Ref.TH.2585-CERN

BEYOND THE SINGLE Z BOSON HYPOTHESIS

G. Ecker ^{*)}

CERN -- Geneva
and
Institute for Theoretical Physics
University of Vienna

A B S T R A C T

The structure of the hadronic neutral current in neutrino interactions is examined in detail with particular emphasis on the isoscalar coupling constants. The present status of the single Z boson assumption is discussed. In order to study violations of factorization the phenomenology of expanded gauge theories $SU(2) \times U(1) \times G$ for arbitrary G is described which are constrained to have the same low energy neutrino neutral current interactions as $SU(2) \times U(1)$. Independent of any further assumptions, various tests for such a gauge structure are possible. The special case of two massive neutral gauge bosons is considered in more detail.

*) Supported in part by "Fonds zur Förderung der wissenschaftlichen Forschung in Österreich".

Ref.TH.2585-CERN

26 October 1978

1. - INTRODUCTION

Recent experimental and theoretical activity has greatly clarified the structure of the neutral current ^{1),2)}. Both model-independent analyses of neutrino hadron neutral current couplings and the observation of parity violation in polarized electron deuteron scattering ³⁾ seem to point almost uniquely to $SU(2) \times U(1)$ as the correct gauge theory of weak and electromagnetic interactions and, even more remarkably, to the simplest version of Salam and Weinberg ⁴⁾. At this stage, it should, however, be emphasized that no real test of the single Z boson assumption has been performed yet. In other words, none of the factorization relations ⁵⁾ following from single Z boson exchange has really been tested. It is the purpose of this paper to review critically the present situation in the neutrino hadron sector on the one hand, with special emphasis on the isoscalar coupling constants. On the other hand, we are going to analyze quantitatively the current status of the single Z boson assumption. That discussion will lead naturally to the consideration of possible extensions of $SU(2) \times U(1)$ as the prototype of a single Z boson theory.

There exists an extensive literature on gauge models with more than one neutral weak gauge boson. Although many of these models have not survived the confrontation with hard experimental facts, there still remains great freedom to adjust the various parameters as data are accumulating. Especially the Higgs sector of many expanded gauge models allows for almost unlimited adaptation and the predictive power of such theories is therefore, on the contrary, rather limited. This unsatisfactory situation leads one to ask the following simple questions : if at least one of the afore-mentioned factorization relations turns out to be violated, will we be able to conclude more than the trivial consequence that there must be at least two massive neutral gauge bosons provided we adhere to the gauge theory prejudice ⁶⁾ of intermediate vector bosons ? Can we make any general predictions at all if we give up the single Z boson exchange ?

Throughout this paper electron muon universality will be assumed. If μe universality is violated more factorization tests for the single Z boson hypothesis are possible which have recently been discussed ⁷⁾ for the purely leptonic neutral current reactions. The considerations of this paper are relevant as long as the neutral intermediate bosons are not produced directly, i.e., only low energy manifestations of the neutral current will be taken into account. This "low energy" domain includes the weak electromagnetic interference effects to be measured at PETRA and PEP energies.

In Section 2 the model-independent determination of neutrino hadron neutral current couplings is briefly reviewed. With our present understanding of the data the isoscalar couplings are somewhat bigger than in the standard model. The origin of this effect and its possible theoretical implications are discussed. The importance of more precise information on the isovector-isoscalar interference terms is emphasized. In Section 3 the present status of the single Z boson hypothesis is analyzed. A single test of factorization is possible at the moment which is, however, weakened by the contradictory results on parity violation in bismuth. The y dependence of the electron deuteron asymmetry can be predicted within the single Z boson scheme. In Section 4 the violation of at least one factorization relation is anticipated. After a short model-independent deliberation we are led to consider expanded gauge theories⁸⁾ of the type $SU(2) \times U(1) \times G$ with arbitrary G where the neutrino neutral current couplings at low momentum transfer are constrained to be the same as for $SU(2) \times U(1)$. The phenomenology of such theories is set up in a general way. Without any further assumptions, definite predictions are made concerning the relation between weak-electromagnetic interference in $e^+e^- \rightarrow \mu^+\mu^-$ and neutrino electron scattering. In principle, it should be possible to decide whether more than two Z bosons are needed from measurements at energies much lower than the mass of the lightest Z boson. In Section 5 the special case where G has only one neutral generator is considered in more detail. The weak contribution to the anomalous magnetic moment of the muon in such theories is discussed. Section 6 contains a summary of the conclusions of this work. In Appendix A the definitions of neutral current couplings and the factorization relations implied by single Z boson exchange are given. In Appendix B the main features of expanded gauge theories à la Georgi and Weinberg⁸⁾ are summarized.

2. - NEUTRINO HADRON NEUTRAL CURRENT COUPLINGS

In the past year, a decisive step forward has been made in determining the neutrino hadron neutral current coupling constants in a completely model-independent way^{1),2)}. Five different experiments on inclusive scattering⁹⁾⁻¹³⁾ on isoscalar targets have produced results in impressive agreement with each other. The observed scaling behaviour is expressed most conveniently in terms of the quark parton model although the analysis of inclusive scattering can be performed more generally where the quark flavours are just labels for the isospin properties of the hadronic neutral current.

Due to the isoscalar target and the inclusive final state the information from inclusive scattering is limited. A crucial new input was provided by a measurement of the pion charge ratios in semi-inclusive pion production in Gargamelle ¹⁴⁾. Those ratios were analyzed ¹⁵⁾ within the parton model to determine uniquely the absolute values of the four chiral coupling constants u_L, d_L, u_R, d_R (see Appendix A for definitions). In the parton model approach the relative signs of these coupling constants are not observable. It was soon recognized ^{16),17)} that the existing data on elastic neutrino proton scattering and exclusive pion production narrow down the various possibilities to essentially two different solutions which are dominantly isovector. Only recently, the remaining VA ambiguity was resolved by a determination ^{18),19)} of the elastic form factors at $Q^2 = 0$. The most sensitive quantity to distinguish between the two solutions turns out ²⁾ to be the magnetic Sachs form factor $G_M(0)$. Unlike the axial form factor $G_A(0)$, it has the additional advantage ²⁰⁾ not to depend on the proton expectation value of the isoscalar axial current which is model-dependent ²¹⁾. Moreover, both $G_E(0)$ and $G_M(0)$ have now been shown ¹⁹⁾ to be quite insensitive to variations of the axial dipole mass M_A . The situation is summarized in Fig. 1 where the two solutions A and B are displayed in the vector coupling plane together with the one standard deviation constraints implied by $G_E(0)$ and $G_M(0)$ ¹⁹⁾. For the coupling constants $\alpha, \beta, \gamma, \delta$ (see Appendix A for definitions), solution A gives

$$\begin{aligned}\alpha &= 0.52 \pm 0.09 \\ \beta &= 0.93 \pm 0.09 \\ \gamma &= -0.30 \pm 0.11 \\ \delta &= 0.10 \pm 0.11\end{aligned}\tag{2.1}$$

Additional arguments, if somewhat less convincing, have been put forward in favour of solution A from a careful study ²²⁾ of elastic neutral to charged current cross-section ratios and from the data on exclusive pion production analyzed ²³⁾ within the Adler model.

Various comments concerning the results expressed in Eq. (2.1) and shown in Fig. 1 are necessary. The existing inclusive data ⁹⁾⁻¹³⁾ were averaged to yield $R = 0.291 \pm 0.008$, $\bar{R} = 0.357 \pm 0.023$ which are only marginally different from the values obtained by the most precise CDHS experiment ¹²⁾. The antineutrino to neutrino charged current cross-section ratio was taken to be $r = 0.48$ for the inclusive analysis in accordance with the high energy measurements ²⁴⁾⁻²⁶⁾ while for the semi-inclusive analysis of pion charge

ratios the low energy value ⁹⁾ of $r=0.38$ was used to account for sea effects. The inclusive analysis is rather insensitive to reasonable variations in r with the over-all tendency of shrinking domains for decreasing r . Even with sea effects taken into account, the uncertainties of the scaling approach to the low energy semi-inclusive data must be kept in mind. Higher energy measurements of semi-inclusive pion production would be of great help to put definite conclusions about the detailed structure of the neutral current on a more solid basis.

