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Abstract

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We review the progress in the understanding of hard scattering processes in QCD.

1. Introduction

During the last few years there have been many experiments concerning hard scattering processes, processes in which there are large energy and momentum transfers. The data from these experiments are usually interpreted within the parton picture, in which hadrons are assumed to consist of constituents, and it is these constituents which participate in the hard scattering [1-4]. To my mind the philosophy of the parton picture is that the strong interactions are described by a quantum field theory, and that the parton model provides an approximation to this field theory for the hard scattering processes. At present, the leading candidate for the theory of strong interactions is quantum chromodynamics. Its property of asymptotic freedom provides an elegant explanation of approximate Bjorken scaling [5,6] in deep inelastic lepton-hadron scattering, moreover the scaling violations can be calculated and are found to agree with the data [see 7-9 and references therein]. It is therefore important to see whether our parton model views concerning other processes, e.g., massive lepton pair production, the inclusive productions of particles with large transverse momenta, etc., are also valid in QCD. Until recently one had no means of calculating QCD predictions for these processes, however, during the last few months there has been considerable progress in understanding this problem. It is this progress that I shall review in this talk.

One would like to make use of the smallness of the running coupling constant to perform a perturbative expansion. In the case of deep inelastic scattering, making use of the powerful theoretical tools of the Operator Product Expansion and Renormalization Group one can indeed

do such an expansion. The "fine structure constant" of strong interactions $\alpha_s(q^2) \equiv \bar{g}^2(q^2)/4\pi$ is however usually taken (from fits to the violation of Bjorken scaling) to be around 0.2 to 0.3 at q^2 values of the order of 10 GeV^2 , which although less than 1 is not very small.

It is therefore a relevant question whether it is sufficient to take the first term in the series in $\bar{g}^2(q^2)$ or whether the higher order terms in the series are large. The only way to answer this question is to calculate these higher order terms; in the case of the non-singlet combination of structure functions, this has been done. I will review the results in Section 2.

Unfortunately for the other hard scattering processes, in particular for those with two hadrons in the initial state, no light cone techniques have been developed and one has to make model assumptions. The approach which we shall take here, is that hadrons can be described by soft wave functions, which cannot be calculated using perturbation theory and which strongly damp the invariant masses and transverse momenta (in the infinite momentum frame, for example) of quarks, gluons and antiquarks. We assume further that hard scattering cross-sections are given by the convolution of these soft hadronic wave functions together with the cross-sections for hard scattering subprocesses involving only quarks, antiquarks and gluons. This assumption *a priori* does not provide a useful prescription, since in principle we need to calculate the cross-section for the subprocess to all orders of perturbation theory, in spite of the presence of a "running coupling constant" $[\bar{g}(Q^2)]$ where Q represents a typical large momentum transfer. This is so, because, although $\bar{g}^2(Q^2) \sim (1/\log Q^2)$ and therefore in appropriate kinematic regions will be small, the integral over the loop momentum

often gives a factor of $\log Q^2$ (this will be explained in detail in subsequent sections). Thus we end up with a power series in $\bar{g}^2(Q^2) \log Q^2$, in which every term is of the same order of magnitude. In order to make QCD predictions we must either be able to sum the series (as in deep inelastic scattering), or to define quantities in which the loop momenta do not give the factor of $\log Q^2$.

An example of a process in which the loop momenta do not give a factor of $\log Q^2$ is the production of jets in e^+e^- annihilation [10]. In this case since we are not looking at an exclusive process, but rather at a suitably defined inclusive one, there are no infra-red divergences. In addition if we define each jet as all the particles of an event whose momenta lie in some cone of non-zero half angle (δ , say) and allow for a fraction of the energy up to some maximum (ϵ , say) to be carried by particles outside of the jets, then the limit in which the masses of the quarks are set equal to zero is non-singular. In the limit where these masses are much smaller than Q^2 this will leave us with two mass scales Q^2 and μ^2 , where μ is the renormalization point. Any $\log Q^2$ terms must therefore be of the form $\log Q^2/\mu^2$, so that by choosing μ^2 to be equal to Q^2 we end up with a series in $\bar{g}^2(Q^2)$ with no $\log Q^2$ terms. There still remain $\log \delta$ and $\log \epsilon$ terms, so that if the first few terms of this series are to represent accurately the jet cross-section, δ and ϵ should not be chosen too small. Explicit examples are given in Reference [10].

In this talk I shall concentrate on processes in which the $\log Q^2$ factors coming from the loop momenta are present^{*)}. One of the most

*) In these processes the limit in which the quark masses are set equal to zero is singular. These masses provide the scale for the logarithms.

interesting of these processes is the inclusive production of a massive lepton pair in hadronic collisions. The usual explanation of this process is due to Drell and Yan [11] and is that a quark from one of the initial hadrons annihilates an antiquark from the other, the resulting massive photon then decays into the observed lepton pair (Fig. 1a). This is represented by the formula

$$\frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2}{3nQ^4} \sum_{\substack{\text{quark} \\ \text{flavours } a}} Q_a^2 \int_0^1 dx_1 dx_2 \delta(x_1 x_2 - \tau) x_1 x_2 \left[G_{q/h_1}(x_1) G_{\bar{q}/h_2}(x_2) + G_{\bar{q}/h_1}(x_1) G_{q/h_2}(x_2) \right] \quad (1)$$

where n is the number of colours and $G_{q/h}(x)$ is the probability of finding a quark q in a hadron h with a fraction x of its longitudinal momentum. In QCD, however, in addition to the simple diagram of Fig. 1a there exist diagrams such as those of Figs. 1b, 1c, and naïvely it would seem that these diagrams would spoil the simple probabilistic picture of Eq. (1). These diagrams are of higher order in the coupling constant but each factor of g^2 is accompanied by a factor of $\log Q^2$ and so they cannot be neglected. We can also ask what is so special about quark-antiquark annihilation. For example the subprocesses $qg \rightarrow \ell^+ \ell^- x$ *) (Figs. 1d and 1e) and $qq \rightarrow \ell^+ \ell^- x$ give equally important contributions ($g^2 \log Q^2$ and $g^4 \log^2 Q^2$, respectively). Moreover, for the subprocess $qq \rightarrow \ell^+ \ell^- x$ we can use only valence quarks and therefore this subprocess may be expected to be important. In fact this was the first mechanism suggested for massive lepton pair production [12]. In this talk (Section 3) we will discover why the Drell-Yan model

*) Throughout this talk q ($\bar{q}$) signifies a quark (antiquark), g signifies a gluon and ℓ a lepton.

