granularity of the detector is determined to a large extent by the constraints of the energy loss measurements.

iii) Unobscured Track Length

The influence of granularity on detector performance can be seen quantitatively as the variation of the number of detector cells occupied by more than one track, hence obscured for the purpose of momentum and energy loss measurements. The unobscured track length is then the total path of a particle through useful cells. The unobscured track length has been investigated for different cell sizes. Plots are shown in fig. 9 as a function of momentum.
Figure 8: Definition of a detection cell
Figure 9: Average unobscured track length vs. particle momentum, for different (azimuthal) cell width [8]

Figure 10: Unobscured track length vs. particle momentum for different magnetic field strength [8]
iv) **Magnetic Field Strength and Detector Radii**

There are two more variables which we have to play with, namely magnetic field strength and detector boundaries.

The magnetic field has an obvious influence on the quality of the momentum measurements and a less intuitive one on the number of low energy primaries and secondaries curling up near the vacuum pipe and on the separation of high energy particles in a jet: in other words on the unobscured track length. For fixed cell size, the unobscured track length has been studied as a function of momentum and magnetic field strength (fig. 10). With the exception of low energy particles, it can be said that the unobscured track length does not change appreciably by varying the field strength.

A field of 1.5 tesla is proposed, which combines manageable engineering problems and stored energy with acceptable momentum resolution [7,8].

The influence of varying detector radii is given in fig. 11.

v) **Pattern Recognition and Momentum Resolutions**

The large number of measuring points per track is beneficial to an efficient pattern recognition and follows, in this respect, the well established trend of most detectors now being constructed or proposed.

In addition to the large number of points, momentum resolution will benefit from measuring precision (high pressure, hence good ionization statistics). We have assumed an error of 200 μ on each measurement. Fig. 12 shows the expected momentum resolution integrated over all track directions.

vi) **Particle Identification by $dE/dx$**

For efficient identifications of all particles in a jet we must have:

a) A fine sampling of the energy loss measurements over the measured track length. A number of samples larger than $\sim 100$ is adequate [5,6].

b) A product of unobscured track length times pressure of the order of $\sim 5$ m . atm or larger [5,6].

We choose an A + 10% CH$_2$ mixture at 4 atm and investigate the efficiency for identifying all particles in a jet. Table II [8] shows how many particles belonging to the same jet have an unobscured track length below a given value $L_{\text{min}}$. For example, if the minimum track length required for identification is 100 cm, 88% of the jets are fully identified, 2%, 9% and 1% of the jets have 1, 2 and 3 unidentified particles respectively, assuming the identification to be momentum independent.
Figure 11: Unobscured track length vs. particle momentum for different inner and outer radii [8]

Figure 12: Momentum resolution vs. particle momentum for different azimuthal cell width [8]
TABLE II

Probability (in %) that a certain number $N$ of particles in a jet have unobscured track lengths smaller than a limit $L_{\text{min}}$ (cm) for a cell size $\Delta r = 1 \text{ cm}$ $\Delta \phi = 1 \text{ cm}$ [8].

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Unobscured track lengths as a function of momentum is shown in fig. 10. Averaging over momenta above 1 GeV/c yields an unobscured track length of 125 cm. This value has been used to compute particle separations [6]. Predictions are shown in fig. 13 and can be summarized as follows: at a three standard-deviation level
- $e\pi$ separation below 28 GeV/c
- $\pi K$ separation between 1.5 and 38 GeV/c
- Kp separation below 1.6 GeV/c.

4.2 Electromagnetic Shower Detector

We will be concerned mostly with the geometrical constraints coming onto the EM shower detector by tight jet structure of the events.

The properties of available or conceivable EM shower detectors have been reviewed in preparation of this Summer Study and are summarized elsewhere [9,10].

The EM shower detector consists of a cylinder (ID = 380 cm), 60 cm thick with end caps 6 m apart. The study of the detector has been concentrated only to the impact of $\gamma$'s and charged particles onto the inner surface of the cylinder and the end caps. The entire field of investigations on the longitudinal and lateral shower development in the magnetic field and the means of separating close-by showers by more refined methods has been left untouched.

We will report here on the studies on the granularity at the inner surface in relation to two problems:
- separation of two $\gamma$'s at their impact point.
- separation of e's from $\pi$'s.

i) $2\gamma$ Separators

We have tracked all $\gamma$'s to the inner surface of the EM shower detector and computed the (shortest) distance between two impact points.