From Fig. 1 solution A is seen to be in rough agreement with the standard model ⁴⁾ although not within one standard deviation. In Fig. 2 solution A is shown in the isoscalar coupling plane. With due caution in view of the above comments one may infer a tendency of the isoscalar coupling constants to be somewhat bigger than in the standard model. It seems worth while to look briefly into the origin of such a possible effect.

Neglecting sea effects in semi-inclusive scattering for the following qualitative arguments we find the following expressions for the pion charge ratios according to the parton model ¹⁵⁾

$$\rho = \frac{\langle \pi^+ \rangle_v}{\langle \pi^- \rangle_v} = \frac{\eta + r_1}{1 + \eta r_1} \quad (2.2)$$

$$\bar{\rho} = \frac{\langle \pi^+ \rangle_{\bar{v}}}{\langle \pi^- \rangle_{\bar{v}}} = \frac{\eta + r_2}{1 + \eta r_2} \quad (2.3)$$

where η is the ratio of integrated quark fragmentation functions ($\eta = 2.8 \pm 0.7$ for the Gargamelle experiment) and r_1, r_2 contain the neutral current couplings [$\xi = 0.15, \bar{\xi} = 0.13$ are defined in Ref. 15]

$$r_1 = \frac{d_L^2 + \xi d_R^2}{u_L^2 + \xi u_R^2} \quad (2.4)$$

$$r_2 = \frac{d_R^2 + \bar{\xi} d_L^2}{u_R^2 + \bar{\xi} u_L^2} \quad (2.5)$$

From the inclusive ratios $R, \bar{R}$ one obtains for $r=0.48$

$$\begin{aligned} u_L^2 + d_L^2 &= 0.271 \pm 0.012 \\ u_R^2 + d_R^2 &= 0.041 \pm 0.015 \end{aligned} \quad (2.6)$$

and therefore a small ratio $(u_R^2 + d_R^2)/(u_L^2 + d_L^2)$. This ratio decreases with r and is even smaller for the CDHS data ¹²⁾ alone.

Equation (2.4) shows that the ratio of left-handed coupling constants d_L^2/u_L^2 is mainly determined by ρ . Experimentally it is found ¹⁴⁾ that $\rho < 1$ which implies $d_L^2/u_L^2 > 1$ in even quantitative agreement with the Salam-Weinberg model. If there were no right-handed couplings at all Eqs. (2.4) and (2.5) would give $r_2 = r_1$ and therefore $\bar{\rho} = \rho$. The experimental value $\bar{\rho} = 1.64 \pm 0.36$, together with the small ratio $(u_R^2 + d_R^2)/(u_L^2 + d_L^2)$, then implies a large value for u_R^2/d_R^2 which is bigger than in the standard model. Expressed in different terms, the large value of $\bar{\rho}$ in connection with the small observed difference $\bar{R} - R$ is responsible for an isovector-isoscalar interference term which exceeds the prediction of the standard model.

In order to confirm the magnitude of the isoscalar couplings one should look at other tests of the isovector-isoscalar interference. In addition to semi-inclusive pion production there exist at least two more experiments which bear on this question. The Gargamelle experiment on single pion production by neutrinos ²⁷⁾ finds a value for the difference $\sigma(\nu p \pi^0) - \sigma(\nu n \pi^0)$ which is essentially two standard deviations away from the standard model when supplemented by the Adler model for pion production ^{23), 28)}. For a model-independent conclusion ²⁹⁾ more antineutrino data would be needed, especially for the difference $\sigma(\bar{\nu} p \pi^0) - \sigma(\bar{\nu} n \pi^0)$ which has not been determined so far ³⁰⁾. On the other hand, a preliminary analysis of double pion production ³¹⁾ yields $\sigma(\nu n \rightarrow \nu n \pi^+ \pi^-)/\sigma(\nu p \rightarrow \nu p \pi^+ \pi^-) = 0.51 \pm 0.21$ which seems rather small compared to the standard model although a detailed theoretical analysis of this process is not available. Let us remark finally that elastic neutrino proton scattering is neither particularly suited nor are the present data ^{18), 19)} precise enough to settle the issue of isoscalar couplings.

What are the possible theoretical implications? Collins, Wilczek and Zee ³²⁾ have recently calculated QCD corrections to the primitive axial-vector current due to heavy quark loops. Starting from the standard model

with $\delta = 0$ they find that the strong interactions induce an isoscalar axial current where $\delta_{\text{ind}}/\beta$ is positive due to the fact that the charmed quark is heavier than the strange quark. With the usual mass values for m_c and m_s they estimate $\delta_{\text{ind}}/\beta$ to be of the order of 10% to be compared with the values given in Eq. (2.1) for solution A.

Another possibility to explain relatively big isoscalar couplings would be to allow for some right-handed non-singlet admixtures within $SU(2) \times U(1)$. Assuming the left-handed quarks to be pure doublets (universality), and neglecting the QCD corrections just mentioned, solution A implies an allowed domain in the plane of right-handed isospin components t_{1R}^u, t_{1R}^d (see Appendix A) as shown in Fig. 3. The standard model with $t_{1R}^u = t_{1R}^d = 0$ is again not within one standard deviation. While t_{1R}^u is perfectly consistent with zero in agreement with the absence ³³⁾ of a high γ anomaly the neutral current data allow for a small negative t_{1R}^d as expected, e.g., in models with an admixture of a right-handed doublet for the down quark. In a different spirit, Fig. 3 may also be used to obtain limits for right-handed mixing angles which for the d quark are much more precise than what one obtains from a study of charged current cross-sections.

Summing up this section, one may conclude that the existing data for neutrino hadron scattering determine a unique solution for the phenomenological neutral current couplings $\alpha, \beta, \gamma, \delta$. This solution resembles the standard model of Salam and Weinberg although small modifications cannot be excluded at the moment. Keeping in mind the theoretical uncertainties in applying the parton model at rather low energies the analysis gives isoscalar couplings bigger than in the standard model. It will be necessary to get more precise information on the isovector-isoscalar interference (mainly in pion production) in order to settle this question.

3. - STATUS OF THE SINGLE Z BOSON ASSUMPTION

In view of the widespread opinion that neutral current experiments have established the validity of $SU(2) \times U(1)$ it must be emphasized that in spite of the large amount of information accumulated in the past five years no real test of the single Z boson hypothesis has been performed so far. In this section the presently possible tests are discussed quantitatively.

A major improvement in our understanding of the structure of the neutral current has been achieved through the observation of parity violation in polarized electron deuteron scattering at SLAC ³⁾. Adopting the valence quark approximation which has been shown ³⁴⁾ to be rather reliable for this purpose, the asymmetry for the scattering of positive and negative helicity electrons on a deuterium target is given ^{2),35)} by

$$A_{ed}(y, Q^2) = \frac{3G_F Q^2}{20\sqrt{2}\pi\alpha} [3\tilde{\alpha} + \tilde{\gamma} + h(y)(3\tilde{\beta} + \tilde{\delta})] \quad (3.1)$$

where $h(y) = (1-(1-y)^2)/(1+(1-y)^2)$. The neutral current couplings are defined in Appendix A and y, Q^2 ($Q^2 > 0$) are the usual variables of inclusive lepton scattering.

