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HADRONIC STRUCTURE FUNCTIONS AND FORM FACTORS
IN LOW ORDER QCD WITH MIGDAL REGULARIZATION

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A B S T R A C T

A regularization scheme for QCD proposed by Migdal for two-point functions is extended to three-point functions. We consider first some general implications of the analytic structure of the three-point function. We then discuss extensively the hadronization of meson and baryon structure functions and form factors, in particular their behaviour for $x \rightarrow 1$ and $|q^2| \rightarrow \infty$, resp.

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1. INTRODUCTION

There is strong evidence that hadrons consist of quarks which behave almost as free particles at small distances. Quantum-chromodynamics (QCD) is a serious candidate for the basic theory of strong interactions, because it is a local gauge field theory with just this property. Since quarks and gluons are not seen as asymptotic states, QCD should also explain this confinement and the excitations of the quark bound states, i.e. the hadronic spectrum. If there are some traces of this property left in perturbation theory around zero fields^{*)}, one could hope to follow these traces and to develop appropriate techniques going beyond ordinary perturbation theory to low order.

Without derivation of confinement, it may be an interesting step to enforce it by some prescriptions, and perform concrete calculations and check internal consistency and consistency with experiment. Effective potential or bag models do so and may be very appropriate for a phenomenological description of hadron physics. But in these models the relation to QCD becomes loose.

It is an -- at least conceptual -- advantage if one can return to QCD in some limit. An approach proposed by A.A. Migdal¹⁾ is of that type. He introduces a quantity R with the dimension of a length, which can be interpreted as an inverse "bag pressure". In contrast to the MIT bag-model²⁾ this quantity is not fixed but renormalized by the gluon interaction. It is suggested to be pushed to infinity in higher order perturbation theory.

Migdal investigates the set of hadronic two-point functions:

$$\Gamma_{ij}(p^2) = \int d^4x e^{ipx} \langle \Omega | T (O_i(x) O_j(0)) | \Omega \rangle \quad (1.1)$$

The $O_i(x)$ are gauge-invariant local operators, which appear, for example, in the operator product expansion of QCD³⁾ [see Eqs. (2.5) and (2.6)]. Because of asymptotic freedom, low-order calculations for Γ_{ij} are supposed to be reliable in the deep Euclidean region. Away from this region they show cuts due to the exchange of quarks and gluons, which should not appear in the final result. Imposing a $1/N_c$ expansion⁴⁾, one expects poles on the real axis to leading order in this expansion, corresponding to stable hadrons. Therefore a new set of meromorphic two-point functions $\hat{\Gamma}_{ij}(p^2)$ is constructed, which gives an optimized approximation to the original expressions $\Gamma_{ij}(p^2)$ in the deep Euclidean region. (This procedure of removing quark and gluon cuts and replacing them by poles is called meromorphization or hadronization in the following.)

*) If instantons are essential for confinement, this may not be the case.

In the case of massless quarks and up to order g^4 it is possible to find an explicit solution for the hadronized expression $\hat{\Gamma}_{ij}(p^2)$. In this approximation conformal invariance guarantees the expression to be diagonal in the rank of the operators O_i , hence particle poles occur for operators with definite rank. If the perturbative expression $\Gamma_{ij}(p^2)$ is replaced by a quotient of polynomials in p^2 of degree M resp. N , and optimal (Padé) approximation at a deep Euclidean point $p^2 = -\mu^2$ is established, one may study the limit $M, N \rightarrow \infty$, $R^2 = MN/\mu^2$ fixed. This yields the following expression for the partial wave contribution to the two-point function with total angular momentum s :

$$\hat{f}_{i,i}^s(p^2) = f_{i,i}^s(p^2) - \frac{1}{Q_i(p^2)} \frac{1}{2\pi i} \oint \frac{dt' Q_i(t') f_{ii}^s(t')}{t' - p^2} \quad (1.2)$$

where f_{ii}^s is the result of perturbation theory to second order; the integral contour is taken around its cuts. The denominator function $Q_i(p^2)$ is given by

$$Q_i(t) = (R\sqrt{t})^{-\nu_i} J_{\nu_i}(2R\sqrt{t}) \quad (1.2a)$$

with $\nu_i = n_i + \tau_i + g^2 \bar{\gamma}_i(n_i)$, where n_i , τ_i , $g^2 \bar{\gamma}_i$ are rank, twist, and anomalous dimension (to order g^2) of O_i ; J_ν is the Bessel function.

The cuts of f_{ii}^s are cancelled by the second term on the r.h.s. of Eq. (1.2), the zeros of the Bessel function in the denominator of the r.h.s. give rise to the (hadron) poles. $\hat{f}_{ii}(s)$ has no essential singularity, since the essential singularity of $Q_i(p^2)$ in the denominator is cancelled by the one of $Q_i(t')$ under the integral. The form of Eq. (1.2) implies the existence of unsubtracted dispersion relations for $\hat{f}_{ii}(s)$. For $p^2 < 0$ the quantity $1/Q_i(p^2)$ decreases exponentially, hence $\hat{f}_{ii}$ rapidly approaches f_{ii} in the Euclidean region.

R is the only parameter with a dimension. Taking $R \rightarrow \infty$ in Eq. (1.2) the series of poles condenses to the original cut. However, in Migdal's approach, R is kept fixed in finite order perturbation theory. Its numerical value as well as that of g^2 is adjusted to the hadron spectrum and a value $R = 2.5 \text{ GeV}^{-1}$ is obtained in Ref. 1. To higher orders in g^2 the matrix $\Gamma_{ij}(p^2)$ is no longer diagonal, and the denominator function $Q_i(p^2)$ becomes a matrix function $Q_{ij}(p^2)$. Including higher order terms in g^2 in the expression $f_{ij}^s(p^2)$, a fit to R is expected to yield larger and larger values. This justifies the above remarks about R as a bag pressure.

Our paper is organized as follows: In Section 2 we extend the method of Migdal to three-point functions. In Section 3 we discuss which cuts have to be hadronized. In Section 4 we present zero-order calculations for mesonic form factor and structure function. In Section 5 we discuss higher-order corrections,

especially those which are supposed to dominate in the region of high q^2 and near $x = 1$. In Section 6 our discussion is extended to baryons. Section 7 contains a discussion of our results.

2. HADRONIZATION OF THE THREE-POINT FUNCTION

In the following we present an extension of the hadronization procedure of Ref. 1 to three-point functions:

$$\tilde{F}_{ijk}(x_1, x_2, x_3) = \langle \Omega | T (O_i(x_1) O_j(x_2) O_k(x_3)) | \Omega \rangle^{(2.1)}$$

This has already been outlined in a preceding letter⁵⁾. In contradistinction to the two-point function, the analytic structures of the three-point function in momentum space

$$F_{ijk}(p_1^2, p_2^2, p_3^2) \delta(p_1 + p_2 + p_3) = \int dx_1 dx_2 dx_3 e^{i(p_1 x_1 + p_2 x_2 + p_3 x_3)} \tilde{F}_{ijk}(x_1, x_2, x_3) \quad (2.2)$$

is rather involved⁶⁾. It will turn out that there are singularities whose hadronization would completely destroy the intuitive picture we have of this procedure. We therefore prefer to leave these cuts unmodified in the hadronized three-point function. We are aware that this is an unpleasant feature which, however, is not specific for this approach. It seems to be a rather typical puzzle for composite particle models in general. In calculations with bound states, anomalous thresholds show up (in potential calculations they constitute the whole effect), and a meromorphization of these singularities would be as unjustified as the conclusion from the nearby anomalous singularity in the form factor of a loosely bound state to a superlight exchanged particle.

