Ref.TH.2518-CERN

PHASE TRANSITIONS IN HADRONIC MATTER ")

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A B S T R A C T

We formulate an equation of state for strongly interacting matter, which leads to a phase transition from massive resonance excitation to ideal gas behaviour. The structural similarity to the Van der Waals equation is discussed, as are extensions to describe hadron to quark matter transitions.

") To appear in Zeitschrift f. Physik
C : Particles and Fields.

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Ref.TH.2518-CERN

23 June 1978

1. - INTRODUCTION

The question of phase transitions in strongly interacting systems has in recent years attracted more and more attention. This attention has three, at present, rather disconnected origins : pion condensation in nuclear matter ¹⁾, transitions from hadron to quark matter ^{2),3)}, and critical behaviour of multihadron systems with resonance excitation ³⁾⁻⁹⁾.

The starting point of the present paper is the statistical description of hadrons with resonance interaction, such as the one given, e.g., by the statistical bootstrap model ¹⁰⁾. However, our results appear to have considerable implications also on transitions from hadron to quark matter ; we shall return to these at the end.

In the statistical bootstrap model, it is assumed that an ideal gas of all possible excited hadronic states correctly describes the effect of the strong interaction on a many-body system ¹¹⁾. The level density of an interacting multipion system then becomes

$$\sigma(P^2) = \delta^{(u)}(P) + \sum_{N=1} \frac{B^N}{N!} \int \prod_{i=1}^N \left\{ d^4 p_i \rho(p_i^2) \right\} \delta^{(u)}\left(\sum_{i=1}^N p_i - P\right) \quad (1)$$

where P denotes the over-all four-momentum of the system, and B a volume parameter : $B = 2\pi V$ makes in the non-relativistic limit of Eq. (1) the grand-canonical level density for pions of mass μ contained in a volume V . The hadronic excitation spectrum is described by

$$\rho(p^2) = \delta(p^2 - \mu^2) + \dots \quad (2)$$

with the dots denoting the higher excitations ; the first term alone would reduce Eq. (1) to the level density of an ideal pion gas. To calculate $\sigma(P^2)$, we must specify $\rho(p^2)$. In the statistical bootstrap model, this is done by imposing the (strong ¹²⁾) bootstrap condition ¹³⁾

$$\rho(p^2) = \delta(p^2 - \mu^2) + \frac{1}{B} [\sigma(p^2) - \delta^{(u)}(p) - B\rho(p^2)] \quad (3)$$

Condition (3) makes the mass spectrum for large p^2 equal to the level density and thus gives it the fastest possible increase with mass. Inserting Eq. (1) into Eq. (3) yields the conventional form of the bootstrap equation

$$\rho(P^2) = \delta(P^2 - \mu^2) + \sum_{N=2}^{\infty} \frac{B^{N-1}}{N!} \int \prod_{i=1}^N \{d^4 p_i \rho(p_i^2)\} \delta^{(4)}(\sum_{i=1}^N p_i - P) \quad (4)$$

The integral equation (4) determines the spectrum of hadronic excitations ; through Eqs. (1) or (3), this in turn fixes the level density of our multi-hadron system.

The solution of the bootstrap equation (4) has the well-known asymptotic form ¹⁴⁾

$$\rho(M^2) = \text{const. } M^{-3} e^{bM}, \quad M^2 = P_\mu P^\mu \quad (5)$$

with $b \equiv \sqrt{b_\mu b^\mu}$ determined by the equation

$$\varphi_0(b) \equiv B \int d^4 p \delta(p^2 - \mu^2) e^{-b_\mu p^\mu} = 2 \ln 2 - 1 \quad (6)$$

We recall here that the linearly exponential rise of the form (5) is in accord with the hadronic excitation spectrum obtained in the dual resonance model ¹⁵⁾. The level density resulting from Eqs. (3) and (5) of course also grows linearly exponential in the total energy - again in accord with statistical Regge resonance ¹⁶⁾ and dual resonance ¹⁷⁾ descriptions.