So far, the asymmetry has only been measured at $y_0 = 0.21$ so that we need at least one more input to check the factorization relations ⁵⁾ implied by single Z boson exchange. This additional information is in principle provided by the experiments aiming to detect parity violation in heavy atoms (Z, N) characterized by a weak charge

$$Q_w(Z, N) = -\tilde{\alpha}(Z-N) - 3\tilde{\gamma}(Z+N) \quad (3.2)$$

In order to extract this quantity from the experimentally measured rotation of polarized laser light in Bi the semi-phenomenological calculations of Khriplovich et al. ³⁶⁾ will be used which are in qualitative agreement with recent effective potential calculations including shielding ³⁷⁾ by the outer electrons. A more disturbing discrepancy now exists on the experimental side. Three different values for $Q_w(\text{Bi})$ have been announced to this date ³⁸⁾. For the following discussion only the two extreme cases will be considered :

$$\text{Novosibirsk }^{39)} : \quad Q_w(\text{Bi}) = -140 \pm 40 \quad (3.3)$$

$$\text{Seattle }^{40)} : \quad Q_w(\text{Bi}) = -5 \pm 18 \quad (3.4)$$

From the factorization relations in Appendix A one gets

$$A_{ed}(y, Q^2) = \frac{3G_F Q^2}{10\sqrt{2}\pi\alpha} [g_A \kappa^{-1}(3\alpha + \gamma) + g_V \kappa^{-1}(3\beta + \delta)h(y)] \quad (3.5)$$

$$Q_W(\text{Bi}) = 2g_A \kappa^{-1} (43\alpha - 627\gamma) \quad (3.6)$$

where g_V, g_A are the neutrino electron couplings and κ characterizes the strength of neutrino neutrino scattering. Solution A for the neutrino hadron couplings, the SLAC asymmetry $A_{\text{ed}}(y_0, Q^2) = (-9.5 \pm 1.6) 10^{-5} Q^2 \text{ GeV}^{-2}$ and the results for $Q_W(\text{Bi})$ can now be used to determine $g_V \kappa^{-1}$ and $g_A \kappa^{-1}$. The one standard deviation domains according to either (3.3) or (3.4) are displayed in Fig. 4. Without using $Q_W(\text{Bi})$ as input the allowed region lies between the two branches of a hyperbola also shown in Fig. 4.

It must be emphasized that in order to arrive at meaningful error estimates it is absolutely essential to take the correlations between the various parameters properly into account. The only safe way to proceed is to map the error ellipsoid in the experimental observables onto the space of the coupling parameters under consideration. This approach has been adopted for all error domains evaluated in this paper.

Let us now turn to the constraints on g_V, g_A implied by neutrino electron scattering. Disregarding the SPS experiment ⁴¹⁾ for $\nu_\mu e \rightarrow \nu_\mu e$, all other existing measurements ⁴²⁾⁻⁴⁴⁾ are in agreement with each other and may be averaged to give

$$\sigma(\nu_\mu e) = (1.35 \pm 0.5) \cdot 10^{-42} \text{ cm}^2 E_\nu \text{ GeV}^{-1} \quad (3.7)$$

$$\sigma(\bar{\nu}_\mu e) = (1.55 \pm 0.7) \cdot 10^{-42} \text{ cm}^2 E_\nu \text{ GeV}^{-1} \quad (3.8)$$

The allowed regions in the g_V, g_A plane from $\nu_\mu e$ scattering are shown in Fig. 5 together with the constraints implied by low energy $\bar{\nu}_e e$ scattering ⁴⁵⁾ which eliminate the ambiguity $g_V \rightarrow -g_V, g_A \rightarrow -g_A$ for $\nu_\mu e$ cross-sections. Although possible in principle, the reactor experiment for $\bar{\nu}_e e \rightarrow \bar{\nu}_e e$ does not resolve the remaining ambiguity $g_V \rightleftharpoons g_A$ (VA ambiguity).

The first factorization test for neutral current couplings then consists in superimposing the $g_V \kappa^{-1}, g_A \kappa^{-1}$ plane on the g_V, g_A plane

and to check whether the cones shown in Fig. 6 intersect the allowed domains from νe scattering. These cones hide our ignorance of the parameter κ which can now in principle be determined from Fig. 6 assuming single Z boson exchange. At the one standard deviation level only the dominantly vector solution for g_V, g_A seems to be consistent with any value of $Q_w(Bi)$ between Seattle and Novosibirsk. On the other hand, the standard model with the right-handed electron in an $SU(2)$ singlet predicts a dominantly axial solution. Moreover, Fig. 6 shows that in order to get consistency (always at the one-standard deviation level) $\kappa \lesssim 0.5$ which would be in contradiction with the value $\kappa \simeq 1$ from neutrino hadron scattering for any $SU(2) \times U(1)$ model with the standard left-handed doublets ^{1), 2), 46)}.

Within the one Z boson hypothesis we can also predict the y dependence of the asymmetry in polarized $e d$ scattering

$$\frac{A_{ed}(y, Q^2)}{Q^2} = \frac{3G_F Q_w(Bi)(3\alpha + \gamma)}{20\sqrt{2}\pi\alpha(43\alpha - 627\gamma)} \left(1 - \frac{h(y)}{h(y_0)}\right) + \frac{h(y)}{h(y_0)} \cdot \frac{A_{ed}(y_0, Q^2)}{Q^2} \quad (3.9)$$

The one standard deviation domains are shown in Fig. 7 for both the Novosibirsk and the Seattle value for $Q_w(Bi)$. The over-all conclusion is that for any $Q_w(Bi)$ between the two cases considered the y distribution is predicted to be falling. This decrease is more pronounced than in the standard model for $\sin^2\theta_w \gtrsim 0.20$.

Both the lack of consistency with the standard model predictions for g_V, g_A and the rather steep y distribution of the ed asymmetry are primarily due to the fact that $Q_w(Bi)/(43\alpha - 627\gamma)$ [Eqs. (3.6) and (3.9)] is smaller than in the Salam-Weinberg model for $\sin^2\theta_w \simeq 0.24$. This quantity depends obviously very sensitively on the isoscalar vector coupling γ whose absolute value is bigger than expected as discussed in Section 2. However, both the uncertainty in the neutrino hadron analysis (Section 2) and in the atomic physics calculations employed to evaluate $Q_w(Bi)$ from the measured parity violation signal in Bi prevent any firm conclusions at the moment concerning the validity of single Z boson exchange, not to speak of the discrepancies in the Bi experiments.

With the coming into operation of the high energy e^+e^- colliding beam facilities PETRA and PEP, information on the neutral current parameters h_{VV} , h_{VA} and h_{AA} (Appendix A) may soon be expected. In principle very clean tests of the single Z boson assumption will then be possible which are free of the complications of hadronic structure. In the remainder of this paper, we shall mainly be dealing with the question what we should expect for the $e\mu$ and e hadron observables in a more general framework than single Z boson exchange.

4. PHENOMENOLOGY OF $SU(2) \times U(1) \times G$

In view of the discussion in Section 3 there is at present no definite need to enlarge the weak and electromagnetic gauge group beyond $SU(2) \times U(1)$ but there is also no evidence whatsoever for the existence of only one massive neutral gauge boson. For the rest of this paper we anticipate that at least one of the factorization relations (A.5) to (A.11) will be shown to be violated by future experiments. The question then poses itself : will we be able to infer more than the trivial conclusion that the number of Z bosons is at least two ?

Let us first tackle the problem in complete generality. For each massive neutral intermediate boson there are seven independent couplings for the fermions relevant to present-day phenomenology : one coupling constant for the left-handed neutrino and one vector and axial-vector coupling each for e , u , d (assuming $e\mu$ universality). On the other hand, there are thirteen observable parameters in the effective Lagrangians defined in Appendix A which are experimentally accessible. While for a single Z boson we therefore obtain the six factorization relations of Hung and Sakurai ⁵⁾, all the experimental information reasonably to be expected in the near future will not suffice to evaluate the primitive couplings for $N \geq 2$ neutral intermediate bosons. It will therefore not be possible to determine the basic structure of the neutral currents for $N \geq 2$ Z bosons until energies are reached where these bosons can be produced directly.

For the time being one has to adopt a more restrictive framework. Even if the standard model needs some small modifications as discussed in Section 2, all neutrino data are perfectly compatible with $SU(2) \times U(1)$. It is therefore very appealing to consider extensions of $SU(2) \times U(1)$ which yield the same low energy neutral current couplings in the neutrino sector.