We will therefore replace by hadron poles only those singularities which in the perturbative expression for the amplitude start above zero (resp. $4m^2$ if we introduce a quark mass m). We shall already see in zeroth order (triangle diagram) that this prescription leaves us with cuts in the time-like region of an hadronized amplitude.

Once we have decided which cuts have to be hadronized, the procedure is very much analogous to that for the two-point function: we replace the perturbative amplitude by a hadronized one following the prescription:

$$\hat{F}_{ijk}(p_1^2, p_2^2, p_3^2) = F_{ijk} - \frac{1}{2\pi i} \left\{ \frac{1}{Q_i(p_1^2)} \int_{-\infty}^{\infty} dt_1 \frac{Q_i(t_1) F_{ijk}(t_1, p_2^2, p_3^2)}{(t_1 - p_1^2)} \right\}$$

$$\begin{aligned}
 & + \frac{1}{Q_j(p_i^2)} \int dt_2 \dots + \frac{1}{Q_k(p_i^2)} \int dt_3 \dots \} \\
 & + \left(\frac{1}{2\pi i}\right)^2 \left\{ \frac{1}{Q_i(p_1^2) Q_j(p_2^2)} \int dt_1 \int dt_2 \frac{Q_i(t_1) Q_j(t_2) F_{ijk}(t_1, t_2, p_3^2)}{(t_1 - p_1^2)(t_2 - p_2^2)} \right. \\
 & \quad \left. + \frac{1}{Q_i(p_1^2) Q_k(p_3^2)} \int dt_1 \int dt_3 \dots + \frac{1}{Q_j(p_2^2) Q_k(p_3^2)} \int dt_2 \int dt_3 \dots \right\} \\
 & - \left(\frac{1}{2\pi i}\right)^3 \frac{1}{Q_i(p_1^2) Q_j(p_2^2) Q_k(p_3^2)} \int dt_1 \int dt_2 \int dt_3 \frac{Q_i(t_1) Q_j(t_2) Q_k(t_3) F_{ijk}(t_1, t_2, t_3)}{(t_1 - p_1^2)(t_2 - p_2^2)(t_3 - p_3^2)}
 \end{aligned}
 \tag{2.3}$$

The contours in the integrals are to be taken around the quark-gluon cuts as indicated above. The denominator functions (matrix functions in higher order) are no longer determined by an optimization but are those taken from the two-point function [Eq. (1.1)]. This is necessary in order to obtain consistent spectral properties. The hadronized three-point function $\hat{F}_{ijk}$ approaches the perturbative one in the deep Euclidean region for all external momenta.

The electromagnetic form-factor as well as the deep inelastic structure function of a hadron are determined by the matrix elements of colour singlet operators:

$$A_{\mu_1 \dots \mu_n}^{q,n}(p_1, p_2) = \langle p_2 | O_{\mu_1 \dots \mu_n}^{q,n} | p_1 \rangle
 \tag{2.4}$$

In leading order in Q^2 the twist-2 SU(3) flavour singlet and octet fermion operators

$$O_{\mu_1 \dots \mu_n}^{q,n} = S \bar{\Psi} \gamma_{\mu_1} \mathcal{D}_{\mu_2} \dots \mathcal{D}_{\mu_n} \frac{\lambda^a}{2} \Psi, \quad \lambda^0 = \mathbb{1}, \lambda^i \quad i=1, \dots, 8
 \tag{2.5}$$

and gluon operators

$$O_{\mu_1 \dots \mu_n}^{A,n} = S \text{Tr} F_{\mu_1}^\alpha \mathcal{D}_{\mu_2} \dots \mathcal{D}_{\mu_{n-1}} F_{\alpha \mu_n}
 \tag{2.6}$$

contribute to the structure function.

The notation is standard: ψ denotes the fermion field operator. Its colour, flavour, and spinor indices are not exhibited explicitly. $\mathcal{D}_\mu$ is the covariant derivative and $F_{\mu\nu}$ is the gluon field tensor.

The form factor is given directly by (2.4) with $n = 1$. The structure function is determined by its moments^{*)}

$$\int_0^1 dx x^{n-2} F_2(x, q^2) = \sum C^{a,n}(q^2) M_n^a \quad (2.7)$$

The coefficient functions $C^{a,n}(q^2)$ can be calculated by renormalization group techniques³⁾. The constants M_n^a (which are independent of q^2) defined by

$$A_{\mu_1 \dots \mu_n}^{a,n}(p, p) = 2^n p_{\mu_1} \dots p_{\mu_n} M_n^a + O(p^2) \quad (2.8)$$

can be calculated in our approach. We shall later discuss a reduced structure function

$$\tilde{F}_2(x) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} dn x^{-n+1} M_n^a \quad (2.9)$$

which does not depend on q^2 at all. It should be compared with the observed structure function at the lower edge of the 'scaling' region.

In order to determine the hadronic expectation values (2.4), one has to calculate the three-point function of operators (2.5) and (2.6) and the local field operators interpolating the hadrons under consideration. After hadronization, at least in p^2 and p'^2 , the hadronic amplitudes are the normalized residues at the corresponding poles: If we suppress the tensor structure, we obtain, for example, for the form factor

$$\begin{aligned} & (Q_{1,2}(M^2) = 0) \\ F(q^2) &= \lim_{p_1^2, p_2^2 \rightarrow M^2} (p_1^2 - M^2)(p_2^2 - M^2) \hat{F}(p_1^2, p_2^2, q^2) / N \\ &= \frac{1}{N} \left\{ \frac{1}{Q'(M^2)^2} \left(\frac{1}{2\pi i} \right)^2 \int_{\epsilon} dt_1 \int_{\epsilon} dt_2 \frac{Q(t_1) Q(t_2) \hat{F}(t_1, t_2, q^2)}{(t_1 - M^2)(t_2 - M^2)} \right. \\ &\quad \left. - \frac{1}{Q(q^2)} \frac{1}{Q'(M^2)^2} \left(\frac{1}{2\pi i} \right)^3 \int_{\epsilon} dt_1 \int_{\epsilon} dt_2 \int_{\epsilon} dt_3 \frac{Q(t_1) Q(t_2) Q(t_3) \hat{F}(t_1, t_2, t_3)}{(t_1 - M^2)(t_2 - M^2)(t_3 - q^2)} \right\} \end{aligned} \quad (2.10)$$

N being the residue of the corresponding hadronic two-point function:

$$N = \lim_{t \rightarrow M^2} (t - M^2) \hat{f}_{i,i}^s(t). \quad (2.11)$$

Note that for $q^2 < 0$ the last term decreases exponentially. The moments of the structure function are obtained by the same procedure.

