CM-P00061705

Ref. TH. 2323-CERN

Archives

UNITARITY AND HADRON SELF-ENERGIES :
THE BREAKING OF SU(6) IN THE MESON SPECTRUM

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ABSTRACT

In this paper the problem of unitarity effects in hadron self-energies is studied. The resulting framework is then applied, within the recently proposed quark geometrodynamical theory, to the (approximate) calculation of mass shifts and SU(6) symmetry breaking patterns of the low-lying mesons. We derive a correct description of the mass splittings in the $(35, L=1)$ $SU(6) \times O(3)$ multiplet, good experimental agreement for the Zweig-forbidden decay $(\phi \rightarrow 3\pi)$ and predict $B(f'(1520) \rightarrow \pi^+\pi^-) \simeq 0.025$.

In the geometrodynamical approach to quarks and hadrons ¹⁾⁻⁵⁾, one starts with a world where all hadronic states (the permissible quark orbits) are non-interacting and therefore stable. The spectrum and the "wave-functions" for hadronic states in this ideal situation have been determined following the very general geometrodynamical laws described in previous works ¹⁾⁻⁵⁾. These results, however, are of importance for the actual physical world only if the interaction among the hadronic states is weak enough to allow a meaningful perturbative treatment of its effects. Thus the whole approach stands or falls depending on the reliability of such a perturbative framework. It has been argued that signals for the existence and accuracy of a perturbative expansion of the hadronic S matrix do indeed exist, and that the usual arguments to demonstrate that "strong" interactions are strong do not apply to the geometrodynamical approach. For in this theory aspects usually ascribed to a strong interaction like Regge trajectories and form factors are already included at the no-interaction level. Thus we have a situation where the rich structure of hadrons is decoupled from the problem of constructing a unitary S matrix of the hadronic world.

As is well known, today we are able to solve the problem of unitarity in a relativistic quantum field theory only through the iterative procedures of perturbation theory : beyond perturbation theory there lies a no man's land, whose conquest nobody can forecast. This is why conventional field theoretical approaches to hadrodynamics have been abandoned at an early stage; the world from which one tried to obtain perturbatively hadronic structure as well as unitarity is, in fact, too distant from the real one.

As a test of the reliability of a perturbative approach to unitarity (but not to hadronic structure !) in the geometrodynamical approach ^{*)}, we present, in this paper, the first calculations of unitarity corrections to the meson spectrum. We concentrate only on the mesons with $L=0$ and 1 , leaving obviously out the pseudoscalar mesons for the reasons discussed in Ref. 4).

The success of these calculations is a further corroboration of the geometrodynamical theory of the hadrons.

1. HADRON SELF-ENERGIES

Let us consider the lowest order amplitude in meson-meson scattering (Fig.1) ³⁾. We can write

*) Discussed at length in Ref. 3).

$$a_s^{(2)}(12 \rightarrow 34) = \sum_i g(34 \leftarrow i) \frac{1}{s - m_{0i}^2} g(i \leftarrow 12) \quad , \quad (1.1)$$

where i denotes any of the meson states comprising the hadron spectrum. At this order, the meson states are stable and as a consequence the amplitude $a_s^{(2)}(12 \rightarrow 34)$ is a sum of poles located at real, positive s . In the next order among many other diagrams we encounter $a_s^{(4)}(12 \rightarrow 34)$ (Fig.2), whose expression is given by :

$$a_s^{(4)}(12 \rightarrow 34) = \sum_{i,j,ab} g(34 \leftarrow j) \frac{1}{s - m_{0j}^2} g(j \leftarrow ab) \Pi_{ab}(s) g(ab \leftarrow i) \frac{1}{s - m_{0i}^2} g(i \leftarrow 12), \quad (1.2)$$

where $\Pi_{ab}(s)$ is the "self-energy function" whose structure shall be discussed shortly. Denoting by $G(12)$ the "column vector" $g(i \leftarrow 12)$, by $G^+(34)$ the "row vector" $g(34 \leftarrow i)$, by $\Pi_{ab}(s)$ the square matrix $g(j \leftarrow ab) \Pi_{ab}(s) \cdot g(ab \leftarrow i)$. and

$$\Delta^{(0)}(s) = \delta_{ij} \frac{1}{s - m_{0i}^2} \quad ,$$

in matrix form we can write :

$$a_s^{(2)}(12 \rightarrow 34) = G^+(34) \Delta^{(0)}(s) G(12) \quad (1.3a)$$

$$a_s^{(4)}(12 \rightarrow 34) = G^+(34) \Delta^{(0)}(s) \Pi(s) \Delta^{(0)}(s) G(12) \quad (1.3b)$$

