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MATTER INSTABILITY IN THE SU(5) UNIFIED MODEL
OF STRONG, WEAK AND ELECTROMAGNETIC INTERACTIONS

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ABSTRACT

Previous estimates for the stability of matter through baryon-number violating nucleon decays are refined by taking into account all possible diagrams that contribute to the process. As a result, we obtain $\tau(p) \sim \tau(n) \sim 10^{33}$ yr, not quite an order of magnitude smaller than previous estimates. Most of the decays will produce mesons, but no direct muon. Muon-producing decays are only $\sim 10\%$ of all decays. Although the theoretical errors are large (about one order of magnitude) we conclude that, if the effect is there, and experiments sensitive to detection of mesons (and not only muons) are devised, an increase of two to three orders of magnitude in the present experimental accuracy may reveal it.

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1. INTRODUCTION

The SU(5) scheme¹⁾ of grand unification of weak, electromagnetic and strong interactions has received some attention recently. In fact, it is not only the most economical grand unified model that contains the Weinberg-Salam-GIM SU(2) $\times$ U(1) theory times colour SU(3), but some of its predictions are in good agreement with experiment^{1),2),3)}. Particularly³⁾, the value of the weak angle $\sin^2\theta_W$ and the mass of the b quark compare well with experimental data. This seems to indicate that, while it is unlikely that SU(5) be a final answer to the unification of all non-gravitational forces, it could very well be a sub-model, or a reasonably good approximation, to a more fundamental theory.

The predictions mentioned depend essentially on Clebsch-Gordan coefficients and a minimal choice of Higgs fields which fix $\sin^2\theta_W$ and relate m_b to m_t at superhigh energy, m_X , where all interactions get unified, plus the behaviour of coupling constants and masses when renormalized by either gluon or ordinary $W^\pm$, Z, photon interactions. In view of this, such predictions do not, perhaps, constitute direct evidence for one of the most appealing aspects of grand unifications, viz., the basic unity of all fermions, quarks and leptons. This property, on the other hand, is essential to obtain one spectacular consequence of the model: baryon number non-conservation and therefore the prediction of proton instability. The success of the tested predictions, and the importance of this last untested one, makes it worth while to have a closer look at matter instability (via baryon-number violating p/n decays) in the SU(5) model.

Proton decay was considered in some detail in Ref. 3); the conclusion reached there was that the lifetime of p was so big ($\tau(p) \sim 10^{37}$ yr) to make the process utterly unobservable. However, when taking correctly into account threshold effects for the renormalization equations that govern m_X , it was shown⁴⁾ that the estimate of Ref. 3) should be modified to $\tau(p) \sim 10^{34}$ yr. The present experimental limit is⁵⁾ $\tau(p) > 2 \times 10^{30}$ yr; the facts that there are proposals for improving this limit, and that theory is no longer so terribly far removed from it, have led us to pay a new visit to the problem of instability of matter, performing as complete a calculation of baryon-number violating p and n decays as possible. After this calculation is made, we find a result for $\tau(p)$ or $\tau(n)$ of

$$\tau(n) \sim 10^{33} \text{ yr.} \quad (1.1)$$

Compared with what one obtains if scaling the calculation of Ref. 3) by taking the new preferred value for m_X , $\tau(p) \sim 3 \times (10^{33}-10^{34})$, we see that our more detailed calculation diminishes the estimate for $\tau(p)$ by not quite an order of magnitude.

There is an important point to be made. The existing experimental limit is for p decays producing muons. Now, these muons can be either produced directly, or through the processes

$$p, n \rightarrow e^+, \nu \pi's$$

$$\quad \quad \quad \searrow \mu \nu.$$

The meson decays are the dominant processes; only $\sim 1/10$ of the time will μ 's be produced directly. Unfortunately, it is very unlikely that the mesons have time to decay before being absorbed in matter^{*)}. In fact, looking for only muons is a very bad way to find proton decay (if, as happens in SU(5), the dominant modes are meson modes), as all in all hardly more than 10% of the decays produce muons. To avoid losing this order of magnitude, experiments should be devised that detect not only muons, but also pions and, if possible, electrons. In this case, the estimate Eq. (1.1) shows that, if the effects are there, an improvement of two or three orders of magnitude in the experimental efficiency may reveal it. If one only looks for muons an extra order of magnitude should be necessary.

