EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH

HEAVY-LEPTON POLARIZATION AND RADIATIVE EFFECTSIN $e^+e^- \rightarrow \tau^+\tau^-$ NEAR THE Z^0 POLEG. Goggi^{*)}

CERN, Geneva, Switzerland

CERN LIBRARIES, GENEVA

CM-P00064004

ABSTRACT

Methods for the experimental determination of τ weak neutral couplings by a measurement of its helicity state in e^+e^- annihilation are discussed. After taking into account substantial corrections due to radiative effects, both leptonic and semileptonic τ decays can provide a sensitive test of the Weinberg mixing angle.

Geneva - 2 November 1978

(Submitted to Nuovo Cimento Letters)

*) On leave from Istituto di Fisica Nucleare, Università di Pavia and INFN, Sezione di Pavia, Italy.

The intent of this letter is to show how the weak neutral couplings of charged heavy leptons can be measured in $e^+e^- \rightarrow \tau^+\tau^-$ near the Z^0 mass with good sensitivity, once QED radiative effects are taken into account. The τ pairs are produced with a forward-backward asymmetry and with a net helicity due to the interference of electromagnetic and neutral-current amplitudes. These effects should be observed to some extent in the next generation of storage rings, where weak contributions are expected to modify the one-photon mechanism for μ -pair production by about 10% at $\sqrt{s} = 30$ GeV. The smallness of the vector coupling though, together with the size of the point cross-section away from the Z^0 pole, is likely to limit the possible experimental information to the angular asymmetry in lepton-pair production, in which case the weak contribution is partially masked by higher order electromagnetic effects.

This picture might radically change with the generation of storage rings following PEP and PETRA, which may have sufficiently high energy to reach the neutral vector boson mass. In this case, simple polarization measurements on the final-state leptons become feasible by using both leptonic and semileptonic decays of τ pairs¹⁾.

Polarization effects would directly establish the existence of a parity-violating mechanism and constitute an unambiguous signal for the existence of a neutral-current amplitude. The determination of the heavy-lepton helicity state provides both a sensitive test of a small vector part in the interaction and represents the only way to determine the relative sign of the vector and axial couplings in the absence of longitudinally polarized beams. As a consequence, a precise test of the assumption of generalized lepton universality would also be possible by comparing the τ couplings with those of the lighter leptons. In this respect the $\tau^+\tau^-$ channel is complementary to the other pure leptonic channels in exploring the products of leptonic couplings of the Z^0 .

In the following, Z^0 effects are computed for convenience using couplings and masses of the Weinberg-Salam (W-S) model²⁾, the main results being given in terms of more general couplings.

We consider the reactions

$$e^+ e^- \rightarrow \tau^+ \tau^- \begin{cases} \rightarrow e^\pm \mu^\mp + \text{neutrals} & (1a) \\ \rightarrow e^\pm h^\mp + \text{neutrals} & (1b) \end{cases}$$

which display very distinctive experimental signatures. In the case of reaction (1b), both two-body and quasi-two-body τ decays can be considered, with h representing the total hadronic system. For $s \gg m_\tau^2$ and summing over the spin orientations of one τ , the cross-section for a given helicity state of the remaining τ in lowest order in the weak and electromagnetic interactions has the form^{3,4)}:

$$\left(\frac{d\sigma}{d\Omega} \right)_\pm = \frac{3}{16\pi} B_\ell B_{\ell,h} \sigma_p \left[F_1 (1 + \cos^2 \theta) + 2 F_3 \cos \theta \mp h_\pm F_4 (1 + \cos \theta)^2 \right], \quad (2)$$

where $B_{\ell,h}$ is the τ leptonic or semileptonic branching ratio, according to the case, σ_p the point cross-section, $h_\pm$ the $\tau^\pm$ helicity and

$$\begin{aligned} F_1 &= 1 + 2g_V^2 \operatorname{Re}(Z) + (g_V^2 + g_A^2)^2 |Z|^2 \\ F_3 &= 2g_A^2 \operatorname{Re}(Z) + 4g_V^2 g_A^2 |Z|^2 \\ F_4 &= 2g_A g_V \operatorname{Re}(Z) + 2g_A g_V (g_V^2 + g_A^2) |Z|^2 \end{aligned} \quad (3)$$

$$Z = \frac{s}{e^2 (s - M^2 + i M\Gamma)},$$

M and Γ being mass and width of the Z^0 , respectively.