⋮

Summing up all amplitudes $a_s^{(2n)}(12 \rightarrow 34)$ we obtain

$$\begin{aligned} \sum_n a_s^{(2n)}(12 \rightarrow 34) &= G^+(34) [\Delta^{(0)}(s) + \Delta^{(0)}(s) \Pi(s) \Delta^{(0)}(s) + \dots] G(12) \\ &= G^+(34) \Delta(s) G(12) \quad , \end{aligned} \quad (1.4)$$

where

$$\Delta(s) = \Delta^{(0)}(s) [1 - \Delta^{(0)}(s) \Pi(s)]^{-1} \quad (1.5)$$

$\Delta(s)$ is the "dressed" propagator matrix and results from the "unitarization" of the "bare" propagator $\Delta^{(0)}(s)$. The procedure we have followed is completely parallel to the derivation of the full propagator in quantum field theory^{*}, so that we are dispensed from an ampler discussion.

2. CALCULATION OF $\Pi_{ij}(s)$

We now analyze the matrix $\Pi_{ij}(s)$ which can be expressed as

$$\Pi_{ij}(s) = \sum_{ab} G_i(ab) \Pi_{ab}(s) G_j^+(ab) \quad (2.1)$$

where the sum runs over all possible quasi-two-particle states. By angular momentum conservation $\Pi_{ab}(s)$ is diagonal in orbital angular momentum l , thus we can write

$$\Pi_{ab}(s) = \sum_l \Pi_{ab}^{(l)}(s). \quad (2.2)$$

$\Pi_{ab}^{(l)}(s)$ depends only on the angular momentum l and on the masses m_a^2 and m_b^2 of the meson states a and b ; and we can set

$$\Pi_{ab}^{(l)}(s) = \Pi^{(l)}(s; m_a^2, m_b^2). \quad (2.3)$$

Furthermore, the imaginary part is given by

$$\text{Im } \Pi^{(l)}(s; m_a^2, m_b^2) = -\frac{1}{8\pi} \left(\frac{2p}{\sqrt{s}} \right)_{ab}^{2l+1} \quad (2.4)$$

where

$$\left(\frac{2p}{\sqrt{s}} \right)_{ab} = \frac{1}{s} \lambda^{1/2}(s, m_a^2, m_b^2) \quad (2.5)$$

and

$$\lambda(a, b, c) = a^2 + b^2 + c^2 - 2ab - 2ac - 2bc$$

is the well-known triangular function. Equation (2.4) defines the "conventionally normalized" self-energy function satisfying the two conditions :

^{*}) See, for instance, Ref. 6).

- i) correct threshold behaviour;
- ii) correct asymptotic behaviour for $p \rightarrow \infty$ (single partial wave unitarity limit).

However, (2.4) does not determine completely $\Pi^{(\ell)}(s; m_a^2, m_b^2)$, as the relevant dispersion relation needs a subtraction.

One might think that we have to introduce an infinite number of subtraction constants [one for each $\Pi^{(\ell)}(s; m_a^2, m_b^2)$], thus taking away any predictive power from the theory. This is, however, not the case : in order to obtain $\Pi^{(\ell)}(s; m_a^2, m_b^2)$ for any ℓ , m_a^2 and m_b^2 we need only one subtraction constant. In fact, if we consider the function

$$G^{(\ell)}(s; m_a^2, m_b^2; m_o^2) = \Pi^{(\ell)}(s; m_a^2, m_b^2) - \Pi^{(0)}(s; m_o^2, m_o^2) \quad (2.6)$$

its imaginary part is given by

$$\text{Im } G^{(\ell)} = -\frac{1}{8\pi} \left[\left(\frac{2p}{\sqrt{s}} \right)_{ab}^{2\ell+1} - \left(\frac{2p}{\sqrt{s}} \right)_{oo} \right] \quad (2.7)$$

and is $O(1/s)$ for $s \rightarrow \infty$; and we can write an unsubtracted dispersion relation for it. This means that, fixing the subtraction constant for $\Pi^{(0)}(s; m_o^2, m_o^2)$, i.e.,

$$\Pi^{(0)}(s; m_o^2, m_o^2) = \gamma(m_o^2) - \frac{s}{8\pi^2} \int_{4m_o^2}^{\infty} \frac{ds'}{s'-s} \left(\frac{2p}{\sqrt{s}} \right)_{oo}, \quad (2.8)$$

