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THE SIX QUARK S WAVE STATES

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A B S T R A C T

We show how the properties of six-quark states change as the number of quark flavours increases. When there are three flavours Jaffe has predicted one stable dibaryon. With four flavours there might be twenty-six if the MIT bag model works in the charmed sector.

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1. - INTRODUCTION

Dibaryon states were for long believed to be of interest mostly to people working in nuclear physics. Ten years ago such states were named exotics by elementary particle physicists, and their existence was something that fell outside the main interests of most people active in subnuclear physics.

This is probably about to change for many reasons. Genuine multi-quark states are now expected for reasons other than dual model considerations ^{1),2)}. Furthermore physicists' confidence ³⁾ in their understanding of interquark forces, has been continuously growing, during the last four years.

Fewer and fewer people doubt that quarks exist, and our confidence grows everyday in the concept of colour charges, which are thought to be the source of the gluon field that propagates the strong interactions. Genuine multiquark states, where not all qqq and $q\bar{q}$ combinations are colour singlets, seem to be observed and the colour degree of freedom would appear to show itself in the number of hadronic states.

Extrapolation of the accumulated knowledge of interquark forces from qqq and $q\bar{q}$ physics seems to explain the properties of multiquark states. Indeed, a bound state of baryons has been predicted ⁴⁾ (from the MIT bag model), and its binding energy might far exceed the binding energies due to nuclear forces. In nuclear physics the greatest binding energy per nucleon is found in He^4 , where the last neutron (proton) is bound by 20.6 MeV (19.8 MeV). This should be compared with the binding energy for the $\Lambda\Lambda$ bound state due to interquark forces in the bag model. Its binding energy ^{4),5)} is predicted to be in the range 30-80 MeV. The only dibaryon state known to be bound by nuclear forces is the deuteron, which has a modest binding energy of 2.226 MeV.

Whereas nuclear forces are explained as coming from the exchange of colourless mesons, interquark forces are believed to originate from the exchange of massless gluons carrying colour.

All mass splittings between s wave qqq baryons and $q\bar{q}$ mesons are due to the colour magnetic interaction term ³⁾, which is completely analogous to the Fermi contact term in atomic physics.

For the interaction between two quarks i and j it takes the form :

$$H'_{ij} = - \sum_{a=1}^8 \mu_i^a \mu_j^a \delta^3(\underline{r}_i - \underline{r}_j) \quad (1)$$

where the eight colour magnetic moment operators for one quark are

$$\mu^a = \sqrt{\frac{2}{3}} \alpha_s f(m) \lambda^a \underline{S} \quad (2)$$

Here α_s is the strong coupling "constant", the eight λ^a 's are the Gell-Mann matrices, which act as colour charge operators, and $\underline{S}$ is the spin operator for the quark. The function $f(m)$ depends on the quark mass ; ideally it would be $f(m) = (g_c/2m_c)$ where g_c could be called the colour magnetic g factor of the quark.

It is convenient to integrate out the spatial part of Eq. (1) to get the interaction term for the colour and spin part of the diquark state. Then

$$H'_{ij} = - A_{ij} \lambda_i^a \underline{\sigma}_i \lambda_j^a \underline{\sigma}_j \quad (3)$$

where the coefficients A_{ij} now are proportional to the square of the wave function at the point $\underline{r}_i = \underline{r}_j$. They still depend on flavour through the quark masses m_i and m_j . Summation over $a=1, \dots, 8$ is implicit in Eq. (2) and in the following. For a system of N interacting quarks

$$H' = \sum_{i,j}^N - A_{ij} \lambda_i^a \underline{\sigma}_i \lambda_j^a \underline{\sigma}_j \quad (4)$$

Multiquark s wave states with baryon number up to six have been studied ⁴⁾⁻⁶⁾ in the case where quarks carry the three flavours which correspond to the u , d and s types of quarks.

There are two reasons for extending the problem to the case where there are more flavours. The first is obvious : the existence of the charmed quark c is now firmly established and there is little doubt that closer analysis of the upsilon states at around 9.5 GeV will confirm the existence of the bottom quark b . The second reason comes from theoretical

considerations ^{7),8)} : the more antisymmetry we have in flavour the stronger are the attractive forces arising from the colour magnetic interaction. When the number of flavours is greater than three we expect to find six-quark states belonging to flavour multiplets that cannot be realized with three flavours. These new multiplets might therefore have so great a mass defect due to the colour magnetic interaction that they could represent bound dibaryons.