Georgi and Weinberg⁸⁾ have shown that this reasoning leads very naturally to gauge groups of the form $SU(2) \times U(1) \times G$ where the neutrino is neutral under the arbitrary group G and where there are no Higgs fields transforming non-trivially under both $SU(2)$ and G . Many gauge models that have been considered in the literature and which are not already excluded by experiment indeed belong to this class. The main question to be answered in this and the next section is whether this framework is restrictive enough to make definite predictions without specifying G or the Higgs sector of the model any further.

The neutral current sector of $SU(2) \times U(1) \times G$ is characterized by neutral generators T_α , gauge coupling constants g_α and coefficients C_α , where $\alpha=0$ refers to $U(1)$, $\alpha=1$ to $SU(2)$ and $\alpha=2, \dots, N$ to the remaining factor G . The electric charge operator is a linear combination of the T_α , i.e.,

$$Q = \sum_{\alpha=0}^N C_\alpha T_\alpha \quad (4.1)$$

and charge normalization implies the relation

$$\sum_{\alpha} p_\alpha^2 = 1, \quad p_\alpha = \frac{e C_\alpha}{g_\alpha} \quad (4.2)$$

Referring to Appendix B for definitions and a summary of Ref. 8) we immediately write down the effective neutral current Lagrangian at low momentum transfer

$$\begin{aligned} \mathcal{L}_{NC} = & - \frac{g_1^2}{2\mu_1^2} [\bar{\psi} \gamma^\lambda (t_1 - p_1^2 q) \psi] [\bar{\psi} \gamma_\lambda (t_1 - p_1^2 q) \psi] - \\ & - \frac{1}{2} \sum_{i=2}^N \frac{g_i^2}{\mu_i^2 C_i^2} [\bar{\psi} \gamma^\lambda (C_i t_i - p_i^2 q) \psi] [\bar{\psi} \gamma_\lambda (C_i t_i - p_i^2 q) \psi] \end{aligned} \quad (4.3)$$

where t_i, q represent the neutral generators T_i and the charge operator Q in the space of fermions ψ and C_1 has been normalized to 1. Because of the assumed neutrality of the neutrino under G only the first part of (4.3) contributes to neutrino interactions. With the identifications

$$p_1^2 = \sin^2 \theta_W \quad (4.4)$$

$$\frac{g_1^2}{8\mu_1^2} = \kappa \frac{G_F}{\sqrt{2}} \quad (4.5)$$

one arrives at the same neutrino hadron and neutrino electron coupling constants (A.12) to (A.17) as for $SU(2) \times U(1)$ adopting the conventional $SU(2)$ assignments for left-handed fermion fields.

Since the generators of $SU(2)$ and G commute and the left-handed fermion fields are grouped into non-trivial representations of $SU(2)$ (doublets) one obtains for $i = 2, \dots, N$

$$t_{iL}^e = t_{iL}^\mu = t_{iL}^\nu = 0 \quad (4.6)$$

$$t_{iL}^u = t_{iL}^d \quad (4.7)$$

Keeping these relations in mind we obtain the following equations for the remaining neutral current observables :

$$\tilde{\alpha} = 2g_A \kappa^{-1} \alpha - \frac{\sqrt{2}\pi\alpha}{G_F} \sum_{i=2}^N \frac{C_i t_{iR}^e}{p_i^2 \mu_i^2} [C_i (t_{iR}^u - t_{iR}^d) - 2p_i^2] \quad (4.8)$$

$$\tilde{\beta} = 2g_V \kappa^{-1} \beta - \frac{\sqrt{2}\pi\alpha}{G_F} \sum_{i=2}^N \frac{(C_i t_{iR}^e + 2p_i^2)}{p_i^2 \mu_i^2} C_i (t_{iR}^u - t_{iR}^d) \quad (4.9)$$

$$\tilde{\gamma} = 2g_A \kappa^{-1} \gamma - \frac{\sqrt{2}\pi\alpha}{G_F} \sum_{i=2}^N \frac{C_i t_{iR}^e}{p_i^2 \mu_i^2} [C_i (2t_{iL}^u + t_{iR}^u + t_{iR}^d) - \frac{2}{3} p_i^2] \quad (4.10)$$

$$\tilde{\delta} = 2g_V \kappa^{-1} \delta + \frac{\sqrt{2}\pi\alpha}{G_F} \sum_{i=2}^N \frac{(C_i t_{iR}^e + 2p_i^2)}{p_i^2 \mu_i^2} C_i (2t_{iL}^u - t_{iR}^u - t_{iR}^d) \quad (4.11)$$

$$h_{VV} = g_V^2 \kappa^{-1} + \frac{\pi \alpha}{\sqrt{2} G_F} \sum_{i=2}^N \frac{(C_i t_{iR}^e + 2p_i^2)^2}{p_i^2 \mu_i^2} \quad (4.12)$$

$$h_{VA} = g_V g_A \kappa^{-1} - \frac{\pi \alpha}{\sqrt{2} G_F} \sum_{i=2}^N \frac{C_i t_{iR}^e}{p_i^2 \mu_i^2} (C_i t_{iR}^e + 2p_i^2) \quad (4.13)$$

$$h_{AA} = g_A^2 \kappa^{-1} + \frac{\pi \alpha}{\sqrt{2} G_F} \sum_{i=2}^N \frac{C_i^2 t_{iR}^{e2}}{p_i^2 \mu_i^2} \quad (4.14)$$

For the hadronic couplings there is in general much freedom in choosing the quark representations under G . The parameter most sensitive to G is $\tilde{\delta}$ because δ is known to be small from neutrino hadron scattering [Eq. (2.1)]. Moreover, for the dominantly axial solution for g_V, g_A (Fig. 5) both $\tilde{\beta}$ and $\tilde{\delta}$ should be dominated by the G couplings while for the vector solution, on the contrary, $\tilde{\alpha}$ and $\tilde{\gamma}$ are especially sensitive to G . A model-independent discussion how to best determine $\tilde{\alpha}, \tilde{\beta}, \tilde{\gamma}, \tilde{\delta}$ has recently been given by Bjorken ³⁴).

For the rest of this paper we shall concentrate on the purely leptonic sector. Before discussing definite tests of $SU(2) \times U(1) \times G$ let us briefly review the standard proposals ⁴⁷⁾ to determine h_{VV}, h_{VA}, h_{AA} in $e^+e^- \rightarrow \mu^+\mu^-$ for energies $\sqrt{s} \ll M_{Z_1}$, where M_{Z_1} is the mass of the lightest Z boson :

a) total cross-sections :

$$\frac{\sigma_{\text{tot}} - \sigma_{\text{QED}}}{\sigma_{\text{QED}}} = - \frac{G_F S}{\sqrt{2} \pi \alpha} h_{VV} ; \quad (4.15)$$

b) forward-backward asymmetry :

$$\frac{\sigma(\theta) - \sigma(\pi - \theta)}{\sigma(\theta) + \sigma(\pi - \theta)} = - \frac{G_F S}{\sqrt{2} \pi \alpha} h_{AA} \frac{2 \cos \theta}{1 + \cos^2 \theta - P^2 \sin^2 \theta \cos 2\varphi} ; \quad (4.16)$$

c) longitudinal muon polarization :

$$P_{\mu^-} = -P_{\mu^+} = \frac{G_F S}{\sqrt{2} \pi \alpha} h_{VA} \left[1 + \frac{2 \cos \theta}{1 + \cos^2 \theta - P^2 \sin^2 \theta \cos 2\varphi} \right] \quad (4.17)$$

where θ, φ specify the direction of the outgoing μ^- and P is the transverse polarization of the e^+e^- beams.

At present we only have an upper limit on the forward-backward asymmetry from a recent experiment ⁴⁸⁾ at SPEAR which yields

$$h_{AA} < 2 \quad \text{with 90\% confidence.} \quad (4.18)$$

implying

$$\mu_i^2 p_i^2 > C_i^2 t_{iR}^{e^2} \cdot (26 \text{ GeV})^2 \quad (4.19)$$

for all $i = 2, \dots, N$.

