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FINAL REPORT ON THE CERN MUON STORAGE RING
INCLUDING THE ANOMALOUS MAGNETIC MOMENT AND
THE ELECTRIC DIPOLE MOMENT OF THE MUON,
AND A DIRECT TEST OF RELATIVISTIC TIME DILATION

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ABSTRACT

A comprehensive description of the Muon Storage Ring and its operation is given, and the final results of the experiment are presented and discussed. The anomalous magnetic moments of positive and negative muons are found to be $a_{\mu+} = 1165911(11) \times 10^{-9}$ and $a_{\mu-} = 1165937(12) \times 10^{-9}$ giving an average value for muons of $a_{\mu} = 1165924(8.5) \times 10^{-9}$. The electric dipole moments were also measured with the results $D_{\mu+} = (8.6 \pm 4.5) \times 10^{-19}$ e•cm and $D_{\mu-} = (0.8 \pm 4.3) \times 10^{-19}$ e•cm. Under the assumption of the CPT theorem these yield a weighted average of $D_{\mu} = (3.7 \pm 3.4) \times 10^{-19}$ e•cm. Finally the time transformation of Special Relativity is shown to be valid to $(0.8 \pm 0.7) \times 10^{-3}$ at $\gamma \approx 29.3$. All the errors quoted here are one standard deviation and contain both statistical and systematic effects.

1. HISTORICAL INTRODUCTION AND
PRINCIPLES OF THE EXPERIMENT

The gyromagnetic ratio g of a particle or atomic system is a measure of its magnetic moment $\mu = g(e/2mc)(j\hbar)$, where m is the mass of the particle concerned and $(j\hbar)$ its angular momentum. Following Uhlenbeck and Goudsmit [1] it was believed that $g = 2$ for the electron, a value which also arises naturally from the Dirac equation [2]. Later it was realized [3] that the quantum fluctuations in the electromagnetic field could induce an extra magnetic moment, so that $g = 2(1 + a)$, where a is the anomaly; similar effects were observed in the fine structure of atomic hydrogen [4], in the Lamb shift [5], and in the magnetic moments of neutral atoms [6].

For a free electron, a polarized beam cannot be produced by a Stern-Gerlach system because of the Uncertainty Principle [7], but Crane and co-workers [8] showed that electrons could be polarized and analysed by Mott scattering through 90° by a heavy element. By introducing a magnetic field between polarizer and analyser they were able to measure g .

It was then realized [9] that if the spin $\vec{s}$ and the momentum vector $\vec{p}$ were both perpendicular to the magnetic field $\vec{B}$, the spin precession frequency $\omega_s = g(e/2mc)B$ and the orbit frequency $\omega_c = eB/\gamma mc$ would be equal in non-relativistic approximation if $g = 2$. Accordingly, the change of beam polarization $(\vec{s} \cdot \vec{p})$ with time would be a sensitive measure of the anomaly $a \equiv (g - 2)/2$; this $(g - 2)$ principle was shown to hold also for relativistic electrons [10]. Employing this idea, a whole series of measurements of gradually increasing accuracy were carried out at Ann Arbor, culminating in the determination of a to 3.5 parts per million (ppm) [11], in agreement with theory. Subsequently, the $(g - 2)$ experiment for the electron has been surpassed by separate measurements of ω_c , ω_s and the spin-cyclotron beat frequency $\omega_a = \omega_s - \omega_c$ for single electrons held in an electromagnetic trap at liquid-helium temperature [12], giving a to 0.2 ppm.

The motivation for these accurate measurements of α for the electron is that they form an excellent test of the theory of quantum electrodynamics (QED). However, in 1956, Berestetskii [13] pointed out that a similar measurement for the muon would test QED to much higher q^2 (or shorter distances) because the natural scale of q^2 is given by the mass of the particle; Schwinger [14], on the other hand, saw such a measurement as a test for the existence of new fields coupled to the muon and capable of explaining the μ -e mass difference. Each field would make its own contribution to the anomaly. In 1957, parity violation was discovered [15] and the muon was found [16] to be born polarized. Moreover, in its decay it shows its polarization direction through the anisotropic emission of decay electrons. With a ready-made polarizer and analyser available, a $(g - 2)$ experiment for muons appeared possible, and in 1958 a group at CERN started to study the problem. Since that time, three different muon $(g - 2)$ experiments [17-19] have been performed at CERN, and several review articles [20] tracing the evolution of this sequence have been published. This paper reports in detail on the last experiment.

The principle of the muon $(g - 2)$ measurement is the same as for the electron. Polarized muons must be trapped in a magnetic field $\vec{B}$ so that they make many turns, and the evolution of $(\vec{s} \cdot \vec{p})$ with time must be measured. The angle θ between the spin $\vec{s}$ and the momentum vector $\vec{p}$ changes according to

$$\frac{d\theta}{dt} = \omega_a = \alpha \left(\frac{e}{mc} \right) B, \quad (1.1)$$

where $\alpha \equiv (g - 2)/2$ is now the muon anomaly. This equation is easily derived in the non-relativistic case, but, as indicated above, it is in fact valid over the whole range of particle velocity [10,21]: ω_a is independent of particle energy.

The value of (e/mc) for the muon is related to its precession frequency at rest, $\omega_s = g(e/2mc)B$, so

$$\frac{\omega_a}{\omega_s} = \frac{\alpha}{1 + \alpha}. \quad (1.2)$$

In practice, ω_s is not determined in the same apparatus but is calculated from the ratio of muon to proton magnetic moments $\lambda \equiv \mu_\mu/\mu_p$, together with a measurement of the magnetic field B in terms of the proton precession frequency f_p . Then

$$\alpha = \frac{f_\alpha}{\lambda f_p - f_\alpha}, \quad (1.3)$$

where $f_\alpha = \omega_\alpha/2\pi$.

The constant λ is known [22] from muon spin precession at rest and from the hyperfine Zeeman effect in muonium to an accuracy of about 1 ppm.

Relative to the electron, the experiment is made more difficult by the limited lifetime τ_0 of the muon (2.2 μsec); this means that the total time available for the measurement is of this order, which imposes a natural scale for the uncertainty in frequency of $1/\tau_0$. For a statistical record of N counts with signal amplitude A in an exponentially decaying source, the error is given approximately by

$$\frac{\Delta f}{f} = \frac{\sqrt{2}}{2\pi A f \tau N^{1/2}}. \quad (1.4)$$

For this reason, in the later (g - 2) experiments the muon lifetime τ is lengthened by Einstein time dilation, first to 27 μsec [18] and then in the present experiment to 64 μsec .

Other factors contributing to the accuracy may be derived from eq. (1.4): a high magnetic field (high f), high muon polarization P, and high sensitivity to muon spin direction (both contributing to A), and a large number of recorded events. The low phase-space density of available muon beams implies that large magnets must be used.

In the first experiments [17], muons from the CERN Synchro-cyclotron (SC) were injected into almost circular orbits which slowly drifted through the field of a large bending magnet (B = 1.6 T) under the influence of a carefully shaped

gradient field. On emerging from the magnet the muons were stopped in a field-free absorber, and their final spin direction was determined by recording the decay electrons in forward and backward counter telescopes. With the storage time ranging from 2 to 8 μsec , only two cycles of the anomalous precession frequency f_a were recorded with a total of 936,000 events; $(g - 2)$ was measured to a precision of 0.4% and found to be in agreement with theory. This established the muon as a point-like heavy electron and was at the time the best test of QED at short distances. The possibility of finding a clue to the μ -e mass difference, or a departure from QED, rested on increasing the accuracy of the measurement.

To obtain higher accuracy it was essential to increase the number of $(g - 2)$ cycles observed, by increasing either B or the time for which the signal was observed. It was decided to dilate the muon lifetime by using highly relativistic muons from the CERN Proton Synchrotron (PS); this implied a radically new technique for storing the particles and for studying their polarization. A muon storage ring, 5 m in diameter, was constructed [18] with $B = 1.7$ T, and a short pulse (10 nsec) of high-energy protons was directed onto a target inside the ring, producing pions. Their decay in flight inside the storage volume gave birth to muons; helped by the change of momentum and direction in the decay, some of these were inflected (by the weak interaction!) onto permanently stored orbits with momentum 1.3 GeV/c. Because a wide range of pion momenta could contribute, the muon polarization P was only 0.26.

When the muon decayed in flight the decay electron almost always had a lower momentum, and about half of them missed the pole-pieces and emerged on the inside of the ring, where they could be detected by energy-measuring shower detectors consisting of layers of lead and plastic scintillator. The requirement that the electron had high energy in the laboratory ($E > E_{\min}$) ensured that in the muon rest frame it must have been emitted within a small range of angles close to the direction of motion. Therefore as the muon spin rotated, the counting rate was modulated according to

$$N_e = N_0 \exp \left(-\frac{t}{\gamma \tau_0} \right) [1 - A \cos (2\pi f_a t + \phi)] , \quad (1.5)$$

where $\gamma \equiv (1 - \beta^2)^{-\frac{1}{2}}$, $\beta \equiv v/c$, and the amplitude of the modulation is $A = PA_e(E_{\min})$, which is a function of the selected electron energy threshold $E_{\min}$. Transforming the Michel spectrum for muon decay at rest, it can be shown [23] that per decaying muon

$$N_e = 1 - \frac{5x}{3} + x^3 - \frac{x^4}{3} \quad (1.6)$$

and

$$AN_e = \frac{x}{3} - x^3 + \frac{2x^4}{3} , \quad (1.7)$$

where x is the ratio of $E_{\min}$ to the maximum possible electron energy, 1.3 GeV. To obtain the best statistical accuracy [see eq. (1.4)] the threshold was chosen to maximize $N_e A^2$, in this case implying $E_{\min} = 780$ MeV.

In calculating the value of α from eq. (1.3), the mean magnetic field seen by the muon population must be supplied, or equivalently the corresponding mean proton precession frequency $\bar{f}_p$. A difficulty arose here due to the field gradient in the ring which was needed to focus the muons vertically; it was necessary to know the mean radius and preferably the detailed radial distribution of the circulating muon population. This was obtained from the rotation frequency $f_{\text{rot}} = \beta c / 2\pi r_e$, where r_e is the equilibrium radius of the muon. As the injection pulse width (10 nsec) was much less than the rotation period $1/f_{\text{rot}} \sim 52$ nsec, the muons were initially bunched in azimuth, and at early times the counting rate in each counter was modulated at the rotation frequency. From an analysis of this bunch structure, it was possible to determine not only $\bar{r}_e$ but also the distribution function of muons in radius $N(r_e)$. The accuracy in radius was limited to ± 3 mm, contributing an error of 160 ppm in $(g - 2)$.

The result of this experiment had an error of 270 ppm and disagreed with theory by 1.7 standard deviations. This stimulated the theorists to calculate the contributions of light-by-light scattering to the α^3 term (see below), thus removing the discrepancy [24].

The muon storage ring [19] described in this paper was built to improve on this result, and to that end incorporated the following design features:

- i) The muons are stored in a uniform magnetic field with vertical focusing provided by electric quadrupoles. With a proper choice of muon energy (see below), ω_a is then, to first order, independent of equilibrium radius and it is no longer necessary to know the radial distribution of muons accurately.
- ii) A momentum-selected pion beam produced outside the magnet is injected into the ring by means of a pulsed inflector. The stored muons then come from almost forward decay and are highly polarized ($P \sim 0.95$).
- iii) The pions from the external primary proton target are collected over a wide solid angle and matched to the acceptance of the ring (smaller angles, but larger area) by appropriate beam optics giving a tenfold increase in intensity over the previously used internal target.
- iv) Less background is generated by the injected beam, and decay-electron detectors can be placed all round the inside of the ring, improving the count rate.
- v) The relativistic factor γ is increased from 12 to 29, increasing the lifetime proportionally.

A plan view of the storage ring is given in fig. 1; it consisted of 40 bending magnets shaped to fit together exactly and all magnetized by common coils to provide a field of 1.47 T. The useful aperture is 8 cm (vertically) $\times$ 12 cm (horizontally), the mean radius 7 m, and the central momentum 3.094 GeV/c. The vertically focusing electric quadrupoles are situated inside a vacuum chamber in the magnet aperture and occupy 0.72 of the total circumference. This combination of electric and magnetic fields is approximately equivalent to a magnet with a weak focusing magnetic gradient, the effective field index being

$$n_{\text{eff}} = \frac{2V}{E} \frac{r_0^2}{\Delta r^2}, \quad (1.8)$$

where E is the particle energy in electron volts, and V is the voltage applied to the side electrodes relative to the centre of the aperture, which is $2\Delta r$ in width. In our apparatus $r_0 = 7$ m, $\Delta r = 6$ cm, $E \approx 3$ GeV, and so voltages of the order of 10 kV are required to give $n_{\text{eff}} = 0.1$. In practice, somewhat higher voltages are needed because the electrodes do not extend all the way round the ring.

The motion of the muon spin in the combined magnetic field $\vec{B}$ and electric field $\vec{E}$ (both perpendicular to $\vec{\beta}$) must now be considered. The angular frequency ω_a of the spin relative to the momentum vector for muons of velocity $\vec{\beta}c$ is shifted from the value $(e/mc)a\vec{B}$ [eq. (1.1)] and is given by [21]

$$\omega_a = \frac{e}{mc} \left\{ a\vec{B} - \left[a - \frac{1}{(\gamma^2 - 1)} \right] \vec{\beta} \times \vec{E} \right\}; \quad (1.9)$$

$$2\pi f_a = |\omega_a|.$$

It will be seen that in general the spin motion in the horizontal plane is influenced by the radial component of the electric quadrupole field. This varies from one side of the aperture to the other, and in general introduces a correction which depends on the position of the muon orbit. This difficulty is avoided if one chooses $\gamma = (1 + 1/a)^{1/2} = 29.304$. This so-called 'magic' value of γ corresponds to a muon momentum of 3.094 GeV/c and fixes the magnetic field at 1.47 T for an orbit radius of 7 m. The coefficient of $\vec{\beta} \times \vec{E}$ in eq. (1.9) is then zero so that the spin motion is independent of the electric field.

Clearly only muons of exactly the correct orbit radius in the uniform magnetic field can have $\gamma = \gamma_{\text{magic}}$; this is arranged to be at the centre of the aperture. The over-all shift in ω_a averaged over the aperture is then only a few parts per million and can be calculated.

The muons required are highly relativistic ($\gamma = 29.304$), which brings the added advantage that their lifetime is dilated from 2.2 μsec to 64 μsec . Since the relative precession frequency ω_a is independent of γ this means that more

periods of the $(g - 2)$ modulation can be observed and the accuracy of the measurement is improved. An accurate test of Einstein time dilation is a subsidiary result of this experiment [25].

The formulation of eq. (1.7) neglects the possibility of an electric dipole moment (edm) for the muon. If the muon possessed such a moment equal to $f(\hbar/2)(e/2mc)$, the motional electric field induced in the muon rest frame $\vec{E}^* = \gamma\vec{\beta} \times \vec{B}$ would add an extra precession of the spin about an axis perpendicular to $\vec{B}$. The result is that the plane of precession is no longer horizontal but tilted at an angle δ given by $\tan \delta = \beta f/2\alpha$, and the precession frequency is increased by the factor $\sec \delta \sim (1 + \delta^2/2)$. This tilt gives rise to an oscillating vertical component of the muon polarization [26], and may be detected by recording separately the electrons which strike the counters above and below the mid-plane of the ring. An experiment to measure the muon edm [27] by means of this effect was carried out in parallel with the $(g - 2)$ measurement. The results and the extent to which an edm would affect the $(g - 2)$ frequency are discussed below.

2. THEORETICAL PREDICTIONS

In parallel with the experimental programmes to measure α_e and α_μ , many theorists have contributed to the calculation of the theoretical values [28], and in so doing have developed new techniques for QED calculations. As this work has recently been summarized in an excellent review article [29], we refer here only to the more recent papers which influence the current predictions.

The main contribution to α_μ^{th} comes from QED and is expressed as a series in (α/π) :

$$\begin{aligned} \alpha_\mu^{\text{th}} (\text{QED}) = & 0.5(\alpha/\pi) + 0.76578(\alpha/\pi)^2 + 24.45(\alpha/\pi)^3 + 135(\alpha/\pi)^4 \\ & + 420(\alpha/\pi)^5 + \dots, \end{aligned} \quad (2.1)$$

where α is the fine structure constant.

Whereas the first two terms are obtained exactly from analytical results, in the sixth order (α^3) there remain some diagrams which must be calculated by

numerical integration [30,31] implying a small residual error in the coefficient. The α^4 and α^5 values are estimates [32,33] obtained by inserting electron loops in the sixth-order diagrams. Taking [34] $\alpha^{-1} = 137.035987(29)$ yields the value of $\alpha_\mu^{\text{th}}(\text{QED})$ given in table 1.

