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THE TRIPLE POMERON COUPLING AND HIGH ENERGY DEUTERON BREAK-UP

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A B S T R A C T

We analyze the triple Pomeron contribution to deuteron break-up in high energy hadron-deuteron collisions and show that although this contribution potentially has a distinctive signal in its dependence on the longitudinal momentum transfer to the proton and neutron, it is somewhat masked by the other major contributions. We point out, however, an interference dip structure at moderate momentum transfers which appears to be quite sensitive to the presence of the triple Pomeron contribution. Furthermore, the latter effect will also be quite sensitive to the relative pieces of the various contributions. Other complications are pointed out.

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The existence of a triple Pomeron coupling (PPP) leads to several very distinctive features in high energy hadron interaction. First there is the familiar high mass diffractive dissociation observed in proton-proton collisions for example. Although these processes account for only about 10% of the total pp cross-section, they give a characteristic $(1-x)^{-1}$ behaviour in the single particle inclusive cross-section, from which the strength of the triple Pomeron coupling can be deduced ¹⁾. Secondly, high-mass diffractive dissociation leads to inelastic shadowing in hadron-deuteron collisions. Although the latter effect at $p = 500 \text{ GeV/c}$ is expected to contribute only 1 mb ^{2),3)} to the total cross-section, using the PPP coupling obtained in the single particle inclusive measurement, it is distinguished by its logarithmic increase with energy, which should be quite pronounced at ISR energies.

Here we discuss another potentially distinctive effect due to the triple Pomeron coupling. In the deuteron break-up interaction

$$h d \rightarrow h p n \quad (1)$$

at high energies, the triple Pomeron coupling leads to contributions (see Fig. 1) with large longitudinal momenta of the proton and of the neutron. These large Δ_z are possible due to the large energy (invariant mass squared) M^2 across the upper Pomeron [see Eq. (7)]. In this process, in contrast with the other two examples mentioned above, the PPP effect is a true exchange effect and not only a representation of the large mass diffractive dissociation. The deuteron break-up reaction thus offers a rather unique means of testing the validity of the PPP exchange mechanism.

We estimate below the magnitude of the PPP contribution to (1). Since over much of the phase space we expect this contribution to be small, we shall also discuss a suitable set of variables in which the large longitudinal momentum transfer effects would be more clearly distinguishable. However, the deuteron wave function falls off so slowly that the single Pomeron exchange (P) and the double Pomeron exchange (PP) (see Fig. 2) together tend to mask the possible signal of the PPP contribution. We will nevertheless see that there is a region in which the P and PP+PPP terms interfere destructively and where the triple Pomeron exchange can have a significant influence.

The single and double scattering contributions to (1) are shown in Fig. 2 and, treating the deuteron as a non-relativistic system, we have

$$A^{(s)} = \sqrt{4m} \left[\phi(\vec{p}_n) A_{hp}(s, t) + \phi(\vec{p}_p) A_{hn}(s, t) \right] \quad (2)$$

and

$$A^{(D)} = \frac{1}{\sqrt{m}} \int \frac{d^3q}{(2\pi)^3} \phi(\vec{q}) A_6 \quad (3)$$

where $s = (\vec{p}_n + \frac{1}{2}\vec{p}_D)^2$ and m is the nucleon mass.

The single scattering contribution (2) is easily evaluated given the deuteron wave function ϕ . We thus turn our attention to the double scattering contribution (3) which contains both the PP and PPP terms (Fig. 2c).

When the q integral in (3) is performed, several types of singularities in A_6 are encountered, of which the most important are the nearby ones shown in Fig. 3.

We shall restrict our attention only to the most important contributions at high energies. We first introduce a convenient set of variables for describing high energy deuteron break-up, namely

$$s, t, t_p, t_n, x \quad (4)$$

where

$$t_p = \left(\frac{1}{2} P_D - p_p \right)^2; \quad t_n = \left(\frac{1}{2} P_D - p_n \right)^2; \quad x = \frac{1}{2} (x_p - x_n) \quad (5)$$

with

$$x_p = p_p^{(\omega)} / \frac{1}{2} P_D^{(\omega)} \quad ; \quad x_n = p_n^{(\omega)} / \frac{1}{2} P_D^{(\omega)} \quad (6)$$

