NON-PERTURBATIVE EFFECTS ON FACTORIZATION IN
HIGH MOMENTUM PROCESSES

John Ellis, Mary K. Gaillard^{*)} and Wojtek J. Zakrzewski^{**)}

CERN -- Geneva

ABSTRACT

It is pointed out that the usual demonstrations within QCD perturbation theory of factorization at high momentum transfers are incomplete when non-perturbative instanton effects are incorporated in a dilute gas approximation. The apparent violation of factorization does not vanish at large Q^2 .

^{*)} Laboratoire de Physique Théorique et Particules Élémentaires,
Laboratoire associé au CNRS, Orsay, France.

^{**)} On leave of absence from the Department of Mathematics,
University of Durham, Durham, England.

Recent studies in QCD perturbation theory have shown that the momentum transfer dependences of many hard scattering processes can be factorized¹⁾. These results are necessary ingredients in the derivation of an improved version of the parton model, in which one may talk of universal Q^2 dependent distributions of quarks and gluons with hadrons, and of universal Q^2 dependent fragmentation functions of partons into hadrons. However, the derivation of such a factorizable QCD parton model is as yet incomplete for two reasons. The first is that even within QCD perturbation theory only the evolution at high Q^2 of cross-sections has been shown to factorize, and the starting conditions at low Q^2 have not yet been shown to factorize²⁾. A related point is that non-perturbative effects have not been studied, and one might expect them to be very important in fixing the low Q^2 boundary conditions. In this paper we investigate³⁾ factorization in the presence of non-perturbative effects. We take into account instantons in a dilute gas approximation⁴⁾, which should at least be valid for sufficiently small instantons. We find³⁾ an apparent violation of factorization which does not vanish asymptotically at large Q^2 . Cross-sections do not in general factorize when calculated in a single background instanton field. The problem is compounded when the contributions from different instanton configurations and the traditional perturbative vacuum are added together. The phenomenological content of this observation can be seen by considering generic hard scattering processes involving two hadrons, such as $e^+e^- \rightarrow h_1(X_1)h_2(X_2) + \text{any}$, $\nu p(X_1) \rightarrow \mu + h(X_2) + \text{any}$ or $h_1(X_1) + h_2(X_2) \rightarrow \mu^+\mu^- + \text{any}$, where X factors in final states refer to momentum fractions of hadrons in different parton jets, and X factors in initial states refer to momentum fractions of partons in different colliding hadrons. Writing the cross-sections as $\sigma(X_1, X_2, Q^2)$ and taking moments with respect to X_1 and X_2 , the perturbative analyses tell us that

$$\int_0^1 dX_1 X_1^n \int_0^1 dX_2 X_2^m \sigma(X_1, X_2, Q^2) \underset{Q^2 \rightarrow \infty}{\sim} (\ln Q^2)^{-\gamma_n} (\ln Q^2)^{-\gamma_m} A_{nm} \quad (1)$$

where the γ_n, γ_m are anomalous dimensions and the Q^2 dependence is explicitly seen to factorize. Our point is that the A_{nm} do not necessarily factorize when non-perturbative effects are taken into account. Independent experimental tests of these two different aspects of factorization would seem to be very topical.

Our understanding of non-perturbative effects in QCD is not yet sufficient for us to assert that factorization does indeed break down, rather we find that within the limited techniques⁴⁾ available to us a problem exists. However, it