*) Mass-dependent terms are suppressed.

Only the last term of the r.h.s. of Eq. (2.3) contributes to a purely hadronic amplitude (coupling constant, decay amplitude), since we have to go to the residues of the hadron poles in all three variables.

3. SELECTION OF THE CUTS TO BE HADRONIZED

In this section we illustrate how the cuts which have to hadronized should be selected. As a criterion we chose the existence and position of the singularities before hadronization. We only hadronize those singularities which are present in the perturbative expression and which are simultaneously situated above zero (resp. $4m^2$). This may lead to singularities in the time-like region, since by integration over the mass variables and analytic continuation one may pick up new singularities, as will be seen explicitly in Section 4.

We illustrate the selection criteria by the example of the zero order contribution to the form factor. The singularities occurring there are those of the triangular diagram of Fig. 1. In order to obtain the singularity in p_1^2 we put lines 2 and 3 on the mass shell. The resulting discontinuity still has singularities in p_2^2 and p_3^2 which can be obtained by putting line 1 on the mass shell. The double discontinuity thus obtained, viewed as discontinuity over p_1^2 and p_2^2 , may still have singularities in p_3^2 , and the question is whether to meromorphize these too. In order to investigate this, we locate the singularity by putting all internal lines on the mass shell, which yields the following condition:

$$|p_3^2 + p_2^2 - p_1^2| \leq \frac{\sqrt{p_1^2 - 4m^2}}{\sqrt{p_1^2}} \sqrt{(p_3^2 + p_2^2 - p_1^2) - 4p_3^2 p_2^2}$$

This means: if we hadronize in p_1^2 and p_2^2 by integrating in Eq.(2.3) over positive values of t_1 and t_2 , p_2^2 has to be non-positive, otherwise the above condition cannot be fulfilled. On the other hand, if we want to hadronize a positive cut of p_3^2 , then p_1^2 or p_2^2 has to be negative. We conclude that the zeroth order (Fig. 1) cannot be hadronized in all variables.

In Section 4 we shall see that when we continue the zeroth order amplitude, hadronized in p_1^2 and p_2^2 to positive values of p_3^2 , we will encounter a singularity which does not have its origin in the physical input, i.e. the perturbative expression. One can only speculate that in higher order calculations with all the implications on the hadronization procedure, unwanted singularities will disappear.

We next consider the contribution to second order in QCD (Fig. 2). One contribution to the multiple singularities comes from putting lines 4, 5, 1, 2, and 3

on the mass shell. It can easily be seen that this leads to conditions at least as stringent as those for the simple triangle diagram. But putting 3, 6, 5, 1, and 2 on mass shell gives rise to a triple discontinuity with all external variables p_1^2 , p_2^2 , and p_3^2 at positive values (see Appendix). So we conclude that the diagrams of the type of Fig. 2 are the lowest ones to be hadronized in all three external variables. They will therefore contribute to hadronic decay amplitudes and give rise to Regge pole corrections to the form factor and the structure function (see Section 5).

4. ZEROth ORDER CALCULATIONS: MESON STRUCTURE FUNCTION AND FORM FACTOR

We now calculate matrix elements of the Fermion operators (2.5) (between single-meson states) in zeroth order (Fig. 1). This corresponds to a bag calculation with no gluon corrections.

For the meson we use the interpolating local field (colour zero contractions are always understood):

$$O_M(x) = \bar{\psi}(x) \Lambda^M \Gamma^M \psi(x) \quad (4.1)$$

where Λ^M and Γ^M determine the flavour and spin structure, respectively. We consider scalar or pseudoscalar mesons, hence $\Gamma^M = \mathbb{1}$ or γ_5 *).

In order to get the perturbation theory expression to be hadronized later, we have to calculate the Δ diagram of Fig. 1. The resulting amplitude is

$$\Delta^{\mu_1} \dots \Delta^{\mu_n} \Gamma_{\mu_1 \dots \mu_n}^{(0)}(p, p', q) = \quad (4.2)$$

$$\frac{i}{(2\pi)^4} \int d^4k \frac{\{ \Gamma^M(k+m) \Delta (2\Delta \cdot k)^{n-1} (k-q+m) \Gamma^N(k-p+m) \}}{(k^2 - m^2) ((k-q)^2 - m^2) ((k-p)^2 - m^2)}$$

We have given the quark a finite effective mass in order to study its effects. Although the wave function in Migdal's approach has been worked out only for $m = 0$ using conformal invariance, a meromorphization as explained in the preceding section is still meaningful in the massive case, with some Q-function not yet known explicitly.

It is convenient to work in light-cone coordinates $k_{\pm}, \vec{k}_{\perp}$ with

$$k_{\pm} = (k_0 \pm k_3)/\sqrt{2}$$

*) For $m = 0$ the present formalism is chiral symmetric. Breaking of chiral invariance has still to be investigated. For $m \neq 0$ there are mass terms in the traces which change sign going from $\mathbb{1}$ to γ_5 couplings.

We evaluate Eq. (4.2) at $q^2 < 0$ and choose as coordinate frame $q_- = 0$, $p_\perp = 0$. We also choose $\Delta_+ = \Delta_\perp = 0$. q_+ is given by

$$q_+ = \frac{p^2 + q^2 - p'^2}{2p_-}$$

and

$$q^2 = -q_\perp^2$$

We introduce $x = k_-/p_-$ and a new transverse variable $\vec{\ell}_\perp$ by

$$\vec{k}_\perp = \frac{1}{2}(1-x)\vec{q}_\perp + \vec{\ell}_\perp$$

The k_+ integration is performed by closing the contour around the pole at $(k-p)^2 - m^2 = 0$. The result is

$$\Delta^{\mu_1} \dots \Delta^{\mu_n} T_{\mu_1 \dots \mu_n}^{(0)} = \quad (4.3)$$

$$(2\Delta_+ p_-)^{n-1} \int_0^1 \frac{dx d^2 \ell_\perp}{2(2\pi)^3} \frac{x^{n-1}}{x^2(1-x)} \frac{\text{Tr}}{(p^2 - u_1)(p'^2 - u_2)}$$

with

$$u_{1,2} = \frac{(\vec{\ell}_\perp \pm \frac{1}{2}(1-x)\vec{q}_\perp)^2 + m^2}{x(1-x)} \quad (4.4)$$

Using

$$2k_+ p_- = p^2 - \frac{k_\perp^2 + m^2}{1-x}$$

we find for the trace

$$\text{Tr} = 4\Delta_+ p_- \left\{ +\frac{1}{4}(1-x)q^2 + \frac{\ell_\perp^2 + m^2}{1-x} + c \right\} \quad (4.5)$$

($c = 0, -4 \times m^2$ for $\Gamma^{\mu_1 \dots \mu_n} \gamma_5, 1$ resp.)