A further contribution comes from the hadronic vacuum polarization in the electromagnetic field around the muon. The virtual photon propagators are modified by the creation of hadron pairs and 1^- resonant states. Fortunately the coupling of hadrons to virtual photons has been measured in e^+e^- colliding beam experiments, and from the cross-section $\sigma_{e^+e^- \rightarrow \text{hadrons}}(t)$ through one virtual photon the hadronic contribution to α_μ^{th} can be calculated by dispersion theory

$$\alpha_\mu^{\text{th}}(\text{hadrons}) = \frac{m_\mu^2}{4\pi^3} \int_{4m_\pi^2}^{\infty} \sigma_{e^+e^- \rightarrow \text{hadrons}}(t) K(t) dt, \quad (2.2)$$

where $t = (\text{total energy})^2$ and

$$K(t) = \int_0^1 \frac{x^2(1-x)dx}{m_\mu^2 x + t(1-x)} \rightarrow \frac{1}{3t} \text{ at large } t. \quad (2.3)$$

The main error in the calculation comes from the imperfect knowledge of the cross-section between the $\pi^+\pi^-$ threshold (280 MeV) and the ρ resonance (760 MeV), and also the continuous background between resonances. The most recent calculations [35,36] based on the latest experimental data are consistent with the value shown in table 1, which also includes an estimate of higher-order effects [36].

The renormalizable gauge theories of weak interactions give a specific contribution [37] to α_μ^{th} , arising from the virtual emission of massive weak bosons. The value given in table 1 corresponds to the prediction of the Weinberg-Salam [38] model.

Adding these known effects gives the theoretical prediction for α_μ also shown in table 1. As mentioned above, this number is sensitive to departures from the rules of QED at high q^2 , to unknown interactions of the muon, and to a

variety of other possibilities. The confrontation of the experimental and theoretical values therefore allows limits to be set to a range of theoretical speculations concerning the electromagnetic, strong, and weak interactions. These will be discussed in further detail in Section 7.

3. REALIZATION OF THE EXPERIMENT

We have seen that the present ($g - 2$) experiment consists of trapping a polarized muon sample with the magic momentum (3.094 GeV/c), such that it circulates in a uniform magnetic field. This sample decays with the time-dilated muon lifetime, and the evolution of the longitudinal component of the polarization can be observed by recording the arrival time of high-energy decay electrons. Such a selection results in a time spectrum of counts which is modulated at the relative spin precession ($g - 2$) frequency.

In this section we will discuss the various components of the apparatus and how they were organized, firstly to achieve the aim of storing a large and stable muon population with high initial polarization, and secondly to detect and record the ($g - 2$) signal. This discussion is organized to cover the basic elements of the experiment which are: a pion beam and inflector; a uniform magnetic field region; electric quadrupole focusing; electron detectors; accurate electronic timing.

General views of the storage ring are shown in figs. 2 and 3, and the various experimental parameters which have been mentioned are listed in table 2 together with others which we will discuss below.

3.1 The beam and inflector

The stored muon intensity and the initial polarization both depend critically upon the quality of the pion beam and the successful operation of the pulsed inflector. The aim was to bring pions, of slightly higher momentum (3.11 GeV/c) than the central value for the ring, into the storage region and as nearly tangential to it as possible.

To create the pions, fast ejected protons (1 to 4 RF bunches per pulse, bunch width 10 nsec, spacing 105 nsec) from the CERN PS were guided to and focused upon a 150 mm long copper target. The use of an external target allowed pions to be collected over ± 11.5 mrad horizontally and ± 21 mrad vertically and transformed into an almost parallel beam before passing into the pulsed coaxial inflector which brought the particles to the injection point adjacent to the muon storage region. At this point the beam has a full width of 14 mm horizontally and 40 mm vertically, while its divergence is ± 6 mrad in the horizontal plane and ± 2.8 mrad in the vertical plane. The beam-line elements were tuned to the momentum of 3.11 GeV/c with an acceptance $\Delta p/p$ of $\pm 0.75\%$ (± 23 MeV/c) and the emittance of the beam was matched to the acceptance of the ring.

The secondary beam profile was monitored with multiwire proportional chambers (MWPCs) and the absolute intensity of each pulse of particles was measured by means of a calibrated ion chamber. The positions of these diagnostic devices are indicated in fig. 1. The production of particles from the target was also monitored, but this was done with an uncalibrated ion chamber. The ion chamber readings were recorded for each PS pulse, as was the size of the signal from an electrostatic pick-up device placed in the proton beam. This latter gave a measure of the proton beam intensity, although its primary purpose was to provide a reference pulse to initiate the timing sequence for the muon decays.

At the calibrated ion chamber the intensity of the secondary beam was measured to be 2.8×10^6 positive particles per extracted RF bunch, for an incident momentum of 22 GeV/c at the target and a total circulating current of 1.8×10^{12} protons (20 RF bunches) in the PS. For the negative polarity the intensity with the same PS conditions was 1.8×10^6 particles.

The positron-to-pion ratio in the positive beam was measured with a differential Čerenkov detector and found to be 19% for the 150 mm copper target. This measurement can be combined with theoretical estimates, based on the thermodynamical

model, to give the approximate particle compositions for beams of both polarities. In the positive case these are: 65% pions; 22% protons; 12.5% positrons; 0.5% kaons, while in the negative case the beam is dominantly pions (78%) and electrons (22%) with kaons and antiprotons at a negligible level. We shall discuss below the extent to which the presence of particles other than pions in the beam interfered with the experimental measurements.

The role of the pulsed inflector was to cancel the effect of the storage ring magnets on the incoming beam, and this requirement had to be balanced against the need to keep the storage volume free from perturbing fields.

A computer simulation of the injection had established that the stored muon intensity fell off sharply as the position at which the pions left the inflector was moved away from the storage region. This program was also used to find the optimum dimensions and direction of the pion beam at the injection point. Part of the problem in this optimization was to squeeze the pion beam into the storage aperture within the small azimuthal region left free of focusing electrodes.

The great technical difficulty of this method of injection is outweighed by the increased pion flux and the high initial longitudinal polarization of the stored muons. The constraints enforced by the momentum and emittance of the beam and the acceptance of the ring meant that the polarization was approximately 95%, and this was borne out by the observed magnitude of the modulation (asymmetry) of the decay electron spectra. This high polarization was independent of the muon equilibrium radius; the mean initial phase of the muon spin only varied by about 5 mrad across the aperture, and consequently any possible asymmetric muon losses could cause no significant shift in the measured spin precession frequency ω_a .

The inflector is shown in fig. 4; it is essentially a pulsed coaxial line with the pion beam passing down a channel between inner and outer conductors. Also shown in fig. 4 is a schematic of one of the crowbar discharge circuits. A parallel arrangement of 16 of these units provides the inflector with a current pulse which rises to a peak value of 300 kA in a time of 12 μ sec. At this peak

value the resulting field in the inflector channel is equal and opposite to the main storage ring field of approximately 1.5 T, and thus the pion beam passes through undeflected. The pulse-to-pulse jitter of the maximum value of the inflector current was of the order of one part in a thousand.

3.2 The storage ring magnet

For a given momentum the maximum accuracy in ω_α is obtained by working at the highest magnetic field. The actual field of 1.47 T was chosen to meet this requirement at reasonable cost, without jeopardizing good field uniformity.

As can be seen from figs. 1, 2, and 3, the 40 C-shaped bending magnets, each about 1 m long, fit together to form a regular polygon with open sides facing the centre of the ring. The pole gap is 14 cm high and 38 cm wide. Each magnet is individually supported on a ring-shaped concrete foundation.

The four big concentric coils of the storage ring are connected in series, and the 2 MW power was provided by a conventional rectifier unit with active filter, the current being stable to five parts in 10^5 . The water-cooled copper windings were split into five sections per coil, the sections being supported independently of the magnets and connected by means of flexible junctions, which allowed for thermal expansion. The temperature of each junction was monitored in a comprehensive security system which also watched over the temperature and pressure of the cooling water. The total weight of the iron yoke was about 480 tons and that of the copper coils about 30 tons.

In the preparation of the magnets, which is described in greater detail elsewhere [39-41], the aim was to provide a field as nearly uniform as possible so that the ω_α of all the muons would be the same. Because the muons sample the field all the way round the ring, this requirement essentially meant that, after averaging in azimuth, the field should be independent of radius. A further aim in the construction of the storage ring was that the field should be stable and reproducible to a few parts per million (ppm).

Studies of reproducibility and uniformity were made on a half-scale model of two adjacent magnets and, on the basis of these, an initial pole profile was designed. In the second stage of the development of the magnets a full-scale sector of four blocks was constructed. This system was first used to define an improved pole profile for the remainder of the blocks and also to study the placing of iron shunts on the yoke in order to reduce the azimuthal variations of the field. Subsequently, each of the 40 blocks was placed in this bank of four and shimmed by grinding iron from the pole faces with a specially developed machine [40]. The field in the magnet gap was measured, and the amount of iron to be removed in order to minimize the deviations from uniformity was calculated in a straightforward way, neglecting saturation and treating the shims as combinations of currents. This approximate method worked very well, and the calculated effect of cutting shims agreed with the measured changes in the magnetic field at the level of one part in 10^6 of the full field.

The final grinding of the magnets and adjustment of the iron shunts were carried out after the construction of the complete ring. The success of this procedure is indicated by the vertical field map shown in fig. 5. This map is a contour line plot for the storage aperture obtained by averaging a three-dimensional map in azimuth. The measured points lie on a square grid of 1 cm spacing, and the lines, which connect up points of equal field strength, are obtained by interpolation. The interval between contours is 2 ppm, which is equivalent to about 3 μ T. These measurements were carried out by a specially developed [42] nuclear magnetic resonance (NMR) system which will be described below.

3.2.1 Stabilization of the magnetic field

To ensure the stability and reproducibility of the field, each of the 40 magnet blocks was stabilized separately [42] by a control system which had an NMR probe and a pick-up coil as its sensor. The signals from these devices were used to determine the current automatically through additional compensating coils which were wound around the yoke of individual magnets close to each pole tip. The block diagram of the stabilization system for one magnet is shown in fig. 6. The

NMR probe contained a flat coil for modulating the field in a small plastic tube, which was shrunk around a cylindrical RF coil of 2 mm diameter and 4 mm length and which contained a 0.1 molar solution of NiSO_4 . The applied field modulation (B_{mod}) was a symmetric 30 Hz triangular waveform with a peak-to-peak amplitude of 0.8 mT, and the system operation was such that the proton resonance frequency for $B_{\text{mod}} = 0$ equalled the frequency of a crystal oscillator. The pick-up coil had 100 turns of 0.22 m² area and was placed on the pole face between the edge shims. The signal from the pick-up coil dominated the response to changes in the magnetic field at frequencies above 1 Hz, while the NMR probe counteracted the slower drifts. By using eight different crystal frequencies for the NMR control probes, making special local shimming at the probe positions and adjusting the iron shunts, the mean fields in all blocks were as closely equalized as possible. This was done in order to minimize all compensating currents; furthermore, during the operation of the ring, the current through the main coils was adjusted automatically so as to keep the average of the 40 compensating currents close to zero.

To bring the magnet into the same operating condition, a special switching-on procedure was adopted; without it the field shape would not have been reproducible, and although the field values at the 40 locations of the fixed frequency stabilizer probes were the same, temporary eddy currents or hysteresis in the yoke could so modify the distribution of magnetization in the iron that the over-all average field in the storage region could change by as much as 50 ppm. The procedure consisted of three rapid (50 A/sec) up and down cycles of the main current to a value some 10% higher than the operating point ($I_0 \sim 4.5$ kA), followed by slow (1 A/sec) oscillations about I_0 of a few percent in amplitude and gradually decreasing. This cycling of the storage ring magnets was controlled automatically and took about half an hour to complete, at which point the stabilization system automatically switched on.

3.2.2 Mapping and monitoring the magnetic field

Between groups of data-taking runs, the whole vacuum chamber was removed and a full map made of the magnetic field with a system of eight NMR probes mounted

on a measuring machine driven around the ring under computer control. The probes were connected to automatic magnetometers, and the measurements could be made and recorded whilst the machine was moving at speeds of up to 6 cm/sec without significantly distorting the map. About 250,000 points were measured throughout the storage volume on a mesh 1 cm vertical by 1 cm radial by 2 cm azimuthal. Because the magnet had to be switched off when mounting and dismounting the vacuum chamber, careful tests had to be carried out on the reproducibility of the field. As a result of the cycling procedure described above, the field in the magnet gap followed a predictable pattern; the average value rose by some 5 ppm over the first two days after switching on and then remained constant to within ± 1 ppm. Thus the full maps of the magnetic field truly reflected the field during the data-taking, subject to some small systematic effects such as the influence of the vacuum chamber and the electrodes. The influence of these materials was measured with an NMR probe mounted on a long arm, over which each vacuum chamber section could be moved. With the probe in a fixed position in the magnetic field the effects of different parts of the chamber and electrodes were readily detected. The resulting over-all contribution to the average field value was less than one part per million.

During the data-taking runs the magnetic field was monitored in 37 of the magnets. This was done with small NMR probes [42] which could be driven into the muon storage region along a radial line in the median plane without breaking the vacuum. The 37 probes were connected, via multiplexers, to the same eight magnetometers used in the field mapping. The whole procedure was controlled by a computer program and only took 10 minutes. The 400 points at which these plunging probe measurements were made were located within the coordinate system of the full field map by survey and the relationship between the two sets of readings determined. This allowed the drift in the mean field to be followed throughout the periods between full-scale maps.

The probes used in mapping, monitoring, and stabilizing the magnetic field were calibrated with respect to a special probe of well-defined geometry and

susceptibility correction [43]. The cylindrical water (+0.1 molar NiSO_4) sample of this probe was perpendicular to the magnetic field and long enough to approximate well the known demagnetization factor of an infinitely long transverse cylinder. The effective free proton resonance frequency for the calibration probe was obtained by applying the standard corrections for diamagnetic shielding and paramagnetic Ni^{++} ions as well as the correction for the demagnetization due to specimen shape. Further steps in the calibration procedure included measuring the small effect of the material of the mapping machine on the field at the position of the mapping probes; both the direct effect and that due to the influence on the stabilization system were examined.

The crystal clock which was used to measure the proton resonance frequencies was also used to time the muon decays. As the experimental value of the anomaly depends upon the ratio of these two measurements, the effect of slow drift in the clock frequency was largely cancelled out.

The radial component of the magnetic field was measured with a Hall-plate pendulum^{*)} and found to be at the level of 10^{-4} T, sufficient to cause the median plane of the muon orbits to undulate with respect to the geometrically central plane of the storage region over a range of about 2 mm. This shift was confirmed by measurements on the muon distribution, as discussed below.

3.3 Electric quadrupole focusing and vacuum system

The electric quadrupoles [44] were constructed in eight sectors, each one covering four magnets. Each unit consisted of a section of aluminium vacuum chamber complete with titanium electrodes mounted inside. These latter were held in position by stand-off insulators of sintered alumina, which material was also used in the electrical feedthrough. The eight units were arranged around the ring in two groups of four, leaving two symmetrical regions without focusing electrodes. One of these regions was necessary for pion injection, while the other was included

*) We are indebted to D. Lehm for this measurement.

largely to minimize the closed orbit distortion. The cross-section of the quadrupole and vacuum chamber is shown in fig. 7. The vacuum, which was normally better than 10^{-7} Torr, was maintained by ion getter pumps, thereby avoiding any deterioration of the high-voltage behaviour of the electrodes due to oil vapour. The gas load of the large number of insulators and feedthroughs was kept small by using indium seals throughout. The plunging probes were driven in via an intermediate vacuum, which meant that the main pressure increased to only about 10^{-6} Torr during their operation.

The vacuum chamber without electrodes, situated on the opposite side of the ring from the inflector, was of a special construction with a thin inner window (0.8 mm Al). This chamber gave much more direct access to the stored muons in the sense that the decay electron trajectories and energies were much less modified by bremsstrahlung and scattering processes than in the case of the normal chamber with electrodes.

The shape of the electrodes approximates to the ideal case of the pair of hyperbolae which would be required to give a perfect quadrupole field, and was calculated by successive approximations to minimize the variations in electric field gradient throughout the storage region. The presence of the vacuum chamber affects the electric field, and so for a given geometry the uniformity of the field gradient depends upon the chosen ratio of potentials applied to the two pairs of electrodes. The ratio which minimized the contribution of other multipoles, in particular the octupole, was found by calculation and later adjusted by measuring the muon losses. The absolute value of the applied potential which determined the field gradient or field index for the storage ring, was also determined in the study of muon losses which is described below.

It had previously been demonstrated in a test electrode system that it was not possible to use steady electric fields for both muon polarities [44] because trapped low-energy electrons produced too much ionization in the residual gas.