Fig. 14 shows a plot of the distance for any two $\gamma$'s originating from the same jet and for the two $\gamma$'s decaying from a $\pi^0$ in the jet.

By knowing the average multiplicity of $\pi^0$ per jet $<n_{\pi^0/jet}> \approx 4.2$ it appears that each jet has $\sim 2$ $\gamma$'s within a distance of 10 cm. If the analyses of the jet requires a detailed knowledge of the parent $\pi^0$, a finer subdivision should be attempted. For instance 4 cm would give 4.4% of unresolved $\pi^0$'s per jet.
Figure 13: e-π, π-K and K-π separation expressed in standard deviations as a function of momentum [6].

Figure 14: Cumulative distribution of the distance of the impact points of two photons: a) belonging to the same jet (solid line) and b) originating from the same neutral pion (dashed line).
ii) π/e Rejection

The capability of distinguishing relatively low energy e's, from π's allows the study of less common decay of the parent particles in the jet and the search for prompt electrons. It gives also a handle to discriminate genuine jets from complicated cascades.

At the top electron energy π/e separation provides a tool for a proper normalization.

- π and γ Hitting the same EM Shower Detector Cell

We have tracked both γ’s and charged pions to the inner surface of the EM Shower Detector. For a fixed cell size a scatter plot of the energy of the γ vs. the momentum of the pion hitting the same cell allows an estimate of the probability of confusing a γ-π pair with an electron (figs 15 and 16), once the errors on energy and momentum are known. By assuming \( \sigma_E/E = 0.1/\sqrt{E} \) and \( \sigma_p/p^2 = 10^{-3}(\text{GeV/c})^{-1} \) one obtains a π/e rejection ratio of \( 10^{-2} \) for a granularity of 4 cm and \( 4.10^{-2} \) for a granularity of 8 cm.

- Confusion of Hadronic and Electromagnetic Showers

We have not calculated the shower development and we rely on recent experimental results based on comparison of shower profiles for π⁻ and e⁻ at 4 GeV/c [11]. For an electron efficiency of 82.8% a π/e rejection of \( 1/410 \) can be obtained.

- Further Rejection by dE/dx Measurements

As seen from fig. 13 π/e separation by energy loss measurements is efficient up to 28 GeV at the 3σ level. This gives an additional factor of \( 10^{-2} \) in rejection which can be combined with each of the figures given above. Proper care should be taken for the correlations given by the momentum measurement common to the different methods.

It appears that an overall rejection of \( 10^{-4} \) per track is not out of reach.

4.3 Detector Layout

The considerations of sections 4.1 and 4.2 on charged particle and electromagnetic shower measurements fix the main parameters of the jet detector.

Performance of the additional equipment listed at the beginning of section 4 has not been worked out in the course of the study, but only dimensions have been roughly guessed.
Figure 15: Scatter plot of the energy of a photon $E_\gamma$ vs. the momentum of a charged pion $p_\pi$ belonging to the same event for 100 events. The distance of the impact points on the EM shower detector inner surface is selected to be smaller than 8 cm.

Figure 16: Scatter plot of the energy of a photon $E_\gamma$ vs. the momentum of a charged pion $p_\pi$ belonging to the same event for 100 events. The distance of the impact points on the EM shower detector inner surface is selected to be smaller than 4 cm.
Figure 17: Layout of the LEP jet detector: a) in a vertical plane containing the beams, b) in a vertical plane perpendicular to the beams.
A layout of the jet detector is shown in fig 17. A few numbers may give a feeling of its size:

- Magnetic field strength: 1.5 T
- Magnetic stored energy: \( \sim 130 \text{ MJ} \)
- Weight of iron for return yoke and forward calorimeters: \( \sim 2000 \text{ T} \)
- Weight of EM shower detector: \( \sim 130 \text{ T} \)
- Number of channels of electronics for central detector: \( \geq 6000 \)
- Overall length: \( \geq 10.5 \text{ m} \)
- Overall width: \( \geq 10 \text{ m} \)
- Depth of pit required below beam height: \( \geq 5.5 \text{ m} \)

Acknowledgements

This report is the result of the work by many people during the preparation of the Summer Study: I feel embarrassed for not giving them, individually, the credit they deserve, but I am afraid of being incomplete. I want, however, to single out the most extensive contributions: by H. Grote on particle tracking and by D. Drijard on jet models.
REFERENCES