may be an artefact of our method of analysis. First, we only consider the effects of instantons⁵⁾, and our second important restriction is that we work in a dilute gas approximation⁴⁾ which is surely not valid for large instantons. We are therefore reduced to trying to take into account the effects of a restricted class of small non-perturbative configurations and piously hoping that the configurations we have omitted do not restore factorization. Another crucial assumption is that we will work in Minkowski space. This may seem foolhardy, since non-perturbative tunnelling calculations are conventionally formulated in Euclidean space. However, we notice that ways have been found^{6),7)} of recovering in Minkowski space the conventional Euclidean WKB results for QCD tunnelling probabilities, both at the classical level⁶⁾ and at the level of one closed loop vacuum fluctuations⁷⁾. On the other hand, no general proof exists that perturbation calculations around a background non-perturbative field configuration can be performed directly in Minkowski space, although this is clearly true of the simplest fermion loop contributions to e^+e^- annihilation⁸⁾. Continuing our list of crucial assumptions, we must confront the problem that we want to discuss processes involving physical hadrons, and we have little conception of their non-perturbative dynamics. We therefore use a toy model³⁾ which we hope will not lead us astray: we use internal scalar "parton" fields ϕ and mimic hadrons by external "scalar photon" sources $J = \phi^+ \phi$. For the present we do not need to discuss higher order perturbation theory diagrams in a background instanton field: our points are illustrated by the simple Born diagrams. We will analyze the regions of configuration space dominant in the parton model, and hope that the scalar photon model does not mislead us as to the structure in those regions. Our explicit calculations will concern the Drell-Yan process, but similar analyses and conclusions apply to $\nu + N \rightarrow \mu + \text{hadron} + \text{any}$, and to $e^+e^- \rightarrow 2 \text{ hadrons} + \text{any}$.

In the parton model^{9),10)} a deep inelastic cross-section

$$W(q,p) \equiv \int d^4x e^{iq \cdot x} \langle p | J^+(x) J(0) | p \rangle \quad (2)$$

can be written as

$$W(q,p) = \int d^4x e^{iq \cdot x} \Delta(x) \langle p | \phi^+(x) \phi(0) | p \rangle \quad (3)$$

where $\Delta(x)$ is the absorptive part of the scalar field propagator and

$$B(x \cdot p, x^2, p^2) \equiv \langle p | \phi^+(x) \phi(0) | p \rangle \quad (4)$$

is a bilocal operator matrix element. It is well known that the Bjorken limit of (2) is controlled by the region $x^2 \rightarrow 0$ of phase space. We are accordingly interested only in $B(x \cdot p, 0, p^2)$. The Fourier transform of this with respect to $x \cdot p$ is just the conventional parton distribution

$$q(X_{Bj}) = \int d(x \cdot p) e^{iX_{Bj}(x \cdot p)} B(x \cdot p, 0, p^2) \quad (5)$$

We could use the LSZ formalism to reduce into Eq. (3) the sources of the target $|p\rangle$, and a priori there is no restriction on the locations y and z of these sources relative to the variables $(x, 0)$ of the bilocal $B(x \cdot p, 0, p^2)$. If we consider as an example of a process involving two hadrons, the Drell-Yan reaction $h_1 + h_2 \rightarrow \text{scalar photon} (\rightarrow \mu^+ \mu^-) + \text{any}$, the cross-section is

$$\overline{W}(p_1, p_2, q) \equiv \int d^4x e^{-iq \cdot x} \Delta^+(x) \langle p_1 p_2 | J^+(x) J(0) | p_1 p_2 \rangle \quad (6)$$

where $\Delta^+(x)$ is the $\mu^+ \mu^-$ contribution to the absorptive part of the time-like scalar "photon" propagator. Within the parton model^{9),10)}, the cross-section (6) can be written as

$$\begin{aligned} \overline{W}(p_1, p_2, q) = \int d^4x e^{-iq \cdot x} \Delta^+(x) & \langle p_1 | \phi^+(x) \phi(0) | p_1 \rangle \\ & \times \langle p_2 | \phi(x) \phi^+(0) | p_2 \rangle \end{aligned} \quad (7)$$

and for large q only $x^2 \rightarrow 0$ is relevant so that

$$\overline{W}(p_1, p_2, q) = \int d^4x e^{-iq \cdot x} \Delta^+(x) B(x \cdot p_1, 0, p_1^2) \overline{B}(x \cdot p_2, 0, p_2^2) \quad (8)$$

As before, if we were to reduce in the hadrons h_1 and h_2 their sources would have arbitrary locations relative to the current locations $(x, 0)$.