In this paper we shall make a systematic classification of all six-quark s wave states, regardless of the number of flavours. The multiplets can be represented by a finite number of Young tableaux : four, if there are two flavours, seven, if there are three flavours and nine, if there are more. Our work will also suggest that the same dynamics that leads to a AA bound state (that is those in the MIT bag model) will lead to many new bound dibaryons when the number of flavours exceeds three.

In the next Section we shall make some purely group theoretical considerations on six-quark multiplets. In Section 3 we shall treat the colour magnetic interaction in the flavour symmetric limit, and show that we get 26 dibaryon bound states when the number of flavours is four, and 1 when the number is three. In Section 4 we shall show that the dynamics in the MIT bag ⁹⁾ plays an essential rôle in producing bound states when the flavour symmetry is broken. A more pragmatic way of deciding whether states are bound or not will be shown to be inconclusive.

2. - SIX-QUARK MULTIPLETS

As each quark transforms under the fundamental representation of $SU_F^{\text{flavour}} = SU_F^f$, of $SU_3^{\text{colour}} = SU_3^c$ and of $SU_2^{\text{spin}} = SU_2^s$ it is convenient to regard the group SU_T^{fcs} as

$$SU_T^{fcs} \supset SU_F^f \otimes SU_3^c \otimes SU_2^s$$

Here $T = F.3.2$ and F is the total number of flavours. T is then the number of degrees of freedom that a quark at rest can have. Because of the Pauli principle, an N quark s wave state must be assigned to the completely antisymmetric representation of the SU_T^{fcs} group of dimension $1/6 N(N-1)(N-2)$. In addition, the state should be a colour singlet.

For a six-quark state one knows, from a simple study of the Clebsch-Gordan series (inner products) of the symmetric group, that to the Young tableaux describing an SU_3^c singlet must correspond the SU_{2F}^{fs} tableau . The relevant SU_F^f representations can then be obtained from the decomposition of the SU_{2F}^{fs} representation above with respect to $SU_F^f \otimes SU_2^s$.

However, it is most instructive to use the colour spin group SU_6^{cs} . Unlike the more familiar spin flavour group SU_{2F}^{sf} this group does not depend on the number of flavours. And, moreover, the colour magnetic Hamiltonian ²⁾ can be expressed in terms of 24 of the generators of SU_6^{cs} . For each representation of SU_6^{cs} with a definite Young tableau we obtain the Young tableau for the corresponding flavour representation by exchanging rows and columns. This SU_6^{fs} Young tableau is independent of the number of flavours (but of course not the dimension of the SU_F^f representation that corresponds to it).

Of the eleven Young tableaux of SU_6^{cs} that can be constructed with six boxes (six quarks) only nine can be associated with a physical state because this has to be a colour singlet. The reduction of SU_6^{cs} to $SU_3^c \otimes SU_2^s$ in the Appendix shows that the 1050 dimensional and 462 dimensional representations represented by the Young tableaux and do not contain any colour singlet. One can intuitively understand this because there is not enough colour antisymmetry in the corresponding states to make a colourless state.

In Table I we show the Young tableaux of these nine representations of SU_6^{cs} together with their dimensions and the value of the quadratic Casimir operator of the colour spin group. In the same Table we also show the tableaux for the corresponding flavour representations, the spin associated with them and their dimensions for various numbers of flavours. A zero for the multiplicity of a flavour representation means that for that specific number of flavours it is impossible to make a sufficiently antisymmetric flavour combination corresponding to that Young tableau.

3. - DIBARYONS FOR FLAVOUR SYMMETRIC COLOUR MAGNETIC FORCES

It is instructive to study the effect of the colour magnetic Hamiltonian (4) in spin colour space in its flavour symmetric limit. The effects of the colour magnetic moments are the same irrespective of whether we have u, d, s, c, t or b quarks and the diagonalization of the colour magnetic interaction (4) is very easy. In an N quark state with quantized colour spin cs one gets ⁷⁾

$$H' |N; cs, s\rangle = \bar{A} \left(8N + \frac{4}{3} s(s+1) - \frac{1}{2} C_6 \right) |N; cs, s\rangle \quad (5)$$

where C_6 is the quadratic Casimir operator for colour spin. An obvious question to ask is, of course, whether this symmetry limit is of any physical relevance. We know that symmetry breaking is important - for example it explains the $\Lambda\Sigma$ mass difference of 75 MeV. It is, however, less important than was believed at first, when the $\psi \eta_c$ mass difference was predicted to be 30 MeV instead of the observed 250 MeV. The reason is that we go from the Hamiltonian (1) to (3) by integrating over the spatial part of the wave function. When the quark mass increases, the colour magnetic moment (2) is expected to decrease like m^{-1} . On the other hand the increased mass leads to a smaller system in which the spatial wave function is more concentrated. As the coefficients A_{ij} in (4) are dependent both on the colour magnetic moments and on the square of the wave function we have two compensating effects and the flavour symmetric approximation of Eq. (4) is not as poor as might be expected. Using the value of the Casimir operator given in Table I, it is trivial to find the eigenvalues of the colour magnetic interaction [Eq. (5)] for six-quark states when $N=6$.