[Eq. (1)] is so special, and why the diagrams of Figs. 1b-1e, or other subprocesses do not invalidate it.

Another interesting process is the production of particles or jets with large transverse momenta in hadronic collisions. The usual parton model philosophy [3] concerning these processes is that an elastic large angle scattering takes place between two "constituents" of the initial hadrons, one of the resulting constituents then fragments forming the trigger particle. This is represented by Fig. 2 and by

$$\begin{aligned}
 E_c \frac{d\sigma}{d^3p_c} (A+B \rightarrow CX) &= \sum_{a,b:c,d} \int_0^1 dx_a dx_b \frac{dx_c}{x_c^2} \cdot \\
 &\cdot G_{a/A}(x_a) G_{b/B}(x_b) \tilde{G}_{C/c}(x_c) S(s'+t'+u) \cdot \\
 &\cdot \left. \frac{s'}{\pi} \frac{d\sigma}{dt'} (ab \rightarrow cd) \right| \begin{array}{l} s' = x_a x_b s \\ t' = x_a/x_c t \\ u' = x_b/x_c u \end{array}
 \end{aligned} \tag{2}$$

where $\tilde{G}_{C/c}(x_c)$ is the probability of constituent c to fragment into particle C , C carrying a fraction x_c of the longitudinal momentum of c . Dimensional counting rules have been derived from which the asymptotic behaviour of the cross-section corresponding to each subprocess (i.e., choice of a, b, c, d) can be calculated. We will see (Section 5) that within the context of QCD we can predict both the normalization and asymptotic behaviour of the cross-section.

From our studies of the processes mentioned above, and from some other processes (such as the production of massive lepton pairs with large transverse momentum, or the production of jets in deep inelastic scattering, which will be discussed in Section 6), we conclude that there

is a surprisingly simple ansatz with which one can adapt the predictions of the parton model to those of QCD. This ansatz is presented at the end of Section 6, and is expected to be valid up to logarithmic corrections. In Section 7 we will demonstrate how to calculate these corrections in a light cone dominated process, the scattering of a proton on a longitudinally polarized photon, and we will try to generalize this to other processes.

2. Higher order corrections to deep inelastic structure functions and sum rules

The operator product expansion and the renormalization group enable us to predict the violations of Bjorken scaling in deep inelastic structure functions. Moreover for combinations of structure functions which are governed by the non-singlet operators, such as $F_2^{\text{ep}} - F_2^{\text{en}}$ or F_3^{VA} where A represents an isoscalar target, the predictions are particularly simple when expressed in terms of the moments of these structure functions. For the remainder of this section we will restrict ourselves to such combinations of structure functions^{*)}.

Defining

$$F_i^n(q^2) = \int_0^1 dx x^{n-1} F_i(x, q^2) \quad (3)$$

the predictions for the q^2 behaviour of $F_i^n(q^2)$ is given by

$$F_i^n(q^2) = F_i^n(q_0^2) \frac{C_i^n(\bar{g}^2(q^2))}{C_i^n(\bar{g}^2(q_0^2))} \exp \left[- \int_{\bar{g}^2(q_0^2)}^{\bar{g}^2(q^2)} \frac{\delta_n(g')}{\beta(g')} dg' \right] \quad (4)$$

*) A detailed study of higher order corrections to singlet combinations of structure functions is presently being carried out.

where $C^n(\bar{g}^2)$ is a calculable power series in $\bar{g}$ and $\gamma_n(g)$, $\beta(g)$ are the anomalous dimensions of the relevant twist two operator and the standard beta function respectively. We write

$$\gamma_n(g) = \gamma_n^0 \frac{g^2}{16\pi^2} + \gamma_n^1 \frac{g^4}{(16\pi^2)^2} + \dots \quad (5)$$

and

$$\beta(g) = -\beta_0 \frac{g^3}{16\pi^2} - \beta_1 \frac{g^5}{(16\pi^2)^2} + \dots \quad (6)$$

Since $\bar{g}(q^2)$ decreases logarithmically with q^2 , Eq. (4) implies that asymptotically,

$$\begin{aligned} F^n(q^2) &= \text{const} \cdot [\bar{g}^2(q^2)]^{\gamma_n^0/2\beta_0} \\ &= \text{const} \cdot \left[\frac{1}{\log q^2/\Lambda^2} \right]^{\gamma_n^0/2\beta_0} \end{aligned} \quad (7)$$

where Λ is a parameter to be fitted to the data. The determination of Λ is equivalent to the normalization of the coupling constant at one value of q^2 . From the q^2 behaviour of the moments, one can invert to obtain the q^2 behaviour of the structure functions themselves. In Fig. 3 we present a comparison of these predictions with the SLAC-MIT electro-production data [13], at several values of x . A lot of new and interesting data concerning scaling violations, in deep inelastic neutrino and muon scattering, will be presented during this symposium [14, 15].

One can now ask whether with the value of $\bar{g}^2(q^2)$ obtained from the lowest order fit [i.e. from Eq. (7)], the next term in the expansion

of Eq. (4) in $\bar{g}^2(q^2)$ is negligible, that is whether it was meaningful to compare (7) to the data at all. Expanding (4) one finds [9]

$$F^n(q^2) = \text{const.} \cdot [\bar{g}_0^2(q^2)]^{\delta_0^n/2\beta_0} \cdot \quad (8)$$

$$\cdot \left[1 + \frac{\bar{g}_0^2(q^2)}{16\pi^2} \left(B_n + \frac{\delta_1^n}{2\beta_0} - \frac{\beta_1 \delta_0^n}{2\beta_0^2} - \frac{\beta_1 \delta_0^n}{2\beta_0^2} \ln(\ln(q^2/\Lambda^2)) \right) \right]$$

where

$$\bar{g}_0^2(q^2) = \frac{1}{\left(\frac{\beta_0}{16\pi^2} \ln q^2/\Lambda^2 \right)} \quad (9)$$

and B_n is the coefficient of $\bar{g}^2(q^2)/16\pi^2$ in C^n .