all other $\Pi^{(\ell)}$'s are uniquely determined. One can in fact write :

$$\Pi^{(\ell)}(s; m_a^2, m_b^2) = \lim_{\Lambda \rightarrow \infty} \left\{ \gamma(m_o^2) - \frac{1}{8\pi^2} \int_{s_{ab}}^{\Lambda} \frac{ds'}{s'-s} \left(\frac{2p}{\sqrt{s}} \right)_{ab}^{2\ell+1} + \frac{1}{8\pi^2} \int_{4m_o^2}^{\Lambda} \frac{ds'}{s'} \left(\frac{2p}{\sqrt{s}} \right)_{oo} \right\} \quad (2.9)$$

and the limit $\Lambda \rightarrow \infty$ is well defined. Fixing for convenience m_o^2 as the ρ mass [$m_o^2 \simeq 0.6 \text{ GeV}^2$], the expression (2.9) can be expressed in terms of elementary functions and of the constant $\gamma(m_o^2)$, which we shall determine once and for all by fitting (approximately) the ω - ρ mass difference.

3. PERTURBATIVE MASS SHIFTS AND WIDTHS

Following our perturbative philosophy, we can neglect in (1.5) the mixing among states belonging to different Regge trajectories (radial recurrences), as such mixings will be suppressed by "energy denominators" which are substantially bigger than the self-masses. Making a diagonal approximation, (1.5) becomes

$$\Delta(s)_{ij} \simeq \delta_{ij} \frac{1}{s - m_{\delta i}^2 - \Pi_{ii}(s)} \quad (3.1)$$

which yields a propagator of polar form; such a pole lies on the second Riemann sheet and its location is given by :

$$\text{Re} = m_i^2 = m_{\delta i}^2 + \text{Re} \Pi_{ii}(m_i^2) \quad (3.2)$$

$$\text{Im} = m_i \Gamma_i = - \text{Im} \Pi_{ii}(m_i^2) ; \quad (3.3)$$

leading to the mass shift

$$\delta m_i = \frac{1}{2m_i} \text{Re} \Pi_{ii}(m_i^2) , \quad (3.4)$$

and the width :

$$\Gamma_i = - \frac{1}{m_i} \text{Im} \Pi_{ii}(m_i^2) . \quad (3.5)$$

Thus the mass shift from the symmetric quark model value $[SU(6) \times O(3)]$ of a given hadronic state is given by the real part of the self-energy function, while its imaginary part produces its width according to (3.5).

In the no-mixing approximation (2.1) can be written

$$\begin{aligned} \Pi_{ii}(s) &= \sum_{\ell} \sum_{ab} G^{(\ell)}(i \rightarrow ab) \Pi^{(\ell)}(s; m_a^2, m_b^2) , \quad (3.6) \\ G^{(\ell)}(i \rightarrow ab) &= G_i^{(\ell)}(ab)^2 ; \end{aligned}$$

and $G_i^{(\ell)}(ab)$ can be expressed, by means of the quark model and $SU(3)$ symmetry as :

$$G_i^{(\ell)}(ab) = \begin{cases} g_+^{(\ell)} \text{Tr}(\lambda_i \{a, b\}) \\ g_-^{(\ell)} \text{Tr}(\lambda_i [a, b]) \end{cases}, \quad (3.7)$$

depending on whether the product of the charge conjugation eigenvalues of the three multiplets is $+1$ (g_+) or -1 (g_-). Through (3.6) and (3.7) we can calculate the mass shift and the width of any hadronic state; in practice however, we will only be able to compute the contributions of the low-lying states. In order to make sure that such contributions correctly describe the hadronic mass shifts, we must have control on the contributions of the states with large m_a or m_b .

It turns out that these contributions are quite harmless. Firstly, in terms of $SU(3)$, due to the insensitivity of $\Pi^{(\ell)}(s; m_a^2, m_b^2)$ to breaking at large masses, we can perform the sums over the intermediate states a and b and obtain :

$$\begin{aligned} \sum_{ab} \text{Tr}(\lambda_i \{a, b\}) \text{Tr}(\lambda_i^+ \{a^+, b^+\}) &= 2 [3 \text{Tr}(\lambda_i \lambda_i^+) + \text{Tr}(\lambda_i) \text{Tr}(\lambda_i^+)] \\ \sum_{ab} \text{Tr}(\lambda_i [a, b]) \text{Tr}(\lambda_i^+ [a, b]) &= 2 [3 \text{Tr}(\lambda_i \lambda_i^+) - \text{Tr}(\lambda_i) \text{Tr}(\lambda_i^+)] \end{aligned} \quad (3.8)$$

Exchange degeneracy of high mass states leaves only the term $\text{Tr}(\lambda_i \lambda_i^+)$, which imparts an equal shift to all the members of an $SU(3)$ multiplet.