Hadronization in p^2 , p'^2 can now easily be done by applying the operation

$$\frac{1}{(2\pi i)^2} \int \frac{dp^2}{p^2 - M^2} Q(p^2) \int \frac{dp'^2}{p'^2 - M^2} Q(p'^2) \quad (Q(M^2) = 0)$$

and interchanging the order of integration. We end up with

$$\Delta^{j_1} \dots \Delta^{j_n} \tilde{F}_{j_1 \dots j_n}^{(0)}(q^2) = \frac{(2\Delta p)^n}{(2\pi)^3} \int_0^1 x^{n-1} dx \int \frac{d^2 \ell_\perp}{x(1-x)} \left\{ + \frac{(1-x)q^2}{4x} + \frac{\vec{\ell}_\perp^2 + m^2}{x(1-x)} + \frac{c}{x} \right\} \frac{Q(u_1) Q(u_2)}{(M^2 - u_1)(M^2 - u_2)} \quad (4.6)$$

The quark lines emerging from bound states are on mass shell after performing the k_+ , p^2 , p'^2 integrations. $Q(u_i)$ plays the role of the Weinberg infinite momentum frame vertex function (to be introduced via sidewise dispersion relations in a covariant framework). The zeros of $Q(u_i)$ cancel the quark poles. This is also the mechanism observed in two-dimensional QCD in the light-cone gauge⁷⁾.

In order to get form factor or structure function, the expression (4.6) has to be normalized as described in Section 2. In the massless case the consistency of this normalization procedure can be explicitly checked. Using

$$Q(u) = (R\sqrt{u})^{-\nu} \gamma_\nu(2R\sqrt{u}) \quad (4.7)$$

one can show the identity

$$\frac{1}{16\pi^2} \int_0^\infty du \frac{2u}{(M^2 - u)^2} Q^2(u) = \frac{Q'(M^2)}{16\pi^2} \int_0^\infty du \frac{2u}{(M^2 - u)} Q(u) \equiv N^{-1} \quad (4.8)$$

Hence the form factor is one at zero momentum transfer, and sum rules for the structure function are fulfilled.

For massive quarks the function $Q(u)$ is not known. In order to get some orientation we shall use the expressions (4.7) also for the case of massive quarks, i.e. we explicitly use the asymptotic behaviour:

$$Q(u) \underset{u \rightarrow \infty}{\sim} (R\sqrt{u})^{-(\nu + 1/2)} \sin(2R\sqrt{u} + \beta) \quad (4.9)$$

The (reduced) structure function $\tilde{F}_2(x)$ [compare Eq. (2.9)] is obtained from Eq. (4.6) by Mellin transformation and taking the limit $q^2 \rightarrow 0$,

$$\tilde{F}_2(x) = N/(2\pi)^3 \sum e_q^2 \times \int \frac{d^2 \ell_\perp (\vec{\ell}_\perp^2 + m^2)}{x^2(1-x)^2} Q^2\left(\frac{\vec{\ell}_\perp^2 + m^2}{x(1-x)}\right) / (M^2 - \frac{\vec{\ell}_\perp^2 + m^2}{x(1-x)})^2 \quad (4.10)$$

The difference between massless and massive theory is now clearly seen. For massless quarks, we can rescale the integration variable and we find

$$\tilde{F}_2(x) = x \sum e_q^2 \quad (4.11)$$

i.e. non-vanishing as $x \rightarrow 1$.

With finite quark masses, however, we have a damping for $x \rightarrow 1$ since the argument of the Q-function now becomes large. Using the asymptotic form (4.9) we obtain

$$\tilde{F}_2(x) \underset{x \rightarrow 1}{\sim} (1-x)^{\nu+1/2} \quad (4.12)$$

with $\nu = 1$ for scalar (and vector) mesons.

The reason for this non-continuous behaviour in the quark mass lies in the corresponding behaviour of the quark cut which gives rise to the hadron poles by meromorphization. Its branch-point at $m^2/x(1-x)$ recedes far behind the hadron pole at $m^2 \neq 0$ but stays at zero in the massless case.

In Fig. 3 we plot $\tilde{F}_2(x)$ and $F_2(x, Q^2)$ at a high q^2 value ($q^2 = -300 \text{ GeV}^2$). The moment equation (2.7) determines the q^2 -dependence. For the latter case we have used the lowest order (zeroth order in $\bar{g}$) expressions for $c_n(q^2)$ with $\Lambda = 0.3 \text{ GeV}$, $Q_0 = 1.8 \text{ GeV}^2$ ⁸⁾. As Fig. 3 clearly shows, mass effects are less and less important with increasing $|q^2|$.

The quark transverse momentum distribution in a meson is obtained from Eq. (4.10) simply by dropping the $d^2 \ell_\perp$ integration. It is essentially power-behaved, with

$$\left(\frac{1}{\ell_\perp^2} \right)^{1+\nu+1/2}$$

at large $\ell_\perp$. The x -dependence of the average quark transverse momentum is shown in Fig. 4. Phenomenological consequences are discussed in Ref. 9.

The meson form factor is obtained from Eq. (4.6) with $n = 1$. We find

$$F(q^2) = \left(\sum e_q \right) \frac{N}{(2\pi)^3} \int_0^1 dx \int \frac{d^2 \ell_\perp}{x(1-x)} \left\{ \frac{(1-x)q^2}{4x} + \frac{\ell_\perp^2}{x(1-x)} + \frac{c}{x} \right\} \frac{Q(u_1)Q(u_2)}{(k^2 - u_1)(k^2 - u_2)} \quad (4.13)$$

with $u_{1,2}$ defined in Eq. (4.4).

Again, there are significant quark mass effects. For $m^2 = 0$ we find

$$F(q^2) \underset{q^2 \rightarrow \infty}{\sim} 1/q^2$$

$F(q^2)$ falls at least as

$$1/(q^2)^{1/2 + 5/4}$$

in the massive case*).

A comparison with pion form factor data will be given in Section 5, together with the hard gluon contribution.

We close this section with some remarks on the analyticity properties of the three-point function after hadronization. For simplicity this is done only for the massless case. Substituting in Eq. (4.13)

$$\alpha = -\frac{(1-x)}{x} q^2, \quad \beta = \frac{q_1^2}{x(1-x)}$$

we obtain

$$F(q^2) = (\sum e_q) \frac{W}{(2\pi)^3} \int_0^\infty d\alpha \frac{(-q^2) 4\alpha}{(4\alpha - q^2)^2} \int_0^\infty d\beta \int_0^{2\pi} d\vartheta (\beta - \alpha) \frac{Q(\alpha + \beta + \sqrt{\alpha\beta} \cos \vartheta)}{\{M^2 - (\alpha + \beta + \sqrt{\alpha\beta} \cos \vartheta)\}} \frac{Q(\alpha + \beta - \sqrt{\alpha\beta} \cos \vartheta)}{\{M^2 - (\alpha + \beta - \sqrt{\alpha\beta} \cos \vartheta)\}} \quad (4.14)$$

So far this has been done under the assumption $q^2 < 0$. Equation (4.14) allows now a continuation in q^2 , and we clearly see the cut from 0 to ∞ .

Since we know that this cut in q^2 has not been present in our dynamical input (compare the discussion in Section 3) it is rather a consequence of our hadronization procedure in p^2 and p'^2 , and therefore should not be hadronized.