From the charge normalization condition (4.2) and relation (4.4) this bound also implies

$$\mu_i^2 > \frac{C_i^2 t_{iR}^{e^2}}{\cos^2 \theta_W} \cdot (26 \text{ GeV})^2 \quad (4.20)$$

For the relation between μ_i^2 ($i = 1, \dots, N$) and the physical Z boson masses we refer to Appendix B.

To simplify the notation let us define two $N-1$ dimensional vectors $\vec{k}_V, \vec{k}_A$ by

$$k_V^{i-1} = \left(\frac{\pi \alpha}{\sqrt{2} G_F} \right)^{1/2} \frac{C_i t_{iR}^e + 2 p_i^2}{p_i \mu_i} \quad (i = 2, \dots, N) \quad (4.21)$$

$$k_A^{i-1} = - \left(\frac{\pi \alpha}{\sqrt{2} G_F} \right)^{1/2} \frac{C_i t_{iR}^e}{p_i \mu_i} \quad (4.22)$$

Taking into consideration the VA ambiguity in νe scattering we furthermore define three observables D_1, D_2, D_3 which are invariant under the replacement $g_V \rightleftharpoons g_A$ and which are constructed to measure the violation of the factorization relations

$$D_1 = h_{VV} + h_{AA} - g_V^2 - g_A^2 \quad (4.23)$$

$$D_2 = h_{VA} - g_V g_A \quad (4.24)$$

$$D_3 = h_{VV} h_{AA} - h_{VA}^2 + 2g_V g_A D_2 \quad (4.25)$$

Using Eqs. (4.12)-(4.14), (4.21) and (4.22) we find

$$D_1 = (g_V^2 + g_A^2)(\kappa^{-1} - 1) + \vec{k}_V^2 + \vec{k}_A^2 \quad (4.26)$$

$$D_2 = g_V g_A (\kappa^{-1} - 1) + \vec{k}_V \vec{k}_A \quad (4.27)$$

$$D_3 = \kappa^{-1} (g_A^2 \vec{k}_V^2 + g_V^2 \vec{k}_A^2) + 2g_V g_A (\kappa^{-1} - 1) (g_V g_A - \vec{k}_V \vec{k}_A) + \vec{k}_V^2 \vec{k}_A^2 - (\vec{k}_V \vec{k}_A)^2 \quad (4.28)$$

It is known ^{1),2),46)} from neutrino hadron scattering that κ is very close to 1 for both solutions A and B discussed in Section 2. This can be understood theoretically ^{*)} if the G singlet Higgs fields all transform as doublets under $SU(2)$. For our purposes it is sufficient to accept the experimental result and we set $\kappa = 1$ in the following.

In that case D_1, D_2, D_3 take the simple form

$$D_1 = \vec{k}_V^2 + \vec{k}_A^2 \quad (4.29)$$

$$D_2 = \vec{k}_V \vec{k}_A \quad (4.30)$$

$$D_3 = g_A^2 \vec{k}_V^2 + g_V^2 \vec{k}_A^2 + \vec{k}_V^2 \vec{k}_A^2 - (\vec{k}_V \vec{k}_A)^2 \quad (4.31)$$

^{*)} Even for a single Higgs doublet in $SU(2) \times U(1)$ κ will in general differ from 1 due to radiative corrections. We assume that such corrections are negligible compared to $(\vec{k}_V^2 + \vec{k}_A^2)/(g_V^2 + g_A^2)$ even for $SU(2) \times U(1) \times G$.

and their non-vanishing would be direct evidence for an enlarged gauge group. More precisely, we obtain the following predictions for the violation of factorization relations in the purely leptonic sector.

Two restrictions follow from positivity :

$$1) \quad D_1 \geq 0 \quad ; \quad (4.32)$$

$$2) \quad D_3 \geq 0 \quad . \quad (4.33)$$

$\vec{k}_V^2, \vec{k}_A^2$ may be determined up to a possible twofold ambiguity as solutions of a quadratic equation which implies two further necessary and sufficient conditions for the reality and positivity of solutions :

$$3) \quad 4(D_3 + D_2^2) \leq (g_V^2 - g_A^2)^2 + 2D_1(g_V^2 + g_A^2) + D_1^2 \quad ; \quad (4.34)$$

$$4) \quad \text{either} \quad D_1 \geq |g_V^2 - g_A^2|$$

$$\text{or} \quad D_3 + D_2^2 \leq D_1 \max(g_V^2, g_A^2) \quad . \quad (4.35)$$

Finally, the solutions obtained in this way must fulfil the Cauchy inequality

$$5) \quad \vec{k}_V^2 \vec{k}_A^2 \geq D_2^2 \quad (4.36)$$

where the strict inequality would be unambiguous evidence for more than two Z bosons. As mentioned in the beginning of this section, such a conclusion is impossible to draw from low energy measurements in a completely model-independent way as long as ep universality is fulfilled.

A final comment concerns the feasibility of the proposed tests. The situation in νe scattering (Fig. 5) together with Eqs. (4.12) to (4.14) implies that h_{VA} and either h_{VV} or h_{AA} will be very sensitive to the additional group factor G because for $SU(2) \times U(1)$ they are predicted to be small compared to the remaining parameter h_{AA} or h_{VV} , respectively. Since the standard model predicts a forward-backward asymmetry of -8% at $\sqrt{s} = 30$ GeV, measurements of this asymmetry, of the total cross-section for $e^+e^- \rightarrow \mu^+\mu^-$ and of a longitudinal muon polarization to a few

per cent at PETRA energies are needed to make the tests practicable. Let us finally emphasize once more that all five tests described above can be performed without resolving the VA ambiguity in νe scattering.

5. - A SPECIAL CASE : TWO Z BOSONS

The general discussion of $SU(2) \times U(1) \times G$ has shown the possibility to decide in principle whether $N=2$ massive neutral gauge bosons are sufficient to explain the assumed violation of the factorization relations. In this last section we are going to consider briefly the case $N=2$ which includes $SU(2) \times SU(2) \times U(1)$ and $SU(2) \times U(1) \times U(1)$.

Diagonalizing the mass matrix of the neutral gauge bosons (see Appendix B) one gets for the masses of the physical Z bosons

$$M_{Z_1, Z_2}^2 = \frac{\mu_1^2}{2} \left(1 + \frac{p_1^2}{p_0^2}\right) + \frac{\mu_2^2}{2} \left(1 + \frac{p_2^2}{p_0^2}\right) \mp \frac{1}{2} \left\{ \left[\mu_1^2 \left(1 + \frac{p_1^2}{p_0^2}\right) - \mu_2^2 \left(1 + \frac{p_2^2}{p_0^2}\right) \right]^2 + \frac{4\mu_1^2 \mu_2^2 p_1^2 p_2^2}{p_0^4} \right\}^{1/2} \quad (5.1)$$

with p_1^2, μ_1^2 determined from neutrino scattering and $p_0^2 = 1 - p_1^2 - p_2^2$. From the purely leptonic reactions one can then determine μ_2^2, p_2^2 and $C_2 t_{2R}^e$ up to a multiplicative factor [Eqs. (4.12) to (4.14)]. Because of this scale factor the Z boson masses cannot be predicted in terms of low energy measurements, but using the inequality (B.12) one can derive at least an upper bound for the mass of the lighter Z boson

$$M_{Z_1}^2 \leq \frac{\mu_2^2}{p_2^2 + g^2 \Theta_W} \quad (5.2)$$

This upper bound will be useful if it turns out to be lower than the general bound ⁸⁾ (see Appendix B)

$$M_{Z_1}^2 \leq \frac{\mu_1^2}{\cos^2 \Theta_W} = M_{Z_0}^2 \quad (5.3)$$

where M_{Z_0} is the mass of the single Z boson for $SU(2) \times U(1)$.

