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A QCD ANALYSIS OF THE $\langle p_T^2 \rangle$ BEHAVIOUR OBSERVED IN
RECENT SEMI-INCLUSIVE NEUTRINO DATA

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ABSTRACT

We present a comparison of QCD predictions with recent semi-inclusive data from the BEBC neutrino experiment on neon. We find, in particular, that the invariant hadronic mass, W , is a much more convenient variable than Q^2 to put in evidence the variations of the hadron $\langle p_T^2 \rangle$ predicted by QCD. The results of the analysis are mildly encouraging for QCD. There are points though which need further clarification from experiments.

Results from a recent neutrino experiment performed with BEBC¹⁾ show a rise of the average transverse momentum $\langle p_T \rangle$ of the hadron secondaries for increasing values of Q^2 (the square of the lepton momentum transfer). The effect is observable only for incident neutrinos, and not for antineutrinos, possibly because of insufficient statistics (Figs 1a, 1d). The experiment was run with a narrow-band beam impinging on a neon-hydrogen mixture. Two other experiments, one at CERN²⁾ and the other at Fermilab³⁾, using wide-band beams on hydrogen do not see the effect. The Q^2 interval studied by them does not extend over 50 GeV².

The data have been reviewed at the Oxford conference by Stenger⁴⁾, who has also made a first comparison with QCD (Quantum Chromodynamics) expectations. A point apparent in Ref. 4) [see also Ref. 5)] is that, because of finite energy effects, the rise of $\langle p_T^2 \rangle$ with Q^2 predicted by QCD happens to be relatively slow at currently available energies. This is also illustrated in Fig. 2a^{4),5)} and 2b.

In this note we show that in order to remove finite energy effects a most natural variable to use in this context is the invariant hadronic mass W . Helped by this fact we then make a detailed comparison of QCD predictions with the semi-inclusive BEBC neon data.

The celebrated QCD expectation for a linear rise of $\langle p_T^2 \rangle$ with $\alpha_s(Q^2)Q^2$, where $\alpha_s(Q^2)$ is the strong interaction effective coupling constant, holds theoretically only at fixed x and y ⁶⁾. If the incident beam energy E is kept fixed, the increase of Q^2 changes x and y and the dependence of $\langle p_T^2 \rangle$ on it becomes more complicated (Figs. 2a and 2b). As a numerical calculation shows, the $\langle p_T^2 \rangle$ behaviour with respect to the invariant hadronic mass W , as predicted by QCD, is instead practically independent of energy (Figs 2c and 2d). This can be intuitively understood thinking of the two-body nature of the underlying (first perturbative order) QCD processes generating the hadron transverse momentum^{*)}. Theoretically, W^2 is as good as Q^2 to set a scale. From Figs. 2c and 2d one gathers that it is a much more convenient variable to use phenomenologically.

*) Indicating, in the QCD parton subprocess, by q , p_1 , p_2 and p_3 the four-momenta of the current, the initial parton, and the two final partons, respectively, and defining $x_p = Q^2/2(p_1 q)$ and $z_p = (p_1 p_2)/(p_1 q)$, one has (from two-body kinematics) $(p_{T,p})^2 = Q^2((1-x_p)/x_p)^p(z_p(1-z_p)) = W_p^2(z_p(1-z_p))$, where $p_{T,p}$ is the common transverse momentum of the final partons, and W_p is the c.m. energy of the subprocess. The $z_p(1-z_p)$ factor implies that no transverse momentum can be developed if one of the two final partons takes all the available energy.

Let us now come, more specifically, to the semi-inclusive BEBC neon data¹⁾. To order $\alpha_s(Q^2)$ three different QCD processes can contribute to the generation of the transverse momentum of the final hadron:

- A) $"W" + \text{quark} \rightarrow \text{gluon} + \text{quark}$
 $\quad \quad \quad \downarrow$
 $\quad \quad \quad \text{hadron} + X$
- B) $"W" + \text{quark} \rightarrow \text{quark} + \text{gluon}$
 $\quad \quad \quad \downarrow$
 $\quad \quad \quad \text{hadron} + X$
- C) $"W" + \text{gluon} \rightarrow \overline{\text{quark}} + \text{quark}$
 $\quad \quad \quad \downarrow$
 $\quad \quad \quad \text{hadron} + X$

The three processes involve different structure and fragmentation functions for the initial and final partons, respectively. For the quark distribution in the isoscalar target we use, in the calculation, a valence-sea parametrization⁷⁾:

$$q(x) = v(x) + \xi(x)$$

$$\bar{q}(x) = \xi(x)$$

with

$$x v(x) = 2.20 \sqrt{x} (1-x)^{3.5}$$

$$x \xi(x) = 0.25 (1-x)^{6.7}$$

which provides a reasonable fit to the x distribution for v and $\bar{v}$ at $E \simeq 100$ GeV as measured in the CDHS experiment⁸⁾. The absolute normalization is fixed requiring the gluons to carry $\frac{1}{2}$ of the momentum of the target nucleon. We vary, however, the x distribution to study the impact of scaling violations and experimental uncertainties, especially in the sensitive small x region, using the available preliminary information on the CDHS data⁸⁾ as a guideline. For the fragmentation of the quark into $\pi^\pm$ we take the simple parametrization by Sehgal⁹⁾:

