Ref.TH.2478-CERN

THE AUXILIARY FIELD STRUCTURE IN CHIRALLY EXTENDED SUPERGRAVITY

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A B S T R A C T

The auxiliary fields for Einstein supergravity with axial gauge coupling are those of Maxwell-Einstein supergravity. The gauge algebra is an irreducible extension of the gauge algebra of Einstein supergravity, so that the complete system is a gauge theory with an extra local chiral invariance, rather than a matter coupling.

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Ref.TH.2478-CERN

31 March 1978

Recently the minimal set of auxiliary fields was found which must be added to the gauge action and transformation rules of second order supergravity in order that its gauge algebra closes ¹⁾. It consists of one scalar field S , one pseudoscalar field P , and one axial vector field A_μ . The same set suffices for pure supergravity in de Sitter space where an extra cosmological and a masslike term are present in the action ²⁾. For the locally supersymmetric Maxwell-Einstein system with spin content $(2, \frac{3}{2}) + (1, \frac{1}{2})$ one needs, in addition to the three auxiliary fields of the $(2, \frac{3}{2})$ gauge action, the scalar auxiliary field D which is already needed in the flat-space globally supersymmetric spin $(1, \frac{1}{2})$ system in order that its global algebra closes ²⁾. Although it seems that the auxiliary field structure of supergravity finally begins to be understood, more case studies are needed before one can claim that this problem is solved. For reasons to be discussed shortly, we consider below the spin $(2, \frac{3}{2}) + (1, \frac{1}{2})$ system in which the physical spin 1 field B_μ is coupled to the gravitino and the spin $\frac{1}{2}$ field χ by means of an axial coupling. This forces the parity of B_μ to be odd and adds to the action two extra terms: an off-diagonal masslike term of the form $\bar{\psi}\chi$ and a term linear in the auxiliary field D of the spin $(1, \frac{1}{2})$ system.

The great advantage of auxiliary fields is that they enable one to write the transformation rules of the gauge fields as independent from the matter fields. Thus one can construct new invariant actions by simply adding invariant subactions. This seemed to us to hold out a promise for the solution of a serious problem in supergravity: the enormous size of the cosmological constant in models where the internal coupling constant is of the order of the electromagnetic or weak coupling constant. The basic observation was that the cosmological constant of pure supergravity in de Sitter space has a negative sign ³⁾ while the cosmological constant of the $(2, \frac{3}{2}) + (1, \frac{1}{2})$ system with chiral coupling, obtained by eliminating the auxiliary field D , has a positive sign ⁴⁾. If the introduction of auxiliary fields would make it possible to add both systems without losing local supersymmetry, then one could cancel the cosmological terms by a suitable choice of the free parameters which are in front of each of the cosmological terms.

Our results are disappointing and interesting at the same time. They are disappointing because it has not been possible to add both systems without destroying local supersymmetry. The reason is that the "matter fields" B_μ and χ appear in the transformation rules of the gauge fields S, P, A_μ .

Thus the combination $(2S + \kappa \bar{\psi}_\mu \sigma^{\mu\nu} \psi_\nu)$ which produces the extra cosmological and masslike terms of pure supergravity in de Sitter space upon elimination of S , is no longer separately invariant. Our results are interesting because the appearance of the matter fields B_μ and χ in the transformation rules of the gauge fields is needed to close the gauge algebra, so that the matter fields B_μ , χ and D are really gauge fields on a par with e_μ^a , ψ_μ , A_μ , S and P . In this respect our results agree with the observation of Breitenlohner ⁵⁾ that the Fayet-Iliopoulos term ⁶⁾ $2D + i\kappa \bar{\chi} \gamma_5 \gamma \cdot \psi$ should be considered as part of the gauge action.