Of course, SU(5) is not the only scheme that unifies weak, electromagnetic and strong interactions. Some other grand unified schemes also feature proton decay; others do not⁶⁾. While evidence for the instability of matter would give an important backing to grand unifications, it is true that lack of the effect would only rule against a very specific set of models. Nevertheless, the process is sufficiently important that we feel that efforts should be devoted to improving the present limits on nucleon lifetimes.

2. NUCLEON DECAY

The SU(5) model¹⁾ contains 24 vector bosons. Of these, 8 are gluons, 4 are the familiar photon, Z , $W^\pm$, and the remaining 12 are $X_{4/3}^i$, $\bar{X}_{4/3}^i$; $Y_{1/3}^i$, $\bar{Y}_{1/3}^i$. These are superheavy objects with charges $\pm 4/3$, $\pm 1/3$, in three colours each ($i = 1, 2, 3$). They carry both baryon and lepton numbers and,

*) One can estimate that only a few percent of the pions would decay. We are grateful to Prof. Michaelis for information on this point.

since quarks and antiquarks are in the same SU(5) multiplet, their existence implies the possibility of quark-quark conversion into antiquark-antilepton and, therefore, baryon number violating nucleon decays. These decays may proceed through the elementary processes depicted in Fig. 1. Of these, only those in Fig. 1(B) with $\bar{\nu}_e, e^+$ as final state leptons were considered in Ref. 3). Here we add decays with $\bar{\nu}_\mu, \mu^+$ and thus strange particles, in the final state, which may proceed through diagrams (Bi), (A) [but only to order $\sin^2\theta_C$ through (Bii)], decays through (A), and decays through (C) which may take place inside the meson field of a nucleus.

If we make the reasonable assumption that $m_X \approx m_Y$ then the part of the Lagrangian that involves X, Y particles may be approximated by an effective four-fermion interaction

$$\begin{aligned} \mathcal{L}_{\text{eff}} = & \frac{4G}{\sqrt{2}} \epsilon_{ijk} \{ \bar{u}_{kL}^c \gamma_\mu u_{jL} [\bar{e}_L^+ \gamma^\mu d_{iL} - \bar{e}_R^+ \gamma^\mu d_{iR} \\ & + \bar{\mu}_L^+ \gamma^\mu s_{iL} - \bar{\mu}_R^+ \gamma^\mu s_{iR}] \\ & + \bar{u}_{kL}^c \gamma^\mu d_{jL} [-\bar{e}^+ \gamma_\mu u_{iL}] \\ & + \bar{u}_{kL}^c \gamma^\mu d_{jL} [\bar{\nu}_e^c \gamma_\mu d_{iR} + \bar{\nu}_\mu^c \gamma_\mu s_{iR}] \} + h.c., \\ G \equiv & \frac{g_{GU}^2}{4\sqrt{2} m_X^2}, \end{aligned} \quad (2.1)$$

where g_{GU} is the coupling constant at grand unification energies, $g_{GU}^2/4\pi \approx 0.022$. The calculations will be made for elementary processes (cf. Fig. 1) assuming quarks in the initial state to have constituent masses, $m_q \simeq m_p/3$, and final state quarks to be massless (see below for details on the treatment of final states).