It is interesting to see how the qualitative properties of the interaction change when the number of flavours increases. The lowest value H' can have is

- $48\bar{A}$ with one flavour ;
- $\frac{8}{3}\bar{A}$ with two flavours ;
- $-24\bar{A}$ with three flavours ;
- $-\frac{88}{3}\bar{A}$ with four or more flavours.

Thus, for one or two flavours the colour magnetic interaction leads to a repulsion ; to minimize the energy the six quarks will always separate into groups such that, when they are in the same space point, the wave function will be small. This property will lead to a decrease of $\bar{A}$, as mentioned before, and will reflect itself in a repulsive-core effect between the two baryons. It is to be noted that the state with the smallest energy in the two-flavour case has the same quantum numbers as the deuteron (which is the only dibaryon bound by nuclear forces).

With three flavours the properties of the s wave interquark forces change in two of the possible new flavour multiplets. These are an eight-dimensional and a one-dimensional flavour multiplet where the colour magnetic force is attractive. As we increase the number of flavours to four the two flavour multiplets mentioned above will have dimensions 64 and 10, respectively. And, moreover, there are two additional new possible flavour representations which are of dimension 10 and 6, and have corresponding colour spin representations of dimension 840 and 1134. In these new states there are at least one c , one s , one u and one d quark. The addition of more flavours increases the multiplicity of the flavour multiplets already known but does not lead to any new colour spin representation. The maximal mass defect due to the colour forces is reached with four flavours.

It is natural to believe that the short-range force is attractive between two baryons that have the same quantum numbers as a six-quark state with negative values of the colour magnetic interaction. But this is not sufficient to obtain dibaryon bound states. The six-quark colour singlet state will generally decay into two baryons, each consisting of three quarks with total colour zero. In the three-quark colour singlet state with $J^P = \frac{1}{2}^+$ (the ordinary baryons) the value of H' of Eq. (4) will be $-8\bar{A}'$. The coefficient $\bar{A}'$ will be equal to $\bar{A}$ if the wave function overlap is assumed to be the same in both the three-quark and six-quark system.

The criterion for stability of the six-quark system is, of course, that the mass of the six-quark state be smaller than the masses of all the three-quark states it can decay into. To decide the stability we therefore have to use a mass formula for the multiquark system.

The most simple is to write (for quarks in s wave)

$$M = \sum_i \bar{m}_i + \langle H' \rangle \quad (6)$$

where $\bar{m}_i$ are the effective masses of the quarks in the system and $\langle H' \rangle$ is defined by Eq. (4). The values of these effective quark masses are supposed not to change when considered in a three- or a six-quark states, and the stability criterium will be :

$$\langle H' \rangle_{6q} < 2 \langle H' \rangle_{3q} = -16\bar{A} \quad (7)$$

Using Eq. (5), we see that the binding energy is

$$E_B = \bar{A} \left\{ \frac{1}{2} C_6 - 64 - \frac{4}{3} s(s+1) \right\}$$

Thus the six-quark states falling into the 490, 840 and 1134 dimensional representation of SU_6^{CS} will be bound with a binding energy of $8\bar{A}$, $8\bar{A}$ and $40/3\bar{A}$, respectively.

The MIT bag model gives another mass formula for an N quark system, which is well approximated as ⁹⁾⁻¹¹⁾

$$M = m_p \left[\frac{2.043 N + \delta_s + \delta_c - 1.84 + 0.0975 \bar{A}^{-1/4}}{3.5} \right]^{3/4} \quad (8)$$

Here m_p is the proton mass and $\delta_s = 0.872 \times$ number of strange quarks in the bag, $\delta_c \approx 7.691 \times$ number of charmed quarks in the bag.

We are well aware of the fact that the bag model is not really intended for quarks as massive as charmed quarks and that it is risky to use it in the charmed sector. As long as we consider only systems with the minimal number (0 or 1) of charmed quarks, the bag model leads to the stability of the same colour-spin multiplets as the mass formula (6). The binding energies are of course different and must be calculated individually for each case.