The relevant question now is whether the second term in the square bracket on the right-hand side of (8) is small compared with 1. Indeed for the non-singlet case all the parameters on the right-hand side of (8) have been calculated. In particular, the calculation of β_1 can be found in Refs. [16,17], that of γ_1^n in Ref. [18] and that of B_n in Ref. [19]. The values of

$$B_n + \frac{\delta_1^n}{2\beta_0} - \frac{\beta_1 \delta_0^n}{2\beta_0^2} - \frac{\beta_1 \delta_0^n}{2\beta_0^2} - \frac{\beta_1 \delta_0^n}{2\beta_0^2} \ln \ln q^2/\Lambda^2$$

for the structure function F_2 for low values of n are presented in Table 1 (where $q^2 \sim 10 \text{ GeV}^2$ and $\Lambda \sim 0.5 \text{ GeV}$ has been taken. The $\ln \ln q^2/\Lambda^2$ term varies very slowly with q^2 in this range).

For F_1 the numbers corresponding to the entries in Table 1 are very similar. Since $\bar{g}^2(q^2)/16\pi^2$ is 2-3% we notice that the corrections for the higher moments are not negligible. The relatively large

numbers in Table 1 indicate that we are still a long way from asymptopia. In spite of this the best fits (for details see Ref. [19]) to the deep inelastic structure functions using (8) are hardly distinguishable from those obtained using just the lowest order formula (7); the effect of the second term in the square brackets on the right-hand side of (8) can essentially be reproduced by using a smaller value of Λ in (8) than in (7).

From the parameters which are needed to construct Table 1, we can deduce what deviations from parton model sum rules we would expect in QCD [20]. These correspond to low moments, and so these deviations turn out not to be too large. The Adler sum rule is exactly obeyed

$$\int_0^1 \frac{dx}{x} [F_2^{\bar{\nu}p} - F_2^{\nu p}] = 2 \quad (10)$$

whereas the Gross-Llewellyn Smith and Bjorken sum rules are expected to be 10-20% below their parton model values in the q^2 range of Refs. [13-15]. The approach to the parton model values for these two sum rules is given by

$$\int_0^1 dx [F_3^{\bar{\nu}p} + F_3^{\nu p}] = 6 \left[1 - \frac{12}{(33-2f) \ln q^2/\Lambda^2} \right] \quad (11)$$

and

$$\int_0^1 dx [F_1^{\bar{\nu}p} - F_1^{\nu p}] = 1 - \frac{8}{(33-2f) \ln q^2/\Lambda^2} \quad (12)$$

where f is the number of quark flavours. It would be very interesting to have some experimental confirmation of (or contradiction with) Eqs. (11) and (12).

Another interesting prediction which can be made is for F_L , the longitudinal structure function. This structure function is zero in the parton model whereas in QCD, it is of the order of $\bar{g}^2(q^2)$. Moreover, there are now no new parameters (Λ having been determined using F_1 or F_2) so that we can calculate both the normalization and the behaviour with q^2 ; we present a comparison [9] of these predictions with the data of Ref. 13 in Fig. 4 ^{*)}. Although the error bars in the data are large, the data lies consistently above the predicted curves. Within QCD if one tries to explain the data, one is limited to saying that either higher order terms in $\bar{g}^2(q^2)$ are very large [which would invalidate the use of a perturbation expansion in $\bar{g}^2(q^2)$], or else the contributions of higher twist operators are large for F_L [21]. If it is the latter, then the data for F_L should fall with q^2 at least as fast as $1/q^2$.

We have seen in this section that in the range of q^2 presently accessible experimentally higher order corrections are relevant, and must be carefully considered. This should also be borne in mind when predicting the asymptotic cross-sections for other hard scattering processes, such as the production of massive lepton pairs in hadronic collisions, or the production of particles and jets with large transverse momenta.

3. Deep inelastic lepton hadron scattering

In the previous section we have seen how the use of the operator product expansion and the renormalization group enable us to make predictions for the violations of Bjorken scaling in deep inelastic

^{*)} We assume that for these large values of x , the main contribution comes from the non-singlet operators.

lepton-hadron scattering. In this section we will study the same process with a different approach, which is certainly less rigorous, but on the other hand is also applicable to processes which are not light cone dominated. The basic assumption is that asymptotically hard scattering cross sections can be written as a convolution of soft hadronic wave functions (which are process independent), with the perturbatively calculated cross section for the "hard-subprocess" which involves only quarks and gluons, and which can be studied perturbatively. Thus, for example, in the case of deep inelastic scattering we assume that we can write (see Fig. 5)

$$F(x, q^2) = \sum_{\text{partons}} \int_x^1 d\bar{x} \int d^2k_1 dk^2 f(\bar{x}, k_1, k^2) \cdot \sigma_{\text{parton}}\left(\frac{x}{\bar{x}}, k^2, q^2\right) \quad (13)$$

where f is related to the square of the soft wave function and σ_{parton} is the cross section for deep inelastic scattering with the parton as the target. We will now study σ_{parton} in perturbation theory.

There are three mass scales relevant to deep inelastic scattering; firstly a large mass scale e.g. q^2 or $p \cdot q$ (we will denote this by q^2), secondly a mass scale of the order of the masses of the partons which we denote by p^2 and finally there is the renormalization point μ^2 . Calculating the contributions of the Feynman diagrams to σ_{parton} , one finds logarithms of these masses. The $\log p^2$ terms will be of interest to us, particularly in the following sections; these terms can arise from two types of singularities: a) the usual infrared divergences, such as for example the bremsstrahlung of a soft gluon from a quark which is nearly on mass shell. These divergences appear as $\int d\omega/\omega$

where ω is the energy of the gluon. It is found that these infrared divergences cancel between diagrams with real gluons and those with virtual gluons in a similar way to the Bloch-Nordsieck mechanism in QED [22,23]; b) mass singularities: these occur because of the simple fact that two massless particles moving parallel to each other in some frame have invariant mass zero. Thus the region of phase space in which a quark radiates a hard gluon with a momentum almost parallel to its own and an almost on-shell quark also leads to a $\log p^2$ term. Mass singularities occur via integrals such as $\int d(\cos \theta)/(1 - \cos \theta)$, where θ is an angle between the momenta of two particles or propagators.

When all the Feynman diagrams of a particular order in perturbation theory are summed, one finds that there is at most one logarithm per loop. In individual diagrams however, there may be both an infrared divergence and a mass singularity corresponding to a particular loop.