5. HIGHER-ORDER CORRECTIONS

In the preceding we have studied the meson form factor and structure function in zeroth order (bag) approximation. At large q^2 (or $x \rightarrow 1$, respectively) this contribution strongly depends on the behaviour of the Q-functions at large argument (in the case $m^2 \neq 0$). There are higher order contributions, however, where Q(u) enters with moderate values of the argument only. The large momentum transfer is

*) Because of the complicated overlap of oscillations in Eq. (4.13) we give only an upper limit.

mediated by hard (hot) gluons¹⁰⁾. These hard gluon terms will dominate the 'geometrical' or bag contribution at large momentum transfer if $Q(u)$ is sufficiently damped at large u .

For the meson form factor at large q^2 we have to calculate the graphs of Fig. 5. These are the lowest-order diagrams for which both the incoming and the outgoing quark pair may have small relative transverse momentum, independent of how large q^2 is.

Keeping only leading terms in q^2 we find, after integration over the + components of the loop momenta,

$$\Gamma^{(2)}(p, p', q) = 2 \left(\frac{1}{2\pi} \right)^2 g^2 C(\mathcal{G}) \int_0^1 \frac{dy}{z} \int \frac{d^2 \tilde{k}_\perp}{y(1-y)} \int_0^1 \frac{dz}{z} \int \frac{d^2 \tilde{\ell}_\perp}{z(1-z)} \quad (5.1)$$

$$\frac{1}{(1-y)^2(1-z)q^4} \frac{1}{p^2 - \frac{\tilde{k}_\perp^2 + m^2}{y(1-y)}} \frac{1}{p'^2 - \frac{\tilde{\ell}_\perp^2 + m^2}{z(1-z)}} \text{Tr}$$

where the group weight $C(\mathcal{G})$ is

$$C(\mathcal{G}) = \frac{N_c^2 - 1}{2N_c}$$

for $SU(N_c)$ ($N_c = 3$).

The two graphs give identical contributions. The leading terms of the trace are

$$\begin{aligned} \text{Tr} &\sim 8p_- m^2 q^2 (1-y) \\ &\sim 8p_- \{ m^2 q^2 (1-2z)(1-2y)(1-y) + (y-1)q^2 \tilde{k}_\perp \cdot \tilde{\ell}_\perp \} \end{aligned} \quad (5.2a)$$

$$q^2 \rightarrow -\infty \quad (5.2b)$$

for couplings of type γ_5 , 1 resp. Hadronization in the meson channels p^2 and p'^2 can easily be done now.

For a consistent normalization of order g^2 , both the vacuum expectation value of the three-point function and the residue of the two-point function should be known to that order. However, in our case the $O(g^2)$ three-point function is supposed to dominate the zeroth order term for large q^2 . Hence normalization can be performed by dividing the hadronized expression (5.1) by the non-normalized zeroth order form factor. We obtain

$$F(q^2) \underset{q^2 \text{ large}}{\sim} (\sum e_q) \frac{\alpha_s}{\pi} C(\mathcal{G}) \frac{G^2}{G_0^2} \frac{m^2}{-q^2} \quad (5.3)$$

with

$$G = \int_0^1 dy \frac{f(y)}{1-y} \int_{m^2/y(1-y)}^\infty dv \frac{Q(v)}{v-m^2} \quad (5.4)$$

where $f(y) = 1, 1 - 2y$ for $\gamma_5, 1$ couplings, resp.

and

$$G_0^2 = \int_0^1 dy \int_{m^2/y(1-y)}^{\infty} dv \frac{v Q^2(v)}{(v - m^2)^2} \quad (5.5)$$

Note that G is divergent for $m^2 = 0$. α_s is defined as

$$\alpha_s = g^2/4\pi \quad (5.6)$$

In a systematic higher-order calculation, the dependence of the coupling constant g on the renormalization point μ has, of course, to be taken into account. In the approach used here, the "bag coupling" g_B is introduced together with a renormalization point $\mu = 1/R$. Since in case of renormalization group invariance only the pair (μ, g) determines the physics, it could be changed to any other equivalent pair for convenience (e.g. better convergence). Calculating the perturbation theoretical input $[F_{ijk} \text{ of Eq. (2.3)}]$ to meromorphization, it seems natural to use the same pair (g, μ) and stay in the same order both for numerator and denominator. This leads to a well-defined series in one coupling constant^{*)}. But since numerator and denominator should both be invariant under the renormalization group, one could also have used separate expansions with different sets (μ, g) . Since in our low-order calculations renormalization does not show up we by-pass this question and always use the bag coupling g_B .

There are also other contributions of the graphs of Fig. 5 corresponding to the various possibilities of closing integration contours around the propagator poles. These different possibilities correspond to the various intermediate states in "time" ordered perturbation theory. In this way the different bound state vertices enter, in particular the $q\bar{q}$ -gluon vertex. Higher order contributions to Q of course also contain gluon effects, and hence also the behaviour for large arguments is modified. However, in the foregoing calculation we have restricted ourselves to the lowest order contribution which dominates for $|q^2| \rightarrow \infty$ for the case that Q (to the order of g^2) is strongly damped.

For the meson structure function near $x = 1$ we have to evaluate the graph of Fig. 6. Only the time-ordering shown in Fig. 5 leads for $x \rightarrow 1$ to a configuration where both incoming (and also outgoing) quarks carry a finite fraction of the meson momentum in some infinite momentum frame. This is equivalent to keeping the argument of the Q -functions finite, and therefore their contribution is supposed to dominate near $x = 1$. Three-loop graphs with a three-gluon coupling do not survive in this limit. Hence our results are the same as for an Abelian theory (apart from the fact that the whole procedure is based on asymptotic freedom).

*) Indeed, for example, the normalization of the form factor $[m = 0, \text{ Eq. (4.13)}]$ can be checked to require the same order g^2 in the numerator and the denominator.

The leading term is found to be

$$\Delta^{\mu_1} \dots \Delta^{\mu_n} \Gamma_{\mu_1 \dots \mu_n}^{(4)}(p, p', q^2=0) \underset{n \text{ large}}{\sim} \frac{(2\Delta p)^{n-1}}{(2\pi)^9} g^4 C^2(\varphi) \quad (5.7)$$

$$\times \int_0^1 \frac{dy}{z} \int_0^1 \frac{dz}{z} \int_0^1 x^{n-1} \frac{dx}{z} \int \frac{d^2 k_1}{1-x} \int \frac{d^2 \ell_1}{y(1-y)} \int \frac{d^2 r_1}{z(1-z)}$$

$$\times \frac{1}{p^2 - \frac{\ell_1^2 + m^2}{y(1-y)}} \frac{1}{p'^2 - \frac{r_1^2 + m^2}{z(1-z)}} \left(\frac{1}{k_1^2 + m^2} \right)^4 \frac{\text{Tr}}{(1-y)(1-z)}$$

with

$$\text{Tr} \sim 16 \Delta p (1-x)^{-1} \{ k_1^2 + m^2 \} \left(m^2 (k_1^2 + m^2) + \left(\frac{\ell_1^2 + m^2}{y(1-y)} \right) \left(\frac{r_1^2 + m^2}{z(1-z)} \right) - m^2 \left(\frac{\ell_1^2 + m^2}{y(1-y)} + \frac{r_1^2 + m^2}{z(1-z)} \right) \right) \quad (5.8a)$$