These low-energy electrons were trapped, vertically, by the electric potential, and while spiralling up and down the magnetic field lines they slowly drifted

along a side electrode in the direction $\vec{E} \times \vec{B}$. At the end of the focusing section the field shape was such that they passed around the electrode and returned along the reverse side. During these slow circuits around the electrode the trapped electrons, with mean energy of 1 keV, built up the ionization of the rest gas, which eventually caused a voltage breakdown even if the vacuum were better than 10^{-6} Torr. The finite time required for this process meant that the danger could be greatly reduced by operating in a pulsed mode.

Voltages in the form of flat-topped pulses, about 1 msec long, were applied to the electrodes by means of an arrangement of spark gaps. Two sets of electrodes were fed from a simple pulse box which contained a 10 nF storage capacitor. These latter were originally made up from 2.7 nF (40 kV) ceramic capacitors, but after some time the mechanical shocks accompanying each pulse loosened the contacts of the individual capacitors; they were therefore replaced by a single capacitor consisting of a roll of cable (110 m of RG213/u).

At the beginning of the experiment the low-voltage pulser was made with spark gaps which could not be regulated below ± 5 kV. This was too high to minimize the higher multipole field components, but the resulting muon losses were too low to affect the $(g - 2)$ frequency. For the lifetime measurements, however, this mode of operation was inadequate, and the spark gaps were replaced by cascaded thyristors ($5 \times$ BTW47/1600M), which allowed the true optimum level to be found and the muon losses to be minimized.

The operation of the full electrode system of the storage ring brought with it some problems which were not present in the test set-up. For the μ^+ polarity, both high and low voltages could be switched on together before the injection of pions and switched off together some 700 μ sec later, while for the μ^- polarity breakdown ensued unless the low-voltage crowbar was delayed by 16-32 μ sec after the high-voltage pulse had been terminated.

3.3.1 Scraping

For the measurement of the muon lifetime it was essential to reduce the late time muon losses to a minimum. This was done by shifting the muon orbits at early

time in order to "scrape off" those muons most likely to be lost. The orbits were shifted by applying voltages asymmetrically to each pair of electrodes. While one member of the pair was pulsed normally with 1 μ sec rise-time, the other was fed with a pulse which had a 10 μ sec rise-time. For the side electrodes the asymmetry of the voltages was reversed between the two groups of focusing units on either side of the ring. Thus the muon orbits were shifted radially outwards on one side of the ring and radially inwards on the other; to first order they were just shifted sideways with their shape remaining the same. The bottom electrodes, all the way round the ring, received the slower pulse, and so the median plane of the muon orbits was initially low and returned to its normal position with the 10 μ sec time constant. The pulses were switched on about 6 μ sec before the arrival of the pion bunch. The whole question of particle orbits, muon losses, and the effect of scraping is discussed more fully in Section 4.

3.4 Detectors

The decay electrons were detected by a system of 22 shower counters spaced around the inside of the ring. Each counter was made up of 13 sheets of plastic scintillator, 1 cm thick, interspersed with 0.5 cm thick sheets of lead. This sandwich was viewed, via an air light-guide, by a 5-inch photomultiplier (58 AVP). The energy resolution of the counters was measured in an electron beam and found to follow the approximate expression $\Delta E/E \sim 0.3/\sqrt{E}$ (FWHM), where E is the electron energy in GeV. The linearity of the pulse height as a function of electron energy was also checked and an approximate energy calibration obtained. It should be emphasized, however, that these tests were carried out in very clean conditions compared with the actual experimental environment of the counter. For example, the large initial flash of particles when the beam arrived at the storage ring clearly perturbed the detectors. To reduce this effect and to protect the fast electronics from large pulses, the photomultipliers were blanked off during this period. This was done by suppressing the electron avalanche in the tube by temporarily connecting dynode 9 to dynode 7. In spite of this precaution the system still exhibited small gain changes during the muon storage time. These effects and their measurement are discussed below.

As mentioned above, in parallel with the main ($g - 2$) experiment the storage ring was also used for the subsidiary experiments of measuring the muon electric dipole moment (edm) and lifetime. For the edm measurement [27], five of the electron detectors were divided into upper and lower halves by placing pairs of scintillation counters in front. These extra counters were made of 6 mm thick plastic scintillator, and the signals from them were used to label each decay electron, detected by the shower counter, as being above or below the median plane.

In the measurement of the lifetime [25] it was important to carefully examine the late-time loss of muons. Such lost muons tend to emerge on the inside of the ring because their energy is degraded in collisions. To monitor these particles, a detector was placed against the vacuum chamber in a position equivalent to that of the decay electron detectors. The muon detector (see fig. 8) consisted of a series of four scintillation counters, each preceded by lead converters, two radiation lengths thick. Pulse-height discrimination was used to distinguish muons from electrons. Measurements in an electron beam indicated that this method rejected electrons by a factor of 700 or so. However, the normal flux of decay electrons so outnumbered the lost muons that even this rejection was inadequate, and the muon detector had to be supplemented with a five-gap optical spark chamber to improve muon identification.

3.5 Fast electronics, digitron, and on-line system

The unique and most demanding requirement imposed on the electronics in this experiment was the necessary ability to measure precisely a relatively large (~ 100) number of time intervals associated with events occurring within a rather short period (~ 100 μ sec). On the other hand, the total amount of data acquired in one PS cycle was rather low, typically 100-200 events, each event consisting of 24 bits of information. These 24 bits contained 16 bits of time information, 4 bits identifying the counter associated with a given event, and 4 bits giving the pulse-height information. In contrast to the time measurement, the required precision on the pulse-height information was much lower, with a rather rough measurement being quite satisfactory.

The required high precision of the time measurement necessitated the design and construction of special-purpose electronics, referred to by (g - 2) tradition as a digitron [45], with its own buffer memory to serve as an intermediate data storage during the short burst of data-taking. The pulse-height information was obtained by feeding the analog counter signals into several discriminators, whose thresholds were set in such a way as to span the range of decay electron energies. This solution, involving almost no dead-time, was preferred to the use of analog-to-digital converters (ADCs) since the basic requirement was not high precision but the ability to work at very high (~ 300 kHz) instantaneous rates.

For the main (g - 2) experiment there were effectively 20 sources of signals, since two of the set of 22 electron detectors were treated as extra counters and the signals from each of them added to those of another member of the set. The 20 sources were divided into five equal groups; the members of each group being spaced at approximately equal intervals around the ring.

The fast electronics associated with each digitron and its group of four counters is shown schematically in fig. 9. To overcome base-line shifts and other low-frequency effects, a 5.5 nsec clip-line was applied to the output of each counter. After attenuation the pulses were split resistively so that both timing and height information could be obtained from them (see fig. 9).

The timing signals were fed to a time-quantizer which placed the randomly occurring pulses into exact positions relative to a free-running, crystal-controlled, 100 MHz clock. This process is sometimes called derandomizing, and it ensures that in the incremental timing process no clock pulse is lost, or counted twice. All four counters in the group were fed to the same time-quantizer module, which gave out a counter label and a stop pulse. These signals, together with the pulse-height labels, were fed into the digitron, as can also be seen in fig. 9. The discriminator thresholds and the attenuator settings were chosen so as to correspond to an electron energy of approximately 700 MeV for the "time" discriminator, and 1000, 1500, 2000, 2500 MeV for the four pulse-height discriminators.

The operation of the digitron circuit, performing the actual time measurement, has been described in detail elsewhere [45]. Each digitron has four 16-bit scalers and four 8-bit latches controlled by a recirculating 4-bit shift register initially loaded with a 1000-bit pattern. Upon receipt of a start signal provided by the electrostatic pick-up in the external proton beam, the first scaler (i.e. the one corresponding to the first bit in the recirculating register) starts counting the 100 MHz crystal oscillator pulses. A muon decay, giving rise to an electron with a measured energy exceeding the threshold of the "time" discriminator, will now generate the following sequence of events. After passing through the time quantizer, the signal causes the recirculating register to rotate by one place, resulting in the active scaler being stopped and the next one beginning to count. Simultaneously, the four counter-label bits and the four pulse-height bits are strobed into the active latch. Then a transfer of the 16-bit incremental time information is initiated, together with the contents of the latch, into a 200-word 24-bit MOS buffer that forms an integral part of each digitron and serves as an intermediate memory buffer between the data-generating hardware and the computer.

The transfer time to the MOS buffer is 360 nsec and, as there are four scalers, two more events occurring during this time can be accepted. Thus very high instantaneous data rates can be handled. A fourth signal arriving during this 360 nsec period would be lost since it requires the first scaler to start again. These so-called queuing losses were very low in the actual experiment and their effect on the frequency measurement is well understood (see Section 5.1.1).

In this mode of operation the incremental time between stops is recorded, and the absolute time of an event is the sum of all previous recorded times. It was necessary to check that no clock pulses were lost in this process, and this was done by means of a marker pulse which arrived at a known time after the start (2^{16} counts of the 100 MHz clock). The sum of the incremental times between the electrostatic pick-up pulse and the marker pulse was checked for every storage period (PS burst). At the end of the storage period the recorded data were transferred via a CAMAC system to the PDP 11/20 computer.

In all, six digitrons were used, five in the timing of the signals from the (g - 2) counters and one for processing the signals from the edm counters and the lost-muon detector. In the edm experiment the "up" and "down" labels were derived by forming a coincidence between the shower counter and the scintillators in front. The time of the event was determined by the arrival of the stop pulse from the shower counter, thus avoiding possible systematic effects due to differences between "up" and "down" coincidence circuits. The data recorded in this sixth digitron were also transferred to the computer at the end of each storage period.

Because of the small amount of data read in each cycle, the main emphasis in designing the software was to get immediate feedback on the over-all performance of the experiment, to verify the proper functioning of the key hardware after every PS burst, and to allow the relatively flexible manipulation of previously acquired data.

The tasks performed by the on-line program fall into four different categories:

- a) acquisition of data,
- b) checks of the integrity of data,
- c) updating of histograms,
- d) responding to physicists' requests.

The normal state of the program was a dormant one: awaiting either an interrupt from the PS, or a special command from the operator. About 3 msec before ejection, a signal was sent from the PS to the computer. Following this interrupt signal, the program would zero necessary registers and go into a wait loop of 80 msec. During that time, muons were stored and decayed in the ring, and auxiliary data, like proton beam intensities, were converted to digital form. At the end of this period, the appropriate CAMAC registers contained beam intensity information, data for all six digitrons, and further bits identifying possible hardware malfunctions.

The data from each PS burst were required to pass a series of tests before they would be accepted as satisfactory:

- i) the intensity of the injected pion beam as measured in the ionization chamber had to be above a certain minimum level to avoid PS bursts with the ejection failure;
- ii) all the storage-ring hardware had to have functioned properly;
- iii) there were no faults associated with any of the digitrons. More specifically: there was no obvious malfunction; the sum of the integral times to the marker pulse was correct; and the crystal-controlled clock output missed no pulses.

The failure level due to any of the above causes was quite low, amounting to about 0.1% of all pulses, except for the electric quadrupole faults which occurred at the 1-2% level.

The data passing all the tests were written on magnetic tape for subsequent off-line analysis and were also used in a variety of histograms to monitor the performance of the apparatus. To obtain extensive histogramming capability ($\sim 100,000$ words) the histograms were kept on disk, being updated every few PS pulses from a small core buffer which stored the incremental information.

A facility was built into the software which enabled the operator to readily create, delete, or clear an arbitrary histogram, to add and subtract several of them, and to display them on a graphics terminal merely by typing a short command string. Typically, the data were accumulated in fine-grained time histograms for each counter at early times ($\leq 20 \mu\text{sec}$) to be used in cyclotron frequency analysis, as well as coarser-grained time histograms over the full time-scale to look at the $(g - 2)$ modulation.

As well as the main $(g - 2)$ program, the computer was used for a digitron test routine and magnetic field measurements. The digitron test program [45] carried out a thorough check of the scalers and the general operation of the

digitron. The absolute calibration and linearity of the digitrons were checked by a so-called "Synchronous Random Phase" method [46] and found to be such as to cause no significant systematic error.

The magnetic field measurements were run by two different programs: one for the plunging probe measurements, which were made about once a day during data taking, and the other for the complete field survey by the mobile mapping machine. In both cases the data were recorded on magnetic tape.

4. OPERATIONAL CONDITIONS OF THE EXPERIMENT

Having described the components of the present storage ring experiment, we will now examine the optimization of their operation and the various tests which were made to investigate possible systematic effects. In doing this we shall follow roughly the same logic as before, starting with the beams and inflector and following through to the processing of the signals.

4.1 Setting up and operation of the pion beam and inflector

In setting up the storage ring, the proton beam was first trimmed to give maximum production from the target as indicated by the uncalibrated ion chamber, then the pion beam line itself was tuned to maximize the reading of the calibrated ion chamber just upstream from the inflector. In the first experimental run, however, the pion trajectories were followed inside the storage region by means of small scintillation counters which could traverse the aperture in the median plane. The observed radial distribution of the pions at different azimuthal positions around the ring was in good agreement with the predictions of a Monte Carlo injection program. The points at which injected particles left the storage volume were also calculated with this program, and their azimuthal distribution is shown in fig. 10. To a good approximation all the particles in the beam came out in this fashion with the exception of those pions which decayed (10%). On the first complete assembly and testing of the storage ring, decay electrons from circulating muons were observed.

Once the appropriate values for the currents of the beam-line magnets had been established, setting up for the data-taking became a straightforward matter

with only fine adjustments necessary in order to obtain the maximum number of decay electron stops per PS burst. With 2.8×10^6 positive particles injected into the storage ring, some 350 muons were trapped.

As a result of the spill-out of particles, immediately following injection, the counters were subjected to a high initial flux; they were designed to detect electrons (positrons) of a few GeV energy, and hence this component of the beam gave rise to an extremely large pulse at this time. The large pulse was followed by a slowly diminishing tail of pulses consistent with the decay of muons stopped in various parts of the storage ring, including the counters themselves. Independent measurements of the neutron flux in the vicinity of the storage ring indicated that these particles were produced by the incoming beam in sufficient quantity to be a contributory factor to the tail of background pulses. By 5 μ sec after injection the background had diminished to an extremely low level ($\sim 10^{-5}$ of the initial count rate). The effects of occasional delayed ejection from the PS were suppressed by means of a pulsed magnet in the latter stage of the pion beam line, and any remaining data with spurious background spikes were rejected.

Another background arising from the particular composition of the incident beam occurred when the electric scraping was applied in the μ^+ polarity. The varying electric field meant that some particles which were scattered at the aperture limits could move onto stable orbits. Any positrons (or electrons in the μ^- polarity) did not remain trapped for long, since they lost energy by synchrotron radiation. This latter caused the orbit radius to decrease by ~ 2 mm per turn. When protons became stored, however, some remained in the ring until the focusing field was switched off 650 μ sec after injection. The protons occupied a region of phase space very close to the aperture edges, and their steady loss throughout the storage period gave a background in both the decay electron and lost muon detectors. The relative proton detection efficiency of these two systems was measured by delaying the switching off of the electric field until about 1 msec after injection. This is nearly 16 muon lifetimes, and thus only protons were left in the ring to emerge because of the lack of vertical focusing. The ratio

of counts in the two systems then gave the efficiency ratio directly. The proton background in the electron counters was in fact very low. The resulting correction to the lifetime is small; it is given in Section 6.1.2. No equivalent problem arose in the negative polarity.

4.2 Electric field, muon orbits, and field index

The orbits of the trapped muons reflected the twofold symmetry of the focusing system, in which two diametrically opposite sectors, of 36° each, were without electrodes. Although the central orbit was unaffected by this structure, the off-momentum closed orbits were distorted towards the aperture limits in the sectors without focusing electrodes. The total radial excursion of this distortion was about 2 mm for a momentum 0.5% above (or below) the central value. Thus with respect to an "ideal" ring with continuous focusing, the momentum acceptance was reduced by only 2 or 3%.

The discontinuous nature of the focusing had a much larger effect on the amplitude of the horizontal betatron motion. The over-all envelope of these oscillations had maxima in the centre of each electrode-free section and minima in the centre of each focusing section. This modulation reduced the horizontal phase space to 78% of the value that it would obtain for the "ideal" case.

The available vertical phase space was reduced by only 3 to 4%, and in this case the maxima of the envelope for the betatron oscillations were in the centre of the electrode sections.

The relative acceptance of the storage ring with respect to the "ideal" case diminished as the average field index increased, and this partly counteracted the normal trend of the absolute acceptance which increases with the field index. Thus the over-all acceptance of the muon storage ring was a fairly flat function of field index, as can be seen in fig. 11. The choice of field index or focusing strength was then not a matter of intensity but one of orbit stability and late-time muon losses.