I would now like to demonstrate that the results of Section 2 can be obtained by summing the leading logarithms of perturbation theory. Again for simplicity we will consider a non-singlet combination of structure functions, the extension to general combinations of structure functions complicates the technical details but does not alter the essential features. Using the notation of Section 2 we have

$$F^n(q^2) = \langle p | O^n | p \rangle C_n(\bar{g}^2(q^2)) \left[\frac{\bar{g}^2(q^2)}{\bar{g}^2(\mu^2)} \right]^{\gamma_0^n/2\beta_0} \quad (14)$$

where $\langle p | O^n | p \rangle$ is the matrix element of the relevant twist-two operators. From Eq. (14) we can recover the leading logarithms of the perturbation series in $g^2(\mu^2)$. Keeping only leading logarithms

[i.e. neglecting $g^2(\mu^2)$ with respect to $g^2(\mu^2) \log q^2/\mu^2$] we have from the solution of the equation for the running coupling constant [5,6]

$$\left[\frac{\bar{g}^2(q^2)}{g^2(\mu^2)} \right]^{\gamma_0^n/2\beta_0} \simeq \left[\frac{1}{1 + \beta_0 g^2(\mu^2) \ln q^2/\mu^2} \right]^{\gamma_0^n/2\beta_0} \quad (15)$$

C^n is a power series in $\bar{g}^2(q^2)$ with no logarithms, and therefore in our leading logarithm approximation, we keep only the first term which is a constant.

The requirement that F^n be independent of the renormalization point μ implies that in the leading logarithm approximation^{*)}

$$\langle p | 0^n | p \rangle = \langle p | 0^n | p \rangle_{\text{bare}} \left[1 + \beta_0 g^2(\mu^2) \ln p^2/\mu^2 \right]^{\gamma_0^n/2\beta_0} \quad (16)$$

Thus if we calculate $F^n(q^2)$ by using ordinary perturbation theory in $g^2(\mu^2)$, and keeping the leading logarithms in $\log q^2/\mu^2$ and $\log p^2/\mu^2$ to each order we would obtain for the sum to all orders

$$F^n(q^2) \propto \left[\frac{1 + \beta_0 g^2(\mu^2) \ln p^2/\mu^2}{1 + \beta_0 g^2(\mu^2) \ln q^2/\mu^2} \right]^{\gamma_0^n/2\beta_0} \quad (17)$$

Convoluting (17) with the soft hadronic wave function gives

*) $\langle p | 0^n | p \rangle$ in the leading logarithm approximation can also be evaluated directly.

$$\frac{\int_0^1 dx x^{n-1} F^n(x, q^2)}{\int_0^1 dx x^{n-1} F^n(x, q_0^2)} = \left[\frac{\bar{g}^2(q^2)}{\bar{g}^2(q_0^2)} \right]^{\delta_0^n/2\beta_0} \quad (18)$$

which is the rigorous result. We notice that we have obtained the correct result for the violations of Bjorken scaling in spite of neglecting interactions involving "spectator quarks". Thus we find that the first term in the "renormalization group improved" perturbation series in $\bar{g}^2(Q^2)$ is equivalent to summing the leading logarithms in the perturbation series in $g^2(\mu^2)$. The leading logarithms come from the region of integration in which all gluons carry a transverse momentum which is much smaller than Q^2 .

Of course, it is only because we have a light cone expansion that we can have complete confidence in the result of summing leading logarithms. Nevertheless even in the absence of a rigorous technique the leading logarithms may still yield the correct asymptotic result. Indeed in the case of massive QED, the correct asymptotic behaviour of the structure functions was first derived using leading logarithm techniques by Gribov and Lipatov [24], and only later rederived using the light cone expansion by Christ, Hasslacher and Mueller [25].

Recently there have been a number of calculations reproducing (17) using diagrammatic techniques [26-28]. For higher order diagrams it has been useful to work in axial gauges [26]. In these gauges one obtains an effective factor of $\sqrt{1 - \cos \theta}$ from each vertex, so that in many diagrams [such as the interference term of Fig. 6a)] there is no mass singularity. In the diagram of Fig. 6b) there are two propagators with momentum $p-k$ giving rise to a factor $1/(1 - \cos \theta)^2$,

so that the two factors of $(1 - \cos \theta)^{\frac{1}{2}}$ are not enough to eliminate the mass divergence. This is a general feature of mass singularities in axial gauges, the leading logarithms come from a simple set of diagrams; the ladder diagrams with dressed propagators and vertices.

4. Massive lepton pair production

We now consider the inclusive production of a pair of leptons with large invariant mass in hadronic collisions, the Drell-Yan process [11]. Sample diagrams which contribute to this process are shown in Fig. 1.

Let us denote the lowest order contribution to cross-section for the subprocess $q\bar{q} \rightarrow \ell^+ \ell^- x$ (that corresponding to Fig. 1a) by σ_0 . Then when all the radiative corrections of order g^2 are calculated (including those of Fig. 1b and Fig. 1c) in the leading logarithm approximation one finds [29] that these corrections are equal to

$$a(\tau) g^2 \left[\log Q^2/p_1^2 + \log Q^2/p_2^2 \right] \quad (19)$$

where $\tau \equiv Q^2/s$. It turns out that $a(\tau)$ is a very significant function, it is exactly the same function which appears in the order g^2 corrections to deep inelastic scattering on a quark. The structure function of a quark is proportional to

$$S(x-1) + a(x) g^2 \log Q^2/p^2 + O(g^4 \log^2 Q^2/p^2) \quad (20)$$

Thus, to this order at least, it seems that the deviations from the naïve Drell-Yan picture are intimately related to the violations of Bjorken scaling in deep inelastic scattering. In other words, the

diagrams which spoil the simple Drell-Yan picture (such as those of Figs. 1b and 1c) are related to those which are responsible for the violation of Bjorken scaling (such as those of Figs. 6a and 6b). In particular to this order we can write a modified Drell-Yan formula

$$\frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2}{3n_q^4} \sum_{\substack{\text{quark} \\ \text{flavours} \\ a}} Q_a^2 \int_0^1 dx_1 dx_2 x_1 x_2 S(x_1 x_2 - \tau) \cdot$$

$$\cdot \left[G_{q/n_1}(x_1, Q^2) G_{\bar{q}/n_2}(x_2, Q^2) + G_{\bar{q}/n_1}(x_1, Q^2) G_{q/n_2}(x_2, Q^2) \right] \quad (21)$$

where the G distribution functions are the appropriate linear combinations of experimentally determined deep inelastic structure functions.