$$\sim 16 \Delta p (1-x)^{-1} \{ k_1^2 + m^2 \}^2 \left(m^2 (1-2y)(1-2z) - \ell_1 \cdot r_1 \right) \quad (5.8b)$$

for γ_5 , 1 coupling resp. After hadronization and normalization this leads to the structure functions

$$\tilde{F}_2(x) \underset{x \rightarrow 1}{\sim} \frac{\sum e_q^2}{4} \left(\frac{\alpha_s}{\pi} \right)^2 C^2(\varphi) (1-x)^2 / G_0^2 \times$$

$$\times \begin{cases} \{ G^2 + \frac{1}{2} \tilde{G}^2 - G \tilde{G} \} \\ \{ G^2 \} \end{cases} \quad (5.9a)$$

$$(5.9b)$$

with the same effective couplings G , G_0^2 defined already in Eqs. (5.4) and (5.5) and with

$$\tilde{G} = \frac{1}{m^2} \int_0^1 dy \int_{1-y}^1 \frac{dv}{y} \frac{v Q(v)}{v - m^2}$$

The $(1-x)^2$ behaviour at large x is due to spin effects; $(1-x)$ would be obtained for scalar quarks.

In a proper analysis of the structure function near $x = 1$ in terms of momentum sum rules the answer is not as simple. If the constants $C_n(q^2)$ (2.7) are taken to lowest order in the effective coupling constant $\bar{g}$, the effective power of $(1-x)$ depends on q^2 (see Ref. 8). Using the expressions for C_n of Ref. 8, the matrix elements Γ should be evaluated with the same parameters for Q_0 and the coupling constant as used there, since the normalization of the operators O_n and the calculation of the C_n are not independent. Very near to $x = 1$, also higher order terms of the type $g^2 \log(1-x)$ are expected to contribute substantially. The (non-uniform) convergence of the sum (2.7) depends on the range of x and n under consideration.

A typical transverse quark distribution is $(1/k_{\perp}^2 + m^2)^2$. The corresponding average quark transverse momentum is $\pi/2 m^*$.

In the case of the wave function Eq. (4.7) the contribution (5.9) to the structure function will not dominate the bag contribution (4.10) which behaves like $(1-x)^{1.5}$. This is different for the meson form factor. In this case the hard-gluon contribution certainly takes over at sufficiently large q^2 . Since we do not only predict the power in q^2 but also the normalization, we can study the question when this asymptotic behaviour sets in. In Fig. 7 we show the zeroth order contribution to the form factor (γ^5 coupling), together with data from Ref. 11. At low q^2 a very good description is obtained with $m = 300$ MeV (which is a reasonable value for a constituent quark mass). At q^2 around -2 GeV the bag contribution starts falling below the data. The dashed line shows the result when the hard-gluon contribution is included (this can be done only at relatively large q^2 since the calculation involves approximations valid only in that limit). $\alpha_s/\pi \sim 0.25$ has been used. This value is found in Ref. 1 from the discussion of an eigenvalue problem. It is not much below the (bag) value $\alpha_s/\pi \approx 0.6$ found from a fit to the spectrum. A stronger fall-off of the "bag" form factor (zeroth order) would require a larger value of α_s/π than we have used to fit the data. For large values of $-q^2$ the higher order contributions are expected to be contained in the effective coupling $\bar{g}(q^2)$. However, the application of renormalization group ideas to not purely light-cone dominated processes is not very well understood up to now.

The second order QCD corrections to the triangle diagram not only give rise to hard-gluon corrections but also influence the structure function and the form factor near $x = 0$ and $q^2 = 0$ resp. Since the second order contributions

$$g^2 \tilde{\Gamma}_n(M^2, M^2, q^2)$$

have a cut in q^2 which is to be hadronized according to our previous considerations (see also Appendix), we have to replace it by

$$\hat{\tilde{\Gamma}}_n(M^2, M^2, q^2) = \tilde{\Gamma}_n(M^2, M^2, q^2) - \frac{1}{2\pi i Q_n(q^2)} \oint_{\text{cont}} \frac{\tilde{\Gamma}_n(M^2, M^2, t') Q_n(t')}{t' - q^2} dt' \quad (5.10)$$

where the contour is only to be taken over "physical" cuts, i.e. those present in the perturbative expression for p^2 , $p'^2 \geq 0$.

We see immediately that the second term on the r.h.s. of Eq. (5.10) does not influence the behaviour of the form factor for large negative q^2 because of the rapid exponential damping due to the factor $1/Q_n(q^2) \approx \exp(-2R\sqrt{-q^2})$. Hence in the limit of large $(-q^2)$ it is consistent to consider only the hard gluon contribution.

*) This value in the limit $x \rightarrow 1$ plays a special role. According to Ref. 9 it gives the $q^2 \rightarrow -\infty$ asymptotic value of $\langle k_{\perp} \rangle$ also at finite $x < 1$.

The situation is a little bit more involved for the structure function. The behaviour of the structure function near $x = 1$ is determined by the behaviour of the moments for $n \rightarrow \infty$. In this limit we have:

$$Q_n(q^2)/Q_n(0) \rightarrow 1$$

and hence

$$\hat{\Gamma}_n^{\sim}(H_1^2, H_2^2, 0) \xrightarrow{n \rightarrow \infty} \tilde{\Gamma}_n^{\sim}(H_1^2, H_2^2, 0) - \frac{1}{2\pi i} \frac{1}{Q_n(q^2)} \int_{\mathcal{C}} dt' \frac{\tilde{\Gamma}_n^{\sim}(H_1^2, H_2^2, t')}{t'}$$

i.e. we are left only with those contributions not to be hadronized in the q^2 channel. Hence in the limit $x \rightarrow 1$ only those contributions are of importance.

Near $x = 0$ the structure function is governed by the right-most singularity of M_n in the n plane [compare Eq. (2.9)]. Because of

$$1/Q_n(0) \approx \Gamma(v+1) \quad (5.12)$$

with

$$v = n + g^2 \bar{\gamma}(n) \quad (5.13)$$

Eq. (5.10) has a set of poles in n . Using Eq. (2.9) we find

$$\tilde{F}_2(x) \underset{x \rightarrow 0}{\sim} x^{1-\alpha_0} \quad (5.14)$$

where α_0 is the intercept of the Regge trajectory with the quantum numbers of the operator $O_{\mu_1, \dots, \mu_n}^{a, n}$ and satisfies

$$\alpha_0 + g^2 \bar{\gamma}(\alpha_0) = -1 \quad (5.15)$$

$g^2 \bar{\gamma}(n)$ is the anomalous dimension (to order g^2) of $O_{\mu_1, \dots, \mu_n}^{a, n}$.

6. BARYON FORM FACTOR AND STRUCTURE FUNCTION

In this section we present an approach to baryon vertices.