The longitudinal polarization of the produced heavy leptons is given by⁴⁾

$$P_{L\pm}(\cos \theta) = \frac{\mp F_4 (1 + \cos \theta)^2}{F_1 (1 + \cos^2 \theta) + 2F_3 \cos \theta} \quad (4)$$

Averaging expression (4) over the polar aperture of any forward-backward symmetric apparatus we get for $s = M^2 \gg \Gamma^2$

$$\langle P_L \rangle_\pm = \mp \frac{2\zeta}{1 + \zeta^2}, \quad \zeta = g_A/g_V \quad (5)$$

from which it is clear how a polarization measurement yields direct information on the ratio and relative sign of the coupling constants and, indirectly, on the value of the Weinberg mixing angle.

The short lifetime of the τ and its polarization-dependent weak decays, for which a pure V-A interaction is assumed, make it a unique probe of parity-violating effects; the intrinsic decay asymmetry is in fact transferred by the Lorentz boost into a distortion of the decay spectrum, after integration over the unobserved decay angle. It is then the final-state momentum of the detected particles which is correlated with the τ spin. Neglecting final-state masses the laboratory differential momentum spectrum of the decay lepton (hadron) has the form⁵⁾

$$\frac{dn_{\ell,h}}{dx} = f_{\ell,h}(x, P_L) = a_{\ell,h}(x) \mp \langle P_L \rangle_{\pm} b(x) \quad (6)$$

where x is the normalized lepton (hadron) momentum $x = 2p/\sqrt{s}$ and a, b are polynomials in x .

Owing to the large energy fraction carried by neutrinos in τ decay, which prevents any kinematical constraint, the function $f(x, p_L)$ and all the polarization estimators which can be constructed with it are very sensitive to undetected radiation originating from higher order QED terms. In fact, the radiation spectrum of photons emitted from the initial or final states generates a continuous transformation of the τ -decay distribution and in turn of the polarization estimator. These radiative corrections become particularly important in the presence of a sharp resonant peak⁶⁾; in the case of Z^0 production corrections have to be applied to the resonant, the QED, and the weak-electromagnetic interference terms. In the narrow-width approximation, which certainly applies to the $\tau\tau$ case^{*)}, the total cross-section becomes

$$\sigma_{\tau\tau\gamma}(W) = \sum_{i=1}^3 \sigma_i(W) C_i(W) \left[1 + \delta_i(W) \right], \quad W = \sqrt{s} \quad (7)$$

where C_i and δ_i represent the infrared and ultraviolet correction factors, respectively, for the three components of the cross-section⁷⁾.

The cross-section (7) for process (1a) is shown in Fig. 1 for two values of the Weinberg angle. Considerable distortion of the resonance is generated by

*) If $\Delta\omega$ is the maximum energy radiated within the energy resolution of the experiment $\Gamma \ll \Delta\omega \approx \sqrt{s}$.

radiative corrections; the relative effects on the total and partial cross-sections are shown in Fig. 2. As an example, the peak cross-section is reduced by a factor

$$\left. \frac{\sigma_{z\gamma}(W)}{\sigma_z(W)} \right|_{W=M} \approx \left(\frac{\Gamma}{M} \right)^{\beta_i}, \quad \beta_i = \frac{4\alpha}{\pi} \left(\ln \frac{M}{m_e} - \frac{1}{2} \right).$$

The main contribution to the probability of finding process (1) accompanied by an energy loss ω in $d\omega$ is given⁷⁾ in the resonant case by

$$dP(\omega) \approx \left(\frac{2\omega}{W} \right)^{\beta_i + 2\beta_{int} + \beta_f} \left\{ \beta_i \frac{4y^2 + \Gamma^2}{4(y-\omega)^2 + \Gamma^2} + 2\beta_{int} \frac{4y(y-\omega) + \Gamma^2}{4(y-\omega)^2 + \Gamma^2} + \beta_f \right\} \frac{d\omega}{\omega}, \quad (8)$$

where $y = W - M$ measures the deviation from resonance and the β factors control the radiation process from the initial and final states and the interference between them. As can be expected, the typical bremsstrahlung-like spectrum, which is obtained by letting $\Gamma \rightarrow \infty$ in expression (8), is considerably modified by the presence of the resonance, which provides a strong damping of the radiation spectrum.