We now turn our attention to other subprocesses which contribute to massive lepton pair production. As an example let us consider qg scattering for which some lowest order diagrams are shown in Figs. 1d, 1e. When one calculates the contribution to the cross-section for the subprocess from the diagrams of Figs. 1d and 1e one finds [30] that the result is proportional to

$$g^2 (1 - 2\tau(1-\tau)) \ln Q^2/p_z^2 \sigma_0 \quad (22)$$

Again the coefficient of $g^2 \sigma_0$ is highly significant. It is the probability of finding an antiquark in a gluon with a fraction τ of its longitudinal momentum^{*}). When the distribution functions are measured in deep inelastic scattering, they include the effects of gluons

^{*}) The moments of $1-2\tau(1-\tau)$ with respect to τ are thus proportional to one of the off diagonal elements in the anomalous dimension matrix of the lowest twist quark and gluon operators.

through diagrams such as those of Fig. 6c and 6d. Equation (22) implies that the contribution to the distribution functions from these diagrams is related to the contribution to the cross-section for lepton pair production from diagrams such as those of Figs. 1d, and 1e. Thus even these diagrams are included in the right-hand side of Eq. (21). In other words the dominant contribution from the subprocess $qg \rightarrow \ell^+ \ell^- x$ can be expressed as the probability of finding an antiquark in the gluon times the cross-section for this antiquark to annihilate the incident quark.

A similar feature has been found in all other subprocesses which have been studied (e.g., $qq \rightarrow \ell^+ \ell^- x$, $gg \rightarrow \ell^+ \ell^- x$ [30]). In each case the leading logarithmic terms can be absorbed into the distribution functions of the incident hadrons, so that the asymptotic behaviour of the cross-section is given by (21), where the G 's are extracted from linear combinations of experimentally measured deep inelastic structure functions. Thus the dominant contribution can be represented by Fig. 7 where the shaded blocks represent these non-scaling distribution functions.

The factorization of the leading mass singularities into the initial quark and antiquark distribution functions is not just a property of the lowest order cross-sections; indeed, it has recently been shown [26] that this factorization holds in all orders of perturbation theory. In particular, in axial gauges, it is only the ladder diagrams (with self-energy and vertex corrections) which contribute to the leading mass singularities, just as was the case for the structure functions.

We can now understand what is so special about the Drell-Yan formula. In its modified form [Eq. (21)] it includes not only the contributions from $q\bar{q}$ annihilation but also the dominant contributions from the other subprocesses. Since the scaling violations in the distribution functions are only logarithmic, the use of the original Drell-Yan formula [Eq. (1)] over a limited Q^2 range should be a good approximation.

5. Inclusive production of particles and jets with large transverse momenta

Deep inelastic scattering and massive lepton pair production both involve an off-shell photon, in this section we study a purely strong interaction process, the production of particles and jets with large transverse momenta in hadronic collisions. The mechanism usually believed to be responsible for this process has been described in the Introduction, and can be summarized by Eq. (2) and Fig. 2. To see what, if anything, QCD can teach us about this process, let us study a concrete example $\text{quark (A)} + \text{quark (B)} \rightarrow \text{quark (C)} + \text{anything}$, up to order g^6 in the cross-section and as before in the leading logarithm approximation [31]. For simplicity we start with the case $x < 1$, where $x = (2E_c/\sqrt{s})$, in which case only the inelastic diagrams of Fig. 8 contribute. We notice that even in this order there are diagrams with a three-gluon vertex.

There are a number of kinematic regions which contribute to the leading logarithmic behaviour. All these contributions can be interpreted in an elegant way. From the region where k is parallel to $p_A(p_B)$ we obtain a contribution to $E_c(d\sigma/d^3p_c)$ of the form $\log s/p_A^2, (\log s/p_B^2)$ which can be interpreted as being the convolution of the probability of finding a quark in quark A(B), and the Born term for quark-quark

elastic fixed angle scattering (Figs. 9a and 9b). In other words the coefficient of these logarithms is just the function "a" of Eqs. (19) and (20) (in spite of the presence of the three-gluon vertex). From the region where k is parallel to p_C , we obtain a contribution which can be interpreted as the convolution of the Born term for q - q elastic fixed angle scattering, and the probability for one of the resulting quarks to fragment into the observed one + anything (Fig. 9c). The fragmentation function is just that which would be calculated from $e^+e^- \rightarrow qX$ and is related to the distribution function (in this order at least) by the Gribov-Lipatov reciprocity relations [32]. The final dominant contribution comes from the region in which k balances the transverse momentum of p_C , and this can be interpreted as the probability of finding a gluon in quark B with a fraction of the longitudinal momentum of $B^{*})$ convoluted with the Born term for gq elastic fixed angle scattering (Figs. 9d, 9e, and 9f).

At $x = 1$ in addition to the contributions from the diagrams of Fig. 8, one must also include the diagrams which contribute to the elastic qq scattering amplitude. The leading $\log s/p^2$ contributions can also be absorbed into the initial distribution and final fragmentation functions. In addition to these $\log s/p^2$ terms, after renormalization there will also be $g^6(\mu^2) \log s/\mu^2$ terms. These terms are exactly those which, when combined with the $g^4(\mu^2)$ contribution from the Born term give the first two terms in the expansion of $g^4(s)$, the running coupling constant.

Thus we conclude that the cross-section for $qq \rightarrow qX$ to order g^6 in the cross-section and in the leading logarithmic approximation can be written in the form (Fig. 9)

*) The moments of this probability are, of course, proportional to an off-diagonal element in the anomalous dimension matrix for the lowest twist quark and gluon operators.

$$\begin{aligned}
 & E_c \frac{d\sigma}{d^3p_c} (A + B \rightarrow CX) \\
 &= \sum_{a,b:c,d} \int_0^1 dx_a dx_b \frac{dx_c}{x_c^2} g_{a/A}(x_a, Q^2) g_{b/B}(x_b, Q^2) \cdot \\
 & \cdot \tilde{g}_{c/c}(x_c, Q^2) \delta(s'+t'+u') \frac{s'}{u} \frac{d\sigma^{(B)}}{dt'}(a+b \rightarrow cd) \left| \begin{array}{l} s' = x_a x_b s \\ t' = x_a/x_c t \\ u' = x_b/x_c u \end{array} \right.
 \end{aligned}
 \tag{23}$$

where $d\sigma^{(B)}/dt'$ is the Born term contribution to the elastic cross-section for qg or qq scattering, calculated using the running coupling constant and Q^2 represents a typical large invariant mass squared (we assume $s \sim t \sim u \sim Q^2$). In the leading logarithm approximation we cannot distinguish between $\log s$, $\log t$ and $\log u$. It is interesting to see how the different subprocesses are interrelated, starting with quark-quark scattering, we have found that we also need to consider quark gluon scattering.