Baryon (local) operators^{*)} are of the type

$$(\psi_a^k(x) C^T \Gamma \psi_b^l(x)) (\psi_c^m(x))_\alpha \in abc \quad (6.1)$$

*) To be discussed in more detail in a forthcoming publication by H.G. Dosch and M.G. Schmidt.

with colour contraction ϵ^{abc} and flavour quark field indices k, ℓ, m . Γ indicates the set of Dirac tensors. For excited states, covariant derivatives have to be included. The question of mixing (and diagonalization) of these operators is difficult. Remark that in quark models for baryons one normally starts from a non-relativistic approximation, whereas here -- with massless quarks -- we have just the complementary case. Hence there are problems with parity doubling and the identification of the components of Dirac spinors (there are no big and small components). Nevertheless one can separate classes of operators corresponding to different spin and parity and diagonalize them in the two-point function.

A typical operator interpolating nucleons is

$$(\psi_a^T C^T \gamma_5 \psi_b^n)(\psi_c^P)_\alpha \epsilon^{abc} \quad (\text{proton}) \quad (6.2)$$

$$(\psi_a^T C^T \gamma_5 \psi_b^P)(\psi_c^n)_\alpha \epsilon^{abc} \quad (\text{neutron})$$

For the calculation of the structure coefficients (2.8) and of the form factor in zeroth order, we have now to consider the graph of Fig. 8.

We again conveniently calculate in light-cone coordinates with $q_- = 0$, $p_- = 0$, k_+ , r_+ , p^2 , and $(p+q)^2$ integration (unnormalized), set the quark lines on mass shell, and then the amplitude results as $[\hat{v} = (1-x)v, \quad \ell_\perp = k_\perp + \frac{1}{2}(1-x)q_\perp, \quad r'_\perp = r_\perp + vk_\perp]$

$$\begin{aligned} \Gamma_n^{(0)}(M^2, M^2, q^2) &= \frac{(2\Delta_+ p_-)^n}{(2\pi)^6} \int_0^1 \frac{dx}{2x^2(1-x)} \int d^2 \ell_\perp \int_0^1 \frac{dv}{2} \int \frac{d^2 r'_\perp}{v(1-v)} \\ &\cdot Q_v \left(\frac{(\ell_\perp - \frac{1}{2}(1-x)q_\perp)^2 + m^2(1-x) + \tilde{m}^2 x}{x(1-x)} \right) / \left\{ M^2 - \frac{(\ell_\perp - \frac{1}{2}(1-x)q_\perp)^2 + m^2(1-x) + \tilde{m}^2 x}{x(1-x)} \right\} \\ &\cdot Q_v \left(\frac{(\ell_\perp + \frac{1}{2}(1-x)q_\perp)^2 + m^2(1-x) + \tilde{m}^2 x}{x(1-x)} \right) / \left\{ M^2 - \frac{(\ell_\perp + \frac{1}{2}(1-x)q_\perp)^2 + m^2(1-x) + \tilde{m}^2 x}{x(1-x)} \right\} \\ &\cdot \sum \text{traces} / 2\Delta_+ p_- \end{aligned} \quad (6.3)$$

Here

$$\tilde{m}^2 = \frac{\pi_\perp'^2 + m^2}{v(1-v)} \quad (6.4)$$

is the effective mass of the (spectator) diquark system. The sum over traces is indicated in Fig. 9 with the different possible contractions. Also spin averaging of the outer baryons is understood. The complicated traces are better done with an algebra program. The structure is very similar to Eq. (4.3) but with further integrations over the effective mass $\tilde{m}$ and v (and an angular variable possibly appearing in the traces). For $m^2 = 0$ the normalization $N(0)$ should be again

such that $F_-(q^2 = 0) = 1$ as a consequence of Eq. (2.6). The index of Q_γ is $\nu = d - 2 - 1/2$ in the baryon case due to spinors multiplying invariant amplitudes, and hence $\nu = 2$ in the zeroth order for baryons. For $m^2 = 0$, the leading contribution of the traces remains finite for $|q^2| \rightarrow \infty$ or $x \rightarrow 1$ resp., and we obtain

$$F_-(q^2) \sim \left(\frac{1}{q^2}\right)^2 \quad (m=0) \quad (6.5)$$

$$\hat{F}_2(x) \sim (1-x) \quad (m=0) \quad (6.6)$$

which is easy to understand in comparison with the meson result: the change comes from a rescaling of the $\tilde{m}^2$ integration $\tilde{m}^2 \rightarrow \tilde{m}^2/(1-x)$.

For $m^2 \neq 0$ we have to take into account that there are traces with big k_+ and r_+ now in the limits $|q^2| \rightarrow \infty$ and $x \rightarrow 1$ resp., and we have

$$F_-(q^2) \sim \left(\frac{1}{q^2}\right)^{\nu/2 + 5/4 - 1/2} \quad (m \neq 0) \quad (6.7)$$

$$\hat{F}_2(x) \sim (1-x)^{\nu + 1/2 - 1} \quad (m \neq 0) \quad (6.8)$$

It is interesting to note that now $\tilde{m}^2 \geq m^2/\nu(1-\nu)$ sets the scale. Therefore deviations from the $m = 0$ case can set in earlier in the asymptotic limit $x \rightarrow 1$.

The γ -algebra in the traces and part of the integration has to be done on the computer since it is very lengthy. Numerical results for the baryons in zeroth order and the hard-gluon corrections will be given elsewhere after a careful analysis.

7. DISCUSSION

Given a function $Q_\gamma(p^2)$ whose zeros correspond to the hadron poles in the hadronic two-point function, we present a well-defined procedure to substitute quark and gluon cuts in the external mass variables by these hadron poles. There are problems with cuts of the hadronized three-point function which cannot be traced back to singularities of the perturbative expression at positive values of the external mass variables. We argued that these cuts should not be hadronized. The question of what happens to them is open to speculation; they should not be present if maximal analyticity holds.

In the limit $x \rightarrow 1$ for the structure functions and $q^2 \rightarrow -\infty$ for the form factor the meromorphization of the "current" channel (q^2) turns out to be unimportant. Hence only meromorphization in the two (external) hadron channels is required; it can be performed without difficulties. Even for values of x not too near to 1 and finite values of q^2 resp. these calculations make sense as parton model type calculations. A fit to the pion form factor for small- q^2 is very satisfactory. Our formulae show an interesting connection between the spectrum and the parton model vertex function, the latter being given by the $Q(p^2)$, up to normalization.

It turns out that our results are very sensitive to the value of the effective (constituent) quark mass m , in particular near $x = 1$ and for $q^2 \rightarrow -\infty$ resp. The results for $m = 0$ are in obvious disagreement with experiments. For $m \neq 0$ the quark cut recedes to large values for $x = 1$ and $q^2 \rightarrow -\infty$ resp. and the arguments of the functions $Q(p^2)$ become large. Unfortunately the $Q(p^2)$ are known only for $m = 0$. We nevertheless used the expressions obtained by Migdal also for the case $m \neq 0$. We then get a behaviour of the (q^2 -independent) structure function as $(1-x)^{3/2}$ near $x = 1$ and for the form factor as $(-q^2)^{-7/4}$ for mesons. If strong damping is assumed for large arguments of $Q(p^2)$ (in the cases of the structure function it must be stronger than that of Migdal's Q -functions) the (geometrical) zeroth order contributions will be dominated by contributions of higher order in α_s . The lowest order graphs contributing under these circumstances in the limit $x \rightarrow 1$ and $q^2 \rightarrow -\infty$ resp. are the hard gluon graphs known from the dimensional counting analysis. We not only reproduce in the framework of the model the well-known powers but also give the absolute normalization of these contributions. This allows us to decide in which range of variables these graphs are expected to be important. For the pion electromagnetic form factor even quantitative agreement can thus be obtained.