Thus in the flavour symmetric limit of the colour magnetic interaction, while there is but one bound state in the three-flavour situation there are 26 with four flavours (and 644 with six flavours).

We now leave the situation of beautiful symmetry and turn to the difficulties of physical reality.

3. - FLAVOUR SYMMETRY BREAKING OF THE COLOUR MAGNETIC INTERACTION

A detailed explicit study of the SU_3^f symmetry breaking effects on the colour magnetic part of the Hamiltonian has been provided by Aerts, Mulders and de Swart ⁵⁾. It is straightforward to generalize their formalism to the case where quarks have more than three flavours. As we have already seen that four flavours give us all the multiplets of SU_6^{cs} that can be physical dibaryons (i.e., colour singlets) we shall here limit ourselves to the flavour group SU_4^f .

To study the flavour symmetry breaking shown in Eq. (4) we rewrite the summation over the quark-quark interaction in the following way

$$\begin{aligned}
 \langle H' \rangle = & \sum_{i>j} -A_{ij} \lambda_i^\alpha \sigma_i^\alpha \lambda_j^\alpha \sigma_j^\alpha = A_{nc} \left\{ \sum_{nn} + \sum_{nc} + \sum_{cc} \right\} \\
 & + A_{sc} \left\{ \sum_{ss} + \sum_{sc} + \sum_{cc} \right\} + A_{ns} \left\{ \sum_{nn} + \sum_{ns} + \sum_{ss} \right\} \\
 & + (A_{nn} - A_{ns} - A_{nc}) \sum_{nn} + (A_{ss} - A_{ns} - A_{sc}) \sum_{ss} \\
 & + (A_{cc} - A_{cn} - A_{sc}) \sum_{cc}
 \end{aligned} \tag{9}$$

Here we have separated the summation over the quark-quark interaction into parts, i.e., when we sum over pairs of non-strange, non-charmed quarks in $\sum_{nn}$, over pairs of one charmed (strange) quark and one non-strange, non-charmed quark in $\sum_{nc} (\sum_{ns})$ and so on. $\sum_{nn}$ is a short-hand notation for

$$\sum_{i>j} -\lambda_i^\alpha \sigma_i^\alpha \lambda_j^\alpha \sigma_j^\alpha$$

where i and j refer to all non-strange non-charmed quarks in the system and the other Σ 's are similarly defined.

Using the permutation symmetry of the states we can re-express the colour-spin tensor of Eq. (9) in terms of second-order Casimir operators of the relevant subgroups SU_6 , SU_4 , SU_3 and SU_2 of the SU_8^{fs} (flavour-spin) group. We now study the expression for $\langle H' \rangle$ in Eq. (9) term by term.

In the first parenthesis on the right-hand side of Eq. (9), we have a summation over all quark pairs which do not include any strange quarks. In the second parenthesis, there is a summation over all except the charmed quarks, and so on. Now, as this summation is over something flavour symmetric it follows that

$$X^{cn} = \sum_{nn} + \sum_{nc} + \sum_{cc} = \sum_{i>j} - \lambda_i^\alpha \sigma_i^\alpha \lambda_j^\alpha \sigma_j^\alpha = 8N_{nc} + \frac{4}{3}J^2 - \frac{1}{2}C_6^{cs} + \frac{1}{2}C_3^c \quad (10)$$

where N_{nc} is the number of non-strange and charmed quarks in the 6q state, $C_3^c(C_6^{cs})$ is the Casimir operator for the colour (colour-spin) of this same subset of quarks and J^2 is the Casimir operator of their spin [with eigenvalues $j(j+1)$].

Due to the connection between colour-spin and flavour imposed by the Pauli principle, we can also express the sum X^{ns} in terms of the Casimir operators of the flavour spin C_6^{fs} and flavour C_3^f of the same group of (non-strange) quarks. We find that

$$X^{cn} = \frac{1}{2} N_{nc}^2 - N_{nc} - \frac{1}{4} C_6^{fs} + C_3^f + \frac{4}{3} J^2 \quad (11)$$

To evaluate the contribution of this expression to the colour magnetic energy we must know the weight with which different representations of flavour-spin, flavour and spin occur in a definite state of the complete six-quark system.