The techniques of Ref. [26] can be extended to this process, so that the results presented above can be generalized to higher orders in perturbation theory. We feel that the result is significant and very suggestive of an ansatz for calculating cross-sections for inclusive production at large transverse momenta. The ansatz is that the cross-section is given by (23) where now we sum over all combinations of quarks and gluons. This is represented in Fig. 10 where the dotted line represents a quark or a gluon and the shaded blobs represent non-scaling distribution or fragmentation functions. The "B" implies we should take the Born term, calculated using the running coupling

constant. Similar prescriptions have already been used by many authors [e.g. Refs. 33-37]. Thus if we believe we know the distribution and fragmentation functions (including those for gluons !) we are able to predict not only the behaviour but also the normalization of large $p_{\perp}$ distributions.

The above results also suggest that the cross-sections for "jet" production at large transverse momenta can be reliably calculated within the present framework; (up to order g^6 in qq scattering this is a simple consequence of the calculation presented above). All the $\log p_A^2$ and $\log p_B^2$ terms can be absorbed into the initial distribution functions as above, leaving us with a series in $\bar{g}^2(Q^2)$ whose coefficients are calculable and depend on the kinematic parameters of the jet.

6. Some multijet processes

In the previous sections we have seen several examples of processes in which parton model ideas proved to be approximately valid also in QCD. Although, unlike in the parton model, transverse momenta are not bounded in QCD, the leading logarithms in these processes come from a region of integration in which all the quarks and gluons have a transverse momentum relative to the initial particles or trigger particle which is much smaller than Q^2 , a typical large invariant of the process. The only dynamical damping in the transverse momentum is due to the propagators, which tend to behave like $1/k_{\perp}^2$, but even this gentle damping is sufficient to reproduce the parton-like results. In this section I would like to present some examples where the most naïve parton model expectations are not fulfilled, but nevertheless where definite QCD predictions can still be made.

We start by considering σ_L , the longitudinal cross-section in deep inelastic scattering. It is proportional to the $p^\mu p^\nu$ projection of $W^{\mu\nu}$, where $W^{\mu\nu}$ is the usual tensor of deep inelastic scattering [2]. As an example consider the scattering on a quark of momentum p , the diagram of lowest order which contributes to σ_L (in the limit where we neglect p^2/q^2 terms) is shown in Fig. 11. The numerator of this diagram in the Feynman gauge is proportional to (we average over the quark spins)

$$\begin{aligned} & \text{Tr} [\not{p} \gamma^\lambda (\not{p}-\not{k}) \not{p} (\not{p}+\not{q}-\not{k}) \not{p} (\not{p}-\not{k}) \gamma_\lambda] \\ & \sim [(p-k)^2]^2 p \cdot (q-k) \end{aligned} \quad (24)$$

Thus we see that both the factors of $1/(p-k)^2$ from the propagators are cancelled and there is no damping of the transverse momentum. The usual parton model derivation [1] of the relation between σ_L and the average transverse momentum squared of a quark

$$\frac{\sigma_L}{\sigma_T} = \frac{4 \langle k_\perp^2 \rangle}{(-q^2)} \quad (25)$$

relies, among other things, on the assumption that the transverse momentum is damped. It is therefore no surprise that (25) is not valid in QCD. Even in lowest order of perturbation theory it is not valid, since in lowest order

$$F_L \propto q^2 x \quad (26)$$

whereas

$$\frac{\langle k_{\perp}^2 \rangle F_T}{(-q^2)} \propto g^2 (7 - 2x + 4x^2) \quad (27)$$

Nevertheless, the invalidity of (25) does not prevent us from being able to make predictions for σ_L or $\langle k_{\perp}^2 \rangle$. In fact, σ_L can be calculated using renormalization group techniques [7,9], here we shall consider the transverse momentum distribution of a quark jet (which cannot be studied by such techniques). Some sample lowest order diagrams which contribute to the production of a quark jet with large transverse momentum are shown in Figs. 12a and 12b. From similar considerations to those of the previous sections, it seems that the dominant contributions from higher order diagrams (e.g. from Figs. 12c and 12d) will again be absorbed into the non-scaling quark and gluon distribution functions. Thus we have for the transverse momentum distribution of a quark jet.

$$\begin{aligned} \frac{d\sigma_T}{dk_{\perp}^2}(x, k_{\perp}, q^2) = \int_x^1 y dy \left[g_{q/p}(y, q^2) \frac{d\sigma_T^{(0)}}{dk_{\perp}^2}\left(\frac{x}{y}, k_{\perp}, q^2\right) \right. \\ \left. + g_{g/p}(y, q^2) \frac{d\bar{\sigma}^{(0)}}{dk_{\perp}^2}\left(\frac{x}{y}, k_{\perp}, q^2\right) \right] \end{aligned} \quad (28)$$

where $d\sigma^{(0)}/dk_{\perp}^2$ and $d\bar{\sigma}^{(0)}/dk_{\perp}^2$ are the contributions from the lowest order diagrams to the cross-section for $q\gamma^* \rightarrow qX$ and $g\gamma^* \rightarrow qX$, calculated using the running coupling constant. Equation (28) is represented pictorially in Fig. 13. The production of jets with large transverse momentum in deep inelastic scattering has been studied using Eq. (28) by Altarelli and Martinelli [38].

Another interesting process is the inclusive production of massive lepton pairs with large transverse momenta. The lowest order diagrams for this process are shown in Fig. 14. Again the indications are that the higher order corrections can be absorbed into the quark, antiquark or gluon distribution functions, so that we have

$$\begin{aligned} \frac{d\sigma}{dk_{\perp}^2 dQ^2} = & \int y_1 dy_1 y_2 dy_2 \left\{ \left[G_{q/n_1}(y_1, Q^2) G_{\bar{q}/n_2}(y_2, Q^2) + \right. \right. \\ & G_{\bar{q}/n_1}(y_1, Q^2) G_{q/n_2}(y_2, Q^2) \left. \right] \frac{d\sigma^{(q)}}{dk_{\perp}^2 dQ^2} (q\bar{q} \rightarrow \ell^+ \ell^- X) + \left[G_{g/n_1}(y_1, Q^2) \cdot \right. \\ & \cdot G_{q/n_2}(y_2, Q^2) + G_{q/n_1}(y_1, Q^2) G_{g/n_2}(y_2, Q^2) \left. \right] \frac{d\sigma^B}{dk_{\perp}^2 dQ^2} (qg \rightarrow \ell^+ \ell^- X) + \\ & \left. \left[G_{g/n_1}(y_1, Q^2) G_{\bar{q}/n_2}(y_2, Q^2) + G_{\bar{q}/n_1}(y_1, Q^2) G_{g/n_2}(y_2, Q^2) \right] \frac{d\sigma^B}{dk_{\perp}^2 dQ^2} (\bar{q}g \rightarrow \ell^+ \ell^- X) \right\} \end{aligned} \quad (29)$$