The question, How are higher order contributions to be handled with renormalization group techniques? deserves further investigation. In this paper we have restricted ourselves to low-order contributions corresponding to typical "bag" calculations, and the concept of the running coupling constant is not exploited.

For baryons we encounter considerable technical difficulties, both because of the more involved structure of the local fields interpolating baryons and because of the more complicated structure of the perturbative expressions. However, there are no principal difficulties and a numerical analysis is in preparation.

The question of the implications of the analytic structure of the three-point function deserves further study. Ultimately one should be able to treat general decay matrix elements and to understand the structure of four- and higher-point functions.

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APPENDIX

In this appendix we briefly review the analytical properties of diagrams which are relevant to our problem.

We first consider the triangle diagram, Fig. 1. Starting with the expression:

$$T(s, t, u) = i \int d^4k \left[(k^2 - m^2) ((k+q)^2 - m^2) ((k-p)^2 - m^2) \right]^{-1} \quad (A.1)$$

with $s = p^2, t = q^2, u = (p+q)^2,$

We calculate the absorptive part by Cutkosky rules ($u, t < 0, s \geq 4m^2$):

$$\begin{aligned} \text{Disc}_s T(s, t, u) &\equiv \frac{1}{2i} \{ T(s+i\epsilon, t, u) - T(s-i\epsilon, t, u) \} \\ &= 4\pi^2 \int d^4k \frac{\delta(k^2 - m^2) \delta((k-p)^2 - m^2)}{(k+q)^2 - m^2} \end{aligned} \quad (A.2)$$

We go to the frame $\vec{p} = 0, p_0 = \sqrt{s}$ and obtain

$$\begin{aligned} \text{Disc}_s T(s, t, u) &= \Theta(s - 4m^2) \pi^2 \sqrt{s - 4m^2} \\ &\cdot \int d\Omega \frac{(u - s + t) - (\sqrt{s - 4m^2} \sqrt{(u - s + t)^2 - 4ut} \cos \theta) / \sqrt{s}}{(u - s + t) + (\sqrt{s - 4m^2} \sqrt{(u - s + t)^2 - 4ut} \cos \theta) / \sqrt{s}} \end{aligned} \quad (A.3)$$

As long as t and u are both negative, the integration over $\cos \theta$ may be performed without special prescription, since the integrand never becomes singular. If, however, we hit a singularity on the $\cos \theta$ integration, we must give an ϵ prescription.

This is the case if

$$|u - s + t| \leq \left| \frac{\sqrt{s - 4m^2}}{\sqrt{s}} \sqrt{(u - s + t)^2 - 4ut} \right| \quad (A.4)$$

From this inequality it follows immediately that ut has to be negative; hence if we are interested in singularities for positive u , then t has to be negative, and we obtain the double discontinuity in u :

$$\begin{aligned} \text{Disc}_u \text{Disc}_s T(s, t, u) &= \\ &\frac{4\pi^3 \sqrt{s}}{\sqrt{(u - s + t)^2 - 4ut}} \Theta(u - u_1) \Theta(u_2 - u) \Theta(-ut) \Theta(s - 4m^2) \Theta(u) \end{aligned} \quad (A.5)$$

with

$$u_{1,2} = s - t \left(\frac{s}{2m^2} - 1 \right) \pm \sqrt{(-s \cdot t)(s - 4m^2)(-t + 4m^2)} / 2m^2 \quad (A.6)$$

If t reaches 0 from below, the cut shrinks to the point 0 and appears again for negative u values, if $t \geq 4m^2$.

Hence we do not have triple discontinuities for $s, t, u > 0$. This is just the result obtained by cutting lines (i.e. putting them on mass shell) in all these channels.

We now apply this procedure of cutting lines to the next order vertex diagram, Fig. 2. Cutting lines 2, 3, 1, 4, and 5 leads to the same condition as the 0-order diagram, since the condition of 1, 4, and 5 to be on the mass shell is just the same as in that case. If, however, we cut 2, 3, 15, and 6, we can show that this leads to a triple discontinuity with $u, s, t > 0$. We consider for simplicity the massless case

This gives the conditions:

$$\begin{aligned} k^2 = 0 : |\vec{k}| &= \pm k_0, \quad l^2 = 0 : |\vec{l}| = \pm l_0 \\ (k-l)^2 = 0 : 2k_0 l_0 - 2|\vec{k}||\vec{l}|\cos\vartheta_{kl} &= 0, \cos\vartheta_{kl} = \pm 1 \\ (k-p)^2 = 0 : k_0 &= \sqrt{s}/2 \\ (l-q)^2 = 0 : l_0 &= t\sqrt{s} [(u-s-t) - \sqrt{(u-s-t)^2 - 4st} \cos\vartheta_{eq}]^{-1} \end{aligned}$$

which have a real solution for $(u-s-t)^2 > 4st$. If we give the internal particle a small mass, then it follows from

$$(k-l)^2 = 0 = 2m^2 - 2k_0 l_0 + 2|\vec{k}||\vec{l}|\cos\vartheta \quad \text{that} \quad k_0 l_0 > 0, \text{ i.e. } l_0 > 0$$

hence we have two branches of solutions:

$$\begin{aligned} t < 0 : (u-s-t) &< -2\sqrt{st} \\ (u-s-t) &> 2\sqrt{st} \end{aligned} \tag{A.7}$$

It is the latter one in which we are interested, since it can give us a triple discontinuity with all variables > 0 .

Figure captions

Fig. 1 : Triangle-diagram. This represents the zeroth order contribution to the meson form factor and the structure function.

Fig. 2 : Vertex corrected diagram giving rise to Regge contributions (i.e. cuts to be hadronized in all three mass variables).

Fig. 3 : Meson structure function in lowest order in g^2 :

a) low q^2 ($q^2 = q_0^2 = -1.8 \text{ GeV}^2$)

— — — $m_R = 0$
 ----- $m_R = 0.3$
 _____ $m_R = 0.75$

b) large q^2 ($q^2 = -300 \text{ GeV}$, $\lambda = 0.3$ is used in the computation).

Fig. 4 : Average quark transverse momentum as function of x .

Fig. 5 : Lowest-order hard-gluon contribution to the meson form factor.

Fig. 6 : Lowest-order hard-gluon contribution to the meson structure function.

Fig. 7 : Meson form factor in comparison with data (Ref. 11)

———— zeroth order contribution only
 ----- hard-gluon graphs included.

Fig. 8 : Lowest-order baryon diagram.

Fig. 9 : Possible spin arrangements [compare Eq. (6.1)] at the baryon vertices. Current acts a) on a proton quark, b) on the neutron quark.

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5

Fig. 6

Fig. 7

Fig. 8

a)

b)

Fig. 9