The evaluation of the sum over all non-charmed quarks X^{ns} given by the third parenthesis of Eq. (9) is completely analogous,

$$\begin{aligned} X^{sn} &= 8N_{ns} + \frac{1}{2}C_3^c + \frac{4}{3}J^2 - \frac{1}{2}C_6^{cs} \\ &= \frac{1}{2}N_{ns}^2 - N_{ns} - \frac{1}{4}C_6^{fs} + C_3^f + \frac{4}{3}J^2 \end{aligned} \quad (12)$$

The Casimir operators now act on the states formed by the subgroup of quarks that do not carry charm, and $N_{n,s}$ is the number of u, d or s quarks.

We turn now to the summation over pairs where there is no u or d quark. The answer in terms of colour-spin operators is always the same :

$$X^{sc} = \sum_{ss} + \sum_{sc} + \sum_{cc} = 8N_{s,c} + \frac{1}{2}C_3^c + \frac{4}{3}J^2 - \frac{1}{2}C_6^{cs} \quad (13)$$

The relevant flavour symmetry group for s and c quarks is SU_2^f , which we shall call the K spin group (by analogy with isospin for SU_2^f with u and d quarks). The corresponding flavour spin group is SU_4^{fs} with Casimir operator C_4^{fs} . In terms of the K spin and C_4^{fs} we can then re-express X^{sc} as follows

$$X^{sc} = \frac{3}{4}N_{s,c}^2 - N_{s,c} - \frac{1}{4}C_4^{fs} + \frac{4}{3}J^2 + 4K^2 \quad (14)$$

In exactly the same manner (simply by replacing K spin with isospin) we get for the summation over non-strange quarks

$$\begin{aligned} X^{nn} = \sum_{nn} &= 8N_n + \frac{1}{2}C_3^c + \frac{4}{3}J^2 - \frac{1}{2}C_6^{cs} \\ &= \frac{3}{4}N_n^2 - N_n + \frac{1}{4}C_4^{fs} + \frac{4}{3}J^2 + 4I^2 \end{aligned} \quad (15)$$

The Casimir operators C_3^c , C_6^{cs} and C_4^{fs} are now evidently determined by the state in which the non-strange quarks lie. The summation over all cc pairs and ss pairs is extremely simple, as there is no flavour group and so

$$\begin{aligned} X^{ss} = \sum_{ss} &= 8N_s + \frac{1}{2}C_3^c + \frac{4}{3}J^2 - \frac{1}{2}C_6^{cs} \\ &= \frac{3}{2}N_s^2 - N_s - \frac{2}{3}J^2 \end{aligned} \quad (16)$$

$$X^{cc} = \sum_{cc} = 8N_c + \frac{1}{2}C_3^c + \frac{4}{3}J^2 - \frac{1}{2}C_6^{cs} = \frac{3}{2}N_c^2 - N_c - \frac{2}{3}J^2 \quad (17)$$

From these formulae (10)-(17) one can now calculate the colour magnetic interaction. It is necessary to know the fractional parentage coefficients of the six-quark states decomposed into linear combinations of states where the operators X are diagonal. The calculation of these coefficients is an extremely lengthy undertaking, and as far as we know it has not been completely done even in earlier studies of six-quark systems (when the quarks have only three flavour degrees of freedom).

Luckily we can proceed a long way without knowing the value of the fractional parentage coefficients, because we can compute the upper and lower limits of $\langle H' \rangle$ with a given set of values for the coefficients A_{ij} . Generally the binding energy of $\langle H' \rangle$ will decrease when we use realistic values of $\{A_{ij}\}$ but so often will the contribution of $\langle H' \rangle$ to the mass defect of the decay products.

In Table II we given the values of H' for the twenty $\frac{1}{2}^+$ baryons that are the lightest possible decay products of our six-quark states. As can be seen, some of these states have a colour magnetic interaction dependent only on A_{nn} . The coefficients A_{ij} are expected ³⁾ to decrease with increasing quark masses (and phenomenologically this is the case ¹²⁾) and so the baryon states where $\langle H' \rangle$ depends only on A_{nn} will be the lightest baryons with fixed quark content. From the masses of the non-charmed baryons we can find values of A_{ij} using the mass formulae (6) or (8). We then use these values to compute the value of $\langle H' \rangle$ for each six-quark state and see if the resulting state is stable against (strong) decays into two baryons.

We go first to the mass formula (6) which is linear in the number N of quarks. It usually gives heavier multiquark states than the mass formula of the MIT bag model, Eq. (8), which depends on $N^{\frac{3}{4}}$. It is thus more difficult to present arguments supporting six-quark bound states using formula (6) than using formula (8).