The $(-)$ components in (6) are the usual $p^{(-)} = 1/\sqrt{2} (E - p_z)$ variables. The incident hadron beam is assumed to be moving in the $+z$ direction. In terms of these variables, we have

$$s_{pn} = (p_p + p_n)^2 = 4m^2 + t - 2(t_p + t_n) \quad (a)$$

and, in the deuteron rest frame

$$M^2 = s(x - q_z/m) + O(1) \quad (b) \quad (7)$$

with

$$t_p = -\vec{p}_p^2; \quad t_n = -\vec{p}_n^2, \quad (c)$$

and

$$|\vec{p}_p - \vec{p}_n|^2 = -t + \frac{(t_p + t_n)^2}{4m^2} \quad (d)$$

If we neglect the dependence of A_6 on the small deuteron internal momenta $\vec{q}_\perp$, we can replace (3) by

$$A^{(D)} \simeq \frac{1}{\sqrt{m}} \int_{-\infty}^{+\infty} \frac{dq_z}{2\pi} \phi(q_z) A_6 \quad (8)$$

where

$$\phi(q_z) = \int \frac{d^2 \vec{q}_\perp}{(2\pi)^2} \phi(\vec{q}) \quad (9)$$

From (7) we see that the q_z integral runs along the right and left singularities of A_6 in M^2 . These singularities give rise to a contribution to (8) which can be expressed in terms of the same amplitude whose discontinuity is measured in inclusive cross-sections. Assuming A_6 is crossing symmetric and using (7a), we can write

$$A^{(D)} \simeq \frac{\sqrt{m}}{4\pi s} \int_{M_{th}^2}^{\infty} dM^2 \operatorname{disc}_{M^2} A_6(M^2) \cdot \left\{ \phi\left[m\left(\frac{M^2}{s} - x\right)\right] + \phi\left[m\left(\frac{M^2}{s} + x\right)\right] \right\} \quad (10)$$

Equation (10) is only approximate because we have neglected the real parts of A_6 . The latter are not measured in the inclusive process and have in general a complicated structure ⁴⁾. The q_z integration contour in Eq. (8) cannot be rotated so as to express $A^{(D)}$ only in terms of $\operatorname{disc}_{M^2} A_6$, due to the singularities in the wave function ϕ ^{2), 5)}. However, the contribution from

the real part of A_6 will play an important rôle in filling in dips due to the destructive interference between $A^{(S)}$ and $A^{(D)}$. The latter contribution can be computed via the integral

$$\text{Real } A^{(D)} \simeq \frac{\sqrt{m}}{2\pi s} \int_{-\infty}^{+\infty} dM^2 \text{Re } A_6(M^2) \phi\left[m\left(x - \frac{M^2}{s}\right)\right] \quad (11)$$

An important contribution to (10) arises from the elastic intermediate state ($M^2 = m^2$) :

$$A^{(PP)} \simeq i \frac{\sqrt{m}}{s} A_{hp}(s, t_p) A_{hn}(s, t_n) \phi(mx). \quad (12)$$

The triple Pomeron contribution (Fig. 1) is

$$A^{(PPP)} \simeq -is 8\pi\sqrt{m} G_{PPP}(t, t_p, t_n) \cdot \int_{M_0^2/s}^{\bar{w}} \frac{dW}{W} \left\{ \phi\left[m\left(W-x\right)\right] + \phi\left[m\left(W+x\right)\right] \right\} \quad (13)$$

where G_{PPP} is the PPP coupling defined in Ref. 1), generalized to the non-forward case.

The limits of the W integral are the ones given by the limits of validity of the PPP form, and we shall take $\bar{W} = 0.2$ and $M_0^2 = 1 \text{ GeV}^2$. The final state rescattering contribution of Fig. 3b is

$$A^{(RS)} \simeq - \frac{A_{hp}(s, t)}{\sqrt{m}} \int \frac{d_3 \vec{q}}{(2\pi)^3} \frac{\phi(\vec{q}) A_{pn}(s_{pn}, \bar{t})}{[(\frac{1}{2}P_0 + q + 4)^2 - m^2]} + (p \leftrightarrow n) \quad (14)$$