Secondly, calling $D_i^{(\ell)}(ab)$ the contribution of the diagram in Fig. 3, from our definition of the coupling constant $G_i^{(\ell)}(ab)$ we have

$$G_i^{(\ell)}(ab)^2 = \left(\frac{M}{2p}\right)^{2\ell} D_i^{(\ell)}(ab) \quad (3.9)$$

where

$$\left(\frac{M}{2p}\right) = \frac{m_i^2}{\chi^{1/2}(i, a, b)}. \quad (3.10)$$

For large m_b^2 we can compute from the geometrodynamical rules ³⁾

$$D_i^{(\ell)}(ab) \simeq \frac{1}{|m_b^2 - m_i^2|} \delta_{R^2}^4\left(\frac{m_a^2}{2}\right) \delta_{\ell\ell_i} \quad (3.11)$$

where $\delta_{R^2}(z)$ is the "fat- δ -function" defined in Ref. 1). Equation (3.11), together with (3.9) and (3.10), implies that the high mass contributions affect only the quark states in s wave ($\ell=0$) and that the magnitude of such terms cannot be calculated due to the divergence (logarithmic) of the sum over the massive states b. Thus high mass states induce a splitting between the singlet and the triplet $L=0$ states of the general (approximate) form

$$\pi_3(s) - \pi_1(s) \simeq A + B \ln s/s_0, \quad (3.12)$$

where B can be, with some labour, computed and A is a constant to be fitted experimentally. It is interesting to note that the structure of the contribution (3.12) resembles the effect of "gluon exchange" advocated in quark models to explain hyperfine splitting⁷⁾.

Summarizing the results of this section, we can compute hadronic mass shifts by use of (3.6) and (2.9), and restricting ourselves to the low-lying states. High mass states affect only the $L=0$ mesons and induce a triplet-singlet splitting whose magnitude is given approximately by (3.12).

4. APPLICATION TO THE LOW-LYING MESON STATES

In this section we calculate the contributions to the mass shifts of the S and P meson states, by taking as intermediate states the S states only. According to the discussion in Section 3, inclusion of higher states should change very little our results; however, in the future we hope to be able to take into account also higher states.

For the subtraction constant we choose the value

$$\gamma(m_p^2) = -.25. \quad (4.1)$$

4-a $\underline{\underline{^3S_1 \rightarrow ^1S_0 + ^1S_0 \ (\ell=1)}}$

The coupling can be written as :

$$\frac{1}{2} G^{(1)}(^3S_1 \rightarrow ^1S_0 + ^1S_0) \text{Tr}(\lambda_i [\lambda_a, \lambda_b]) \text{Tr}(\lambda_i^\dagger [\lambda_a^\dagger, \lambda_b^\dagger]) \quad (4.2)$$

and we determine $G^{(1)}$ by the $\rho \rightarrow \pi\pi$ width :

$$\Gamma(\rho \rightarrow \pi\pi) = \frac{1}{M_\rho} \frac{1}{8\pi} \left(\frac{2p}{M}\right)^3 G^{(1)} \times 2 = .15 \text{ GeV}. \quad (4.3)$$

The mass shifts for the various numbers of the vector meson multiplets are reported in Table I.

4-b $\underline{\underline{^3S_1 \rightarrow ^3S_1 + ^1S_0 \ (\ell=1)}}$

We determine $G^{(1)}(^3S_1 \rightarrow ^3S_1 + ^1S_0)$ from the coupling constant $g_{\omega\rho\pi}$, by use of the relation

$$G^{(1)}(^3S_1 \rightarrow ^3S_1 + ^1S_0) = \frac{M_\omega^4}{24} g_{\omega\rho\pi}^2, \quad (4.4)$$

and setting $g_{\omega\rho\pi} \simeq 1.8/m_\pi$, as given by the $\omega \rightarrow 3\pi$ width; the resulting mass shifts appear in Table I.

The intermediate state $^3S_1 + ^3S_1$ has been neglected due to the smallness of the coupling, as can be estimated by the procedures of Ref. 4).