As the masses of charmed baryons are not yet known we have to go to charmed mesons to determine A_{nc} , A_{sc} and A_{cc} . The D^*D , F^*F and $\psi \eta_c$ mass differences lead to $A_{nc} \simeq 6.7$ MeV, $A_{sc} \simeq 5.6$ MeV, and $A_{cc} \simeq 11.7$ MeV if we use mass formula (6). The NA mass difference gives $A_{nn} \simeq 17.0$, if we take the A mass from the real part of its pole position (1211 MeV), and $A_{nn} \simeq 18.3$ if $M_A = 1232$ MeV. The Σ - A mass difference gives $A_{ns} = A_{nn} - 7.3$ MeV. From the mass difference Ω^-N we estimate the value of $A_{ss} \simeq 7$ MeV.

With the use of these parameters (we choose $A_{nn} = 18.3$ MeV) we can now try to decide if the six-quark states can be stable against strong interaction decay. In most cases we cannot say anything either way, as the diagonalization of the colour magnetic interaction proves to be too complicated. The limits that can be set on the mass defect without knowing the fractional parentage coefficients for the six-quark state are so wide that no conclusions about stability can be reached. In some cases, however, we are more lucky. One place is the $\Lambda\Lambda$ -like state of Jaffe and its counterpart $C_0 C_0$, if the strange quarks are replaced by charmed ($s \rightarrow c$). We then find the following extrema for the binding energy :

$$\begin{aligned} -E_1 &= \frac{32}{3} (A_{nn} - 2A_{ns}) + \frac{8}{3} A_{ss} \\ -E_2 &= 12(A_{nn} - 2A_{ns}) + 4 A_{ss} \end{aligned}$$

With the above coefficients, $\Lambda\Lambda$ is bound with a binding energy between 21 and 26 MeV. The $ududcc$ state is not bound ; it lies between 30 and 105 MeV above $C_0 C_0$ threshold.

For the multiplets that can only be realized with four flavours the situation is much more complicated. Here it is expected that mixing between the colour-spin representations 1134 and 896 with spin 1 may be important as their mass defects due to H' are not very different in the flavour symmetric limit. But just to compute the mean value of H' in one of the colour-spin representations is hard. We have to settle for upper and lower limits which are often around 100 MeV. The case where we have the best limits is for $\langle 1134 | H' | 1134 \rangle$ when the quark content is $ududsc$. Then $E_2 \leq \langle H' \rangle \leq E_1$ with

$$\begin{aligned} E_1 &= -12(A_{nc} + A_{ns}) - 4A_{nn} - \frac{4}{3} A_{sc} \\ E_2 &= -8(A_{nc} + A_{ns}) - \frac{16}{3} A_{nn} - 8A_{sc} \end{aligned}$$

Clearly, with our values of the parameters A_{ij} there is no proof that this $ududsc$ state cannot decay into $C_0 \Lambda$. The numerical values of E_1 and E_2 are $E_1 = -293$ MeV, $E_2 = 284$ MeV. These values should be compared with the combined mass defect of C_0 and Λ which is $-16A_{nn} \approx -292$ MeV.

We therefore see that the C_0^A state can be from 1 MeV below threshold to 6 MeV above threshold. It is very probable that a complete treatment, solving the eigenvalue problem for H' with the above values of A_{ij} , would give an unambiguous bound state. It would be better still if charmed baryons (when they are discovered) were to show that we have underestimated A_{nc} and A_{sc} . To regard this exercise as a proof of the stability of dibaryon states would, of course, be silly. The mass formula (6) is only approximate and relatively small variations of the coefficients A_{ij} can transform the bound dibaryon states into low-lying resonance states. Only a complete theory of multiquark states or experimental observation can prove their stability.

The MIT bag model comes up with a clean prediction : there should be a $\Lambda\Lambda$ bound state at 2200 MeV or below. In the same model (applied in the charmed sector of multiquark states) the binding energies of the six-quark states belonging to the 1134 colour-spin representation are of the order of 200 MeV.

4. - CONCLUSIONS

In this article we have shown how the effect of dibaryon forces in the s wave system changes when we go from states with few flavours to states built up from many flavours. The "flavour antisymmetry principle" states that the more flavour antisymmetric the N quark wave function the stronger the colour magnetic attraction. We have shown that with just four flavours we can obtain the maximally possible mass defect due to colour magnetism, and that in this case there may be 26 bound dibaryons.