Just as in a previous paper³⁾ we used scalar photon-photon scattering as a toy model for deep inelastic scattering (see Fig. 1) we now use scalar photon + photon $\rightarrow$ photon + any as an analogue for the Drell-Yan reaction (see Fig. 2). In the presence of non-perturbative effects a general amplitude

$$\langle A | B \rangle = \frac{\langle A | B \rangle_0 + \langle A | B \rangle_1 + \dots}{\langle 0 | 0 \rangle_0 + \langle 0 | 0 \rangle_1 + \dots} \quad (9)$$

where the subscripts $0, 1, \dots$ refer to different topological sectors of the functional integral. In the approximation of a dilute gas of instantons, the expression (9) is

$$\langle A|B \rangle \approx \langle A|B \rangle_0 + [\langle A|B \rangle_1 - \langle 0|0 \rangle_1 \langle A|B \rangle_0] + \dots \quad (10)$$

where the contributions to the amplitude are evaluated with zero or one instanton background fields, and one must integrate over the instanton size and position. If we define P_0 and P_1 respectively to be the four-point scalar loops of Fig. 1 without and with the background instanton field, the amplitude relevant to deep inelastic scattering is

$$P \approx P_0 + \int d\rho D(\rho) \int d^4t [P_1 - P_0] + \dots \quad (11)$$

where ρ and t are the instanton size and position and $D(\rho)$ is the conventional instanton density function⁴⁾. Similarly if L_0 and L_1 are the corresponding six-point loops of Fig. 2, the amplitude relevant to the Drell-Yan process is

$$L \approx L_0 + \int d\rho D(\rho) \int d^4t [L_1 - L_0] + \dots \quad (12)$$

The usual factorization hypothesis would require that in the light-cone limit $x^2 \rightarrow 0$ the amplitude (12) satisfies

$$\begin{aligned} L(y_1, z_1, y_2, z_2; x) &= P(y_1, z_1; x) P(z_2, y_2; x) \\ &\approx P_0^{(1)} P_0^{(2)} + \int d\rho D(\rho) \int d^4t \left\{ P_0^{(1)} [P_1^{(2)} - P_0^{(2)}] \right. \\ &\quad \left. + P_0^{(2)} [P_1^{(1)} - P_0^{(1)}] \right\} + \dots \end{aligned} \quad (13)$$

where $P^{(i)} \equiv P(y_i, z_i)$ and the terms neglected in (13) are of higher order in the dilute gas approximation and comparable to those neglected in Eqs. (9) to (12).

In the absence of a background field the massless scalar loops in configuration space are

$$P_0(y, z; x) = \frac{1}{x^2} \frac{1}{(x-y)^2} \frac{1}{(y-z)^2} \frac{1}{z^2} \quad (14)$$

and

*) It may well be that our arguments about the breakdown of factorization extend beyond this approximation, because of the need to sum over different topological sectors in Eq. (9).

$$L_0 = \frac{1}{(x-y_1)^2} \frac{1}{(y_1-z_1)^2} \frac{1}{z_1^2} \frac{1}{z_2^2} \frac{1}{(y_2-z_2)^2} \frac{1}{(x-y_2)^2} \quad (15)$$

which satisfy factorization. In the presence of an instanton the six-point function for massless scalar fields is^{*)}

$$L_1 = L_0 \frac{1}{4} \text{Tr} \left\{ F(x-t, y_1-t) F(y_1-t, z_1-t) F(z_1-t, -t) \right. \\ \left. \times F(-t, z_2-t) F(z_2-t, y_2-t) F(y_2-t, x-t) \right\} \quad (16)$$

where¹¹⁾

$$F(x, y) \equiv \left(1 + \frac{\tau \cdot x \tau^\dagger \cdot y}{x^2 y^2} \right) \left(1 + \frac{p^2}{x^2} \right)^{-\frac{1}{2}} \left(1 + \frac{p^2}{y^2} \right)^{-\frac{1}{2}} \quad (17)$$