4-c $\underline{\underline{^3P_2 \rightarrow ^1S_0 + ^1S_0 \ (\ell=2)}}$

The coupling constant $G^{(2)}(^3P_2 \rightarrow ^1S_0 + ^1S_0)$ is determined by the formula

$$\Gamma(^3P_2 \rightarrow \pi\pi) = \frac{1}{M_f} \frac{1}{8\pi} \left(\frac{2p}{M}\right)^5 G^{(2)}(^3P_2 \rightarrow ^1S_0 + ^1S_0) \times 3. \quad (4.5)$$

The various contributions are shown in Table II.

4-d $\underline{\underline{^3P_2 \rightarrow ^1S_0 + ^3S_1 \ (\ell=2)}}$

According to the discussion in Ref. 4), this coupling is quite small and the decay $A_2 \rightarrow 3\pi$ must be ascribed mostly to direct 3π production. We therefore neglect it.

4-e $\frac{{}^3P_0 \rightarrow {}^1S_0 + {}^1S_0 \ (\ell=0)}{=====}$
From

$$\Gamma(\epsilon \rightarrow \pi\pi) = \frac{1}{M_\epsilon} \frac{1}{8\pi} \left(\frac{2P}{M} \right) G^{(0)}({}^3P_0 \rightarrow {}^1S_0 + {}^1S_0) \times 3, \quad (4.6)$$

we compute $G^{(0)}({}^3P_0 \rightarrow {}^1S_0 + {}^1S_0)$ by setting $M_\epsilon \cong 0.8 \text{ GeV}$ and $\Gamma(\epsilon \rightarrow \pi\pi) \simeq 0.5 \text{ GeV}$. The calculation is reported in Table II.

4-f $\frac{{}^1P_1 \rightarrow {}^1S_0 + {}^3S_1}{=====}$

The decay $B \rightarrow \omega\pi$ is predominantly in s wave and we derive the coupling constant from the relation

$$\Gamma(B \rightarrow \omega\pi) \simeq \frac{1}{M_B} \frac{1}{8\pi} \left(\frac{2P}{M} \right) G^{(0)}({}^1P_1 \rightarrow {}^1S_0 + {}^1S_0) \times 2. \quad (4.7)$$

4-g $\frac{{}^3P_1 \rightarrow {}^1S_0 + {}^3S_1}{=====}$

The A_1 , as is well known, is quite elusive. However, in the geometrodynamical approach we estimate its width into $\rho\pi$ to be around 100 MeV; this fixes, in the usual way, $G^{(0)}({}^3P_1 \rightarrow {}^1S_0 + {}^3S_1)$, and we obtain the numbers reported in Table II.

5. COMPARISON WITH EXPERIMENT

In order to compare with experiment ^{*)}, we could start from the zeroth order spectrum, and correct it by use of the mass shifts in Tables I and II. This, however, requires a slight adjustment of the parameters used in Ref. 4); therefore we believe it more meaningful to compare with experiment a set of mass differences which are unaffected by such adjustments.

For the vector multiplet we obtain

$$m_\rho - m_\omega \simeq -30 \text{ MeV} \quad (\text{Exp. } -10 \pm 3 \text{ MeV}), \quad (5.1)$$

*) Most of the experimental information is taken from Ref. 8).

and we can compute, from the $\omega\phi$ mass matrix element :

$$\Gamma(\phi \rightarrow \rho\pi) \simeq 430 \text{ KeV} \quad (\text{Exp. } 600 \pm 60 \text{ KeV}), \quad (5.2)$$

which in view of the approximate nature of our calculation is quite a remarkable result.

Going to the 2^{++} mesons, we obtain :

$$m_f - m_{A_2} \simeq -30 \text{ MeV} \quad (\text{Exp. } -40 \pm 10 \text{ MeV}) \quad (5.3)$$

and from the $f-f'$ mass matrix element, we easily get :

$$\Gamma(f' \rightarrow \pi\pi) \simeq 1 \text{ MeV}. \quad (5.4)$$

This leads to the branching ratio $B(f' \rightarrow \pi\pi) \simeq 2.5\%$, which has been recently experimentally determined to be $(1.7 \pm 0.5)\%$ ⁹⁾.