The establishment of the stored muon population involved considerable losses at early time. Most muons were borne onto orbits with betatron amplitudes large

enough to intersect the electrodes or other obstructions and were thereby removed. The longest time involved in the betatron oscillations ($\sim 2.5 \mu\text{sec}$) is the orbit precession period in the horizontal plane, but losses were seen to occur at much later times. This phenomenon is at least partly due to the complicated geometry of the obstructions (e.g. the shape of the focusing electrodes) and the azimuthal variation in the betatron oscillation envelope mentioned above. In the absence of scraping, the amount of loss after $32 \mu\text{sec}$ was typically about 3%, which corresponds to the fraction of phase space occupied by orbits approaching within about 0.5 mm from the nominal aperture limit. The actual limit is not sharply defined all the way round the ring, and the transverse motions of the particles are not simple Lissajou figures; thus it is not unreasonable that a muon can describe many turns before it arrives at an obstruction.

Another mechanism for particle losses in cyclic machines is that of non-linear resonances [47]. Field imperfections drive resonances in the betatron motion; this causes the amplitude of these oscillations to breathe, or to grow continually, depending on the proximity to the resonant conditions. In either case particles will be lost. The further away the field index is from the resonance value, the shorter is the period of the breathing and the quicker the associated losses are completed. There are essentially two steps in the minimization of these losses: one is to choose a field index value well away from dangerous resonances; and the other is to ensure that the harmonic components of the field which drive the nearest resonances are as small as possible. Within the operating range of the muon storage ring the two broadest resonance-free regions were centred on the field index values of 0.135 and 0.185. The lower region was chosen by virtue of its lower operating voltage for the electrodes. Test runs were made at different voltages and for different voltage ratios between the two pairs of electrodes of the quadrupole. The equivalent field index range was 0.1 to 0.16. The muon losses were monitored with the detector described in Section 3; and during this exploration of the region of stable operation, conditions were found under which large late-time losses were observed. These late losses were possibly due

to the second azimuthal harmonic of the octupole component of the field. For non-ideal ratios of the electrode voltages there was an octupole component in the electric field which had a strong second harmonic due to the azimuthal arrangement of the electrodes. For minimum losses the optimum voltages were found to be 24 kV for the side electrodes and 3.7 kV for the top and bottom electrodes.

The effect of the electric scraping on the muon orbits derives principally from two sources: one is the displacement of the orbit due to the superposition of a dipole electric field, and the other is the changing of the focusing strength. There are, of course, higher-order effects in the electric field, but these two, due to dipole and quadrupole components, dominate. A two-dimensional plot of the distorted equipotentials at the time of injection is shown in fig. 12.

In the vertical direction the median plane is some 3 mm low at the start and returns to the central position with a time constant of 10 μ sec. During this period the amplitude of the vertical oscillations contracts owing to the increasing strength of the focusing. The combination of these effects means that the vertical motion of the particles is ultimately clear of the aperture limits by approximately 6 mm. This picture is complicated by the presence of a small radial component of the magnetic field and the consequent distortions of the median plane. Thus the final clearance between the stored muon orbits and the vertical aperture limits is not uniform in azimuth.

The effect of scraping in the horizontal direction is influenced by the large modulation of the envelope of the betatron oscillation which has been discussed above. A simplified calculation indicates that the central closed orbit should be shifted sideways by about 12 mm. This is of the same order as the horizontal shift in the centre of the distorted quadrupole field shown in fig. 12, but the orbit calculation has to include the fact that the shift is inwards on one side of the ring and outwards on the other and that there are two electric field-free sections. It is important to note, however, that this excursion is maximum in the centre of the electrodes section where the betatron amplitude envelope is

minimum. The waist in this envelope function is also about 10 mm and so the extent of the scraping in the horizontal direction is considerably lessened. For the horizontal case the focusing is slightly stronger at injection than it is later, and so the betatron amplitudes increase slightly as the closed orbits return to their normal positions. The net outcome of these effects is that the narrowing of the muon population is much less than the shift in orbits would imply, and this is supported by the fast rotation analysis, which indicated an equivalent ideal phase-space horizontal aperture of ± 55 mm without scraping and ± 50.5 mm with.

4.3 Performance of the counters

The influence of the operating environment on the performance of the detectors was carefully checked. The magnetic field inside the iron counter housings was only 5 μ T and had a negligible effect on the tubes. This low value was due largely to the inner pair of coils on the storage ring, which effectively decoupled the magnet field from any material mounted in the central area of the ring.

The largest perturbation in the performance of the counters arose from the initial flash of particles which struck them every time the beam was injected into the storage ring. Blanking off the phototubes during the injection period greatly reduced the effect but did not completely remove the gain changes during the muon storage time. Such gain variations led to changes in the energy acceptance of the detection system as a function of the time elapsed after injection. The resulting modification to the counting rate was very gradual compared to the 4.3 μ sec period of the $(g - 2)$ modulation, and consequently this effect was less important for the $(g - 2)$ frequency than for the measurement of the muon lifetime. The crucial point as far as the former is concerned is that any instrumental time measurement error should be independent of the elapsed time. Such time-slewing may be partly due to the above-mentioned variations in the system gain, but can also follow from changes in the propagation time of pulses in the photomultiplier and fast electronics due to sagging voltages and paralysis effects which follow from the enormous instantaneous counting rates experienced during and just after the initial flash.

The gain changes and time-slewing were both measured in an experiment using light-emitting diodes (LED). The former were found to be at the 1-2% level, and the results were used to apply a small gain correction in the data analysis, as described in Section 5, while the effect of time-slewing was shown to be negligible. These measurements were carried out with the injection and beam conditions as for normal data-taking.

The diodes were mounted in the wall of the air light-guide of each shower detector and were driven by avalanche pulse generators. A small fraction of the output pulse from each generator was fed directly to the timing discriminator to give the reference time for the time-slewing measurement. The time interval between this reference signal and the arrival of the LED pulse, which had been propagated through the photomultiplier and fast electronics, was extracted off-line from the recorded data. It was found to be constant for all counters to within ± 0.1 nsec for elapsed times greater than 20 μ sec after the initial flash.

During the LED experiment the gain for all counters and associated electronics was sampled throughout the muon storage period by triggering the avalanche pulse generators at some eleven fixed times between 7 and 580 μ sec after injection. The signals from the LEDs were distinguished from the decay electron counts by their fixed timing pattern.

The measurement of the relative gain was achieved by adjusting the size of the avalanche generator output so that all counter pulses, which originated from LED signals, were above the threshold of the timing discriminator, while only half of them exceeded a second threshold 3 dB higher. In this arrangement the upper threshold is near the maximum of the LED pulse-height spectrum where the sensitivity to gain changes is the greatest. The absolute sensitivity depends upon the shape of the LED pulse-height spectrum and the characteristics of the discriminator. The relationship between the system gain and the ratio of count rates from the two discriminators was found in a separate calibration experiment. In practice, all four pulse thresholds, arranged in 0.5 dB steps, were used to cover the maximum in the LED pulse-height spectrum thus allowing the gain change to be measured simultaneously at two or more levels -- a useful internal consistency check.

The LED experiment was repeated several times during the total data-taking period of the $(g - 2)$ frequency measurement, and the gain curve for each counter was obtained with a precision of about 0.1% per point. The magnitude of the variations clearly depended upon the size of the initial flash seen by the counter, but typically the relative gain rose by 2% or 3% during the first 100 μsec after injection and then fell throughout the remainder of the storage period by about 1%. A more complete discussion of the precision and reproducibility of the measurements is given in Section 6.1.

4.4 Optimization of the $(g - 2)$ signal

The energy thresholds for the timing and pulse-height discriminators were chosen to optimize the information on the $(g - 2)$ frequency. The statistical error in the frequency is inversely proportional to the asymmetry (A) of the modulation in the decay electron time spectrum multiplied by the square root of the number of decay electrons detected (N) [see eq. (1.4)]. Thus for a single threshold, the value for which NA^2 is maximum should be chosen. However, the variation in A and N with threshold means that the error in the frequency can be reduced by about 10% if the same data are broken up into several independently analysed pulse-height bands and the weighted average of the frequencies is taken. In this case the thresholds have to be set so as to give the maximum sum of NA^2 for all the bands.

In principle the best values can be calculated from the approximate energy calibration of the counters in the test beam and from the predicted asymmetries and relative intensities for different electron energies. However, this simple picture is modified by the finite resolution of the detectors, the change in system gain between quiescent (test beam environment) and operational conditions, and energy loss of the decay electrons in the titanium electrodes and aluminium vacuum chamber walls. The net result of these effects is a migration of high asymmetry and intensity from the high pulse-height to the low pulse-height bands. This means that thresholds in the experiment were set lower than in the ideal case. For example, in the latter a lowest threshold of about 1500 MeV would be optimum, while

in the experiment the best value was about 700 MeV. Analysis of the data taken in the earlier runs for various threshold settings led to the adoption of the relative attenuations 3 dB, 7 dB, 9 dB, and 11 dB for the pulse-height levels with respect to the timing threshold. The absolute value of the thresholds was maintained during the data collecting by adjusting the counter high voltages so that the numbers of counts in the top two pulse-height bands were in the ratio 2:1. This gave a good working rule whereby the settings were kept at their optimum values.

5. DATA ANALYSIS

The raw data for each detected decay electron consisted of a recorded time in bins of 10 nsec within the range 0 to 655.35 μ sec, and labels for counter and pulse height. In the first step of the off-line analysis these are converted to a three-dimensional histogram HIST(T,C,P): T(1-4096) labels the time bin which has a width of either 160 nsec for the (g - 2) frequency analysis (Section 5.1) or 10 nsec for the analysis of the radial distribution of the muons (Section 5.2); C(1-20) and P(1-5) label the struck counter and pulse-height bands, respectively. The events are checked for consistency of counter and pulse-height information and are only added to this histogram provided

- i) one and only one counter label is set;
- ii) the pulse-height information is consistent. If the discriminator corresponding to a given pulse-height band fires, so must the discriminators of all lower energy bands (see fig. 9, showing the organization of the fast electronics).

Condition (i) rejects about 10^{-3} of the events and is at the level expected from accidental coincidences, while (ii) rejects only 3×10^{-4} of the events. These requirements are the only off-line cuts made on the data, except for selecting time and pulse-height ranges for fits. Typically one histogram is made for each experimental run and written on a data summary tape. The histograms for equivalent runs are then added before final fitting.

5.1 Maximum likelihood fit to the (g - 2) frequency

The time spectrum of the decay electrons is fitted using the function minimization program MINUIT [48]. Denoting the observed spectrum by $H(t_i)$ and the fitted function by $N(t_i)$, the likelihood function L is given by assuming Poissonian errors for $H(t_i)$, by:

$$L = \prod_i \exp [-N(t_i)] \frac{[N(t_i)]^{H(t_i)}}{H(t_i)!} . \quad (5.1)$$

The function actually minimized by MINUIT is

$$\ln L = \sum_i [H(t_i) \ln N(t_i) - N(t_i)] , \quad (5.2)$$

where the constant term $\sum_i \ln H(t_i)$ is dropped and $N(t_i)$ is taken as

$$N(t_i) = N_0 \left\{ L(t_i) \left[\exp (-t_i/\tau) \right] \left[1 - A \cos (\omega_a t + \phi) \right] + B \right\} . \quad (5.3)$$

The function $L(t_i)$ is an empirical correction factor to allow for the effect of muon losses and gain changes at early times. The "ideal" distribution would have $L(t_i) = 1$, and $B = 0$. $L(t_i)$ is parametrized in the form

$$L(t_i) = 1 + A_L \left[\exp (-t_i/\tau_L) \right] . \quad (5.4)$$

The total number of parameters used in the fits is then eight: five (N_0 , τ , A , ω_a , ϕ) from the "ideal" distribution plus three (B , A_L , τ_L). As many of the parameters in the fit are correlated with each other, some care is needed in extracting the frequency by such global fits. The only parameter strongly correlated with ω_a is ϕ (correlation coefficient ~ -0.8), and in the case where the fit to the experimental distribution is good the sensitivity to the empirical parameters B , A_L , τ_L is small. However, attempting to fit data with early-time loss or gain distortions to a simple six-parameter (N_0 , τ , A , ω_a , ϕ , B) function leads to an incorrect value of ω_a . This effect, which was noted in the previous muon (g - 2) experiment [18,49] is presented in fig. 13, where $f_a = \omega_a/2\pi$ is shown for different starting

times of the fit for six- and eight-parameter fits to a subsample of the data. For six parameters the fitted value of ω_α varies with the starting time of the fit, oscillating about the correct value as a function of this time with a frequency of approximately f_α . Fits to Monte Carlo distributions show that these oscillations are symmetrical about the correct frequency. Including A_L and τ_L in the fit removes these oscillations. However, the starting time chosen for fits, presented in table 3, was taken at a zero crossing of the oscillation (21 μ sec) so that the final results, quoted below, will be unchanged within statistical error whether the six- or the eight-parameter fit is used. The mechanism of this frequency shift is easily understood in terms of "phase pulling" of the early ($g - 2$) cycles by the excess counts at early times when the simple six-parameter function is used.

In fig. 14 the variation of f_α with the starting time of the fit is shown for the combined data. The values quoted are weighted averages for the top four pulse-height bands. The upper limit of the fit interval is 650 μ sec. The shaded area within the curves marked $+\sigma$, $-\sigma$ delimits the one standard deviation region for statistical fluctuations about the 21 μ sec starting-time value. The absence of any significant starting-time dependence of f_α indicates that the empirical description of early-time gain and muon loss effects by the chosen form for $N(t_i)$ is adequate. Similar conclusions were reached by examining the starting time and pulse-height dependence of f_α for all individual data sets used in the analysis. No effects inconsistent with statistical fluctuations were observed.

For μ^+ data with scraping the background due to protons was at the level of approximately 10^{-3} at early times but had a negligible effect on ω_α . For all the μ^- data and those for μ^+ without scraping, B was at the very low level of 2×10^{-5} .

As we have noted in the previous section, the asymmetry A varies strongly with the pulse-height band. The fitted asymmetry for decay electrons above four of the energy thresholds is shown in fig. 15 for six runs under comparable conditions. These asymmetries are plotted as a function of that fraction of the total counts which lies above the particular threshold. It should be noted that there

is excellent agreement between μ^- and μ^+ . The variance in the frequency is inversely proportional to NA^2 , and so the data in each pulse-height band were analysed separately thereby avoiding dilution of high-asymmetry data by that with low asymmetry. The data from all 22 counters were added for each pulse-height band, but in order to avoid any low-energy backgrounds and to minimize time slewing the lowest pulse-height band was discarded. The frequency values quoted below are thus the weighted averages for the top four pulse-height bands for each data set.

Figure 16 shows the values of $f_\alpha = \omega_\alpha/2\pi$ for the individual pulse-height bands in the combined data. The consistency of the four values with the weighted mean is excellent ($\chi^2 = 3.36$ for 3 degrees of freedom). The average electron energy in each band is estimated by comparing the counting rate and asymmetry with a Monte Carlo simulation.

Extensive checks of the fitting program were done using Monte Carlo data as input. In addition, a subsample of data was fitted using a simple linearized six-parameter χ^2 minimization program. The result found for ω_α was identical to that given by MINUIT. The parabolic nature of the $\ln L$ surface in the neighbourhood of the likelihood maximum has been checked by plotting out likelihood contours around the maximum. The error on the frequency calculated by MINUIT, assuming a parabolic variation, was found to agree with that given by the $\ln L_{\min} + 0.5$ contour of the exact likelihood function to within 0.8%, while the same contour was symmetrical about the maximum in the frequency to ~ 1 part in 10^4 . Before giving the values found for f_α for the nine sets of data which were analysed separately, some possible systematic corrections to the $(g - 2)$ frequency are discussed. Only one of these is found to be large enough for corrections to f_α to be necessary.

5.1.1 Systematic corrections to f_α

a) Time slewing

The time-slewing effect due to the large counting rates experienced by the counters and fast electronics after injection was directly measured using light diode pulses, as described in Section 4.3. The effect varied considerably from counter to counter, depending on the position relative to the injected beam. The

largest shift seen was about 1 nsec in the first few microseconds after injection. Fitting Monte Carlo data distorted by the measured average time-slewing function indicated a frequency shift of 0.9 ppm for a starting time of 15 μ sec and < 0.5 ppm for a starting time of 21 μ sec, which was the earliest time used in the analysis. No correction is made to the data for this effect.

b) Gain effects

These are included in the empirical function $L(t)$ for the $(g - 2)$ frequency analysis. Provided A_L , τ_L , and τ are adjusted to give a good over-all fit, ω_a is not affected since it is very weakly correlated with these parameters (see discussion in Section 5.1 above).

c) Digitron calibration and linearity

This has been extensively checked using a special test method developed specifically for this purpose [43]. Both the linearity and the absolute calibration of all digitrons used in the experiment were found to be good to a small fraction of a ppm.

d) Queuing and dead-time losses

As mentioned in Section 3.5, scalars in a single digitron cannot store more than three events at a time. This limitation results in queuing losses, which depend upon the counting rate N and the time needed to read out a stored event, i.e. the buffer transfer time ζ . For a constant counting rate the number of counts N in a given time interval has a Poisson distribution, and the modified count rate N' is given by [50]

$$N' = NR(x) , \quad (5.5)$$

where $R(x) = [x + (1/e^{2x} - xe^x)]^{-1}$, with $x = N\zeta$.