The coupling of leptons to weak gauge bosons will also manifest itself in the anomalous magnetic moments of these leptons. The present experimental value for the anomalous magnetic moment a_μ of the muon ⁴⁹⁾ together with calculations ⁵⁰⁾ of QED and hadronic contributions limits the weak contribution to

$$-1.8 \cdot 10^{-8} \leq a_\mu^{\text{weak}} \leq 2.5 \cdot 10^{-8} \quad (5.4)$$

at the 90% confidence level. In order to calculate the corrections to a_μ due to the exchange of Z bosons the effective Lagrangian (4.3) is of course not sufficient but one has to diagonalize the mass matrix to get the primitive μZ couplings. We shall not bother to write down those four coupling constants explicitly, but only note that they depend again on the three quantities μ_2^2 , p_2^2 and $C_2 t_{2R}^e$. Using the model-independent formulae of Leveille ⁵¹⁾, the sensitivity of the Z boson corrections a_μ^Z to these parameters has been investigated.

Without going into details, the results of this calculation can be summarized as follows : although a_μ^Z may be much bigger than the $SU(2) \times U(1)$ value -0.2×10^{-8} , the constraint (4.19) on p_2^2 , μ_2^2 , $C_2 t_{2R}^e$ ensures that the sum of a_μ^Z and the charged boson contribution $a_\mu^{W^-} = 0.4 \times 10^{-8}$ stays within the bounds (5.4) unless the muon is practically neutral under G ($t_{2R}^e \simeq 0$) in which case (4.19) becomes trivial. For $t_{2R}^e = 0$ the experimental limit (5.4) on a_μ^{weak} implies a lower limit on the mass of the lighter Z boson. This lower limit is around 12 GeV for $p_2^2 = 0.3$ and increases with increasing p_2^2 . Z bosons of such low mass would of course be produced directly at PETRA and PEP and Eqs. (4.15) to (4.17) would have to be modified.

Without further assumptions about G and the Higgs content of the theory it is impossible to estimate the total weak contribution to a_μ . Confronting the Z boson contributions alone with the experimental limit (5.4) does not in general lead to any restrictions on the couplings of the muon to the neutral gauge bosons which would be more stringent than the bound (4.19) from the limit ⁴⁸⁾ on the forward-backward asymmetry in $e^+e^- \rightarrow \mu^+\mu^-$. This conclusion should hold a fortiori for $SU(2) \times U(1) \times G$ in general ($N \geq 2$).

Besides p_2^2 , μ_2^2 , $C_2 t_{2R}^e$ from purely leptonic reactions one may, in principle, obtain the remaining unknowns $C_2 t_{2L}^u$, $C_2 t_{2R}^u$, $C_2 t_{2R}^d$ from measurements of parity violating electron hadron interactions where one over-all scale factor ^{*)} remains undetermined. This should give further insight into open theoretical problems such as the possible existence of an underlying left-right symmetry of the weak interactions.

6. - CONCLUSIONS

There can hardly be any more doubt that the unique solution for the hadronic neutral current in neutrino interactions has been found. Although this solution is in qualitative agreement with the standard model of Salam and Weinberg there is still enough room to accommodate sizeable QCD corrections to the isoscalar axial current and/or small right-handed non-singlet admixtures, especially for the down quark. A quantitative analysis which depends crucially on the low energy semi-inclusive pion production data leads to isoscalar coupling constants which are bigger than in the standard model. More precise determinations of the isovector-isoscalar interference terms are needed to settle the issue of isoscalar couplings.

In spite of the success of $SU(2) \times U(1)$ in the neutrino sector there is at present no evidence whatsoever for the validity of single Z boson exchange. The only factorization test which is possible at the moment does not fully agree with $SU(2) \times U(1)$. However, the experimental and theoretical uncertainties concerning the measurements of parity violation in Bi considerably weaken such a conclusion. The same qualification applies to the factorization prediction that the y distribution for the electron deuteron asymmetry should be falling. Both factorization tests are very sensitive to the magnitude of the isoscalar vector coupling.

In order to study possible violations of factorization expanded gauge theories $SU(2) \times U(1) \times G$ with arbitrary G were considered which have the same low energy neutral current neutrino interactions as $SU(2) \times U(1)$. The phenomenology of such theories was discussed without any reference to the Higgs sector and without restricting the fermion representations under G . Various tests for such a gauge structure emerged from a comparison of neutrino electron scattering and weak electromagnetic interference effects

*) This multiplicative factor is, however, constrained by the bound $p_2^2 \leq \cos^2 \theta_w$.

in $e^+e^- \rightarrow \mu^+\mu^-$. Within this framework, it is possible in principle to decide on the basis of low energy experiments whether more than two Z bosons exist.

The case of $N=2$ neutral gauge bosons was discussed in some detail, especially in connection with the weak contributions to the anomalous magnetic moment a_μ of the muon. Comparison between theory and experiment for a_μ does not put any stringent limits on the parameters of the theory. For $N=2$ low energy neutral current phenomenology will determine all parameters of the model up to an over-all scale factor.

Determination of the weak-electromagnetic interference effects at PETRA and PEP will play a decisive rôle in establishing the correct gauge theory of weak and electromagnetic interactions.

ACKNOWLEDGEMENTS

I would like to thank A. Bottino and H. Grosse for useful discussions and J. Ellis for reading the manuscript.

APPENDIX A - DEFINITION OF NEUTRAL CURRENT COUPLINGS AND FACTORIZATION RELATIONS ⁵⁾

With the assumption of eu universality there are 13 observable parameters in neutral current interactions at low momentum transfer ($Q^2 \ll M_Z^2$) if the parity conserving electron quark couplings are neglected :

$$\begin{aligned} \mathcal{L}_{\nu q} = & - \frac{G_F}{\sqrt{2}} \bar{\nu} \gamma^\lambda (1 + \gamma_5) \nu \left\{ \bar{u} \gamma_\lambda [u_L (1 + \gamma_5) + u_R (1 - \gamma_5)] u + \right. \\ & + \bar{d} \gamma_\lambda [d_L (1 + \gamma_5) + d_R (1 - \gamma_5)] d \left. \right\} = - \frac{G_F}{\sqrt{2}} \bar{\nu} \gamma^\lambda (1 + \gamma_5) \nu \left\{ \frac{1}{2} [\bar{u} \gamma_\lambda (\alpha + \beta \gamma_5) u - \right. \\ & - \bar{d} \gamma_\lambda (\alpha + \beta \gamma_5) d] + \frac{1}{2} [\bar{u} \gamma_\lambda (\gamma + \delta \gamma_5) u + \bar{d} \gamma_\lambda (\gamma + \delta \gamma_5) d] \left. \right\} \end{aligned} \quad (A.1)$$

$$\mathcal{L}_{\nu e} = - \frac{G_F}{\sqrt{2}} \bar{\nu} \gamma^\lambda (1 + \gamma_5) \nu [\bar{e} \gamma_\lambda (g_V + g_A \gamma_5) e] \quad (A.2)$$

$$\begin{aligned} \mathcal{L}_{e\mu} = & - \frac{G_F}{\sqrt{2}} \left[h_{VV} (\bar{e} \gamma_\lambda e + \bar{\mu} \gamma_\lambda \mu) (\bar{e} \gamma^\lambda e + \bar{\mu} \gamma^\lambda \mu) + \right. \\ & + 2 h_{VA} (\bar{e} \gamma_\lambda \gamma_5 e + \bar{\mu} \gamma_\lambda \gamma_5 \mu) (\bar{e} \gamma^\lambda e + \bar{\mu} \gamma^\lambda \mu) + \\ & + h_{AA} (\bar{e} \gamma_\lambda \gamma_5 e + \bar{\mu} \gamma_\lambda \gamma_5 \mu) (\bar{e} \gamma^\lambda \gamma_5 e + \bar{\mu} \gamma^\lambda \gamma_5 \mu) \left. \right] \end{aligned} \quad (A.3)$$

$$\begin{aligned} \mathcal{L}_{eq} = & - \frac{G_F}{2\sqrt{2}} \left\{ \bar{e} \gamma_\lambda \gamma_5 e [\tilde{\alpha} (\bar{u} \gamma^\lambda u - \bar{d} \gamma^\lambda d) + \tilde{\gamma} (\bar{u} \gamma^\lambda u + \bar{d} \gamma^\lambda d)] + \right. \\ & + \bar{e} \gamma_\lambda e [\tilde{\beta} (\bar{u} \gamma^\lambda \gamma_5 u - \bar{d} \gamma^\lambda \gamma_5 d) + \tilde{\delta} (\bar{u} \gamma^\lambda \gamma_5 u + \bar{d} \gamma^\lambda \gamma_5 d)] \left. \right\} \end{aligned} \quad (A.4)$$