Now any level density of the form

$$\sigma(M^2) = \text{const. } M^a e^{bM}, \quad a, b = \text{const.} \quad (7)$$

leads to a partition function

$$Z(\tau) \equiv \int d^4 P e^{-\beta_\mu P^\mu} \sigma(P^2) = \frac{1}{\beta} \int dM^2 M K_1(M\beta) \sigma(M^2) \quad (8)$$

which diverges if the temperature $T \equiv (\beta_\mu \beta^\mu)^{-\frac{1}{2}}$ surpasses $T_0 = 1/b$; hence T_0 represents some type of critical temperature for the system ^{3), 4)}. If in Eq. (7) $a \geq -\frac{7}{2}$, then the energy density

$$\varepsilon(T) = \frac{T^2}{B} \left(\partial \ln Z(T) / \partial T \right)_B \quad (9)$$

also diverges as $T \rightarrow T_0$; hence in this case (which includes the statistical bootstrap solution $a = -3$), T_0 is attained only in the limit of infinite energy density and thus constitutes an ultimate temperature of the system ¹⁰⁾. On the other hand, for $a < -\frac{7}{2}$, T_0 corresponds to a finite energy density, and therefore phase transitions become possible ³⁾⁻⁷⁾.

What are the two phases of such systems ? How does the bootstrap condition (3) have to be modified ⁶⁾ to allow phase transitions ? Can one formulate an equation of state for such systems - a hadronic analogue of the Van der Waals equation ? These are the main questions to which the present paper is addressed.

2. - LIMITED FIREBALL EXCITATION

The reason for the existence of an ultimate temperature T_0 in the statistical bootstrap model is the asymptotic equality (or proportionality) between mass spectrum and level density : since new resonance states alone accommodate almost all degrees of freedom, the increase of kinetic energy becomes bounded.

From Eq. (1), the partition function (8) can be written

$$Z(T) = \exp \varphi_p(T) \quad (10)$$

with

$$\varphi_p(T) \equiv B \int d^4p \, e^{-\beta_\mu p^\mu} \rho(p^2) \quad (11)$$

and again $T = (\beta_\mu \beta^\mu)^{-\frac{1}{2}}$. Hence the bootstrap equation (8) becomes

$$2 \ln Z(T) = \varphi_0(T) + Z(T) - 1 \quad (12)$$

It is easily verified that at $T=T_0$, with T_0 determined by Eq. (6), the first derivative of $\varphi_0(T)$ with respect to $Z(T)$ vanishes, while all higher derivatives are finite there. Hence ¹⁴⁾ the leading singularity of $Z(T)$ at $T=T_0$ is a square root branch point

$$Z(T) = Z(T_0) + \text{const.} \sqrt{T-T_0}. \quad (13)$$

The functional form of $Z(T)$ is shown in Fig. 1. The physical region corresponds to the piece between $T=0, Z=1$ and $T=T_0, Z=Z(T_0)$; since the logarithmic derivative of Eq. (13) diverges at $T=T_0$, the temperature T_0 is reached only in the limit of infinite energy density.

To permit phase transitions, we must evidently find a partition function which has a region of singular behaviour, but still allows Z to continue at finite energy density into the region above T_0 . Let us try a general parametrization of the form

$$Z(T) = 1 + \varphi_0(T) + g(T). \quad (14)$$

For

$$\begin{aligned} g(T) &= 2 \int d^4P e^{-\beta P} \sum_2 \frac{B^N}{N!} \int \prod_1^N \{d^4p_i \rho(p_i^2)\} \delta^{(4)}(\sum_1^N p_i - P) \\ &= 2(Z - 1 - \ln Z) \end{aligned} \quad (15)$$