For the observed electron spectrum, however, N is not independent of time, and analytical calculation of the modified count rate is more difficult. However, the period of the $(g - 2)$ oscillations (4.3 μ sec) is considerably greater than the buffer transfer period ζ , which was 0.48 μ sec in early runs and 0.36 μ sec in the later high-intensity runs. Thus a crude estimate of the magnitude of the losses

is given by using the relation (5.5) to distort a spectrum of the functional form given in eq. (5.3). In this case, however, $x = N(t - \Delta) \cdot \zeta$, to allow for the fact that high queuing losses at time t reflect the counting rate not at that time but at an earlier time $t - \Delta$, where the lag parameter Δ is approximately equal to the buffer transfer time ζ .

This simple theoretical simulation of the queuing losses was checked by a special experiment in which the decay electron stops, as well as being fed to the normal five digitrons, were all passed through a fan-in to the sixth digitron to give a very high rate. By comparing the results obtained from the five-digitron data with those from the sixth digitron, the effect of queuing losses on the $(g - 2)$ frequency was measured directly. Two different groups of data were used, one with 3 times and the other with 5.5 times the normal data-taking rate in the sixth digitron. By comparing the observed frequency shift $\Delta f_\alpha / f_\alpha$ (168 ppm) for the highest rate with that obtained by fits to Monte Carlo distributions generated according to the theoretical simulation described above, Δ was found to be 0.85ζ . Both the dependence of the frequency shift on the time interval of the fit, and the observed magnitude of the queuing losses as a function of starting time, were now accurately reproduced by the theoretical simulation. The result for the frequency shift as a function of counting rate is given in fig. 17. Two curves are shown for $\zeta = 0.48$ and 0.36 μsec . It can be seen that the observed frequency shift in the "3 times" data is well predicted by the Monte Carlo simulation when Δ is fixed by the "5.5 times" data. These curves are used to estimate the queuing-loss corrections to f_α given in table 3.

At high instantaneous rates there is an increased probability that two decay electrons arrive within the same 10 nsec time bin, either at a single counter or at two counters of the same digitron. In the first case the second electron count is lost, while in the second case two counter labels are set and the event is rejected off-line. The resulting frequency shift is, in both cases, small; in the high-rate experiment it was found to be about 30 times smaller than that due to queuing losses, and in consequence it was neglected.

5.1.2 Results of fits to the (g - 2) frequency

The values obtained for f_α are presented in table 3. The raw frequency queuing-loss correction (which in the worst case is 1.3 ppm) and the corrected frequencies are given for each run, as well as the total number of decay electrons in the fitted spectra.

5.2 Analysis of the radial distribution of the stored muons

As will be shown in the following section (5.3), the uniformity of the magnetic field in the storage ring was such that the (g - 2) frequency was essentially independent of the distribution of the muons within the storage region. However, a precise value (accuracy $\sim 10^{-4}$) of the mean radius of the muon population is needed in order to derive the muon lifetime at rest from the measurements made in flight (see Section 6.1).

Information on the radial distribution is obtained by analysis of the bunch structure of the muons in the ring at early times. This fast rotation pattern is shown in fig. 18. The parent pion bunch has a width of 10 nsec, a time structure which is inherited by the decay muons. Shortly after injection, as the bunch length is much shorter than the rotation period in the ring (147 nsec), the decay electrons detected by a particular counter will appear in discrete bunches. The delays of the various counters are adjusted so that the bunched data from different counters may be coherently added, simply by combining single counter histograms suitably shifted by an integer number of bin widths (10 nsec).

Two methods of analysis are available. In the first, which is applicable only to non-overlapping bunches, the centroid in time of each bunch is calculated. By fitting a straight line to these values the mean rotation frequency f_{rot} is obtained. The mean radius is then deduced from the average relativistic γ -parameter $\bar{\gamma}$, which is given by

$$\bar{\gamma} = \frac{2\lambda \bar{f}}{g f_{\text{rot}}}, \quad (5.6)$$

where $\lambda = \mu_{\mu}/\mu_p$ is the ratio of the muon and proton magnetic moments; $\bar{f}_p$ is the mean proton magnetic resonance frequency, corrected to vacuum, over the muon orbits; and g is the g -factor of the muon.

More detailed information on the radial distribution can be obtained by making a least squares fit to the shape of the bunches as a function of storage time. The program to perform this analysis was also used in the previous CERN ($g - 2$) experiment [18] where knowledge of the radial distribution was crucial. Although all the muons in the storage ring have essentially the same velocity $\sim c$, the high-energy ones, because of their longer path length, gradually lag behind those of low energy. This results in a spreading out of the bunch as time progresses, so that eventually all bunches overlap. The rate at which this overlap occurs is sensitive to the radial distribution of the muon orbits. The latter may then be found by least squares fitting to the observed time development of the bunches. With only one pion bunch injected, the bunches of detected electrons remain resolved until ~ 10 μsec , and analysis is normally carried out in the time range 5 to 20 μsec or about 100 revolutions. However, the bunch structure does not disappear completely until about 40 μsec after injection. The analysis may also be carried out when two pion bunches (separated by 105 nsec) are injected into the ring. In this case the fit is performed entirely on overlapping bunches, and the simple centroid fitting procedure described above cannot be used.

The statistical error on the mean radius given by the fit is typically 0.1-0.2 mm. A number of systematic checks of the fitting program have been carried out, both by using Monte Carlo data as input and by varying input assumptions and the parameters in the fits to real data. Some examples are given below:

- i) Rate and dead-time effects. These change the shape of the pulses at early times. A rate 10 times higher than the experimental one gave $\Delta\bar{r} < 0.1$ mm.
- ii) Effect of a flat background between bunches. A level five times higher than observed experimentally gave $\Delta\bar{r} < 0.1$ mm.

- iii) Effect of muon losses. Putting in linearly asymmetric losses of 20% with an exponential time constant of 3 μ sec (this loss level is more than 10 times greater than observed experimentally) gave $\Delta \bar{r} = 1.5$ mm.
- iv) Width of input pion pulse. This was given the values 5, 10, 20 nsec;
 $\Delta \bar{r} < 0.1$ mm.
- v) Shape of input pion pulse. Square or triangular. $\Delta \bar{r} < 0.1$ mm.

A further check on the performance of the program is given by comparing the reconstructed radial distribution with that calculated^{*)} by a Monte Carlo injection program, which takes into account the details of the machine lattice and aperture as well as the injection angle, position, and phase space of the incident pion beam. The result of this comparison is shown in fig. 19. As can be seen for the central region, the scraped and unscraped distributions are essentially identical. However, the removal by the scraping of muons from the wings of the distribution, i.e. those at the extremes of the momentum range, is also illustrated by the figure.

For a storage ring with azimuthal symmetry and uniform population in phase space, the distribution of muons as a function of equilibrium orbit radius r is given by curve A in fig. 20. The curve is made up of two equal parabolae with first derivatives zero on the boundaries of the storage region and a cusp in the centre. Curve B in this figure can be used as a guide to the parametrization of the radial distribution, which is used in the orbit tracking programs to calculate the mean magnetic field (see Section 5.3). This curve is still represented by two parabolae, but the central cusp is shifted by δ from the centre of the storage region. To take into account the effect of horizontal scraping, the limits of the distribution are taken at $\pm h$ instead of at $\pm a$, the physical boundaries of the storage region. Denoting $x \equiv r - r_0$, where $r_0 \equiv 700$ cm is the central radius of the storage region, then $\delta = 2\langle x \rangle$ and $h = (10\langle x^2 \rangle - 12\langle x \rangle^2)^{1/2}$. Thus δ and h are completely defined in terms of the mean values $\langle x \rangle$ and $\langle x^2 \rangle$ of the fitted radial

*) We are indebted to W. Lysenko for performing this calculation.

distribution. For the purpose of field averaging, it can be seen by comparing figs. 19 and 20 that such a two-parameter function gives an adequate representation of the data, particularly in view of the extreme uniformity of the magnetic field.

The results found for $\langle x \rangle$, $\sqrt{\langle x^2 \rangle}$, δ , and h for the various radial distribution fits made throughout the experiment are given in table 4. Typical total errors on $\langle x \rangle$ and $\sqrt{\langle x^2 \rangle}$ are estimated at 0.5 mm and 0.1 mm, respectively. For input to the orbit tracking program (Section 5.3) the results for unscraped data were averaged, as were those for the scraped data. The 1974 data, which were unscraped, are treated separately, as here injection conditions were somewhat different, $\langle x \rangle$ and δ being significantly lower than in later data.

5.3 Magnetic field analysis

The determination of the average magnetic field in terms of the equivalent proton magnetic resonance frequency $\bar{f}_p$, for each set of data which was analysed separately for f_a (see table 3), involved three distinct steps:

- i) Using the radial distribution parameters found as described in the previous section, an ensemble average of the full field map, nearest in time to the given run, was made. The effect of the electric field amounts to a correction of the order of 2 ppm and was included in this average.
- ii) Using information from the plunging probe measurements (see Section 3.2.2) a correction was applied to allow for any changes in the magnetic field between the data collection and the making of the full map.
- iii) The measured proton magnetic resonance frequency was corrected to the value in vacuum.

These steps will now be discussed in more detail.

5.3.1 Ensemble averaging of the magnetic and electric fields

Two methods were available for calculating the magnetic field averaged over the stored muon population. In the first, an azimuthal average was taken at each x, y (horizontal, vertical) point of the storage region weighted by the function

$$W(x,y) = \sqrt{a^2 - x^2} \sqrt{b^2 - y^2}, \quad (5.7)$$

where a and b are the horizontal and vertical half-apertures of the storage region ($x = y = 0$ is at the centre of the region). This weighting function corresponds to the time-averaged muon distribution in a rectangular aperture with uniformly filled phase space, and was used when assessing the field uniformity during the shimming of the magnets.

For the ensemble averaging to find the mean field appropriate to a particular data set, a second method was used. This was a full Monte Carlo tracking program, taking into account the structure of the ring, in particular the two electric field-free sections (see Section 3.3). For this the magnetic field was azimuthally averaged only over single magnet blocks; a procedure justified by the small lateral displacement of a muon in traversing a single block. The effect of the electric field was also taken into account by averaging the spin effective magnetic field [49]:

$$B_{\text{eff}}^s = B + \frac{E_r}{\beta} \left[\frac{1 + (1/a)}{\gamma^2} \right], \quad (5.8)$$

where E_r is the radial component of the electric field, which was calculated by relaxation methods from the known electrode geometry and voltages. Typically 300 particles were tracked for 30 turns with 200 field samplings per turn. The agreement between the two averaging methods for the magnetic field was found to be excellent, within 0.5 ppm. This agreement is a consequence of the extreme uniformity achieved in the radial and vertical directions for the azimuthally averaged field (see fig. 5). The effect of the electric field on B_{eff}^s is small (~ -1.5 ppm) for the observed distribution of muon orbits.

Before giving the results of the ensemble averaging for the individual runs, it is of interest to consider in more general terms the sensitivity of $\bar{B}$ to the radial distribution of the muons. This sensitivity can be explored by making large changes to the input distribution of the tracking program. The results of such a study are shown in fig. 21. In all cases the average magnetic field is changed

by less than 2 ppm. This extreme insensitivity to the muon distribution is the most important systematical improvement in the present experiment, compared with the previous one [18] in which the estimated uncertainty of ± 3 mm in $\bar{r}$ led to a 160 ppm error in $\bar{B}$; a systematic error which was comparable to the statistical error in the result. There is some evidence from measurements made in the present storage ring, using movable aperture stops, that the distribution of betatron amplitudes differs somewhat from an ideal phase-space distribution in the sense that large betatron amplitudes are disfavoured. It was also checked, using the tracking program, that such effects gave negligibly small corrections to $\bar{B}$.

The first column of table 5 shows the results of the ensemble averaging for the nine different experimental runs. The results are given in terms of shifts (in ppm) from a reference NMR frequency of 62.781 MHz. The electric field effect is included in the numbers, as is a small (-0.6 ppm) correction due to the effect of vertical betatron motion [51] which is not taken into account by the orbit averaging program.

5.3.2 Time averaging of the magnetic field over the experimental runs

As discussed in Section 3.2.2, there was a "warm up" period of three days or so each time the magnet was switched on, during which the field slowly increased. Data were often taken throughout this period and so it was necessary to correct for the small shift in $\bar{B}$. This was done by taking a weighted average of the plunging probe (Section 3.2.2) magnetic field measurements. The latter were also used to establish the off-set of the plateau value of $\bar{B}$ to that of the nearest full field map used in the ensemble averaging. The combination of the two effects is given in the fourth column of table 5 under "Time averaging". The corrections are at most a few ppm with an estimated error of 1 ppm. More details on this correction are given elsewhere [40,41].

5.3.3 Calibration and other corrections

The reduction of the measured nuclear magnetic resonance frequency $\bar{f}_p^{\text{uncorr}}$ to the equivalent value in vacuum $\bar{f}_p$ is described briefly in Section 3.2.2 and in

more detail elsewhere [43]. These calibration corrections are summarized in table 6. An additional correction arises in principle from the effect of the vacuum tank, vacuum ducts, bellows, and inflector. The net result of these is to increase the average field in the storage region by some 3.3 ppm. However, an equivalent concentration of flux lines also occurs at the stabilization probes (Section 3.2.1). The consequent adjustment of the field by the control system almost completely cancels the above effect. The resultant shift of 0.5 ppm is just equal and opposite to that due to the leakage field from the pulsed inflector. Thus the over-all combined correction is zero within an error of 0.6 ppm. The total calibration correction is then as in the last line of table 6, with the error increased to 0.8 ppm. This is the number quoted in the third column of table 5.

The total correction is given in the last column of table 5 and yields the $\bar{f}_p$ values in table 7.

5.4 Results for $\alpha_\mu \equiv (g - 2)/2$

The values of f_α for the individual runs given in table 3 and the corresponding values of $\bar{f}_p$ from table 5 are presented again in table 7 together with the ratios $R = f_\alpha / \bar{f}_p$ and the corresponding errors. These are purely statistical on f_α and systematic on $\bar{f}_p$. Also quoted in table 7 are the weighted average values of R for μ^+ , μ^- and the combined data. The errors quoted on the weighted average values of R in table 7 are due only to statistical errors (those in f_α). The values of R for the nine experimental runs are plotted, together with the weighted averages in fig. 22. The consistency of the over-all weighted average with the individual runs is good ($\chi^2 = 7.3$ for 8 degrees of freedom). Because the systematic error on $\bar{f}_p$ does not decrease on combining the different data sets, it must be added separately to the final statistical error.

The over-all average value of the frequency ratio is

$$R = 3.707213(27) \times 10^{-3} .$$

The error is made up of a 7.0 ppm statistical contribution from f_a and a 1.5 ppm systematic contribution from $\bar{f}_p$. This number is the essential result of the experiment described here.

To further extract the value of a_μ requires knowledge of the experimental parameter $\lambda \equiv \mu_\mu/\mu_p$. The relation between a_μ , R , and λ is found by combining the expressions for f_a and f_μ (the Larmor precession frequency for muons in the mean magnetic field $\bar{B}$):

$$2\pi f_a = a_\mu \frac{e}{m_\mu c} \bar{B} ,$$

$$2\pi \bar{f}_\mu = 2\pi \lambda \bar{f}_p = \frac{g}{2} \cdot \frac{e}{m_\mu c} \cdot \bar{B} = (1 + a_\mu) \frac{e}{m_\mu c} \bar{B} .$$

Taking the ratio of these equations and solving for a_μ gives

$$a_\mu = R/(\lambda - R) .$$

Three measurements, of comparable accuracy, exist for λ , that of Crowe et al. [22],

$$\lambda = 3.1833467(82) ,$$

that of Casperson et al. [22]

$$\lambda = 3.1833403(44) ,$$

and the more recent measurement due to Camani et al. [22] giving

$$\lambda = 3.1833448(29) .$$

Taking the weighted average value of these measurements, which is

$$\lambda = 3.1833437(23)$$

leads to the following results for the g-factor anomaly:

$$a_{\mu^+} = 1165911(11) \times 10^{-9} \text{ (10 ppm)}$$

$$a_{\mu^-} = 1165937(12) \times 10^{-9} \text{ (10 ppm) ,}$$

and for the average of μ^+ and μ^- ,

$$a_\mu = 1165924(8.5) \times 10^{-9} \text{ (7 ppm) .}$$

This value is slightly shifted from the previously published one [19] owing to the different value of λ used.