The assumption of single Z boson exchange provides the following factorization relations among the 13 coupling constants :

$$h_{VA}^2 = h_{VV} h_{AA} \quad (A.5)$$

$$g_V g_A = \kappa h_{VA} \quad (A.6)$$

$$g_V^2 + g_A^2 = \kappa (h_{VV} + h_{AA}) \quad (\text{A.7})$$

$$\tilde{\alpha} \kappa = 2g_A \alpha \quad (\text{A.8})$$

$$\tilde{\beta} \kappa = 2g_V \beta \quad (\text{A.9})$$

$$\tilde{\gamma} \kappa = 2g_A \gamma \quad (\text{A.10})$$

$$\tilde{\delta} \kappa = 2g_V \delta \quad (\text{A.11})$$

where κ denotes the strength of the neutrino neutrino interaction $[\kappa = 1$ in $SU(2) \times U(1)$ with only Higgs doublets].

Assuming strict universality for the left-handed fields the neutrino hadron and neutrino electron couplings for $SU(2) \times U(1)$ or for expanded gauge theories based on $SU(2) \times U(1) \times G$ are

$$u_L = \kappa \left(\frac{1}{2} - \frac{2}{3} \sin^2 \theta_w \right) \quad (\text{A.12})$$

$$d_L = \kappa \left(-\frac{1}{2} + \frac{1}{3} \sin^2 \theta_w \right) \quad (\text{A.13})$$

$$u_R = \kappa \left(t_{1R}^u - \frac{2}{3} \sin^2 \theta_w \right) \quad (\text{A.14})$$

$$d_R = \kappa \left(t_{1R}^d + \frac{1}{3} \sin^2 \theta_w \right) \quad (\text{A.15})$$

$$g_V = \kappa \left(-\frac{1}{2} + t_{1R}^e + 2 \sin^2 \theta_w \right) \quad (\text{A.16})$$

$$g_A = \kappa \left(-\frac{1}{2} - t_{1R}^e \right) \quad (\text{A.17})$$

APPENDIX B - EXPANDED GAUGE THEORIES ⁸⁾ BASED ON SU(2)×U(1)×G

The neutral current sector of the model is determined by the Lagrangian

$$\mathcal{L}_0 = \sum_{\alpha} g_{\alpha} A_{\alpha}^{\mu} \bar{\psi} \gamma_{\mu} t_{\alpha} \psi - \frac{1}{2} \sum_{\alpha, \beta} \mu_{\alpha\beta}^2 A_{\alpha}^{\mu} A_{\mu\beta} \quad (\text{B.1})$$

with gauge fields A_{α}^{μ} , gauge coupling constants g_{α} , a matrix representation t_{α} of neutral generators T_{α} in the space of fermion fields ψ and a mass matrix $\mu_{\alpha\beta}^2$ of the form

$$\mu_{\alpha\beta}^2 = \begin{pmatrix} \mu_0^2 & \mu_{01}^2 & \dots & \mu_{0N}^2 \\ \mu_{01}^2 & \mu_1^2 & & 0 \\ \vdots & & \ddots & \\ \mu_{0N}^2 & 0 & \dots & \mu_N^2 \end{pmatrix} \quad (\text{B.2})$$

where $\alpha=0$ refers to $U(1)$, $\alpha=1$ to $SU(2)$ and $\alpha=2, \dots, N$ to the remaining factor G . The mass matrix in the G sector has been diagonalized without loss of generality. The crucial assumption of the model is the absence of non-diagonal terms $\mu_{\alpha\beta}^2$ for $\alpha=1, \beta=2, \dots, N$ which is a consequence of the requirement that the spontaneous breakdown of $SU(2) \times U(1) \times G$ occurs through scalar fields which transform trivially either under $SU(2)$ or under G .

The electric charge operator Q is a linear combination of the neutral generators T_{α}

$$Q = \sum_{\alpha} C_{\alpha} T_{\alpha} \quad (\text{B.3})$$

In order to accommodate the photon field $\mu_{\alpha\beta}^2$ must have an eigenvalue zero which implies

$$\mu_0^2 = \sum_{i=1}^N \frac{(\mu_{0i}^2)^2}{\mu_i^2} \quad (\text{B.4})$$

The corresponding eigenvector p is given by

$$p_\alpha = \frac{e c_\alpha}{g_\alpha} \quad (\text{B.5})$$

which is already normalized to 1 due to charge normalization. The orthogonal matrix which diagonalizes $\mu_{\alpha\beta}^2$ is denoted by $u_{\alpha\beta}$ with $p_\alpha = u_{\alpha\alpha}$. The effective neutral current Lagrangian at low momentum transfer is then given by

$$\mathcal{L}_{NC} = -\frac{1}{2} \sum_{\alpha,\beta} (\bar{\psi} \gamma^\mu t_\alpha \psi) (\bar{\psi} \gamma_\mu t_\beta \psi) g_\alpha g_\beta \Delta_{\alpha\beta} \quad (\text{B.6})$$

with

$$\Delta_{\alpha\beta} = \sum_{i=1}^N M_{Z_i}^{-2} u_{i\alpha} u_{i\beta} \quad (\text{B.7})$$

where the M_{Z_i} are the masses of the neutral gauge bosons. As shown in Ref. 8), $\Delta_{\alpha\beta}$ may also be written in the form

$$\Delta_{\alpha\beta} = \sum_{i=1}^N \mu_i^{-2} (\delta_{\alpha i} - p_{\alpha} p_i) (\delta_{\beta i} - p_{\beta} p_i) \quad (\text{B.8})$$

which directly leads to $\mathcal{L}_{NC}$ in Eq. (4.3).

The eigenvalues of $\mu_{\alpha\beta}^2$ are given by the zeroes of the characteristic polynomial $P(\lambda)$ which may be written as

$$P(\lambda) = [\mu_0^2 - \lambda - \sum_{i=1}^N \frac{(\mu_{0i}^2)^2}{\mu_i^2 - \lambda}] \prod_{k=1}^N (\mu_k^2 - \lambda) \quad (\text{B.9})$$

where the summation is over indices i only for which $\mu_{0i}^2 \neq 0$. From (B.9) one immediately derives the following relations between the mass matrix elements μ_i^2 ($i=1, \dots, N$) appearing in the effective Lagrangian (4.3) and the physical Z boson masses

$$\mu_1^2 \leq M_{Z_1}^2 \leq \mu_2^2 \leq \dots \leq \mu_N^2 \leq M_{Z_N}^2 \quad (\text{B.10})$$

where the μ_{ij}^2 are ordered in increasing magnitude. Of particular interest is an upper and lower bound on the mass of the lightest neutral gauge boson Z_1 which follows from (B.10) and the positivity property of $\Delta_{\alpha\beta}$

$$\Delta_{\alpha\beta} \geq \mu_i^{-2} (\delta_{\alpha i} - \rho_\alpha \rho_i) (\delta_{\beta i} - \rho_\beta \rho_i) \quad (\text{B.11})$$

for all $i=1, \dots, N$. This leads to

$$\min \mu_i^2 \leq M_{Z_1}^2 \leq \min \frac{\mu_i^2}{1 - \rho_i^2} \quad (\text{B.12})$$

which generalizes the theorem of Georgi and Weinberg ⁸⁾ that in the expanded gauge theories under consideration at least one massive neutral gauge boson must be lighter than the single Z boson of $SU(2) \times U(1)$.