we recover the bootstrap form (12). If

$$\begin{aligned} g(T) &= \int d^4P e^{-\beta P} \sum_2 \frac{B^N}{N!} \int \prod_1^N \{d^4p_i \delta(p_i^2 - \mu^2)\} \delta^{(4)}(\sum_1^N p_i - P) \\ &= e^{\varphi_0(T)} - 1 - \varphi_0 \end{aligned} \quad (16)$$

we have the other extreme, an ideal pion gas, for which $Z(T)$ grows monotonically with T [Fig. 1, curve (b)]. As intermediate situation, we consider the case in which the mass spectrum $\rho(p^2)$ is such that two excited hadrons (fireballs) plus an arbitrary number of pions provide the dominant contribution :

$$\begin{aligned}
 g(T) &= \alpha \int d^4 P e^{-\beta P} B^2 \int d^4 p_1 \rho(p_1^2) \int d^4 p_2 \rho(p_2^2) * \\
 &\quad \sum_0^{\infty} \frac{B^N}{N!} \int \prod_1^N \{d^4 q_i \delta(q_i^2 - \mu^2)\} \delta^{(4)}\left(\sum_1^N q_i + p_1 + p_2 - P\right) \\
 &= \alpha e^{\varphi_0(T)} \ln^2 Z(T),
 \end{aligned} \tag{17}$$

with α as a constant of proportionality. The resulting equation for the partition function is

$$Z(T) = 1 + \varphi_0(T) + \alpha e^{\varphi_0(T)} \ln^2 Z(T). \tag{18}$$

Requiring $\partial \varphi_0 / \partial Z$ to vanish, we obtain

$$Z(T) = 2 \alpha e^{\varphi_0(T)} \ln Z(T); \tag{19}$$

if $\partial^2 \varphi_0 / \partial Z^2$ is to vanish as well, then

$$Z(T) = 2 \alpha e^{\varphi_0(T)}. \tag{20}$$

Equations (18)-(20) determine critical values for temperature and coupling :

$$\varphi_0(T_c) = (e-2)/2, \tag{21}$$

$$\alpha_c = \frac{1}{2} e^{(2-e/2)}. \tag{22}$$

For $\alpha < \alpha_c$, $\partial \varphi_0 / \partial Z$ never vanishes ; for $\alpha = \alpha_c$, both $\partial \varphi_0 / \partial Z$ and $\partial^2 \varphi_0 / \partial Z^2$ vanish at $T = T_c$, while for $\alpha > \alpha_c$, $\partial \varphi_0 / \partial Z$ vanishes already for some temperature $T' < T_c$, at which point $\partial^2 \varphi_0 / \partial Z^2$ is negative. The three cases are illustrated in Fig. 2.

Consider first $\alpha > \alpha_c$. Then at $T = T' < T_c$, $\partial \varphi_0 / \partial Z = 0$, $\partial^2 \varphi_0 / \partial Z^2 < 0$; hence near $T = T'$,

$$Z(T) = Z(T') + \text{const.} \sqrt{T' - T}, \quad (23)$$

and we have the same behaviour as in the bootstrap model ; $T \rightarrow T'$ only in the limit of infinite energy density. For $\alpha = \alpha_c$, the situation is quite similar, only with a cube root singularity at $T = T_c$, instead of Eq. (23). Thus, for $\alpha \geq \alpha_c$, no phase transitions are possible, and we shall from now on restrict ourselves to $\alpha < \alpha_c$.

For α close to but below α_c , $Z(T)$ increases near $T = T_c$ in a bootstrap-like fashion : the first derivative $\partial \varphi_0 / \partial Z$ there becomes

$$\partial \varphi_0 / \partial Z \sim (1 - \alpha / \alpha_c) \quad (24)$$

and hence $\partial Z / \partial T$ almost diverges, as for $\alpha \geq \alpha_c$ or as in the bootstrap model. Once T is sufficiently large, however,

$$Z(T) \sim e^{\varphi_0(T)} \quad (25)$$

and hence we then have an ideal pion gas. Since $\partial \varphi_0 / \partial Z$ for $T > 0$ never vanishes in the case $\alpha < \alpha_c$, we moreover always have finite energy densities. This is in contrast to the case $\alpha \geq \alpha_c$, where the "ideal pion branch" of $Z(T)$ is unphysical, since $T = T_c$ or $T = T'$ can only be reached as $\epsilon \rightarrow \infty$.