The theoretical significance of these results is discussed in Section 7.

6. MUON LIFETIME AND ELECTRIC DIPOLE MOMENT

6.1 Muon lifetime analysis

The muon lifetime in flight in a circular orbit provides an important check on the time dilation predicted by the special theory of relativity. The eight-parameter fits to the data, using eq. (5.3) as described above, yielded values for the lifetime τ which were found to agree within 1% with the theoretical time-dilated value of 64.43 μsec . Therefore without further discussion we can state that the theory of relativity is confirmed at a γ of 29 to this level of accuracy. However, a more careful analysis of the data, together with some subsidiary experiments, lead to a much more precise comparison, as presented below.

In an ideal experiment, the decay electron counting rate $N(t)$ would be given by

$$N(t) = N_0 e^{-t/\tau} |1 - A \cos(\omega_a t + \phi)|. \quad (6.1)$$

In the actual experiment, as we have seen, deviations from this ideal distribution are caused by losses of stored muons, by variations of the photomultiplier gain accompanying the recovery from the initial flash, and by background counts due to causes other than muon decay. In order to obtain the muon lifetime to an accuracy of better than 1% it is necessary to make very careful measurements of all these effects which systematically distort the recorded time spectrum.

6.1.1 Muon losses

The level of muon losses was dramatically reduced by means of the electric scraping (Section 3.3.1). Losses later than 40 μsec were reduced by a factor of 30; those later than 100 μsec were reduced by a factor of more than 50. The removal of this systematic effect to a level that was essentially negligible

compared with the statistical error, made possible the rather precise lifetime measurement being reported here. The data without electric scraping were, however, also important, as they enabled the muon detector to be calibrated. The relationship between the muon loss counts observed in the telescope and the more general correction function $[1 + F(t)]$ was established by comparing the following two measurements of the muon losses:

- i) Muon losses extracted directly from the decay electron data. This was done by performing a simple exponential fit to the data beyond 300 or 400 μsec after injection, by which time the losses were expected to be at a very low level. The losses at earlier times were then estimated by extrapolation. Plotting the ratio of the decay spectrum (obs) to the curve fitted at late time (fit), the losses are revealed as the excess above the line $\text{obs/fit} = 1$ (see the points marked with crosses in fig. 23).
- ii) Muon losses deduced from the counts recorded in the muon detector. Forgetting, for the moment, the important problems of paralysis effects and misidentified electron background, which will be discussed in full later, the correction function $F(t)$, which is equivalent to the excess shown in the plot of obs/fit discussed above, is given in terms of the lost muon time spectrum $M(t)$ by the integral

$$F(t) = \frac{1}{\epsilon \tau N_0} \int_t^{t_L} M(t) e^{t/\tau} dt . \quad (6.2)$$

The result of this integration, normalized to the obs/fit distribution, is also shown in fig. 23 (filled circles). The ratio ϵ of the detection efficiencies of the muon detector for lost muons, to that of the summed electron detectors for decay electrons, is determined by the relative normalization of the obs/fit and $F(t)$ distributions. It was found to have the value 0.09.

The good agreement in the shape of the function $F(t)$ as derived by the two methods indicates that the muon detector, although it is sampling the lost muons at only one point in azimuth, is doing so in an unbiased manner at different times.

By moving the detector to different positions around the ring, a small variation was seen in the value of ϵ , which can be covered by assigning an error of about 50%. This error had a negligible effect on the lifetime results. It was assumed that the calibration constant was the same for scraped and unscraped losses. The $F(t)$ determined using scraped data and method (i) exhibited no statistically significant excess counts as shown in fig. 23 (open circles). The 0.5% displacement of the flat obs/fit distribution above the value 1.0 comes simply from normalization errors in the late-time fit. Using method (ii) with the same scraped data indicated that the excess counts were at the 0.001 level by 40 μ sec after injection, a thirtyfold decrease as compared to the unscraped data.

The raw spectrum of muon counts $M(t)$ in the muon detector is shown in fig. 24. Notice that after 100 μ sec they have a time distribution consistent with that of decay electrons. In fact the majority ($\sim 80\%$) of such apparent muon counts after 100 μ sec are interpreted as misidentified decay electrons. The main evidence for this interpretation is twofold. Firstly, by assuming that the hypothesis is true and comparing this misidentified electron count rate with the actual electron count rate in the muon detector, the electron rejection of the 'muon' requirement is found to be $\epsilon_R = 1.68 \pm 0.12 \times 10^{-3}$. This number agrees very well with the value 1.48×10^{-3} that was deduced from electron rejection measurements at various momenta in a test beam by properly weighting them according to the decay electron momentum spectrum.

Secondly, a five-gap optical spark chamber was placed immediately downstream from the muon detector during part of the running. This was triggered whenever a lost muon was detected and the time of the event recorded directly on film. Muons, characterized by four or five sparks in line, were found to constitute only 10% of the muon triggers after 100 μ sec. The total number of such events seen was small, 30, and their distribution was consistent with a normal muon decay exponential. Assuming that the events are distributed exactly exponentially, such a muon loss will give a starting-time independent correction to the fitted value of τ of 0.008 μ sec.

Returning again to fig. 24, the muon losses are seen to be made up of two components, one with a long time-scale mentioned above, the other associated with the counts above the fitted exponential at times below 100 μsec . The total estimated muon loss correction is shaded.

The other important systematic effect that must be considered is paralysis of the muon detector due to the initial flash of particles at the time of injection. This effect was much more severe for the muon detector than for the electron detectors. No counts were recorded at all for about 20 μsec after injection, and subsequent recovery was quite slow. This is shown by fig. 25 in which the decay electron distribution $E(t)$ recorded in the muon detector has been weighted by the factor $e^{t/64.4 \mu\text{sec}}$. Recovery from paralysis is indicated by the distribution becoming flat, which occurs after about 50 μsec . At higher beam intensities, recovery from paralysis was delayed until 100 μsec . This varying recovery time for the muon detector accounts for the different starting times used in the fits shown in table 8.

6.1.2 Proton background

As discussed in Section 4.1, the use of electric scraping in the positive polarity led to the storing of protons, which gave a time-varying background $B(t)$ in both the electron detectors and the muon detector.

The delayed switching off of the electric focusing to a time when essentially only protons remained in the ring, enabled the lost proton detection efficiency ratio ϵ_p for these two systems to be measured. Assuming the same efficiency ratio for the protons lost during the muon storage interval, and that the fractional muon losses are the same as in μ^- runs under identical conditions, a background correction term $B(t)$ can be deduced from the observed spectrum of counts in the muon detector. For the μ^+ runs it was found that the muon detector time spectrum after 100 μsec was dominated by lost proton counts. However, the estimated proton background level in the decay electron detectors was quite low, $(3 \pm 1) \times 10^{-3}$ at 32 μsec after injection. Half the error on this estimate comes from counting statistics in the determination of ϵ_p and half from the uncertainty in the azimuthal

distribution of lost protons. The value assigned to this systematic error corresponded to the change in ϵ_p when the azimuthal distribution of lost protons was changed from a uniform one to that actually observed on switching off the electric field at 1 msec. The resulting correction to the lifetime was small, as can be seen in table 8.

6.1.3 Gain effects

The gain correction factor $G(t)$ contributes the largest systematic error to the lifetime measurement, being of the same order as the statistical error. The gain measurement has been described in detail in Section 4.3, but here we will comment of the general accuracy and reproducibility of the results. The calibration of the gain was done independently for each photomultiplier/LED combination, and associated errors were essentially negligible in comparison with the statistical error of 0.1% at each of the 11 points measured for each counter. Averaging the gain change curves for the selected counters led to a reduction in this point-by-point error to the level of 0.03%.

More important than such statistical effects, however, was the stability in the shape of the gain curve over the period of several days which constituted each of the runs shown in table 8. This was checked by performing measurements of the gain both before and after each period of data-taking intended for lifetime analysis. The comparison of two such measurements, taken three days apart, is shown in fig. 26 for the average of the ten counters used to give the third data set of table 8. The agreement between the two sets of measurements is very good, and their general trend is clearly much flatter than the indicated straight line which would give a correction to the fitted lifetime of 0.1%. In view of this, the over-all error in the lifetime due to the gain correction is conservatively estimated as 0.03%. This is the error quoted for each data set in table 8.

6.1.4 Lifetime results

Including all the correction terms, the equation required to fit the lifetime data was

$$N(t) = N_0 \left\{ \left[1 + S \cdot G(t) + F(t) \right] e^{-t/\tau} \left[1 - A \cos (\omega_a t + \phi) \right] + B(t) \right\}. \quad (6.3)$$

The term $S \cdot G(t)$ represents the gain variation; $G(t)$ is the average measured gain curve, at the electron energy threshold, of the data used in the fits. This was chosen to be that of the second pulse-height level. Thus the sum of the counts in the top four pulse-height bands was used in the analysis. The fractional change of the electron counting rate per unit gain change, $S \equiv \Delta N / (N \cdot G)$, is derived from the observed shape of the decay electron spectrum. For the chosen energy threshold, $S = 1.00 \pm 0.02$. $F(t)$ represents the correction due to the lost muons, the derivation of which has been described in detail above, and $B(t)$, which was important only in the case of positive muons, represents counts due to protons which were trapped in the ring by the scraping mechanism and subsequently scattered into the counters by a loss mechanism presumably similar to that causing the loss of muons.

The results obtained for the lifetime for the six separately analysed sets of data are presented in table 8. The raw fitted lifetime is given together with the contribution of various corrections. The latter can be seen to be smaller than the statistical error for all data sets. In particular, the correction due to lost muons is essentially negligible. In the last row the lifetimes are given for the data sets 3, 4, 5 starting at 100 μ sec. It is clear that fitting all data sets starting at this time will give essentially the same result as that quoted below using earlier starting times.

In table 9 are shown the weighted average lifetimes in flight of μ^+ and μ^- , together with the corresponding rest frame lifetimes and the previous best published measurements for the latter [52,53]. The rest frame lifetimes are calculated from the ones measured in flight by dividing by the relativistic time dilation factor

$$\bar{\gamma} = 29.327(4). \quad (6.4)$$

The measurement of $\bar{\gamma}$ has been described in Section 5.2. The quoted error corresponds to an uncertainty of ± 1 mm in the mean radius. The values given in table 9

can be compared (i) down the columns to check the validity of the relativistic time transformation, and (ii) across the rows to check the equality of the μ^+ and μ^- lifetimes:

$$(i) \quad \frac{\tau_0^+ - \tau_{\bar{Y}}^+ / \bar{\gamma}}{\tau_0^+} = (0.2 \pm 0.9) \times 10^{-3} , \quad (6.5)$$

$$\frac{\tau_0^- - \tau_{\bar{Y}}^- / \bar{\gamma}}{\tau_0^-} = (1.4 \pm 1.0) \times 10^{-3} .$$

Taking the mean of these two differences, the relativistic time transformation is shown to be valid to $(0.8 \pm 0.7) \times 10^{-3}$ or a 95% confidence range of $(-0.61 \text{ to } 2.2) \times 10^{-3}$:

$$(ii) \quad \frac{\tau_{\bar{Y}}^+ - \tau_{\bar{Y}}^-}{\tau_{\bar{Y}}^+} = (0.8 \pm 1.0) \times 10^{-3} \text{ (this expt.)} , \quad (6.6)$$

$$\frac{\tau_0^+ - \tau_0^-}{\tau_0^+} = (-0.4 \pm 1.0) \times 10^{-3} \text{ (previous best values) .}$$

The mean of these two differences gives for the relative difference of the lifetimes $(0.2 \pm 0.7) \times 10^{-3}$ or a 95% confidence range of $(-1.2 \text{ to } 1.6) \times 10^{-3}$. Thus the results of this experiment are consistent with the previous best measurements, the relativistic time transformation, and equality of the μ^+ and μ^- lifetimes. A more detailed discussion of the implications of these results is given in our previous publication on the muon lifetime [25].

6.2 Measurement of the muon electric dipole moment

As indicated in Section 1, the existence of an electric dipole moment for the muon would cause the plane of precession of the spin to be tilted away from the plane normal to $\vec{B}$. The tilt angle δ is given in terms of f (the edm equivalent of the g factor) by

$$\delta = \frac{\omega_{\text{edm}}}{\omega_a} = \frac{f\beta}{2a} , \quad (6.7)$$

and it can be measured through its effect on the relative phase of the $(g - 2)$ oscillations in the count rate of the upper and lower halves of the decay electron

detectors. The phase difference comes from the oscillating vertical component of the polarization, which gives a signal in quadrature with the $(g - 2)$ modulation and is of opposite sign in the upper and lower halves of the detectors. The manner of labelling these 'up' and 'down' counts is described in Section 3.4, and clearly a clean separation into upward- and downward-going electrons is not possible. However, the consequent diminution in the magnitude of the phase difference for a given size of edm is only a factor of 0.75. The equation relating the phase difference $\Delta\phi$ between the up and down time spectra to the tilt angle δ was found by Monte Carlo simulation to be $\Delta\phi = 0.33\delta/A$, where A is the asymmetry of the $(g - 2)$ oscillation.

The first step in the off-line analysis was to create time histograms of the edm data. This was done by scanning the events recorded in digitrons 1 to 5 during a single PS burst for stops in any of the shower detectors equipped with an up or down counter pair. The coincident events were then located in the digitron 6 data. From this combined information, four histograms were obtained for each of the edm shower counter systems. The four categories had the common feature that the shower counter signal was always present, either coincident i) with the upper edm counter, ii) with the lower edm counter, iii) with both edm counters, or iv) by itself. Category (iv) comprised those cases where the decay electron entered the end of the shower counter adjacent to the vacuum chamber wall, and also the cases where the energy of the original decay electron was largely carried by a bremsstrahlung photon created in an interaction in the electrode or vacuum chamber wall. For counters situated by the normal vacuum chamber this category formed 43% of the counts, while for those next to the special thin-walled chamber it was only 29%. The cases where a shower started upstream of the detectors, causing both up and down scintillators to fire, category (iii), formed only 5% of the counts.

6.2.1 Data analysis

The fitting of the separate 'up' and 'down' time spectra followed the normal $(g - 2)$ analysis procedure set out earlier, with the exception that the frequency ω_a was held fixed at the accurate value known as a result of the main experiment.

A further difference was that the statistical error in the measurement of the tilt angle was independent of the asymmetry A and so it was of advantage to use all the data above the timing threshold.

The measured phase difference $\Delta\phi$ for each of the seven runs analysed for an edm signal is shown in table 10, with the weighted average values for μ^+ and μ^- data given separately, and then added together to give an over-all figure for the muon. In this latter the CPT theorem is assumed under which an edm for the muon would imply the same value of f (and δ) for muons of each sign.

6.2.2 Systematic errors

The system of data collection was examined for possible sources of a false signal, but only in one instance was a significant effect found. This was the sensitivity of the phase difference to the vertical position of the split between 'up and down' counters. This effect derives from the fact that the decay electron trajectories are curved inwards by the magnetic field. Thus an electron initially emitted outwards travels further before hitting the shower counter than one emitted inwards. The different vertical distributions of these two classes of electrons at the detector mean that the average phase of the recorded "up" or "down" events depends upon the vertical position of the counter split. A simple analytical calculation of the electron trajectories in the magnetic and electric fields indicated that a 1 mm misalignment of this split with respect to the local median plane of the muon orbits would generate a phase difference of 8 mrad. This effect was examined experimentally and the results are shown in fig. 27. The straight line fit to the data gave $\Delta\phi = (7.6 + 1.0) \text{ mrad/mm}$, which is equivalent to a simulated edm of $(3.9 + 0.5) \times 10^{-19} \text{ e}\cdot\text{cm}$ per mm displacement. The ratio of the counts in the 'up' and 'down' scintillators gave a measurement of the relative height of the split with respect to the median plane, and the position at which this ratio was unity was found to be close to that deduced for the median plane from the measurement of the radial component of the magnetic field. However, from one experimental run to another there were small shifts in the vertical position of the median plane over a range of $\pm 0.5 \text{ mm}$ and thus a systematic error of $\pm 2 \times 10^{-19} \text{ e}\cdot\text{cm}$ had to be assigned to this source.

Further evidence for the position of the median plane came from investigations of the variation in the number of muons stored as a function of the vertical position of an aperture stop. This stop was positioned three quarters of the way around the storage ring from the inflector, by which time most of the pion beam has left the storage region. Thus changes in the size of the aperture at this point did not influence the initial muon intensity but only the phase space available for storage. Short runs with different positions of the aperture stop enabled the local centre of gravity of the muon population to be established, and once more it was found to be in good agreement with that deduced from the radial component of the magnetic field.

The scintillation counter pairs were repeatedly inverted during the runs in order to obtain 'up' and 'down' signals unbiased by counter efficiencies. To ensure against phase errors which might occur in the electronics, the timing of both 'up' and 'down' events was taken from the shower counter.