One can also interpret these results from an algebraic point of view. The Poincaré supersymmetry algebra is extended to an algebra with an axial charge which does not commute with the supersymmetry generators. Therefore, the newly obtained gauge algebra is irreducible, unlike the algebra of the Maxwell-Einstein system where the electromagnetic charge commutes with the generators of the Poincaré supersymmetry algebra. Gauging this extended algebra leads to our action. Similarly to $O(n)$ extended supergravity, where all fields form an irreducible set, the axial field B_μ , the spin $\frac{1}{2}$ field χ and the pseudoscalar field D must be combined with the gauge fields of pure supergravity in order that the gauge algebra of supersymmetric and chiral rotations closes. Thus we can obtain a new gauge action on the basis of an extended symmetry group, which upon elimination of the auxiliary fields reduces to the action proposed by Freedman ⁴⁾. For obvious reasons we will refer to this theory as chirally extended supergravity.

We now discuss the model. The action is a sum of the gauge action of supergravity with the auxiliary fields S , P and A_μ , the locally supersymmetric spin $(1, \frac{1}{2})$ system with its auxiliary field D and a chiral coupling of the axial field B_μ to ψ_μ and χ , with a corresponding term linear in D and an off-diagonal masslike term

$$\begin{aligned} \mathcal{L} = & -\frac{e}{2\kappa^2} R(e, \omega) - \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} \bar{\psi}_\mu \gamma_5 \gamma_\nu (D_\rho + i g B_\rho \gamma_5 \psi_\sigma) \\ & + \frac{e}{3} (A_\mu^2 - S^2 - P^2) - \frac{e}{4} F_{\mu\nu}^2(B) + \frac{e}{2} D^2 \\ & + \frac{\kappa e}{4} \bar{\psi}_\mu \sigma^{\alpha\beta} \gamma^\mu \chi \left(\hat{F}_{\alpha\beta}(B) + F_{\alpha\beta}(B) \right) \\ & - e g \kappa^{-2} (2D + i\kappa \bar{\chi} \gamma_5 \gamma \cdot \psi) \end{aligned} \quad (1)$$

We used the conventions of Ref. 2). We stress that the spinor connection field $\omega_{\mu}^{ab}(e, \psi)$, occurring in R and the covariant derivatives D_{μ} , does only depend on the vierbein and gravitino fields e_{μ}^a and ψ_{μ} , respectively ^{1), 2)}. The symbol $\hat{F}_{\alpha\beta}(B)$ is the supersymmetric field strength defined by

$$\hat{F}_{\alpha\beta}(B) = \partial_{\alpha} B_{\beta} - \partial_{\beta} B_{\alpha} + \frac{\kappa}{2} (\bar{\Psi}_{\alpha} \gamma_{\beta} - \bar{\Psi}_{\beta} \gamma_{\alpha}) \chi \quad (2)$$

For $A_{\mu} = S = P = 0$ this is precisely the action of Ref. 4), whereas for $g = 0$ this action reduces to that of the supersymmetric Maxwell-Einstein system ²⁾.