In Figs. 3 and 4, we show the energy density (9) and the pion number density

$$v(T) = (\varphi_0 / B) (\partial \ln Z(T) / \partial \varphi_0(T)), \quad (26)$$

both as functions of T , for various values of $\alpha < \alpha_c$. In both figures we note bootstrap-like behaviour near $T = T_c$: most energy goes into increasing the mass of the two fireballs ; this prevents the temperature to increase much with ϵ and leads to a rapid growth of v . Once we are well above T_c , we find ideal pion gas behaviour. In both figures, the critical curve, where $\alpha = \alpha_c$, is shown for comparison.

Let us now look at the behaviour of $\epsilon(T)$ for some fixed $\alpha < \alpha_c$ in more detail (Fig. 5). The segment of the curve between T_a and T_b does not satisfy one of the basic requirements for a statistical description : the specific heat there becomes negative

$$\partial \epsilon / \partial T = C_v(T) < 0 \quad (27)$$

and hence the entropy is not maximal for such situations. They in fact correspond to a constellation where the two fireballs convert part of their rest mass into an ideal pion gas and hence mainly into kinetic energy. To see this more clearly, we consider the energy carried by the two fireballs. From Eq. (18), the over-all energy density is

$$\epsilon(T) = \frac{T^2}{8Z} \left(\frac{\partial \varphi_0}{\partial T} \right) \left[1 + \alpha e^{\varphi_0} \ln^2 Z + \frac{2\alpha}{Z} e^{\varphi_0} \ln Z \cdot \frac{\partial Z}{\partial \varphi_0} \right], \quad (28)$$

of which the first two terms come from the ideal pion gas component, in contrast to pions in fireball form. Hence,

$$\epsilon_F(T) = \epsilon(T) - \frac{T^2}{8Z} \left(\frac{\partial \varphi_0}{\partial T} \right) \left[1 + \alpha e^{\varphi_0} \ln^2 Z \right] \quad (29)$$

is the mean energy of the fireball component. It is shown in Fig. 6 as function of the temperature, again in comparison with the critical case $\alpha = \alpha_c$, which leads evidently to infinitely massive fireballs. Above T_a , we have for $\alpha < \alpha_c$ a sort of evaporation process, where the mass is converted mainly into kinetic energy.

As is evident from Fig. 3, the instability region of negative specific heat decreases with decreasing α . If we define the value $\alpha = \bar{\alpha}_c$ by Eq. (28) and the conditions

$$(\partial \epsilon(\bar{\alpha}_c) / \partial T)_{T=\bar{T}_c} = 0, \quad (30)$$

$$(\partial^2 \epsilon(\bar{\alpha}_c) / \partial T^2)_{T=\bar{T}_c} = 0, \quad (31)$$

to hold for some $T = \bar{T}_c$, then the variation of $\epsilon(T)$ for $\alpha < \bar{\alpha}_c$ is free from instabilities (see Fig. 7). The resulting numerical value of $\bar{\alpha}_c$ is about $\frac{1}{2}$.

We thus have three distinct region behaviours : for $\alpha \geq \alpha_c$, bootstrap-like behaviour ; for $\alpha_c > \alpha > \bar{\alpha}_c$, a temperature range in which thermodynamical instabilities occur and which separates low temperature resonance excitation from a high temperature ideal gas structure ; finally, for $\alpha < \bar{\alpha}_c$, a conventional statistical picture approaching an ideal gas as $\alpha \rightarrow 0$. It is obviously tempting to consider the intermediate case as a phase transition¹⁸⁾, and we shall do so in Section 3.