It has not escaped our attention that for tribaryons (which are nine quarks) in a relative s state the colour magnetic attraction does not become maximal when the number of flavours is four. It is therefore possible that, for example, SU_6^{cs} multiplets of the type would lead to bound tribaryons.

Their observation is not likely in the near future.

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Spin	SU_F^F representations	Dimensions					SU_6^{cs} representation	Dimension	Values of $C_6(cs)$
		$F = 1$	$F = 2$	$F = 3$	$F = 4$	$F = 6$			
0		1	7	28	84	462		1	0
1		0	5	35	140	1050		35	48
1 and 3		0	1	$\overline{10}$	50	490		175	96
0 and 2		0	3	27	126	1134		189	80
1		0	0	10	70	840		280	96
1 and 2		0	0	8	64	896		896	120
0		0	0	1	$\overline{10}$	175		490	144
0		0	0	0	10	280		840	144
1		0	0	0	6	189		1134	160

- TABLE I -

Flavour and colour spin representations of six q states

Label	Quark content	$\langle H' \rangle$
N(939)	nnn	$-8A_{nn}$
$\Lambda(1115)$	nns	$-8A_{nn}$
$\Sigma(1193)$	nns	$\frac{8}{3}(A_{nn} - 4A_{ns})$
$\Xi(1318)$	nss	$\frac{8}{3}(A_{ss} - 4A_{ns})$
$C_1^{++,+,0}$	$c(nn)_{\text{sym}}$	$\frac{8}{3}(A_{nn} - 4A_{nc})$
C_0^+	$c(nn)_{\text{anti}}$	$-8A_{nn}$
$S^{+,0}$	$c(sn)_{\text{sym}}$	$\frac{8}{3}[A_{ns} - 2(A_{sc} + A_{nc})]$
$A^{+,0}$	$c(sn)_{\text{anti}}$	$-8A_{ns}$
T^0	css	$\frac{8}{3}(A_{ss} - 4A_{sc})$
X_u^{++}, X_d^+	ccn	$\frac{8}{3}(A_{cc} - 4A_{nc})$
X_s^+	ccs	$\frac{8}{3}(A_{cc} - 4A_{sc})$

- TABLE II -

Colour spin contributions for $\frac{1}{2}^+$ baryons

APPENDIX

The calculations done in this paper are greatly simplified with the help of the following tables and formulae.

Reduction of SU(6) representations with respect to SU(3) × SU(2) :

$$\begin{matrix} 175 \\ (2^3) \end{matrix} = (27, 3) + (8, 5) + (1, 7) + (10 + \overline{10}, 1) + (8, 3) + (1, 3)$$

$$\begin{matrix} 280 \\ (3, 1^3) \end{matrix} = (27, 3) + (10, 5 + 3) + (8, 5) + (10 + \overline{10}, 1) + 2(8, 3) + (8, 1) + (1, 3)$$

$$\begin{matrix} 896 \\ (3, 2, 1) \end{matrix} = (35, 3 + 1) + (27, 5) + 2(27, 3) + (10 + \overline{10}, 5) + (8, 7) + (27, 1) + \\ 2(10, 3) + (\overline{10}, 3) + 2(8, 5) + (10, 1) + 3(8, 3) + 2(8, 1) + (1, 5 + 3)$$

$$\begin{matrix} 35 \\ (2, 1^4) \end{matrix} = (8, 1 + 3) + (1, 3)$$

$$\begin{matrix} 189 \\ (2^4, 1^2) \end{matrix} = (1, 1 + 5) + (8, 1 + 5) + 2(8, 3) + (10 + \overline{10}, 3) + (27, 1)$$

$$\begin{matrix} 840 \\ (4, 1^2) \end{matrix} = (35, 5 + 3) + (10, 7) + (27, 5 + 3) + (10, 5) + (27, 1) + 2(10, 3) \\ + (\overline{10}, 3) + (8, 5) + (10, 1) + 2(8, 3) + (8, 1) + (1, 1)$$

$$\begin{matrix} 490 \\ (3^2) \end{matrix} = (\overline{10}, 7) + (27, 8, 5) + (35, 10 + \overline{10} + 8, 3) + (28 + 27 + 1, 1)$$

$$\begin{matrix} 1134 \\ (4, 2) \end{matrix} = (28, 3) + (27, 7) + (35, 5) + (35, 3) + (27, 5) + (10, 5) + (35, 1) + (\overline{10}, 5) + (8, 5) \\ + 2(27, 3) + (10, 3) + 2(8, 3) + (10 + \overline{10} + 8, 1) + (1, 3)$$