This action is invariant under the following local supersymmetry transformations

$$\begin{aligned} \delta e_{\mu}^a &= \kappa \bar{\epsilon} \gamma^a \psi_{\mu} \\ \delta \psi_{\mu} &= \frac{2}{\kappa} (D_{\mu} + i g B_{\mu} \gamma_5 + \frac{i}{2} \kappa A_{\mu} \gamma_5) \epsilon + \frac{1}{3} \gamma_{\mu} N \epsilon \\ \delta B_{\mu} &= -\bar{\epsilon} \gamma_{\mu} \chi \\ \delta \chi &= \hat{F}_{\alpha\beta}(B) \sigma^{\alpha\beta} \epsilon + i D \gamma_5 \epsilon \\ \delta S &= \frac{1}{2} \bar{\epsilon} \gamma \cdot R - \frac{1}{2} \kappa \bar{\epsilon} \gamma^{\mu} N \psi_{\mu} - 2 g \kappa^{-1} i \bar{\epsilon} \gamma_5 \chi \\ \delta P &= -\frac{i}{2} \bar{\epsilon} \gamma_5 \gamma \cdot R + \frac{i}{2} \kappa \bar{\epsilon} \gamma_5 \gamma^{\mu} N \psi_{\mu} - 2 g \kappa^{-1} \bar{\epsilon} \chi \\ \delta A_{\mu} &= \frac{3i}{2} \bar{\epsilon} \gamma_5 (R_{\mu} - \frac{1}{3} \gamma_{\mu} \gamma \cdot R) + \frac{i}{2} \kappa \bar{\epsilon} \gamma_5 N \psi_{\mu} + 2 g \kappa^{-1} \bar{\epsilon} \gamma_{\mu} \chi \\ \delta D &= i \bar{\epsilon} \gamma_5 \gamma^{\mu} (D_{\mu} \chi + i g B_{\mu} \gamma_5 \chi + \frac{i}{2} \kappa A_{\mu} \gamma_5 \chi - \frac{1}{2} \kappa \hat{F}_{\alpha\beta}(B) \sigma^{\alpha\beta} \psi_{\mu} \\ &\quad - \frac{i}{2} \kappa D \gamma_5 \psi_{\mu}) \end{aligned} \quad (3)$$

The symbol N stands for the universal combination

$$N = S - i \gamma_5 P - i \cancel{A} \gamma_5 \quad (4)$$

while R^μ denotes the spin $\frac{3}{2}$ field equation with a chiral coupling both to B_μ and to A_μ

$$R^\mu = e^{-1} \epsilon^{\mu\nu\tau} \bar{\psi}_\nu \gamma_5 \gamma_\tau (D_\rho + i g B_\rho \gamma_5 + \frac{i\kappa}{2} A_\rho \gamma_5) \psi_\tau \quad (5)$$

Note that in the action A_μ does not couple to ψ_μ but that introducing such a coupling in the transformation rules simplifies the expressions for these rules. Again for $g=0$ one finds back the results of Ref. 4), while for $A_\mu = S = P = 0$ one recovers the results of the Maxwell-Einstein system of Ref. 2). However, the complete transformation laws are not simply obtained by chirally covariantizing the transformation rules of Ref. 2) with respect to the local chiral symmetry. The $g \bar{\epsilon} \chi$ terms in δA_μ , δS and δP which are necessary to cancel the variation $\delta \psi_\mu = \frac{1}{3} \gamma_\mu N_\epsilon$ in $-g e i \kappa^{-1} \bar{\chi} \gamma_5 \gamma^\mu \psi$ are new. We stress that the occurrence of these terms would be very disturbing if one would insist on considering the (B_μ, χ, D) system as matter coupled to Poincaré supergravity. However, within the argument presented above there is no inconsistency.

We have explicitly evaluated the commutator algebra of local co-ordinate (G), Lorentz (L), supersymmetry (S) and chiral transformations (A) acting on the fields. We found that this extended gauge algebra still closes without the need for more auxiliary fields beyond A_μ , S , P and D of Maxwell-Einstein supergravity. We have found uniformly

$$\begin{aligned} [\delta_Q(\epsilon_1), \delta_Q(\epsilon_2)] &= \delta_G [2 \bar{\epsilon}_2 \gamma^\alpha \epsilon_1] + \delta_S [-\kappa \bar{\epsilon}_2 \gamma^\alpha \epsilon_1 \psi_\alpha] \\ &+ \delta_L [2 \bar{\epsilon}_2 \gamma^\alpha \epsilon_1 \omega_\alpha^{ab} + \frac{2\kappa}{3} \bar{\epsilon}_2 (N \tau^{ab} + \tau^{ab} N) \epsilon_1] + \delta_A [g \bar{\epsilon}_2 \gamma^\alpha \epsilon_1 B_\alpha] \\ [\delta_Q(\epsilon), \delta_A(\alpha)] &= \delta_Q (i \alpha \gamma_5 \epsilon) \\ [\delta_A(\alpha), \delta_A(\beta)] &= 0 \end{aligned} \quad (6)$$