$$z D(z)_{\text{quark}}^{\pi^\pm} = 0.05 + 1.05 (1-z)^2$$

For the gluon structure function we assume

$$x g(x) = 0.5 (1 + \eta)(1-x)^\eta$$

and vary η from 4 to 6. The effects of this variation are negligible. As to the gluon fragmentation function, there is a general consensus that it should be softer than for quarks. This agrees with the results of theoretical analyses¹⁰⁾. We take, as a basic reference, the simple model for gluon fragmentation of Koller and Walsh¹¹⁾, according to which the gluon first breaks up into a quark-antiquark pair, with an ordinary two-body decay distribution, and then the quarks fragment into pions. This gives:

$$D(\xi')^{\pi^\pm}_{\text{gluon}} = 2 \int_{\xi'}^1 \frac{dz}{z} D(z)^{\pi^\pm}_{\text{quark}}$$

where ξ' is the fraction of the gluon energy taken by the pion. However, to estimate the theoretical uncertainties reported in Fig. 1, we also consider the possibility of a harder fragmentation function with $D(z)^{\pi^\pm}_{\text{gluon}} = D(z)^{\pi^\pm}_{\text{quark}}$.

In order to reproduce the data, one has of course to take into account the so-called "non-perturbative" sources of transverse momentum: the "primordial" transverse momentum k_T of the partons inside the target nucleon, and the transverse momentum p_T^{FR} given to the hadron in the fragmentation of the quark or gluon. We make the customary assumption that dependences in these variables simply factor out and have Gaussian shapes, also in order to make contact with related calculations for other processes. In this way the resulting "non-perturbative" contribution to $\langle p_T^2 \rangle$ becomes independent of other variables, except z . More realistic approaches, using longitudinal phase space models, also give a $\langle p_T^2 \rangle$ contribution approximately constant in Q^2 and W^2 in the intervals of interest¹⁾.

We calculate $\langle p_T^2 \rangle$ making the usual approximation:

$$\begin{aligned} \langle p_T^2 \rangle &\equiv \frac{\int d\Phi \, p_T^2 \frac{d\sigma}{d\Phi}}{\int d\Phi \frac{d\sigma}{d\Phi}} \\ &\simeq \frac{\int d\Phi \, p_T^2 \frac{d\sigma_0}{d\Phi} + \int d\Phi \, p_T^2 \frac{d\sigma_1}{d\Phi}}{\int d\Phi \frac{d\sigma_0}{d\Phi}} \end{aligned}$$

where $d\sigma_0/d\Phi$ and $d\sigma_1/d\Phi$ are the contributions of zero and first order, respectively, in $\alpha_s(Q^2)$ to the differential cross-section $d\sigma/d\Phi$. As is well known, $d\sigma_1/d\Phi$ contains a $1/p_T^2$ divergence, which is eliminated in the integral by the p_T^2 factor, and therefore when calculating $\langle p_T^2 \rangle$ we cannot include in it the effects of k_T and p_T^{FR} which would disturb the removal of the divergence.

In the calculation, the effective coupling constant $\alpha_s(Q^2)$ is kept running: $\alpha_s(Q^2) = 12\pi/(25 \ln(Q^2/\Lambda^2))$, with $\Lambda = 0.74 \text{ GeV}$ ¹²⁾. The cuts appropriate to the BEBC neon data¹⁾ are applied: $E_\mu > 5 \text{ GeV}$, $Q^2 > 2 \text{ GeV}^2$, $W > 4 \text{ GeV}$. Only the data for $z > 0.2$ (where the p_T^2 variation is higher) are considered, so as to avoid the target fragmentation region where the fragmentation model used is not applicable. The CERN narrow-band beam spectra for neutrinos and antineutrinos have been folded in¹³⁾.

In order to calculate dN/dp_T^2 , the effects of k_T and p_T^{FR} on $d\sigma_1/d\Phi$ must also be taken into account. When convoluting $d\sigma_1/d\Phi$ with the k_T and p_T^{FR} distributions, one always receives contributions, even at large p_T , from the region where the internal QCD transverse momentum approaches zero, i.e., where QCD is divergent. Similar problems are met in applications to hadronic production at large p_T and in Drell-Yan processes. In order to cure them, we modify the squares of the quark propagators from $1/(p^2)^2$ into $1/((p^2)^2 + \Lambda_R^4)$, where we take $\Lambda_R^4 = 0.5 \text{ GeV}^4$. Of course, the calculation is meaningful only for p_T such that the contribution from the region in which the QCD cross-section is sensibly modified remains small. While p_T^{FR} appears only in an external convolution on the differential cross-section, the dependence on k_T is more dynamical, since this variable also shows up in the QCD matrix element. To compare with the data, we take $\langle p_T^{FR} \rangle = 0.330 \text{ GeV}$, and $\langle k_T \rangle = 0.848 \text{ GeV}$ -- this three-digit number comes from a fit to Drell-Yan data and has been used to reproduce hadronic production data at large p_T ¹⁴⁾. The resulting "non-perturbative" $\langle p_T^2 \rangle$ for $z > 0.2$ is $\langle p_T^2 \rangle \approx 0.26 \text{ GeV}^2$, and reasonably reproduces the experimental values for ν and (somewhat less for) $\bar{\nu}$ at small W^2 , where the QCD contribution is small (Figs. 1b and 1e).