From Eq. (7b), we see that we are interested in the region $s_{pn} \approx 4m^2$, so that $A_{pn}(s_{pn}, \bar{t})$ can be approximated by the S wave contribution, $A_{pn} \simeq 16\pi m/a_T k^2$, where a_T is the triplet scattering length and $4k^2 = s_{pn} - 4m^2$. The latter amplitude can be taken out of the integral which then can be performed analytically, leading to ⁶⁾

$$A^{(RS)} \simeq i \frac{8\sqrt{m} A_{hp}}{\sqrt{-t} a_T [t - 2(t_p + t_n)]} \cdot \sum_j C_j \ln \left[\frac{\sqrt{-t} + \sqrt{t - 2(t_p + t_n) + 2i\alpha_j}}{-\sqrt{-t} + \sqrt{t - 2(t_p + t_n) + 2i\alpha_j}} \right] \quad (15)$$

where we have assumed a wave function of the form

$$\phi(\vec{q}) = \sum_j \frac{c_j}{\vec{q}^2 + \alpha_j^2} \quad (16)$$

Exchange poles like the one involving the pion in Fig. 3c may give important contributions at high energies. They are of the general triple Regge form and it is argued in Ref. 3) that they give a significant antishadowing effect in the hd total cross-section. Furthermore, Alberi et al. 7) have pointed out their possible important rôle in deuteron break-up. Like the PPP term, the contribution of Fig. 3c and other Regge contributions involve a large M^2 and their study would be very interesting in itself, since a good knowledge of the pion contribution in particular is important in understanding other aspects of hd interactions 3), 8). The latter contribution will be important for $M^2 \sim s$ ($w \sim 1$). The dependence on x will thus be weak and we obtain the approximate form

$$A^{(\pi\pi P)} \simeq i s \cdot 16\pi\sqrt{m} G_{P\pi\pi}(t, t_p, t_n) \int_0^1 dw \cdot w \cdot \phi(mw) \quad (17)$$

Thus $A^{(\pi\pi P)}$ has a similar structure as $A^{(S)}$. We shall include $A^{(S)}$, $A^{(RS)}$ and $A^{(\pi\pi P)}$ in a single term $A^{(P)}$ corresponding to single Pomeron exchange.

Comparing the PPP and the single Pomeron exchange contributions (P) [Eqs. (13) and (2), (15) and (17)], we see that the PPP is characterized by a strong x dependence ($\approx x^{-1}$) at fixed s, t, t_p, t_n , since important contributions come from $w \simeq |x|$ where $\phi(q_z)$ is large. Unfortunately, this signal will be obscured by the similar x dependence of the PP contribution [Eq. (12)], coming from the deuteron wave function. The PP term is distinguished from the PPP term by its flat behaviour as a function of t .

In order to get an idea of the size of the various terms and the resulting cross-sections, we shall use values of A_{hN} appropriate to $s = 1000 \text{ GeV}^2$ and the Hulthén wave function

$$\phi(\vec{q}) = N \left[\frac{1}{\vec{q}^2 + \alpha^2} - \frac{1}{\vec{q}^2 + \gamma^2} \right] \quad (18)$$

where $N = 1.377 \text{ GeV}$, $\alpha = 45.7 \text{ MeV}$ and $\gamma = 6.2 \alpha$.

It turns out that the rescattering term of Eq. (15) is small compared to $A^{(S)}$. At large t_p, t_n the $\pi\pi P$ term will be important, however, the results of any calculation will be model-dependent, and for simplicity we will not include this contribution. For the purpose of making some estimates, we approximate the remaining terms by

$$A^{(P)} \simeq i s e^{bt} \beta^2 N \left[\frac{1}{-t_p + d^2} - \frac{1}{-t_p + \gamma^2} \right] + (P \rightleftharpoons n)$$

$$A^{(PP)} \simeq -i s e^{b(t_p+t_n)} \beta^4 \frac{N}{4\pi} \ln \left[\frac{m^2 x^2 + \gamma^2}{m^2 x^2 + d^2} \right]$$

$$A^{(PPP)} \simeq -i s e^{2.25(t+t_p+t_n)} \beta^3 G_{PPP}(0) \frac{N}{4\pi} \cdot \int_{\gamma_s}^2 \frac{dW}{W} \left[\ln \left(\frac{m^2(W-x)^2 + \gamma^2}{m^2(W-x)^2 + d^2} \right) + (x \rightleftharpoons -x) \right] \quad (19)$$

with $\beta = 10 \text{ GeV}^{-1}$ and $G_{PPP}(0) = 6.25 \text{ GeV}^{-4}$, $b = 6 \text{ GeV}^{-2}$.