All the cross-sections are calculated using the running coupling constant. Equation (29) justifies using the shaded blobs in Fig. 14. An approach similar to Eq. (29) has been used by many authors (see for example [39]). Thus, for example, the dominant contribution for the process $q\bar{q} \rightarrow \ell^+ \ell^- X$, where the lepton pair has a large transverse momentum, is given by the probability of finding a gluon with small transverse momentum in one of the initial quarks convoluted with the cross-section for this gluon to scatter on the other quark.

There is, however, no simple relationship between the transverse momentum of the lepton pair and the transverse momentum of the quarks in deep inelastic scattering. For example in $q\bar{q} \rightarrow \ell^+ \ell^- X$ one finds [39]

$$\langle k_{\perp}^2 \rangle_{\mu^+\mu^-} Q^4 \frac{d\sigma}{dQ^2} \sim Q^2 (1 - \tau^3) \quad (30)$$

which should be compared to (26) and (27). This is clearly to be expected since when the transverse momenta are of the order of q^2 , even in the parton picture, the Bjorken x is no longer equal to the fraction of the longitudinal momentum carried by the quark.

The above discussion applies to transverse momenta whose distribution can be calculated perturbatively and which increase with Q^2 .

In addition there may be a primordial transverse momentum in the hadronic wave function, but this should not increase with Q^2 , and we have no way of calculating it. From phenomenological studies it is estimated to be about two to three times the "usual" hadronic transverse momentum cut-off of 300 MeV [39], and is phenomenologically relevant at present values of Q^2 . However, the authors of Ref. 26, study the interesting (but different) region of phase space where

$$p^2 \ll k_{\perp}^2 \ll Q^2$$

which arguably may be more relevant to present data. The analysis for this region of phase space is considerably more complicated. With that treatment, they find that it is sufficient to take an intrinsic transverse momentum of 300 MeV.

From the results presented in Sections 3 to 6 we conclude that in QCD the correct way to calculate the asymptotic behaviour of hard scattering cross-sections as $Q^2 \rightarrow \infty$ (where Q^2 is a typical large invariant of the process) is given by the following simple ansatz:

1) take the parton model hard scattering formula;

- 2) replace the scaling distribution and fragmentation functions by the appropriate non-scaling ones; in the case of quarks and antiquarks these are measured in deep inelastic scattering, whereas, for the gluon distribution, model assumptions have to be made;
- 3) keep only the contribution of lowest order in the coupling constant (which is non-zero) for the hard scattering subprocess, calculated using the running coupling constant, $\bar{g}(Q^2)$.

7. Beyond leading logarithms

In this section we will go beyond the discussion of the preceding sections and try to deduce how one should calculate the non-leading corrections to processes which are not light-cone dominated, such as those of Sections 4 to 6. With this aim in mind, it is instructive to start by examining F_L , the longitudinal structure function in deep-inelastic scattering, for which we can use either the light cone techniques of Section 2 or the approach of Sections 4 to 6. For simplicity of representation we again restrict ourselves to the non-singlet part of F_L .

The lowest order Feynman diagram for F_L is shown in Fig. 12a. From a leading logarithm analysis analogous to the ones in Sections 4 to 6 we would write

$$F_L(q^2, x, g) = \sum_{\text{quarks}} \int_x^1 dy \, q(y, q^2) F_L^{(0)}(q^2, \frac{x}{y}, \bar{g}(q^2)) \quad (31)$$

where q denotes the (non-scaling) quark distribution function and $F_L^{(0)}$ represents the lowest order $[O(g^2)]$ structure function for the scattering of a quark on a longitudinal photon. We can easily check that (31) is indeed correct since [from a generalization of Eq. (4)]

$$F_L^n(q^2) = C_L^n(\bar{g}^2(q^2)) \exp \left[- \int_{\bar{g}}^{\bar{g}(q^2)} \frac{\delta_n(g')}{\beta(g')} dg' \right] \langle p | O^n | p \rangle \quad (32)$$

and

$$F_T^n(q^2) = C_T^n(\bar{g}^2(q^2)) \exp \left[- \int_{\bar{g}}^{\bar{g}(q^2)} \frac{\delta_n(g')}{\beta(g')} dg' \right] \langle p | O^n | p \rangle \quad (33)$$

so that

$$F_L^n(q^2) = \frac{C_L^n(\bar{g}^2(q^2))}{C_T^n(\bar{g}^2(q^2))} F_T^n(q^2) \quad (34)$$

To lowest order in $\bar{g}$ (for details about calculating the coefficient functions see for example, Refs. 19 and 40)

$$\left. \begin{aligned} C_T^n(\bar{g}^2(q^2)) &= 1 \\ C_L^n(\bar{g}^2(q^2)) &= F_L^{n(0)}(\bar{g}^2(q^2)) \end{aligned} \right\} \quad (35)$$

$[F_L^{n(0)}]$ is independent of q^2 so that (to this order)

$$F_L^n(q^2) = F_L^{n(0)}(\bar{g}^2(q^2)) F_T^n(q^2) \quad (36)$$

By the convolution theorem Eq. (36) implies Eq. (31), and shows that the leading logarithms will factorize to all orders in g .

From Eq. (34) we can also calculate the corrections to Eq. (31).