Finally, the asymmetry A as a function of pulse height was extracted from both 'up' and 'down' data and compared in order to check that the two halves of the shower counter had the same energy response. The magnitude of any false signal coming from this source was estimated at less than $\Delta\phi = 0.1$ mrad -- much smaller than both the statistical and the above-mentioned systematic errors.

6.2.3 Electric dipole moment results

The results of the edm measurement for the seven data-taking periods are listed in the final column of table 10, where one standard deviation statistical errors are quoted. Adding the systematic error in quadrature yields

$$D_{\mu+} = (8.6 \pm 4.5) \times 10^{-19} \text{ e}\cdot\text{cm}$$

$$D_{\mu-} = (0.8 \pm 4.3) \times 10^{-19} \text{ e}\cdot\text{cm} ,$$

and assuming the CPT theorem

$$D_{\mu} = (3.7 \pm 3.4) \times 10^{-19} \text{ e}\cdot\text{cm} .$$

More details of these measurements are given in our previous publication [27] where also the theoretical implications of the results are discussed.

7. COMPARISON OF THEORY AND EXPERIMENT FOR a_μ

The main conclusions to be drawn from this measurement of the anomalous magnetic moment of the muon may be summarized as follows:

- i) The QED calculation of the anomaly is verified up to the sixth order, the experimental uncertainty being equivalent to 5% of this term.
- ii) The hadronic contribution to the anomaly is observed and measured to an accuracy of 20%.
- iii) There is no evidence for a special coupling of the muon. The actual limits imposed by the measurement depend upon the nature of the coupling and are discussed below.

We will now consider each of these points in more detail, setting the present test of theory into the context of that achieved with the two previous CERN experiments [17,18].

Together the three CERN muon ($g - 2$) experiments represent a continuous program of refinement of technique and precision extending over a period of some 18 years. During this time the accuracy of the theoretical calculations has moved in parallel with that of the experimental numbers. The results of the three experiments are:

$$(1) \quad a_\mu = (1162000 \pm 5000) \times 10^{-9} \quad (\text{ref. 17})$$

$$(2) \quad a_\mu = (1166160 \pm 310) \times 10^{-9} \quad (\text{ref. 18})$$

$$(3) \quad a_\mu = (1165924 \pm 8.5) \times 10^{-9} \quad (\text{this expt.})$$

from which it can be seen that there is almost a thousandfold increase in precision between the first and last experiments in the sequence.

The discussion of the three main points listed above will be followed by a brief consideration of the small, weak interaction contribution to the anomaly,

and then finally we will review the limits which may be set to various single 'exotic' effects, including specific types of QED breakdown, by virtue of the close agreement between experiment and 'normal' theory.

7.1 QED

The QED contribution to the anomaly is that which arises from Feynman graphs containing only photons and charged leptons. These graphs are of two types, the first containing either no vacuum polarization insertions or only those insertions where the virtual lepton is identical to the external line, the second containing vacuum polarization loops where the lepton is different from the external line. The first type of graph gives a contribution to the anomaly which is independent of the lepton mass and consequently the same for both electron and muon, while the second type of graph gives a considerable contribution to the muon anomaly but a negligible one to the electron. The leading term, for this latter type, occurs in the fourth order, and in the case of the electron is [54]:

$$\left(\frac{\alpha}{\pi}\right)^2 \frac{1}{45} \left(\frac{m_e}{m_\mu}\right)^2 \approx 3 \times 10^{-12} .$$

Thus, as far as QED is concerned, the difference between the electron and muon anomalies is due entirely to graphs with an external muon line and one or more electron vacuum polarization loop insertions. It is the over-all contribution of these diagrams which is checked by the present measurement, since the sum of the first, mass-independent, type of diagram is checked to great precision by the measurement of the electron anomaly [12]. The gross features of the situation are illustrated in fig. 28, while the coefficients of the perturbation series in (α/π) for a_μ [see eq. (2.1)] as well as the actual contribution to a_μ at each order are given in Table 1.

The leading contribution to a_μ and a_e first calculated by Schwinger [3] is $1/2(\alpha/\pi)$ and determines the order of magnitude of the lepton anomalies as 0.001. The dominant term in the difference between a_μ and a_e comes from a graph with a single electron loop inserted in the Schwinger graph (see fig. 28). All terms of order $(\alpha/\pi)^2$ have been calculated analytically [55] with the result given in

table 1. The largest contribution to $a_\mu - a_e$ at sixth order is given by the six 'light-by-light' scattering graphs [23,31], an example of which is shown in fig. 28. Not all of the 96 graphs which contribute to a_μ in the sixth order have been calculated analytically, so the error quoted on this contribution in table 1 is dominantly the estimated error in numerical computation. For the second- and fourth-order terms the error comes from the uncertainty in α , the fine structure constant. The values through sixth order quoted in table 1 are taken from the review article cited in ref. 29.

The theoretical predictions are compared with the results of the three CERN muon experiments in table 11, where the values of the expression

$$\frac{|\alpha_{\text{exp}} - \alpha_{\text{th}}| + \{\sigma_{\text{exp}}^2 + \sigma_{\text{th}}^2\}^{1/2}}{c_n \left(\frac{\alpha}{\pi}\right)^n} \quad (7.1)$$

are listed for $n = 1, 2, 3$ in each case. This expression gives a measure of how well each experiment checked the terms in the perturbation expansion for a_μ [eq. (2.1)]. As can be seen, the first experiment checked the (α/π) term to 1%, the second experiment checked the $(\alpha/\pi)^2$ term to 13%, while the present experiment checks the $(\alpha/\pi)^3$ term to 5%. This verification of successive terms in the expansion is an important feature of lepton anomaly measurements which is not shared by some other types of QED test [56]. The other aspect, searching for small violations of QED at very short distances in the leading graph, is discussed in Section 7.5 below. The value of α_{th} used in table 11 is that for the total theoretical estimate including the hadronic contribution, although this latter and its error which dominates σ_{th} is only important for the present experiment.

For completeness the theoretical estimates of the eighth- [32] and tenth- [33] order contributions to the anomaly are given in table 1, although they are below the sensitivity of even the latest experiment. Because of the recent spectacular breakthrough in the experimental precision for the electron anomaly [12,57], the computation of the eighth-order term for the electron has become of considerable interest, but for the muon these higher-order terms seem unlikely to be checked

experimentally in the foreseeable future since, even if the experimental precision could be improved by an order of magnitude, the uncertainty in the hadronic contribution would still obscure any more searching comparison between theory and experiment.

7.2 Hadronic vacuum polarization effects

The leading hadronic contribution to the muon anomaly comes from inserting a virtual hadron loop in the Schwinger graph, as shown in fig. 29. The relationship between this graph and the total cross-section for electron-positron annihilation into hadrons has already been discussed in Section 2 and is also indicated in fig. 29. As well as the estimate of the hadronic contribution to a_μ via equations (2.2) and (2.3) using recent storage ring data [35,36], table 1 contains the first estimate of higher-order hadronic contributions [36] which turn out to be of opposite sign to the leading term. Only the present experiment gives a significant test for the presence of the hadronic contribution. Following our analysis of the checks of QED, we use the expression

$$\frac{|a_{\text{exp}} - a_{\text{th}}| + \{\sigma_{\text{exp}}^2 + \sigma_{\text{th}}^2\}^{1/2}}{a_\mu^{\text{had}}} = 0.22 . \quad (7.2)$$

Thus the existence of hadron vacuum polarization is necessary for the agreement between theory and experiment at around the five standard deviation level.

This measurement of hadron vacuum polarization has to be added to the earlier work on the ACO storage ring at Orsay in which the presence of such effects in $e^+e^- \rightarrow \mu^+\mu^-$ in the region of the ϕ mass was established [58]. Hadron vacuum polarization effects have also been observed at SPEAR at centre-of-mass energies in the region of the J/ψ mass [59].

7.3 Anomalous coupling of the muon

A possible explanation of the muon-electron mass difference is that there exists an interaction particular to the muon; an interaction of coupling strength f , say, mediated by a heavy meson of mass M_0 . Such an interaction would contribute to the muon magnetic moment anomaly through a graph of the Schwinger type where

the internal photon line was replaced by a virtual heavy meson line. This contribution is considered in some detail in ref. 28 and is given by

$$\Delta\alpha = \left(\frac{f}{2\pi}\right)^2 \left(\frac{m_\mu}{M_0}\right)^2 L, \quad (7.3)$$

where L takes the following values depending upon the type of coupling

$$L_S = \ln\left(\frac{M_0}{m_\mu}\right) - \frac{7}{12} \quad (\text{scalar})$$

$$L_{PS} = -\ln\left(\frac{M_0}{m_\mu}\right) + \frac{11}{12} \quad (\text{pseudo-scalar})$$

$$L_V = \frac{1}{3} \quad (\text{vector})$$

$$L_A = -\frac{5}{3}. \quad (\text{axial-vector})$$

The experimentally measured and theoretical values of the muon anomaly

$$a_{\text{exp}} = 1165924(8.5) \times 10^{-9}$$

$$a_{\text{th}} = 1165921(8.3) \times 10^{-9}$$

may be used to limit the range of possible values for any extra contribution $\Delta\alpha^*$ to

$$-20 \times 10^{-9} < \Delta\alpha^* < 26 \times 10^{-9} \quad (7.4)$$

at the 95% confidence level.

Using the upper or lower bound of this range as appropriate, limits on the ratio f/M_0 may then be set for the different types of coupling listed above. These are shown in fig. 30 as a function of the mass of the heavy meson M_0 . At a mass of $10 \text{ GeV}/c^2$, the 95% confidence upper limit on f/M_0 is $\sim 5 \times 10^{-3} (\text{GeV}/c^2)^{-1}$ for S, PS and A, and $\sim 1.7 \times 10^{-2} (\text{GeV}/c^2)^{-1}$ for V, corresponding to coupling strengths intermediate between those of the electromagnetic and strong interactions.

It is interesting to note that the S and PS contributions to the anomaly vanish for $M_0 = 189 \text{ MeV}/c^2$, $264 \text{ MeV}/c^2$, respectively, and so nothing can be said concerning the existence of mesons of this type at or near these masses (see fig. 30).

7.4 Weak interaction effects

Reliable calculations of the weak interaction contribution to the muon anomaly [37] became possible for the first time with the advent of renormalizable [61] gauge theories unifying the weak and electromagnetic interactions. In general the weak contribution is critically dependent upon the parameters of the theory, such as the masses of the Higgs and intermediate vector bosons, and in any given theory the measured value of the anomaly may be used to put constraints on these parameters [29,62]. Only in the simplest of such theories, due to Weinberg and Salam [38], are the parameters sufficiently well determined experimentally (from neutrino cross-sections [63]) to give a firm prediction [29] of the expected weak anomaly (see table 1).

Clearly the precision of even the latest experiment is inadequate for testing this prediction, and the remarks made above concerning the possibility of checking the eighth-order QED term apply also here. If, however, the Weinberg-Salam model is not verified experimentally [64,65] the muon anomaly will provide a valuable constraint in the search for the correct model, if this is indeed of the renormalizable gauge type. The value of the weak contributions to the anomaly quoted in table 1 is that for the Weinberg-Salam model but it is also typical of that found in other unified theories with reasonable guesses for the parameters.

7.5 Limits on exotic contributions to a_μ

The range defined by the inequalities (7.4) may be used to set limits on single contributions to the muon anomaly from various other sources including different models for the breakdown of QED. With more than one such contribution, cancellation might occur, thus rendering the discussion inconclusive. However, it is still a useful exercise to see what limits can be placed on the individual contributions. In table 12 the limits set by the present experiment are summarized and compared with those which could be inferred from the previous two CERN ($g - 2$) experiments [17,18].

7.5.1 Modifications of QED

a) Photon propagator

The parametrization used to describe possible modifications to the photon propagator at high q^2 , or short distances, is

$$\frac{1}{q^2} \rightarrow \frac{1}{q^2} \left[1 \pm \frac{q^2}{(\Lambda_Y^\pm)^2} \right], \quad (7.5)$$

where the $\pm$ signs refer to modifications with positive and negative metric [66]. Such a modification leads to a contribution to a_μ of

$$\Delta a = \pm \frac{1}{3} \left(\frac{\alpha}{\pi} \right) \left(\frac{m_\mu}{\Lambda_Y^\pm} \right)^2 \quad (7.6)$$

and the allowed range for Δa^* [see inequalities (7.4)] gives the 95% confidence limits

$$\Lambda_Y^+ \geq 18.2 \text{ GeV} : \Lambda_Y^- \geq 20.7 \text{ GeV} .$$

These limits are comparable to those set for the photon propagator in e^+e^- colliding beam experiments [56]. It should be noted that we are discussing only the photon propagator in the second-order Feynman diagram, since the effect of such modifications in higher-order graphs is negligible. The precision of the muon ($g - 2$) experiment is such, however, that with a mass scale of $\sim m_\mu$ the distances probed are equivalent to those reached in the e^+e^- experiment where $q^2 \approx 8 \text{ GeV}/c^2$.

b) Vertex function

Modifications to the vertex function, unrelated to changes in the fermion propagator, lead to a contribution to the muon anomaly of the same order in α as the photon propagator modification.

The vertex form factor is parametrized as

$$F_\mu(q^2) = 1 \pm \left(\frac{q}{\Lambda_\mu} \right)^2 \quad (7.7)$$

and yields a change in the anomaly of the form

$$\Delta\alpha = K \frac{\alpha}{\pi} \left(\frac{m_\mu}{\Lambda_\mu} \right)^2 . \quad (7.8)$$

The limits imposed on Λ_μ are

$$\Lambda_\mu^+ > 31.6 \text{ GeV and } \Lambda_\mu^- > 36 \text{ GeV} ,$$

the two cases corresponding to $K = +1$ and $K = -1$, respectively.

c) Muon propagator

As first pointed out by Kroll [67], any modification to the fermion propagator leads to a corresponding modification in the vertex function, which cancels out the propagator effect to order (α/π) . This is a consequence of the gauge invariance of the electromagnetic interaction. Such modifications affect the anomaly only at fourth order, and consequently the equivalent cut-off factor Λ is less severely restricted by the experimental results; from the present measurements Λ is limited to values greater than 1.5 GeV.

7.5.2 Heavy leptons

Insertion of a vacuum polarization loop involving a lepton of mass M_L gives a contribution to the anomaly which depends on the ratio M_L/m_μ as shown in fig. 31. This figure has been constructed from published vacuum polarization calculations [55]. The 95% confidence limit set for M_L by the present result is

$$M_L > 210 \text{ MeV}/c^2 .$$

It is noted in passing that the heavy lepton τ [68] with mass $1.8 \text{ GeV}/c^2$ gives a contribution of only

$$\Delta a(\tau) \approx 0.4 \times 10^{-9} ,$$

which is an order of magnitude below the sensitivity of the present experiment.

7.5.3 Muon electric dipole moment

An electric dipole moment for the muon not only causes the plane of the muon spin precession to be tilted through an angle δ (see Section 1), but also increases

the magnitude of the precession frequency by a factor $\sec \delta$. The corresponding increase in the observed anomaly is given by

$$\Delta a(\text{edm}) = 4 \left(\frac{\mu}{\alpha} \right) \left(\frac{L_\mu}{\lambda_\mu} \right)^2, \quad (7.9)$$

where λ_μ is the muon Compton wavelength and

$$eL_\mu = D_\mu \doteq \text{muon edm} . \quad (7.10)$$

The contribution is positive definite, leading to the 95% confidence limit

$$D_\mu < 7.3 \times 10^{-19} \text{ e}\cdot\text{cm} .$$

This may be compared with the directly measured value of the edm (Section 6.2):

$$D_\mu = (3.7 \pm 3.4) \times 10^{-19} \text{ e}\cdot\text{cm} .$$

7.6 Limits on exotic contributions to the electron anomaly a_e

The best published value [12] for the electron anomaly is some 16 times more precise than the value given here for the muon:

$$\begin{aligned} a_e^{\text{exp}} &= (1159652.41 \pm 0.2) \times 10^{-9} \\ a_\mu^{\text{exp}} &= (1165924 \pm 8.5) \times 10^{-9} . \end{aligned}$$

However, the majority of the exotic contributions to the lepton anomaly scale as the mass squared, and so with the assumption of μ -e universality [28] the exotic contributions to the muon and the electron are related by

$$\Delta a_e = \left(\frac{m_e}{m_\mu} \right)^2 \Delta a_\mu . \quad (7.11)$$

Within this framework of μ -e universality the observed agreement between theory and experiment for the muon then ensures the absence of exotic contributions to the electron anomaly at a level 2×10^{-5} below the precision of the muon experiment. Thus the present muon experiment implies that no exotic contributions would be detected in an electron experiment even for a three-thousandfold increase

in the 0.2 ppm level of precision so far obtained for the electron anomaly. The electron anomaly is therefore expected to be a pure QED quality down to a level of precision of about 0.001 ppm at which hadronic effects become significant.