The polarization estimator can be defined as an integral function of the normalized decay spectrum:

$$R_{\ell,h} = \frac{\int_{x_{\ell,h}}^1 f_{\ell,h}(x, P_L) dx}{\int_{x_0}^{x_{\ell,h}} f_{\ell,h}(x, P_L) dx} \quad (9a)$$

where

$$\frac{\partial f_{\ell,h}(x, P_L)}{\partial P_L} = 0 \quad \text{at} \quad x = x_{\ell,h} \quad (9b)$$

In these expressions x_0 is a suitable momentum cut dictated by experimental conditions, such as a lower limit on momentum acceptance owing to background or apparatus design. The value of $x_{\ell,h}$, the point at which $b(x) = 0$ and the differential decay spectrum is independent of polarization, depends through Eq. (6) on

the details of τ decay. It divides the $(x_0, 1)$ interval into two regions, which appear as integration limits in Eq. (9a), where the partial derivative (9b) has opposite signs.

In the presence of radiation, the function $f(x, P_L)$ scales in the reduced variable $x' = x [1 - (\omega/W)]^{-1}$ and the polarization estimator becomes:

$$R_{\ell, h}(\omega, P_L) = \frac{1 - \tilde{f}_{\ell, h}(x'_{\ell, h}, P_L)}{\tilde{f}_{\ell, h}(x'_{\ell, h}, P_L) - \tilde{f}_{\ell, h}(x'_0, P_L)} \quad (10)$$

where

$$\tilde{f}_{\ell, h}(x'_i, P_L) = \int f_{\ell, h}(x, P_L) \theta(1-x) dx \Big|_{x=x'_i}$$

and the θ -function defines the physical limits of integration. Averaging Eq. (10) over the radiation spectrum (8) at fixed y yields the value of the polarization estimator after radiative corrections. For given values of Z^0 parameters M and Γ

$$\bar{R}_{\ell, h}(y, P_L) = \frac{\int_0^{\Delta\omega} R_{\ell, h}(\omega, P_L) \frac{dP(\omega)}{d\omega} d\omega}{\int_0^{\Delta\omega} \frac{dP(\omega)}{d\omega} d\omega}, \quad (11)$$

where the upper limit of the energy loss $\Delta\omega$ is given by the experimental momentum cut $\Delta\omega = (1 - x_0)W$.

In the case of general couplings, with the τ longitudinal polarization as free parameter, Figs. 3a and 4a show the numerical values of Eq. (11) for leptonic^{*)} and semileptonic decays, with $x_0 = 0.05$, $y = 0$, and the Z^0 parameters defined at $\sin^2 \theta_W = 0.20$. In both cases the estimator displays a good sensitivity to the value of the heavy-lepton longitudinal polarization. In realistic conditions, the value of $\bar{R}(y, P_L)$ has to be further integrated over the machine energy resolution:

*) Only the μ -momentum distribution has been considered; the electron spectrum is further modified by radiative effects⁸⁾ in τ decay.

$$\bar{R}(y, P_L) = \int \bar{R}(y', P_L) G(y' - y) dy'$$

$$G(y' - y) = \frac{1}{\sqrt{2\pi} \sigma_W} \exp \left[\frac{-(y - y')^2}{2\sigma_W^2} \right], \quad (12)$$

σ_W being the c.m. energy dispersion. The relevant parameter is the ratio of the dispersion to the Z^0 width⁹⁾; $\sigma_W/\Gamma \ll 1$ for the proposed LEP machine¹⁰⁾. The sensitivity of $\bar{R}_{\ell,h}$ at the Z^0 peak on the resonance width Γ and the deviation from resonance y can be expressed in the form of the logarithmic derivatives

$$S_\Gamma = \left. \frac{d}{d\Gamma} \ln \bar{R}_{\ell,h}(y, P_L) \right|_{y=0}$$

$$S_y = \left. \frac{d}{dy} \ln \bar{R}_{\ell,h}(y, P_L) \right|_{y=0} \quad (13)$$

For the expected values of $\Gamma \approx 0.1$ GeV typical sensitivities of 0.1% per GeV are obtained, as shown in Figs. 3b and 4b, indicating that the proposed estimator is negligibly affected by small uncertainties in the parameters which control the higher order electromagnetic corrections.