A. Muon-like quark decay and three-quark fusion

Using Eq. (2.1) we can perform a calculation entirely similar to that of the muon decay for processes going through diagram (A) of Fig. 1. The result is ^{*)}

$$\begin{aligned} \tau^{(A)}(p \rightarrow 2\pi e^+) &= \tau^{(A)}(n \rightarrow 2\pi e^+) \simeq \frac{192\pi^3}{G^2 m_q^5} \cdot \frac{1}{80 A_s(f)}, \\ \tau^{(A)}(p \rightarrow 2\pi \bar{\nu}_e) &= \tau^{(A)}(n \rightarrow 2\pi \bar{\nu}_e) \simeq \frac{192\pi^3}{G^2 m_q^5} \cdot \frac{1}{16 A_s(f)}. \end{aligned} \quad (2.2)$$

^{*)} We take here that two quarks, and two antiquarks, condense into two mesons; only the lighter ones are considered for these processes.

Here $A_s(f)$ is an enhancement factor due to strong interactions; its value is ~ 5 for $f = 6$ flavours³⁾.

$\tau(N \rightarrow \pi K \nu, \pi K \mu^+)$ are just 1/5 of each. The factor $192\pi^3/G^2 m_q^5$ will be recognized as similar to the one in the muon lifetime. This huge number $192\pi^3$ due to three-body phase space spoils the result: within our method of calculation the contribution of these processes only decreases the nucleon lifetime by $\lesssim 1\%$.

However, it is not clear what the correct calculational procedure is in this case. We have taken initial quarks with an effective mass and final ones with a negligible mass. We could have adopted a different, more naïve procedure: consider an effective Hamiltonian, $M_{\text{eff}} \sim N G \bar{\psi}_p \phi_\pi \phi_\pi \psi_\ell^-$. Then, we could just say

$$\Gamma^{(A)}(p \rightarrow 2\pi \ell^-) \sim \frac{G^2 (\Delta m)^5 N^2}{192\pi^3}.$$

N comes from colour and spinology; we would expect $N \sim 4$ to 9 [cf. Eq. (2.2)]. Δm is the energy release, $\Delta m = m_p - 2 m_\pi \sim 0.65$ GeV. Because this is $\sim 2 m_q$, the ensuing width would give a result ~ 30 times smaller for $\tau^{(A)}$ than Eq. (2.2). Although we still believe that the previous calculation is more reliable, we see that even now this only contributes $\lesssim 30\%$ to the nucleon lifetime.

The three-quark fusion mechanism [Fig. 1(C)] is favoured by phase space on the other hand. Unfortunately, the probability of coherent interaction with the meson field of the nucleus is very small. Quantitative arguments indicate that the inclusion of such processes again only decreases nucleon lifetimes by a few percent. Therefore, most of nucleon decays will be given by the remaining mechanism Fig. 1(B) to which we now turn.

B. Two-quark fusion.

Suppose the Lagrangian responsible for the free annihilation were

$$\mathcal{L} = a_s \bar{\psi}_1^c \gamma_\mu \psi_2 \bar{\psi}_\ell^c \gamma^\mu \psi_3,$$

where a_s is a constant. Then the decay rate for this "s channel" fusion is

$$\Gamma = \frac{4a_s^2}{3\pi} |\psi(0)|^2 m_q^2 A_s(f), \text{ for } J=1, \quad (2.3a)$$

$\Gamma=0$, for $J=0$.

Here J denotes the total spin of the quarks and $\psi(0)$ is the two-quark wave function at the origin. However, if the process goes in the t channel, as described by

$$\mathcal{L} = a_t \bar{\psi}_1^c \gamma_\mu \psi_3 \bar{\psi}_2^c \gamma^\mu \psi_4,$$

then

$$\Gamma = \frac{a_t^2}{3\pi} |\psi(0)|^2 m_q^2 A_s(f), \text{ for } J=1, \quad (2.3b)$$

$$\Gamma = \frac{2a_t^2}{\pi} |\psi(0)|^2 m_q^2 A_s(f), \text{ for } J=0.$$

We apply the Lagrangian Eq. (2.1) to the proton and neutron $SU(6)$ wave functions, use the formulae (2.3) with due modifications for the presence of L and R couplings and find the partial transition rates shown in Table I. The factor A is