As noted previously³⁾, after summing over instantons and anti-instantons the trace may be evaluated using Dirac γ matrices via the substitution $\tau \cdot x \tau^\dagger \cdot y \rightarrow \gamma \cdot x \gamma \cdot y$. Since the expression (16) therefore contains non-trivial dependences on such quantities as $(y_1 - t) \cdot (z_2 - t)$, the total Drell-Yan amplitude (12) cannot factorize into separate functions of (y_1, z_1) and (y_2, z_2) unless these dependences vanish upon integration. This would not in any case suffice for factorization, as we shall see in the special case below.

Since the analysis is cumbersome even in the light-cone limit $x^2 \rightarrow 0$, we shall analyze the short-distance limit $x_\mu \rightarrow 0$. This is related to a moment of the Drell-Yan cross-section, and any negative conclusions concerning factorization will obviously carry over to the more general case. Then, since the propagator at short distances reduces to the free propagator ($F(x, x) = 1$) the four-point scalar loop of Fig 1 is:

*) Strictly speaking, to calculate cross-sections that we are interested in, we should take the appropriate discontinuities in the expressions (15) and (17). It is easy to show that there is no divergence of the integration with respect to the position of instantons when this discontinuity is taken. Furthermore, our arguments about the absence of factorization apply directly to this discontinuity.

$$\begin{aligned}
 P_1(y, z; x) &\rightarrow P_0 \frac{1}{4} \text{Tr} \left\{ F(-t, y-t) F(y-t, z-t) F(z-t, -t) \right\} \\
 &= P_0 \frac{1}{4} \text{Tr} \left\{ A(-t, y-t, z-t) \right\}
 \end{aligned} \tag{18}$$

for $x_\mu \rightarrow 0$ in the presence of an instanton. In the same limit Eq. (16) becomes

$$L_1 \rightarrow L_0 \frac{1}{4} \text{Tr} \left\{ A^{(1)}(-t, y_1-t, z_2-t) A^{(2)}(-t, z_2-t, y_2-t) \right\} \tag{19}$$

In order to simplify further, we symmetrize (19) with respect to (y_1, z_1) and with respect to (y_2, z_2) , corresponding to production of a flavour singlet "photon" in the collision of two flavour singlet "photons". (This would not of course correspond to a standard physical situation such as proton-proton scattering.) Using the Dirac matrix substitution one sees that

$$\begin{aligned}
 &\frac{1}{2} \left\{ A(x, y, z) + A(x, z, y) \right\} \\
 &= 1 - \frac{e^2}{2} \left\{ \frac{(x-y)^2}{(p^2+x^2)(p^2+y^2)} + \frac{(y-z)^2}{(p^2+y^2)(p^2+z^2)} + \right. \\
 &\quad \left. + \frac{(z-x)^2}{(p^2+z^2)(p^2+x^2)} \right\}
 \end{aligned} \tag{20}$$

which contains no γ matrices, so that

$$\text{Tr} \left\{ A^{(1)} A^{(2)} \right\}_{\text{symmetrized}} = \text{Tr} \left\{ A^{(1)} \right\}_{\text{symm}} \text{Tr} \left\{ A^{(2)} \right\}_{\text{symm}}$$

giving factorization of the one-instanton loop

$$L_1 = P_1^{(1)} P_1^{(2)} \tag{21}$$

but not the desired factorization specified in Eq. (13) (except in a multiple light-cone or short-distance limit as would occur in Euclidean space.) Instead we find

$$\begin{aligned}
 L_1 - L_0 &= P_1^{(1)} P_1^{(2)} - P_0^{(1)} P_0^{(2)} \\
 &= \tilde{P}_1^{(1)} P_0^{(2)} + \tilde{P}_1^{(2)} P_0^{(1)} + \tilde{P}_1^{(1)} \tilde{P}_1^{(2)} : \tilde{P}_1 = P_1 - P_0 \quad (22)
 \end{aligned}$$

The last term in (22) destroys factorization.