REFERENCES

- 1) L.M. Sehgal - Invited talk at the International Conference on Neutrino Physics and Astrophysics, Purdue University (1978) (Aachen Preprint PITHA-102 (1978)).
- 2) J.J. Sakurai - Invited talk at the Topical Conference on Neutrino Physics at Accelerators, Oxford (1978) (UCLA Preprint UCLA/78/TEP/18 (1978)).
- 3) G.Y. Prescott et al. - Phys.Letters 77B, 347 (1978).
- 4) S. Weinberg - Phys.Rev.Letters 19, 1264 (1967) ;
A. Salam - Proceedings 8th Nobel Symposium (Almqvist and Wiksell, Stockholm, 1968).
- 5) P.Q. Hung and J.J. Sakurai - Phys.Letters 69B, 323 (1977).
- 6) For a more general framework see J.D. Bjorken - SLAC-PUB-2133 (June 1978).
- 7) G.V. Dass and P. Ram Babu - Tests of the Single Z Boson Hypothesis in Some Purely Leptonic Processes, Indian Institute of Technology Preprints (1978).
- 8) H. Georgi and S. Weinberg - Phys.Rev. D17, 275 (1978) ;
See also R. Gatto and F. Strocchi - Phys.Rev. B18, 2062 (1978).
- 9) J. Blietschau et al. - Nuclear Phys. B118, 218 (1977).
- 10) P. Wanderer et al. - Phys.Rev. D17, 1679 (1978).
- 11) F.S. Merritt et al. - Phys.Rev. D17, 2199 (1978).
- 12) M. Holder et al. - Phys.Letters 72B, 254 (1977).
- 13) P.C. Bosetti et al. - Phys.Letters 76B, 505 (1978).
- 14) H. Kluttig, J.A. Marfin and W. Van Doninck - Phys.Letters 71B, 446 (1977).
- 15) L.M. Sehgal - Phys.Letters 71B, 99 (1977).
- 16) P.Q. Hung and J.J. Sakurai - Phys.Letters 72B, 208 (1977).
- 17) G. Ecker - Phys.Letters 72B, 450 (1978).
- 18) L.R. Sulak et al. - Invited talk at the International Conference on Neutrino Physics and Astrophysics, Purdue University (1978).
- 19) M. Claudson, E.A. Paschos and L.R. Sulak - Paper submitted to the XIX International Conference on High Energy Physics, Tokyo (1978).
- 20) G. Ecker - Nuclear Phys. B123, 293 (1977).
- 21) P.Q. Hung - Phys.Rev. D17, 1893 (1978).
- 22) D.P. Sidhu and P. Langacker - Phys.Rev.Letters 41, 732 (1978).

- 23) L.F. Abbott and R.M. Barnett - Phys.Rev.Letters 40, 1303 (1978) and SLAC-PUB-2136 (June 1978) ;
See also E. Monsay - Phys.Rev.Letters 41, 728 (1978).
- 24) B.C. Barish et al. - Phys.Rev.Letters 39, 1595 (1977).
- 25) P.C. Bosetti et al. - Phys.Letters 70B, 273 (1977).
- 26) J. Steinberger - Lectures at 6th International Winter Meeting on Fundamental Physics, El Paolar, Spain (1978).
- 27) W. Krenz et al. - Nuclear Phys. B135, 45 (1978).
- 28) S.L. Adler - Ann.Phys. 50, 189 (1968) and Phys.Rev. D12, 2644 (1975).
- 29) G. Ecker and R. Fischer - Nuclear Phys. B117, 209 (1976) ;
L. Wolfenstein and D. Wyler - Phys.Rev. D15, 670 (1977).
- 30) D. Erriques et al. - Phys.Letters 73B, 350 (1978).
- 31) N.P. Samios - Private communication to J.J. Sakurai [Ref.2].
- 32) J. Collins, F. Wilczek and A. Zee - Phys.Rev. D18, 342 (1978).
- 33) M. Holder et al. - Phys.Rev.Letters 39, 433 (1977).
- 34) J.D. Bjorken - SLAC-PUB-2146 (June 1978).
- 35) R.N. Cahn and F.J. Gilman - Phys.Rev. D17, 1313 (1978).
- 36) I.B. Khriplovich, V.N. Novikov and O.P. Suchkov - Soviet Phys. JETP 44, 872 (1976).
- 37) C.E. Loving and P.G.H. Sandars - Atomic Shielding and PNC Optical Rotation in Bismuth, Oxford University Preprint (1977).
- 38) S. Weinberg - Rapporteur Talk at XIX International Conference on High Energy Physics, Tokyo (1978).
- 39) L.M. Barkov and M.C. Zolotorev - Pisma ZHETF 26, 379 (1978).
- 40) N. Fortson - Invited Talk at the International Conference on Neutrino Physics and Astrophysics, Purdue University (1978).
- 41) P. Alibran et al. - Phys.Letters 74B, 422 (1978).
- 42) J. Blietschau et al. - Phys.Letters 73B, 232 (1978).
- 43) H. Faissner et al. - Phys.Rev.Letters 41, 213 (1978).
- 44) A.M. Gnops et al. - Phys.Rev.Letters 41, 357 (1978).
- 45) F. Reines, H.S. Gurr and H.W. Sobel - Phys.Rev.Letters 37, 315 (1976) ;
F.T. Avignone, III and Z.D. Greenwood - Phys.Rev. D16, 2383 (1977).
- 46) J. Bernabéu and G. Jarlskog - Phys.Letters 69B, 71 (1977).

- 47) R. Budny - Phys.Letters 55B, 227 (1975) and references therein.
- 48) T. Himel et al. - Phys.Rev.Letters 41, 449 (1978).
- 49) J. Bailey et al. - Phys.Letters 67B, 225 (1977).
- 50) J. Calmet et al. - Revs.Modern Phys. 49, 21 (1977) and references therein.
- 51) J.P. Leveille - Nuclear Phys. B137, 63 (1978).

FIGURE CAPTIONS

- Figure_1** Dominantly isovector solutions A, B from inclusive and semi-inclusive neutrino scattering in the vector coupling plane (α : isovector, γ : isoscalar). Data ¹⁹⁾ from elastic scattering for the form factors $G_E(0)$ (dash-dotted lines) and $G_M(0)$ (dashed lines) (1st dev. limits). Predictions of the standard model for various values of $\sin^2\theta_w$ are also shown.
- Figure_2** Solution A in the isoscalar coupling plane (γ : vector, δ : axial-vector). Predictions of the standard model are indicated for various values of $\sin^2\theta_w$.
- Figure_3** Solution A as a function of right-handed isospin components for u and d quarks. The standard model prediction is $t_{1R}^u = t_{1R}^d = 0$.
- Figure_4** Factorization constraints in the variables $g_V^{\kappa-1}, g_A^{\kappa-1}$. The interior of the hyperbola follows from neutrino data (solution A) and the electron deuteron asymmetry. The additional restrictions implied by measurements ^{39),40)} of parity violation in Bi are also shown.
- Figure_5** Present experimental situation for the neutrino electron couplings g_V, g_A . Accelerator experiments ⁴²⁾⁻⁴⁴⁾ on $\nu_\mu e \rightarrow \nu_\mu e$ determine the regions bounded by dashed lines, the reactor experiment ⁴⁵⁾ on $\bar{\nu}_e e \rightarrow \bar{\nu}_e e$ those bounded by full lines.
- Figure_6** Superposition of Figs. 4, 5 to test the single Z boson hypothesis. The allowed regions N ³⁹⁾ or S ⁴⁰⁾ are shown for $\kappa = 1$; for other values of κ these domains are scaled up ($\kappa > 1$) or down ($\kappa < 1$) to move within the respective cones denoted by dash-dotted lines.
- Figure_7** Predictions for the γ dependence of the electron deuteron asymmetry ³⁾ with the single Z boson assumption on the basis of all semi-leptonic neutral current data. Full lines : Novosibirsk ³⁹⁾; dashed lines : Seattle ⁴⁰⁾.

FIG.1

FIG.2

FIG.3

FIG. 4

FIG.5

FIG.6

FIG.7