$$A = \frac{G^2 |\psi(0)|^2 m_q^2}{3\pi} A_s^{-1}(f). \quad (2.4)$$

Before we can get numbers we have to decide what to do about quarks in the final state. To get the absolute decay rates we just assume, as stated before, massless final state quarks and work in the usual approximation for the sum over final states:

$$\sum_{\text{quarks}} | \text{quarks} \rangle \langle \text{quarks} | \approx \sum_{\substack{\text{physical} \\ \text{hadrons}}} | \text{hadrons} \rangle \langle \text{hadrons} |. \quad (2.5)$$

It only remains to plug in values for G^2 , $|\psi(0)|^2$, $A_s(f)$. Let the gluon coupling constant be given, at energy m , by

$$g_c^2(m)/4\pi = \frac{12\pi}{25 \log(m^2/\Lambda^2)}. \quad (2.6)$$

Taking^{*)} $\Lambda \sim 0.2$ GeV, the refined version of Ross⁴⁾ of previous analyses^{2),3)} gives

$$\begin{aligned} m_X \approx m_Y &\approx (2.5 \pm 1.3) 10^{15} \text{ GeV}, \\ G &\approx 8 \times 10^{-33} \text{ GeV}^{-2} \end{aligned} \quad (2.7)$$

For $|\psi(0)|^2$ we take a mean of existing estimates⁷⁾

$$|\psi(0)|^2 \approx 8 \times 10^{-3} \text{ GeV}^3; \quad (2.8)$$

finally, for $A_s(f)$ we take the six-flavour figure³⁾ with $m_t \sim 15$ GeV, $A_s(f) \approx 5$. Putting all this together we obtain $A^{-1} \approx 5 \times 10^{35}$ yr and hence

$$\begin{aligned} \tau^{(B)}(p) &\approx 0.9 \times 10^{33} \text{ yr}, \\ \tau^{(B)}(n) &\approx 1.1 \times 10^{33} \text{ yr}. \end{aligned} \quad (2.9)$$

Adding to this the results for $\tau^{(A)}$, $\tau^{(C)}$ plus some 4% for Cabibbo suppressed decays and estimated contributions from Higgs-mediated decays ($\sim 1\%$) we get the optimum values for the total lifetimes,

$$\tau^{\text{op}}(p) \approx 0.8 \times 10^{33} \text{ yr}, \quad \tau^{\text{op}}(n) \approx 10^{33} \text{ yr}. \quad (2.10)$$

*) Here we are only concerned with an "optimum" estimate. For errors and uncertainties, cf. Section 3.

These results are relevant for comparison with experiments in which at least pions and, preferably, also positrons are detected. If the experiment is only sensitive to muons then some extra work has to be done before a meaningful comparison can be made. This poses two problems. First, one has to disentangle the different physical processes. To do this correctly would require a detailed analysis of spin and isospin of the final state quarks. However, we would expect that a reliable estimate be obtained by just weighing each individual channel with the relative available phase space. Thus, decays with K get a weight $1/1.44$, decays with ρ, ω get a weight $1/4$ and decays with π 's get $3/4$.

The second, and more important, point is that it is very unlikely that 0.5 GeV pions have time to decay before being absorbed: one can estimate that only a few percent of the pions will produce muons. This means that the effective lifetime for p decay is mostly given by the process $p \rightarrow \mu + K^0$, and therefore (cf. Table I) the Reines-Crouch bound, $\tau_\mu(p) \gtrsim 2 \times 10^{30}$, is to be compared with ten times Eq. (2.10) or