The integration over instanton size in (12) and (13) does not converge for large ρ . Nevertheless these expressions should perhaps be believed for sufficiently small instantons. Of course, in the limit of infinitesimally small instantons factorization would be restored because $\tilde{P}_1 \propto \rho^2$. However, there is no reference in (13) to the momentum scale Q of the produced photon - a priori the relevant sizes of instantons are as large as $O(y_1 - x, z_1, \dots)$ which are likely to be characterized by the masses of the colliding "photons" in our toy model and of physical hadrons in the real world. Any higher order or multi-instanton corrections are damped by extra factors⁴⁾ of $\exp(-8\pi^2/g^2(\rho))$ which fall off much faster than ρ^2 as $\rho \rightarrow 0$. In our toy model we can surely choose external masses where the dilute gas approximation is good but ρ^2 is not negligible. From the point of view of the dilute gas approximation it would seem peculiar if factorization were restored by the inclusion of large scale vacuum fluctuations of size ρ up to $1/m_h$. As the effect makes no reference to the current momentum Q , the apparent breakdown of factorization does not go away as $Q^2 \rightarrow \infty$. The point here is that vacuum fluctuations affect simultaneously both the target and projectile constituents, and that the region over which these effects may take place has a finite limit when $x^2 \rightarrow 0$.

So far we have not discussed quantum fluctuations in a background instanton field. In a flat background field these just generate the conventional $\log Q^2$ factors of QCD perturbation theory and the familiar anomalous dimensions¹⁾. It has been argued^{3),12)} that the logarithms of perturbation theory are identical in the presence of a background instanton field. This therefore suggests that when hard scattering processes are calculated in a background instanton field there will be the same logarithms with the same factorizing dependence on the external legs as in conventional perturbative analyses¹⁾. We therefore anticipate the factorizing dependence on $\log Q^2$ indicated in Eq. (1), with the anomalous dimensions γ_n, γ_m identical with those encountered in normal perturbative calculations. The radiation of the quantum gluons giving these logarithms presumably occurs in a space-time region of size $O(1/Q_0)$ where Q_0 is some initial momentum at which one would specify the initial parton distribution in a perturbative QCD analysis, and beyond which the development in Q^2 is³⁾ adequately described by the normal perturbative evolution equations¹³⁾. Our non-perturbative

analysis suggests the possibility that the initial values to be specified in different hard scattering processes may not factorize into the products of universal parton distributions and fragmentation functions. This is reflected in the moment equations (1) by the possibility that the constants A_{nm} may not factorize in their dependence on n and m .

As emphasized earlier, we do not assert definitively that factorization does indeed break down, merely that in a dilute gas approximation for instantons it seems that the perturbative logarithms of Q^2 may factorize, while factorization of the initial boundary conditions is by no means obvious and requires further exploration. If full factorization does occur, it may well mean that our present feeble dilute gas approximation techniques for estimating non-perturbative effects were indeed inadequate. If full factorization does not occur, it would first be important to check that the violation did not disappear as $Q^2 \rightarrow \infty$, perhaps as $O(1/\ln Q^2)$ as suggested by QCD perturbation theory. If a violation of factorization were found to persist at large Q^2 , it might provide a hitherto elusive^{3),8)} window on non-perturbative effects.

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FIGURE CAPTIONS

- Fig. 1 : Deep inelastic scattering is modelled by the scattering of "scalar photons" and spin 0 partons. The box diagram is to be calculated both with and without a background instanton field.
- Fig. 2 : The Drell-Yan process is modelled by the production of a massive "scalar photon" in the scattering of two essentially real "scalar photons". The loop diagram is to be calculated both with and without a background instanton field.

FIG.1

FIG.2