Before that, however, we want to comment briefly on the generality of our specific partition function (18). It can be shown that $g(T) \sim (e^{\varphi_0} - 1) \ln Z$ provides a gas without instabilities in the physical region. Powers of the type $g(T) \sim Z^{\delta} e^{\varphi_0}$ either also do this (for $\delta < 1$) or result in bootstrap behaviour ($\delta \geq 1$). It thus appears that powers of $\ln Z$, and hence limited fireball production (two, three, or any finite number of fireballs), indeed give the simplest example of the situation we look for.

3. - PHASE TRANSITIONS

Hadronic systems described by our equation of state (18), with $\alpha_c > \alpha > \bar{\alpha}_c$, lead to a negative specific heat in a certain temperature interval ($T_a < T < T_b$ in Fig. 5). They thus violate one of the basic assumptions of statistical physics, in a way which is quite similar to the one encountered in the Van der Waals description of a real gas. In Fig. 8 we compare the P-V behaviour (P : pressure, V : volume) of such systems with the ϵ -T behaviour found in our case. The Van der Waals equation leads to a range of volume, $V_a < V < V_b$, within which the system leads to a negative isothermal compressibility

$$\kappa_T \equiv - \frac{1}{V} \left(\frac{\partial V}{\partial P} \right)_T < 0 \quad (32)$$

for $T < T_c$. In our case, we have instead a range of temperature, $T_a < T < T_b$, within which the system has a negative specific heat

$$C_V \equiv (\partial \varepsilon / \partial T)_V < 0 \quad (33)$$

for couplings $\alpha_c > \alpha > \bar{\alpha}_c$. We thus have the correspondence

$$\begin{array}{ll} P & \longleftrightarrow \varepsilon \\ V & \longleftrightarrow T \\ T & \longleftrightarrow \alpha \\ \mathcal{H}_T & \longleftrightarrow C_V \end{array} \quad (34)$$

between the two cases. Both situation (32) and situation (33) are unphysical, more precisely "unstatistical", in the sense of violating the maximum entropy condition.

In case of the Van der Waals equation, the unphysical behaviour $\kappa_T < 0$ is eliminated by dropping the assumption of homogeneity and introducing two phases : below V_A we have a homogeneous gas, above V_B a homogeneous liquid ; at V_A , condensation begins and we have a inhomogeneous system of co-existing gas and liquid up to V_B (see Fig. 9a).

In our case, the problem can be approached in a very similar fashion. Consider the multiplicity as shown in Fig. 10 : in the fireball region, for T at some value T_A near the peak T_a (see Fig. 5), the vast majority of pions exists in form of fireballs (F), with only a small fraction present as an ideal gas (D). For T at some value T_B sufficiently above the dip T_b , the situation is reversed. We can idealize this by saying that below some T_A , we have a homogeneous bootstrap phase, with all pions contained in the two fireballs, while above T_B , all pions exist in form of an ideal gas ; we shall return shortly to the determination of T_A and T_B . At $T = T_A$, fireball evaporation begins ; the two fireballs give up rest mass, and this is converted into an ideal pion gas. At $T = T_B$, the two fireballs have evaporated down to their pion core, and hence we are left there with an ideal pion gas. The actual case is of course somewhat more complicated, as bubbles of free pions are present already at T_A , and the two fireballs are still somewhat excited even at T_B .

The evaporation energy ϵ_{AB} and the temperatures T_A and T_B can be determined as in the Maxwell construction for the Van der Waals gas. We require ϵ_{AB} to give the correct energy average over the temperature range $T_A \leq T \leq T_B$:

$$\xi_{AB}(\alpha) = \frac{1}{T_B - T_A} \int_{T_A}^{T_B} dT \epsilon_{\alpha}(T), \quad (35)$$

with $\epsilon_{\alpha}(T)$ given by Eq. (28). Clearly, this is equivalent to an equal area rule.