$$\tau_\mu(p) \approx 0.8 \times 10^{34} \text{ yr.} \quad (2.11)$$

3. UNCERTAINTIES AND DISCUSSION

First of all we have the number of flavours. The bare calculation of p decay is made at an energy m_X ; to get to the physical energy of the decay, ~ 1 GeV, one uses the renormalization group equations getting an enhancement factor

$$A_s(f) = \left(\frac{g_c^2(1 \text{ GeV})}{g_{Gu}^2} \right)^{12/(33-2f)}, \quad (3.1)$$

where f is the number of flavours. Actually, Eq. (3.1) assumes that all f quarks have masses $\lesssim 1$ GeV. A more realistic estimate is to take

$$A_s(f) = \left(\prod_{i=4}^f A'_s(i) \right) \hat{A}_s(f) A_4 \quad (3.2a)$$

where

$$\left. \begin{aligned} \hat{A}_s(f) &= \left(\frac{g_c^2(m_f)}{g_{GU}^2} \right)^{12/(33-2f)} , \\ A'_s(i) &= \left(\frac{g_c^2(m_{i+1})}{g_c^2(m_i)} \right)^{12/(33-2i)} , \\ A_4 &= \left(\frac{g_c^2(1\text{GeV})}{g_c^2(m_b)} \right)^{12/25} . \end{aligned} \right\} \quad (3.2b)$$

To get an estimate on Eqs (3.2) we have assumed that quark masses increase by a factor of three: $m_{i+1}/m_i = 3$. In Ref. 3) it is argued that, to maintain the agreement between theory and experiment for the s , b quark masses, at most 8 and (marginally) 10 flavours are allowed. Nevertheless, we will present the results for $f \leq 14$; sixteen flavours are ruled out for p decay alone as one would get lifetimes $\tau(p) \sim 10^{26}$ yr. Using these Eqs (3.2), it is possible to write

$$\tau(N) \big|_{f \text{ flavours}} = \frac{\tau(N) \big|_{6 \text{ flavours}}}{\rho_f} , \quad (3.3a)$$

$$\rho_8 \simeq 1.3, \quad \rho_{10} \simeq 2, \quad \rho_{12} \simeq 3.6, \quad \rho_{14} \simeq 18. \quad (3.3b)$$

Next, we have uncertainties from decay modes not directly estimated. Decays via instantons⁸⁾ are extremely small and shall be neglected. Decays like $q + q + (q') \rightarrow \bar{l} + \bar{q} + (q')$ with final condensation $\bar{q} + (q') \rightarrow 2\pi$ (non-resonating), rather than $\bar{q} + (q') \rightarrow \pi$, are probably phase-space suppressed because

they lead to three body final states. We have already discussed Cabibbo suppressed decays and decays via diagrams of Fig. 1(C). All in all, we took these contributions to decrease τ by $\sim 20\%$, which gave τ^{op} . A conservative error estimate on this effect is to let it vary by a factor 2 from 10% to 40%.

We also have errors in our phase space estimates. We took the two initial quarks (in $q + q \rightarrow \bar{\ell} + \bar{q}$) to be massive, $m_{q,\text{in}} \simeq m_p/3$; the final quark to be massless (except when it was a strange quark). This gives an energy release of $2 m_q \sim (2/3) m_p$. We could have taken the final antiquark to be also massive, which would have given an energy release of $m_q \sim (1/3) m_p$, or we could have scaled by the phase space for the physical process, with energy release (for pionic modes) $m_p - m_\pi$. This may decrease τ by a factor 1/2 or increase it by a factor 4.

Finally, we have the largest sources of errors: $|\psi(0)|$, m_X , g_c (1 GeV). For the first we took the value given in Eq. (2.8), but we could have⁷⁾

$$|\psi(0)|^2 \simeq 4 \times 10^{-3} \text{ GeV}^3 \quad \text{up to} \quad 16 \times 10^{-3} \text{ GeV}^3.$$

For the second the error in Eq. (2.7) gives a possible 5.3 increase to a 1/20 decrease in $\tau(N)$. As to g_c (1 GeV) our choice corresponded to selecting $\Lambda \sim 0.2 \text{ GeV}$ in Eq. (2.6). Increasing (decreasing) Λ between the bounds $0.07 \leq \Lambda \leq 0.45 \text{ GeV}$ increases (decreases) τ by a factor ~ 18 ($\sim 1/4$).