The chiral transformations on the fields are given by

$$\begin{aligned}
 \delta_A \psi_\mu &= i\alpha \gamma_5 \psi_\mu \\
 \delta_A \chi &= i\alpha \gamma_5 \chi \\
 \delta_A B_\mu &= -g^{-1} \partial_\mu \alpha \\
 \delta_A S &= -2\alpha P \\
 \delta_A P &= 2\alpha S \\
 \delta_A A_\mu &= \delta_A e^a_\mu = \delta_A D = 0
 \end{aligned}
 \tag{7}$$

and are easily found by requiring that Eq. (3) be locally chirally invariant.

After having clarified the full auxiliary field structure of chirally extended supergravity we now discuss the cosmological constant. If we eliminate the auxiliary field D then we find a cosmological term ⁴⁾

$$\rho_{\text{cosm}}^{(D)} = -2g^2 \kappa^{-4} e
 \tag{8}$$

In the case of pure supergravity there exists another way to introduce a cosmological term, namely by adding ^{3),2)}

$$\mathcal{L}_S = e \lambda \kappa^{-2} (2S + \kappa \bar{\psi}_\mu \gamma^{\mu\nu} \psi_\nu)
 \tag{9}$$

This term was shown to be separately invariant in pure supergravity where it leads upon elimination of the auxiliary field S to a cosmological term with opposite sign

$$\rho_{\text{cosm}}^{(S)} = 3\lambda^2 \kappa^{-4} e
 \tag{10}$$

By adding $\mathcal{L}_S$ to the action in Eq. (1) we can thus cancel the cosmological term in the resulting action by choosing $3\lambda^2 = 2g^2$. However, since the symmetry of (1) is that of chirally extended supergravity, $\mathcal{L}_S$ is no longer separately invariant, neither under local supersymmetry nor

under local chiral rotations. As we have seen, the root of the problem is that $\mathcal{L}_S$ and Eq. (1) are based on different graded algebra, namely on the ordinary graded Poincaré algebra and on its chirally extended version, respectively. As to the problem of eliminating the cosmological terms for the action, one might try to use the non-linear Akulov-Volkov action ⁷⁾. An alternative is to use the freedom one obtains by adding extra scalar fields. In the $(2, \frac{3}{2}) + (\frac{1}{2}, 0^+, 0^-)$ coupling there exists the freedom to add an arbitrary potential depending on the scalar fields ⁸⁾ which can be chosen such that the cosmological term disappears. It might be interesting to investigate whether the same mechanism could eliminate the cosmological terms in the various kinds of extended supergravities such as the model discussed in this article.

ACKNOWLEDGEMENTS

It is a pleasure to thank S. Ferrara, D.Z. Freedman, M.T. Grisaru and M. Roček for discussions.

REFERENCES

- 1) S. Ferrara and P. van Nieuwenhuizen - CERN Preprint TH. 2463 (1978),
to be published in Physics Letters B.
K. Stelle and P.C. West - ICTP Preprint 77-78/6, to be published in
Physics Letters B.
- 2) S. Ferrara, M.T. Grisaru and P. van Nieuwenhuizen - CERN Preprint TH.
2467 (1978).
- 3) P.K. Townsend - Phys.Rev. D15, 1007 (1977).
- 4) D.Z. Freedman - Phys.Rev. D15, 1173 (1977).
- 5) P. Breitenlohner - Nuclear Phys. B124, 500 (1977).
- 6) P. Fayet and J. Iliopoulos - Phys.Letters 51B, 461 (1974).
- 7) S. Deser and B. Zumino - Phys.Rev.Letters 38, 1433 (1977).
- 8) A. Das, M. Fischler and M. Roček - Phys.Letters 69B, 186 (1977) ;
Phys.Rev. D16, 3427 (1977) ;
E. Cremmer and J. Scherk - Phys.Letters 69B, 97 (1977) ;
J. Polonyi - Preprint Budapest KFKI-1977-93.

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