7.7 CPT theorem

Finally, we summarize a comparison of the results for positive and negative muons, which should be equal according to the CPT theorem.

For the muon anomaly and g-factors we have

$$6 \times 10^{-6} > \frac{a_{\mu^+} - a_{\mu^-}}{a_{\mu}} > -50 \times 10^{-6} \text{ (95\% CL)}$$

$$7 \times 10^{-9} > \frac{g_{\mu^+} - g_{\mu^-}}{g_{\mu}} > -58 \times 10^{-9} \text{ (95\% CL) ,}$$

while for the lifetime measurements in flight the comparison can be expressed as

$$3.0 \times 10^{-3} > \frac{\tau_{\mu^+} - \tau_{\mu^-}}{\tau_{\mu}} > -1.4 \times 10^{-3} \text{ (95\% CL) .}$$

It should be noted that the Lorentz γ -factor (29.3) is the same for both μ^+ and μ^- to a much higher precision than the quoted lifetime errors.

As can be seen in all cases, the exact equality of the properties for positive and negative muons lies within the 95% confidence limits.

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Table 1

Theoretical contributions to the muon g-factor anomaly

<u>QED</u>		
	Coefficient of $\left(\frac{\alpha}{\pi}\right)^n$	Contribution to a_μ ($\times 10^9$)
n = 1	0.5	1161409.84 $\pm$ 0.25
2	0.76578223	4131.77 $\pm$ 0.002
3	24.452 $\pm$ 0.056	306.45 $\pm$ 0.70
4	135 $\pm$ 63	3.9 $\pm$ 1.8
5	420 $\pm$ 30	0.028 $\pm$ 0.002
Total QED		1165852.0 $\pm$ 1.9
<u>Hadronic</u>		
	4th order	70.2 $\pm$ 8.0
	6th order	-3.5 $\pm$ 1.4
Total hadronic		66.7 $\pm$ 8.1
<u>Weak</u>		
	Weinberg-Salam model	2.1 $\pm$ 0.2
Total theoretical value for the anomaly		1165921 $\pm$ 8.3

Notes: The value of α^{-1} is taken to be 137.035987(29) from ref. 32.

The major source of uncertainty is in the hadronic contribution and comes from the experimental error in the measurement of the annihilation cross-section. Other principal sources are in the estimates for the eighth-order QED and sixth-order hadronic contributions, and we have taken the optimistic view that all these errors may be added in quadrature.

Table 2

Parameters of the muon storage ring

Radius	r_0	7.00	m
Vertical semi-aperture	a	4	cm
Horizontal semi-aperture	b	6	cm
Magnetic field	B	1.472	tesla
pole gap		14	cm
pole width		38	cm
Field index (average)	n	0.12	
Applied voltage ^{a)}			
side electrodes		24	kV
top and bottom		-3.7	kV
Muon momentum		3.094	GeV/c
gamma	γ	29.3	
lifetime	τ	64.4	μsec
Revolution frequency		6.81	MHz
time		147	nsec
(g - 2) frequency	f_a	0.2327	MHz
period		4.3	μsec

a) The sign of these voltages is for negative muons and the magnitudes are the actual voltages at the electrodes.

Table 3

Results for the relative spin precession frequency f_a

Period	Sign	Events analysed ($\times 10^6$)	f_a (raw) (MHz)	Q-loss correction (ppm)	f_a (corrected) (MHz)
1974	+	11.8	0.2327448(56)	0.0	0.2327448(56)
1975 A	+	9.5	0.2327438(52)	1.3	0.2327441(52)
B	+	15.3	0.2327467(69)	0.9	0.2327469(69)
C	-	11.7	0.2327601(54)	0.4	0.2327602(54)
1976 A	+	10.9	0.2327477(54)	0.9	0.2327479(54)
B	-	7.2	0.2327530(69)	0.0	0.2327530(69)
C	-	10.2	0.2327437(58)	0.0	0.2327437(58)
D	+	24.8	0.2327503(38)	0.9	0.2327505(38)
E	-	33.0	0.2327534(32)	0.4	0.2327535(32)

Table 4

Parameters of the radial distribution of muon equilibrium orbits obtained from fast rotation measurements made throughout the data collection period. (All entries in mm.)

$\langle x \rangle$	$\sqrt{\langle x^2 \rangle}$	δ	h	Comments
2.6	17.5	5.2	54.6	1974 (Unscraped)
5.9	17.5	11.8	51.4	1975 (Unscraped)
6.0	17.6	12.0	51.9	
6.1	17.6	12.2	51.4	
5.7	17.7	11.4	52.3	
6.5	17.6	13.0	50.9	
6.0	15.3	12.0	43.7	1976 (Scraped)
5.5	15.7	11.0	45.8	
5.2	16.2	10.4	46.6	
6.1	16.1	12.2	46.3	
5.5	16.2	11.0	47.6	

Table 5

Values of $\bar{f}_p$ for the different periods

Entries are given as corrections in ppm to a reference frequency of 62.781 MHz

Period	Sign	Ensemble averaging (error 0.7)	Time averaging (error 1.0)	Calibration	Total correction (error 1.5)
1974	+	8.6	-0.7	23.4(0.8) ^{a)}	31.3
1975 A	+	12.5	-1.5		34.4
B	+	12.5	-3.4		32.5
C	-	17.9	-0.5		40.8
1976 A	+	5.2	3.5		32.1
B	-	6.1	0.4		29.9
C	-	6.3	0.0		29.7
D	+	2.1	5.3		30.8
E	-	9.6	0.0		33.0

a) This correction is the same for all periods.

Table 6

Calibration corrections to f_p^{uncorr} in ppm

Correction	$\Delta f_p / f_p \times 10^6$
Bulk diamagnetic effect of cylindrical water sample	-1.51(0.04)
Effect of 0.1 mole solution of NiSO_4	0.88(0.09)
Diamagnetic screening the water molecule	25.67(0.02)
Calibration of mapping machine probes	-0.1(0.5)
Effect of mapping machine on stabilization probes	-1.5(0.2)
Total	23.4(0.6)

Table 7

Summary of the values of f_a , $\bar{f}_p$, and $R = f_a/\bar{f}_p$

Period	Sign	No. of stops ($\times 10^6$)	f_a (MHz)	$\bar{f}_p$ (MHz)	$R = f_a/\bar{f}_p$ ($\times 10^{-3}$)
1974	+	11.8	0.2327448(56)	62.78297(10)	3.707133(89)
1975 A	+	9.5	0.2327441(52)	62.78316(10)	3.707110(83)
B	+	15.3	0.2327469(69)	62.78304(10)	3.707162(110)
C	-	11.7	0.2327602(54)	62.78356(10)	3.707343(86)
1976 A	+	10.9	0.2327479(54)	62.78302(10)	3.707179(86)
B	-	7.2	0.2327530(69)	62.78288(10)	3.707269(110)
C	-	10.2	0.2327437(58)	62.78287(10)	3.707121(92)
D	+	24.8	0.2327505(38)	62.78294(10)	3.707225(61)
E	-	33.0	0.2327535(32)	62.78307(10)	3.707265(51)
Weighted averages	+	72.3	0.2327475(23)	62.78302(5)	3.707173(36)
	-	62.1	0.2327530(23)	62.78309(5)	3.707256(37)
	Over- all	134.4	0.2327502(16)	62.78306(3)	3.707213(27)

Table 8

Results for the muon lifetime in flight

All entries in μsec

Run	1	2	3	4	5	6
Sign	+	+	-	-	-	-
Starting time of fit	100	100	50	50	90	100
Uncorrected lifetime (statistical error)	64.406(66)	64.295(103)	64.444(55)	64.336(36)	64.366(84)	64.278(60)
μ -loss correction	0.008(8)	0.008(8)	0.013(8)	0.012(8)	0.009(8)	0.008(8)
p-background correction	0.006(2)	0.044(22)	-	-	-	-
Gain correction	0.022(20)	0.016(20)	-0.010(20)	-0.002(20)	0.033(20)	0.020(20)
Fully corrected lifetime (total error)	64.442(69)	64.363(108)	64.447(59)	64.346(42)	64.408(87)	64.306(64)
Fully corrected lifetime (total error) 100 μsec starting time	-	-	64.427(86)	64.347(63)	64.419(95)	-

Table 9

Lifetime results (μsec)

	μ^+	μ^-
Lifetime in flight (this experiment)	64.419(58)	64.368(29)
Lifetime at rest (this experiment)	2.1966(20)	2.1948(10)
Lifetime at rest from previous best measure- ments [52,53]	2.19711(8)	2.198(2)

Table 10

Summary of edm results

Period	Sign	No. of stops ($\times 10^6$)	$\Delta\phi = \phi_{\text{up}} - \phi_{\text{down}}$ (mrad)	Dipole moment ^{a)} ($e \cdot \text{cm}$) $\times 10^{-19}$
1975 B	+	1.1	-14.9 (17.1)	7.7 (8.8) ^{b)}
C	-	1.9	-2.6 (13.2)	-1.3 (6.8)
1976 A	+	2.1	-27.0 (11.5)	14.9 (6.3)
B	-	0.8	+1.6 (18.8)	0.9 (10.3)
C	-	1.5	+1.9 (13.6)	1.0 (7.4)
D	+	2.2	-6.3 (11.8)	3.3 (6.1)
E	-	1.8	+5.5 (13.4)	2.8 (6.9)
Weighted averages	+	5.4	-16.5 (7.4)	8.6 (4.0)
	-	6.0	+1.6 (7.2)	0.8 (3.8)
	Over- all	11.4	-7.2 (5.2)	3.7 (2.7) ^{c)}

- a) The definition of phase difference is such that a negative value implies a positive value for f . Since $D \equiv (f/2)(eh/2mc)$, the dipole moment has the same sign as f for positive muons and the opposite for negative ones.
- b) Errors quoted are 1 standard deviation statistical only.
- c) The consistency χ^2 of this value with the values found for the individual runs is 4.9 for 6 degrees of freedom.

Table 11

Tests of QED from the three CERN muon ($g - 2$) experiments.
The entries are the values of

$$\frac{|a_{\text{exp}} - a_{\text{th}}| + (\sigma_{\text{exp}}^2 + \sigma_{\text{th}}^2)^{1/2}}{C_n (\alpha/\pi)^n}$$

for different members of the perturbation series $a_\mu = \sum_n C_n (\alpha/\pi)^n$.

	n = 1	n = 2	n = 3
First experiment [17]	0.008	2.2	29.0
Second experiment [18]	0.0005	0.13	1.8
This experiment	0.000012	0.0035	0.047

Table 12

Summary of the 95% confidence limits set by the three CERN muon ($g - 2$) experiments on various 'exotic' contributions to a_μ

QED violation parameters (GeV)						Heavy lepton mass (GeV/c ²)	Heavy boson coupling for M ₀ = 10 GeV/c ²					edm (e•cm)
Photon propagator		Vertex function		Muon propagator			M _L	f _S	f _{PS}	f _V	f _A	
First experiment [17]	Λ _γ ⁺	Λ _γ ⁻	Λ _μ ⁺	Λ _μ ⁻	Λ ⁺	Λ ⁻	0.015	0.74	1.2	2.6	1.7	1.1 × 10 ⁻¹⁷
	1.2	0.8	2.1	1.4	0.1	0.06						
Second experiment [18]	3.2	4.7	5.5	8.2	0.3	0.4	0.039	0.28	0.20	0.98	0.28	4.2 × 10 ⁻¹⁸
This experiment	18.2	20.7	31.6	36.0	1.5	1.7	0.210	0.048	0.045	0.17	0.065	7.3 × 10 ⁻¹⁹

Figure captions

- Fig. 1 : Plan view of the muon storage ring and a cross-section through one of the forty magnets. For most of the data taking 22 electron detectors were used. These were distributed around the ring at intervals which were as nearly uniform as possible.
- Fig. 2 : General view of the completed muon storage ring showing its basic structure of 40 magnets set at an angle of 9° with respect to each other.
- Fig. 3 : A view of the inner part of one sector of the ring showing the magnet coil supports and the positions of the decay electron detectors.
- Fig. 4 : a) A horizontal section through the inflector showing the pion channel and adjacent part of the storage region.
b) A vertical section through the inflector.
c) The basic crowbar circuit used to pulse the inflector.
- Fig. 5 : A contour line plot of the magnetic field strength in the muon storage aperture. This map is obtained by averaging a three-dimensional map in azimuth. The interval between the contours of equal field strength is 2 ppm or $3 \mu\text{T}$.
- Fig. 6 : Block diagram of the stabilization system for one magnet showing how the signals from the NMR probe and pick-up coil are combined to act upon the current in the compensating coils.
- Fig. 7 : Cross-section through the quadrupole electrodes and vacuum chamber.
- Fig. 8 : Schema of the muon detector and associated electronics used in the measurement of losses.

Fig. 9 : Schema of the fast electronics used in the detection of decay electrons. The group of four counters associated with digitron 1 are shown at the top. The signals from these counters are divided to provide the timing pulses and the pulse-height information as shown. The start pulse for the timing of the muon decays is derived from an electrostatic pick-up device in the proton beam. The entry of this pulse into the digitron controller is indicated at the bottom right of the diagram.

Fig. 10 : Results of the Monte Carlo injection program:

- a) The emergence of the pion beam from the storage ring as a function of azimuthal angle.
- b) The number of muons born in the storage region and those leaving the aperture as a function of azimuthal angle.

Both curves are expressed as a percentage of the number of pions injected into the ring.

Fig. 11 : The effect of the focusing structure on the acceptance of the storage ring as a function of mean field index.

Fig. 12 : The distorted equipotentials of the focusing field due to the scraping process at the time of injection. The numbers are in kV and refer to the polarity for negative muons.

Fig. 13 : Variation of the $(g - 2)$ frequency f_a obtained when fitting that part of a data subset which occurred after a particular storage time. For the six-parameter hypothesis there are clear oscillations in f_a with the starting time of the fit (20 to 40 μ sec after injection). These oscillations are removed when the eight-parameter hypothesis is used.

- Fig. 14 : The variation of f_{α} with the starting time of the fit for the total data. The weighted averages for the top four pulse-height bands are shown. Since the data used in the fits which start at later storage times consist of smaller and smaller subsets of those data used in the fit which starts at 21 μ sec, the results should agree within the limitation of statistics. The $\pm\sigma$ zone within which these results should lie is shaded, and as can be seen there is no evidence for any systematic error.
- Fig. 15 : The fitted asymmetry as a function of the fraction of the total counts lying above each threshold.
- Fig. 16 : The values of f_{α} for individual pulse-height bands.
- Fig. 17 : The frequency shift due to queuing losses as a function of counting rate for different buffer transfer times $\zeta = 0.48$ and 0.355μ sec.
- Fig. 18 : The fast rotation pattern. This is the count rate at early time which clearly shows the muon bunch rotating around the ring with a period of 147 nsec.
- Fig. 19 : The reconstructed radial distribution of muons for scraped and unscraped data compared with a Monte Carlo simulation of the latter.
- Fig. 20 : Parametrization of the muon distribution.
- Fig. 21 : An illustration of the sensitivity of the average magnetic field $\bar{B}$ to changes in the radial distribution of muons.
- Fig. 22 : The individual values of R for the nine experimental periods and the weighted averages.
- Fig. 23 : The muon losses displayed in terms of the ratio of the experimental data at early time (less than 300 μ sec) to points on a curve extrapolated back from a fit to the data at a late time (greater than 300 μ sec). The points $\times$ give the results with no scraping, the points $\circ$ results with scraping. For comparison with $\times$, the points $\bullet$ show the results obtained by integration [eq. (6.2)] over the lost-muon time spectrum of the muon detector.

- Fig. 24 : The raw spectrum of muon counts in the muon detector. The total estimated muon loss is shaded, the remainder of the spectrum being due to misidentified decay electrons.
- Fig. 25 : The decay electron distribution recorded in the muon detector, weighted by the factor $\exp(t/64.4 \mu\text{sec})$. Notice that the detector takes some 50 μsec to fully recover from the paralysis caused by the initial flash of particles injected into the ring.
- Fig. 26 : Comparison between two gain measurements. The size of the effect required to shift the lifetime by 1% is indicated by the solid line.
- Fig. 27 : The phase difference between 'up' and 'down' ($g - 2$) oscillations as a function of the vertical distance between the counter split and the median plane of the muon orbits.
- Fig. 28 : The principal Feynman diagrams determining the electron and muon anomalous magnetic moments.
- Fig. 29 : The relationship between the virtual hadronic contribution to the muon anomaly and electron-positron annihilation into real hadrons.
- Fig. 30 : The upper limits on the coupling of a heavy neutral boson to the muon.
- Fig. 31 : Contribution to the muon anomaly due to another lepton as a function of the ratio of the lepton to muon masses.

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Fig. 3

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