For the spin 0^{++} mesons, we get the following results :

$$m_s - m_{A_2} \simeq -280 \text{ MeV} \quad (\text{Exp. } -330 \pm 15 \text{ MeV}). \quad (5.5)$$

The ϵ and ϵ' states get mixed and we obtain the following masses

$$\begin{aligned} m_\epsilon &\simeq 890 \text{ MeV} \\ m_{\epsilon'} &\simeq 1350 \text{ MeV}, \end{aligned} \quad (5.6)$$

and their widths into π 's are of the order of $300 \div 500 \text{ MeV}$. A more accurate analysis is certainly called for; however, we note that we have no $S^*(990)$, and that the ϵ meson parameters are very close to the ones found by Lang ¹⁰⁾ to describe the $\pi\pi$ phase shifts up to 1100 MeV (S^* effect included); thus our theory seems to reproduce very well the experimental observation. As for the κ meson, we get :

$$m_{K^{**}} - m_\kappa \simeq 160 \text{ MeV} \quad (\text{Exp. } \sim 170 \text{ MeV}). \quad (5.7)$$

Let us now consider the multiplet 1^{++} . we predict

$$m_{A_2} - m_{A_1} \simeq 90 \text{ MeV}, \quad (5.8)$$

but this prediction may be modified by the inclusion of the direct 3π channel. The $D(1285)$ is predicted at the mass :

$$m_D \simeq 1295 \text{ MeV} \quad (\text{Exp. } 1286 \pm 10 \text{ MeV}) \quad (5.9)$$

and the Q_A meson at (neglecting mixing with Q_B)

$$m_{Q_A} \simeq 1340 \text{ MeV} . \quad (5.10)$$

Experimentally, the situation is quite confused ¹¹⁾.

Finally, for the 1^{+-} multiplet, we obtain the following results

$$m_B \simeq 1195 \text{ MeV} \quad (\text{Exp. } 1228 \pm 10 \text{ MeV}) \quad (5.11)$$

$$m_H \simeq 970 \text{ MeV} \quad (\text{Exp. } 990 \text{ MeV?}) \quad (5.12)$$

$$m_{Q_B} \simeq 1270 \text{ MeV} . \quad (5.13)$$

The success of our analysis suggests quite strongly that the mass splittings within quark multiplets have their origin in the physical couplings among the hadronic states. Even if our calculations are not the whole story, any realistic approach to the hadron spectrum in the quark model must take into account these effects which originate from the incontrovertible fact that hadrons interact with each other. Having this in mind, we cannot see how the recent attempts ⁷⁾ to calculate the fine structure of the hadronic spectrum from gluon-exchange potentials may have any chance to succeed, as they leave aside very important and unavoidable contributions.

The aim of this paper was to show that such effects can be calculated, and that once they are taken into account lead to a spectrum of low-lying mesons which is very close to nature.

It belongs to future work to make more precise calculations by including a larger set of intermediate states, and to extend this analysis to the amazingly rich baryon spectrum ⁵⁾.

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FIGURE CAPTIONS

Figure 1 : The lowest order amplitude $a_s^{(2)}(12 \rightarrow 34)$.

Figure 2 : The diagrams contributing to $a_s^{(4)}(12 \rightarrow 34)$.

Figure 3 : The high mass contribution to the self-energy function.

TABLE I Mass shifts of the vector meson multiplet;
the figures are in MeV.

Particle	$^1S_0 + ^1S_0$	$^1S_0 + ^3S_1$	Total
ρ	-93	0	-93
ω	-11	-52	-63
K^*	-60	+12	-48
Φ	-27	+ 6	-19
$\omega-\Phi$	+20	+ 2	+22

TABLE II Mass shift of the $L=1$ meson states;
the figures are in MeV.

Particle	$^1S_0 + ^1S_0$	$^1S_0 + ^3S_1$	Total	Particle	$^1S_0 + ^1S_0$	$^1S_0 + ^3S_1$	Total
A_2	-14	-	-14	δ	- 290	-	-290
f_0	-43	-	-43	ϵ	- 420	-	-420
K^{**}	-13	-	-13	κ	- 170	-	-170
f'	-25	-	-25	ϵ'	- 280	-	-280
$f-f'$	-16	-	-16	$\epsilon-\epsilon'$	- 210	-	-210
A_1	-	-100	-100	B	-	-130	-130
D	-	-30	-30	H	-	-350	-350
Q_A	-	-100	-100	Q_B	-	-167	-167

$$a_s^{(2)}(12 \rightarrow 34) = \sum_i$$

FIG.1

$$a_s^{(4)}(12 \rightarrow 34) = \sum_{\substack{ij \\ ab}}$$

FIG.2

$$= D_i^{(1)}(ab)$$

FIG.3