The negative compressibility of a Van der Waals gas is directly connected to large (particle number) density fluctuations. In our case, quite similarly, the negative specific heat implies large energy density fluctuations⁴⁾; in the region between T_a and T_b (Fig. 5), the energy fluctuates between fireball and ideal gas form. In contrast, below T_A or above T_B , one or the other form dominates. We are thus also from a fluctuation point of view led to a two-phase picture with a co-existence region.

4. - CONCLUSIONS

We have given an explicit equation of state of hadronic matter, which leads to a second order phase transition in much the same way as the Van der Waals equation provides a first order transition. In the transition region, our system has a negative specific heat, and thus strong energy fluctuations, in contrast to the negative compressibility and the associated density fluctuations in the Van der Waals vase. Condensed water droplets in the vapour become in our picture fireballs in an ideal pion gas.

The ideal pion gas second phase arose because we chose, in Eq. (6) and Eqs. (14)-(17), pions as basic constituents. Our picture does not, however, depend very sensitively on the mass of these constituents^{*)} : if we hold the hadronic range B fixed and let the pion mass μ vanish, the critical parameters change rather little. This allows us to conjecture the case $\mu = 0$ to be an interacting quark gas inside a bag B . With increasing energy, the quark system then passes first through a hadronic regime characterized by strong binding in form of resonance formation, but in the

*) Neither does the statistical bootstrap model¹⁹⁾.

asymptotic limit converts into an ideal quark gas in a bag B . The ultimate temperature T_0 of the statistical bootstrap picture is then replaced by a transition temperature T_A , where the discontinuous evaporation process begins, leading to T_B and an ideal quark gas beyond that (see Fig. 11). The energy ϵ_{AB} would thus characterize the transition between hadron (i.e., resonance excitation) and quark matter.

It would certainly be of great interest to see if an equation of state of a form like the one considered here can be derived from assumptions on quark interaction dynamics or entirely from QCD.

ACKNOWLEDGEMENTS

It is a pleasure to thank B. Combridge and R. Hagedorn for invaluable help with the numerical computations on the CERN SIGMA facility. One of us (T.C.) thanks the Scientific and Technical Research Council of Turkey for financial support.

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FIGURE CAPTIONS

- Figure 1 The partition function $Z(T)$ for the statistical bootstrap model (a) and the ideal pion gas (b).
- Figure 2 The partition function $Z(T)$ for limited fireball production with $\alpha > \alpha_c$ (a), $\alpha = \alpha_c$ (b) and $\alpha < \alpha_c$ (c).
- Figure 3 Energy density vs. temperature for limited fireball production with different $\alpha < \alpha_c$, and for comparison, with $\alpha = \alpha_c$.
- Figure 4 Pion number density vs. temperature for limited fireball production with different $\alpha < \alpha_c$, and for comparison, with $\alpha = \alpha_c$.
- Figure 5 Energy density vs. temperature with $\alpha < \alpha_c$ for limited fireball production.
- Figure 6 Energy of the fireball component vs. temperature in limited fireball production with $\alpha < \alpha_c$ and $\alpha = \alpha_c$.
- Figure 7 Energy density vs. temperature in limited fireball production for $\alpha = \alpha_c$ (a), $\alpha_c > \alpha > \bar{\alpha}_c$ (b), $\alpha = \bar{\alpha}_c$ (c), and $\bar{\alpha}_c > \alpha$ (d).
- Figure 8 Equation of state for Van der Waals (a) and limited fireball (b) systems.
- Figure 9 Coexistence regions for Van der Waals (a) and limited fireball (b) systems.
- Figure 10 Pion number density in the critical region ; the dashed curve is the ideal gas limit.
- Figure 11 Temperature vs. energy density in limited fireball production (a) and statistical bootstrap (b).

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