$$\begin{matrix} 1050 \\ (5, 1) \end{matrix} = (28, 5) + (35, 5) + (35, 3) + (27, 5) + (10, 5) + (27 + 10, 3) + (27, 1) + (\overline{10} + 8, 3) \\ + (8, 1) + (35, 7)$$

$$\begin{matrix} 462 \\ (6) \end{matrix} = (28, 7) + (35, 5) + (27, 3) + (\overline{10}, 1)$$

$$\begin{matrix} 420 \\ (3, 2) \end{matrix} = (21 + 24 + \overline{15} + 6 + \overline{3}, 2) + (24 + \overline{15} + 6 + \overline{3}, 4) + (\overline{15}, 6)$$

$$\begin{matrix} 210 \\ (3, 1) \end{matrix} = (15' + 15 + \overline{6} + 3, 3) + (15, 5) + (15' + 3, 1)$$

$$\begin{matrix} 105 \\ (2^2) \end{matrix} = (15' + \overline{6}, 1) + (15 + 3, 3) + (\overline{6}, 5)$$

For $SU(3)$ representations, we use the following correspondence with the Young tableaux notation :

$$\begin{array}{lll}
 3 = (1) & 21 = (5) & 27 = (4,2) \\
 6 = (2) & 24 = (4,1) & 28 = (6) \\
 15 = (3,1) & 10 = (3) & 35 = (5,1) \\
 15' = (4) & 8 = (2,1) &
 \end{array}$$

Decomposition of the representation (3^2) of $SU(8)$ with respect to $SU(6) \times SU(2)$:

$$\begin{array}{l}
 2520 = (6,2) + (420,2) + (21,3) + (210,3) + (56,4) + (70,2) + (1,1) + (490,1) + (15,1) + (105,1) \\
 (3^2)
 \end{array}$$

Decomposition of the representation (3^2) of $SU(8)$ with respect to $SU(4) \times SU(4)$:

$$\begin{array}{l}
 2520 = (4,60) + (60,4) + (10,45) + (45,10) + (6,20'') + (20'',6) + (20,20) + (20',20') \\
 (3^2) \qquad \qquad \qquad + (1,50) + (50,1)
 \end{array}$$

Note that in terms of Young tableaux, the representations of $SU(4)$ appearing in these decompositions can be written :

$$\begin{array}{ll}
 4 = (1) & 6 = (1^2) \\
 60 = (3,2) & 20 = (3) \\
 10 = (2) & 20' = (2,1) \\
 45 = (3,1) & 20'' = (2^2)
 \end{array}$$

Decompositions of the representations $(3,2)$, $(3,1)$ and (2^2) of $SU(6)$ with respect to $SU(4) \times SU(2)$:

$$\begin{aligned} 420 = & (6,1) + (45,2) + (20,3) + (20'',2) + (20',3) + (10,4) + (20',1) + (10,2) + (6,2) \\ (3,2) & \qquad \qquad \qquad + (4,3) + (4,1) + (1,2) \end{aligned}$$

$$\begin{aligned} 210 = & (45,1) + (20,2) + (20',2) + (10,3) + (6,3) + (10,1) + (4,4) + (4,2) + (1,3) \\ (3,1) & \end{aligned}$$

$$\begin{aligned} 105 = & (20',2) + (20',4) + (10,3) + (6,1) + (20'',1) + (1,1) \\ (2^2) & \end{aligned}$$

We now recall the formulae ⁷⁾ allowing us to calculate explicitly the Casimir operator eigenvalue for a given representation of SU(6) :

$$C_6[R] = \frac{280}{9 N_R} \sum_k d_k (2j_k+1) j_k (j_k+1)$$

where the sum over k covers all SU(3)×SU(2) representations (d_k, j_k) contained in the representation R of dimension N_R and where d_k represents the dimension of the SU(3) representation and j_k the "spin" of the k th pair. For SU(3) :

$$C_3[R] = \frac{32}{3 N_R} \sum_k (2I_k+1) I_k (I_k+1)$$

and this sum covers all the "isospin" multiplets in the R representation of SU(3) ^{*)}.

An analogous formula can be derived for SU(4) using the spin-isospin decomposition SU(4) = SU(2)×SU(2) :

$$C_4[R] = \frac{20}{N_R} \sum_k d_k (2j_k+1) j_k (j_k+1)$$

d_k being the dimension of the isospin SU(2) representation and j_k the spin in the sum over k .

*) Note that this definition of the Casimir operator eigenvalue differs from that of Ref. 5) by a factor 1/4.

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