Because of the difficulties in detecting high energy neutrons, one will measure high energy elastic pd break-up in pd collisions by extracting the neutron elastic part in the inclusive experiment

$$pd \rightarrow pp + X_n \quad (20)$$

The elastic process contributes only near $p_x^2 = p_n^2 = m^2$, and can be written in the form

$$2E_p 2E_{p'} \frac{d\sigma}{d_3 p_p d_3 p_{p'}} = \frac{1}{2s} \frac{1}{(2\pi)^5} \left| \sum_{i=P, PP, PPP} A^{(i)} \right|^2 \delta(p_n^2 - m^2) \quad (21)$$

with

$$P_n = P + P_D - P_P - P'$$

Experimentally, the cross-section measured will be Eq. (21) integrated over p_n^2 near $p_n^2 = m^2$, and we therefore plot this quantity in the following estimates. Using (19) we plot in Fig. 4 the dependence of the longitudinal momentum transfer variable x , for $s = 1000 \text{ GeV}^2$, $|t_p| = |t_n| = 0.15 \text{ GeV}^2$ and $|t| = 0.5 \text{ GeV}^2$, for P , $P+PP$ and $P+PP+PPP$, respectively. We see that the PPP term is likely to have a considerable influence on the position of the dip. However, qualitatively, a renormalized PP term will have much the same effect. Since the PP term is flat in the variable t , whereas the PPP term falls like $\exp(2.25 t)$, one should see how the dip structure changes with t . For comparison we have plotted (dotted curve) the respective contributions for $|t| = 0.1$.

In Fig. 5 we compare the t dependence of P , $P+PP$ and $P+PP+PPP$, respectively. Here the qualitative similarity of $P+PP$ versus $P+PP+PPP$ is disappointing. However, the behaviour in the vicinity of the dips will be important in determining the relative phases.

The combined x and t distributions for $P+PP$ and $P+PP+PPP$ are shown in Figs. 6 and 7, respectively. Again we see that although the triple Pomeron effect is likely to have a significant effect (based on the size of the PPP coupling in inclusive distributions), it is qualitatively not very distinguishable from the effect of the PP term alone interfering with the single-scattering amplitude.

An estimate of the rescattering contribution, Eq. (15), shows that in the above regions of interest, it contributes less than 10% of the single Pomeron exchange term and has a similar x and t dependence. The same is true for the pion exchange term in Eq. (17). Although the latter is likely to be somewhat larger.

We have not shown the effect of different wave functions, but this is another problem in any analysis of the data. A good knowledge of the high momentum tail of the deuteron wave function will also be required and

the effect of the D wave is likely to be important in the vicinity of the dip ⁹⁾. Our analysis shows that accurate high statistics data on high energy pd break-up provides a unique opportunity for extracting valuable information on both the magnitude and the phase of the triple Pomeron coupling. However a careful analysis with a reliable wave function, which includes the D wave, will be required. Furthermore data at different energies will be useful in disentangling double scattering from wave function effects.

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FIGURE CAPTIONS

- Figure 1 The triple Pomeron exchange mechanism in pd break up.
- Figure 2 Single and double scattering in pd break up.
- Figure 3 Contributions to double scattering arising from various singularities in A_6 :
a) ordinary double scattering ;
b) final state rescattering ;
c) particle exchange poles.
- Figure 4 Plot of the inclusive cross-section $pd \rightarrow pp + X_n$ integrated over the neutron pole in mb/GeV^2 as a function of x , for $|t_p| = |t_n| = 0.15 \text{ GeV}^2$ and $s = 1000 \text{ GeV}^2$ for $|t| = 0.05 \text{ GeV}^2$ (solid curves) and $|t| = 0.1 \text{ GeV}^2$ (dotted curves), respectively. The P, P+PP and P+PP+PPP contributions are shown separately.
- Figure 5 Plot of the inclusive cross-section $pd \rightarrow pp + X_n$ integrated over the elastic neutron pole in mb/GeV^2 as a function of t for $x = 0.1$, $|t_p| = |t_n| = 0.15 \text{ GeV}^2$, $s = 1000 \text{ GeV}^2$.
- Figure 6 Two-dimensional plot showing the dependence of the P+PP contribution as a function of x and t .
- Figure 7 Two-dimensional plot showing the dependence of the P+PP+PPP contribution as a function of x and t .

Fig.1 The Triple Pomeron Mechanism

Fig.2 Single and Double Scattering Contribution to
p d Breakup

Fig.3: Contributions to double scattering arising from various singularities in A_6

(a) Ordinary double scattering

(b) Final state rescattering

(c) Particle exchange poles

Fig.4

Fig. 5

Fig. 6

Fig. 7