Let us first discuss the $\langle p_T^2 \rangle$ behaviour. For neutrinos, the calculated $\langle p_T^2 \rangle$ rise with W^2 appears to be in broad agreement with data (Fig. 1b) -- the theoretical curve, though, does not seem to really catch up completely with the rapid experimental increase. The experimental rise of $\langle p_T^2 \rangle$ with Q^2 , although less steep than for W^2 , looks somewhat unexpected theoretically (Fig. 1a). The wide-band beam data on hydrogen^{2),3)}, showing no or little effect versus Q^2 , appear more understandable from the point of view of QCD. Antineutrino data, in spite of the lower average incident energy per event ($\approx 80 \text{ GeV}$ versus $\approx 100 \text{ GeV}$ for neutrinos), should exhibit essentially the same $\langle p_T^2 \rangle$ behaviour expected for neutrinos, and only statistics can excuse them for not doing so (Figs. 1d, 1e). Figures 1c and 1f show that a sizeable increase of $\langle p_T^2 \rangle$ at small x should also be observable experimentally, according to QCD.

As for dN/dp_T^2 , a value of $\langle k_T \rangle = 0.848 \text{ GeV}$ allows the theoretical curve to have some contact with the data^{*)} (Fig. 3). The light dashed curve in Fig. 3 indicates how much, at most, the theoretical curve is allowed to move up, because of the uncertainties in the quark structure function and in the gluon fragmentation distribution. Data have insufficient statistics at p_T large enough to allow a comparison less dependent on the assumed amount of $\langle k_T \rangle$.

*) In order to reproduce the data which have no z cut, we have made the calculation imposing $z > 0.02$, which is, perhaps, beyond the range of validity of the model. This is not really relevant, though -- a cut at $z > 0.02$ gives essentially the same curve for $p_T \gtrsim 1 \text{ GeV}$, since $\langle z \rangle$ approaches 1 as p_T increases.

In conclusion, the invariant hadronic mass, W , proves to be the right variable to discuss the $\langle p_T^2 \rangle$ behaviour predicted by QCD without facing, as is the case of Q^2 , unwanted complications due to the finite experimental energies. The rise of $\langle p_T^2 \rangle$ with W^2 observed in the neutrino data of the BEBC neon experiment is qualitatively reproduced by QCD, although the theoretically expected variation in the W^2 interval of interest is about $\approx 0.1 (\text{GeV}/c)^2$ short of the experimental value ($\approx 0.35 (\text{GeV}/c)^2$). As for the Q^2 dependence, the rise of $\langle p_T^2 \rangle$ with Q^2 in the BEBC neon data is not accounted for by the theory. The comparison with the antineutrino data is inconclusive because of insufficient statistics. Further insight can be provided by the study of the $\langle p_T^2 \rangle$ dependence on x , which is expected to be monotonically increasing at small x . The dN/dp_T^2 distribution can be approximately reproduced by QCD using a level of intrinsic transverse momentum k_T comparable to that found necessary to reproduce Drell-Yan data and hadronic production at large p_T ¹⁴). Because of lack of statistics, the comparison cannot be extended, however, beyond the p_T region where non-perturbative effects remain dominant.

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FIGURE CAPTIONS

- Fig. 1 : a,b,d,e: Comparison of QCD predictions with the behaviour of the hadron $\langle p_T^2 \rangle$ versus Q^2 and W^2 as measured in the BEBC neon experiment¹⁾, for incident neutrinos and antineutrinos. Dashed lines represent the amount of non-perturbative $\langle p_T^2 \rangle$ which results from taking $\langle k_T \rangle = 0.848$ GeV/c and $\langle p_T^{FR} \rangle = 0.330$ GeV/c.
- c,f: QCD predictions for the $\langle p_T^2 \rangle$ behaviour versus Bjorken's x for incident neutrino and antineutrino.
- Fig. 2 : Dependence of hadron $\langle p_T^2 \rangle$ on Q^2 and W^2 as predicted by QCD, for various incident energies of the neutrino and with narrow and wide-band beam spectra folded in. Kinematical cuts etc., are the same as for the analysis of BEBC neon data.
- Fig. 3 : Comparison of BEBC neon data¹⁾ for $N_E^{-1} dN/dp_T^2$ with theoretical predictions. The light dashed curve above the solid line, representing the full theoretical prediction, indicates how much the latter can possibly move up because of theoretical uncertainties.

Fig. 1

$$\nu N \rightarrow \mu^- \pi^\pm + X, z > 0.2 \quad [\text{QCD}]$$

Fig. 2

Fig. 3