Beyond the first term in $\bar{g}$ [i.e. the one given by (35)] the coefficient functions C are both gauge and renormalization prescription dependent, although of course in the ratio C_L^n/C_T^n this dependence cancels. To order $\bar{g}^4(q^2)$ we write

$$\frac{F_L^n(q^2)}{F_T^n(q^2)} = C_L^{n,0} \bar{g}^2(q^2) + C_L^{n,1} \bar{g}^4(q^2) - C_L^{n,0} C_T^{n,1} \bar{g}^4(q^2) \quad (37)$$

where $C_L^{n,0}$, $C_L^{n,1}$ and $C_T^{n,1}$ are given by

$$\left. \begin{aligned} C_T^n(\bar{g}^2(q^2)) &= 1 + C_T^{n,1} \bar{g}^2(q^2) + \dots \\ C_L^n(\bar{g}^2(q^2)) &= C_L^{n,0} \bar{g}^2(q^2) + C_L^{n,1} \bar{g}^4(q^2) + \dots \end{aligned} \right\} \quad (38)$$

and

The $O(\bar{g}^4)$ term in F_L^n/F_T^n is given by

$$C_L^{n,1} - C_L^{n,0} C_T^{n,1} \quad (39)$$

From Eqs. (32) and (33) we can easily deduce how to calculate

$C_L^{n,1} - C_L^{n,0} C_T^{n,1}$ using perturbation theory. Writing structure functions for deep-inelastic scattering on a quark as $F_{L,q}$ and $F_{T,q}^n$ and defining

$$F_{L,q}^n(q^2, q^4) = F_{L,q}^{n,0} q^2 + \alpha g^4 \log q^2/p^2 + F_{L,q}^{n,1} q^4 + \dots \quad (40)$$

and

$$F_{T,q}^{\sim}(g^2, q^2) = 1 + \beta g^2 \ln Q^2/p^2 + F_{T,q}^{\sim,1} g^2 + \dots \quad (41)$$

we can identify

$$C_L^{\sim,0} = F_{L,q}^{\sim,0} \quad ; \quad C_L^{\sim,1} = F_{L,q}^{\sim,1} + a F_{L,q}^{\sim,0} \quad ; \quad C_T^{\sim,1} = F_T^{\sim,1} + a$$

where a is an arbitrary constant depending on the gauge and on the renormalization prescription. a cancels out of the combination

$$C_L^{\sim,1} - C_L^{\sim,0} C_T^{\sim,1} \text{ as it must.}$$

In the above example the operators must be renormalized in the same way when calculating F_L and F_T . For other hard scattering processes we do not have a light cone expansion, so we cannot deduce with a 100% certainty what the correct ansatz is. A possible approach, and one which has been taken [41,42], is the analogous one to the $a = 0$ case above. The procedure is as follows:

- 1) One calculates the lowest two orders of perturbation theory for the moments of the cross section for the hard scattering subprocess in the form

$$\sigma_0^{\sim} (1 + g^2 b^{\sim} \ln Q^2/p^2 + C^{\sim} g^2) \quad (42)$$

where Q^2 is the large mass scale in the problem.

- 2) In the modified parton model hard scattering formulae [such as (21)] where non scaling distribution and fragmentation functions are used, one writes for the moments of the subprocess cross-section

$$\sigma_0^{\sim} (1 + (C^{\sim} - F_T^{\sim,1}) \bar{g}^2(q^2) + \dots) \quad (43)$$

Equation (43) is clearly correct for the example of F_L above. In general C^n is gauge dependent, as is F_T^n , the difference of the two should be gauge invariant (it is easy to check that this is indeed the case for the processes of Sections 4 to 6). Since, in the present approach interactions with "spectator" quarks are neglected, at the level where such interactions would be important, gauge dependence would appear; however we expect this to be in terms which violate naïve scaling rules by powers of Q^2 .

It is straightforward to generalize the discussion of this section to $O(\bar{g}^6)$ and higher terms.

8. Conclusions

From the results presented in this talk we see that, at last we can confidently make quantitative predictions for processes involving strong interactions. The ansatz presented at the end of Section 5 provides the first term in an expansion in the running coupling constant, and although the higher order corrections may be non-negligible, we can calculate these corrections. It is particularly satisfying that the beautiful probabilistic picture of the parton model is preserved also in QCD.

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Table 1

Values of the coefficient of $\bar{g}^2(q^2)/16\pi^2$
on the right hand side of Eq. (8) for F_2

n	Coefficient of $\bar{g}^2/16\pi^2$
1	0
2	5.88
3	11.09
4	15.56
5	19.46
6	22.94
7	26.08
8	28.94
9	31.57
10	34.02

Figure captions

- Fig. 1 : Sample diagrams which contribute to massive lepton pair production.
- Fig. 2 : Parton model description of the inclusive production of particles with large transverse momenta.
- Fig. 3 : Fit to $\nu\omega_2$ using the lowest order prediction of QCD. The data are from Ref. 13.
- Fig. 4 : Comparison of the ratio $R \equiv \sigma_L/\sigma_T$ to the data of Ref. 13. The solid curve includes target mass corrections, whereas the dashed curve does not.
- Fig. 5 : Diagrammatic representation of Eq. (13).
- Fig. 6 : Sample diagrams contributing to the deep inelastic structure functions of a quark (a) and (b), and a gluon (c) and (d).
- Fig. 7 : The dominant contribution to massive lepton pair production in QCD. The shaded blobs represent, experimentally determined, non-scaling distribution functions.
- Fig. 8 : Lowest order inelastic diagrams which contribute to the process $qq \rightarrow qX$.
- Fig. 9 : Symbolic representation of the dominant contribution from the diagrams up to order g^6 for the process $qq \rightarrow qX$. The shaded blobs represent the non-scaling distribution and fragmentation functions.
- Fig. 10 : Dominant contribution to $E_c(d\sigma/d^3p_c)$. The -.-.-. line represents quarks and gluons, the shaded blobs represent non-scaling distribution and fragmentation functions and the "B" implies we take the Born term contribution calculated using the running coupling constant.

Fig. 11 : Lowest order diagram which contributes to the longitudinal cross-section of a quark.

Fig. 12 : Sample diagrams which contribute to the production of a quark jet with large transverse momentum.

Fig. 13 : The dominant contribution to the production of a quark jet with large transverse momentum. The shaded blobs represent non-scaling distribution functions and the "B" implies we take the lowest non-zero contribution for the subprocess, calculated using the running coupling constant.

Fig. 14 : Dominant contribution to the production of a massive lepton pair with large transverse momentum. The shaded blobs represent non-scaling distribution functions.

(a)

(b)

(c)

(d)

(e)

~~~~~ gluon
 ~~~~~ photon
 ——— hadron
 ——— quark

FIG.1

FIG. 2

FIG.3

FIG.3 cont.

FIG.4

FIG.4 cont.

FIG.4 cont.

FIG. 5

(a)

(b)

(c)

(d)

FIG.6

FIG.7

FIG.8

FIG.9

$$E_c \frac{d\sigma}{d^3p_c} (A+B \rightarrow CX) = \sum_{\text{quarks gluons}}$$

FIG.10

FIG.11

FIG.12

FIG.13

FIG.14