It is clear from the previous considerations that heavy-lepton-pair production in e^+e^- annihilation at the mass of the weak neutral vector boson allows a simple determination of final-state lepton helicity. From a general viewpoint the value of the longitudinal polarization arising from weak-electromagnetic interference allows a model-independent determination of both ratio and relative sign of the vector and axial weak couplings. To this end both leptonic and semi-leptonic τ decays can provide, to a different extent, good analysing power and selective experimental signature. Although such a measurement is, in principle, conceivable even at the PEP/PETRA upper energy, the quantity $\langle P_L \rangle \cdot \sigma$ increases roughly as $(g_V^2 + g_A^2)(M/\Gamma)^2$, i.e. by a factor of about 400, at the Z^0 peak. In this case a precision of the Weinberg mixing angle of $\Delta \sin^2 \theta_W \approx 0.003$ could be achieved within practical running times (~ 500 h) at the luminosities expected⁹⁾ for LEP.

The evaluation of radiative correction factors is essential at these energies, in order to extract from the experimental data detailed physical parameters. It has been shown that the resulting polarization estimator displays only minimal sensitivity to parameters such as Z^0 width or beam energy calibration and dispersion.

Useful discussions with L. Camilleri, J.H. Field and K. Gaemers are gratefully acknowledged. Thanks are due to M. Jacob for discussions and careful reading of the manuscript.

REFERENCES

1. G. Goggi, Reports ECFA/LEP/2 (July 1978) and ECFA/LEP/4 (August 1978) (CERN, unpublished).
J.E. Augustin, Report ECFA/LEP/29 (August 1978) (CERN, unpublished).
2. S. Weinberg, Phys. Rev. Letters 19, 1264 (1967), and 27, 1688 (1971).
A. Salam, *in* Elementary particle physics (Ed. N. Svartholm) (Almqvist and Wiksell, Stockholm, 1968).
3. R. Budny, Phys. Letters 45B, 340 (1973) and 55B, 227 (1975).
4. J.H. Field and B. Richter, *in* CERN 76-18 (1976).
5. Y.S. Tsai, Phys. Rev. D 4, 2821 (1971).
S.V. Golovkin, M.I. Grochev, V.I. Gridasov, V.P. Daushin, R.I. Djelyadin, A.M. Zaitsev, V.F. Kostantinov, V.I. Kotov, V.P. Kubarovsky, L.G. Landsberg, V.M. Leontiev, V.A. Mukhin, T.I. Petrunina and Yu.S. Khodyrev, Nuclear Instrum. Methods 138, 235 (1976).
6. G. Pancheri, Nuovo Cimento 60A, 321 (1969).
7. M. Greco, G. Pancheri-Srivastava and Y. Srivastava, Phys. Letters 56B, 367 (1975), Nuclear Phys. B101, 234 (1975) and references therein.
8. T. Kinoshita and A. Sirlin, Phys. Rev. 107, 593 (1957).
9. See for instance: J.D. Jackson and D.L. Scharre, Nuclear Instrum. Methods 128, 13 (1975).
10. The LEP Study Group, Design study of a 15 to 100 GeV e^+e^- colliding beam machine (LEP), CERN/ISR-LEP/78-17 (CERN, August 1978).

Figure captions

- Fig. 1 : Radiatively corrected cross-sections for $e\mu$ events near the Z^0 pole for two values of the Weinberg mixing angle. The pure QED term, including radiative contributions, is shown for comparison.
- Fig. 2a : Ratio of the total cross-section, as in Fig. 1, to the total uncorrected cross-section for $\sin^2 \theta_W = 0.20$.
- Fig. 2b : Effect of radiative corrections on the separate QED, interference and resonant contributions to the total Z^0 cross-section.
- Fig. 3a : Dependence of the estimator $\bar{R}_\ell$ on the longitudinal heavy-lepton polarization for leptonic τ decays at the Z^0 peak ($y = 0$). The dashed line shows for comparison the same quantity without radiative corrections. A scale with equivalent values of $\sin^2 \theta_W$ in the W-S model is also shown.
- Fig. 3b : Sensitivity of the radiatively corrected value of $\bar{R}_\ell$ to the width Γ and the deviation y from the Z^0 pole position as a function of Z^0 width [see text and Eqs. (13)].
- Fig. 4a : Dependence of the estimator $\bar{R}_h$ on the longitudinal heavy-lepton polarization for semileptonic τ decays at the Z^0 peak ($y = 0$). The dashed line shows for comparison the same quantity without radiative corrections. A scale with equivalent values of $\sin^2 \theta_W$ in the W-S model is also shown.
- Fig. 4b : Sensitivity of the radiatively corrected value of $\bar{R}_h$ to the width Γ and the deviation y from the Z^0 pole position as a function of Z^0 width [see text and Eqs. (13)].

Fig. 1

Fig. 2

Fig. 3

Fig. 4