It is not very easy to compose all these uncertainties, as they appear multiplicatively and also some of them are strongly correlated. For example, m_X and Λ are strongly tied and their variations give opposing effects. A very conservative error estimate is obtained by presenting separate results for each value of the total number of flavours, f , and adding statistically all other uncertainties (except those for m_X , Λ whose correlation is taken into account) for the logarithm of $\tau(N)$. In this way we get τ_f^{op} (the optimum value for f flavours) and $\tau_f^{\text{min}}, \tau_f^{\text{max}}$. τ_f^{min} is some 10 times smaller than τ_f^{op} , τ_f^{max} about 10 times bigger. Probably, realistic errors should only be half as big. The results are collected in Table II.

From these results a number of conclusions may be drawn. First of all, by comparing Table II with the implications of the Reines-Crouch experiment, it is clear that the lowest estimates are already in the range of present experiments. An increase by two orders of magnitude in the experimental precision would either reveal an effect or chop off a good part of Table II.

The theoretical uncertainties are large. To draw unambiguous conclusions more precision is necessary in the theoretical determination of a few key parameters: g_c (1 GeV) , m_X , $|\psi(0)|^2$.

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TABLE I

Elementary processes and rates.

Elementary process	Rate in proton (a)	Rate in neutron (a)	Physical processes
$uu \rightarrow e^+ \bar{d}$	80	0	$p \rightarrow \pi^0, \eta e^+$ $p \rightarrow \rho^0, \omega e^+$
$uu \rightarrow \mu^+ \bar{s}$	32	0	$p \rightarrow K^0 \mu^+$
$ud \rightarrow e^+ \bar{u}$	380	380	$p \rightarrow \pi^0, \eta e^+$ $n \rightarrow \pi^- e^+$ $p \rightarrow \rho^0, \omega e^+, n \rightarrow \rho^- e^+$
$ud \rightarrow \bar{\nu}_e \bar{d}$	76	76	$p \rightarrow \pi^+ \bar{\nu}_e, n \rightarrow \pi^0, \eta \bar{\nu}_e$ $p \rightarrow \rho^+ \bar{\nu}_e, n \rightarrow \rho^0, \omega, \bar{\nu}_e$
$ud \rightarrow \bar{\nu}_\mu \bar{s}$	4	4	$p \rightarrow K^+ \bar{\nu}_\mu, n \rightarrow K^0 \bar{\nu}_\mu$
$dd \rightarrow \bar{\nu}_e \bar{u}$	0	16	$n \rightarrow \pi^0, \eta \bar{\nu}_e$ $n \rightarrow \rho^0, \omega \bar{\nu}_e$

(a) Rates in units of A, defined in Eq. (2.4) of the text.
Suppression factors due to phase space are not included.

TABLE II

Lifetimes for decays of nucleons. To get $\tau^{\max}$ ($\tau^{\min}$) multiply (divide) by 10. The muon-producing efficiency is $\sim 15\%$ for p decay, $\sim 10\%$ for n decay.

Number of flavours	6	8	10	12	14
Proton : $\tau_f^{\text{op}}(p)$	0.8×10^{33}	6×10^{32}	4×10^{32}	2.2×10^{32}	4×10^{31}
Neutron : $\tau_f^{\text{op}}(n)$	10^{33}	6.7×10^{32}	5×10^{32}	2.5×10^{32}	4.5×10^{32}

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FIGURE CAPTIONS

Figure 1 : Elementary processes contributing to N decay.

(A)

(Bi)

(Bii)

(C)

FIG.1