GLUEBALL SINGULARITY, FLAVOUR LOOPS AND THE HARARI - FREUND PICTURE

P. Aurenche

Centre de Physique Théorique
Ecole Polytechnique
91128 Palaiseau Cedex - France
(Equipe de Recherche du CNRS No. 174)

and

L. González Mestres

CERN -- Geneva

ABSTRACT

We discuss several points concerning the structure of the cylinder term in the dual topological expansion. Using dual field theory as a guide, we concentrate on the possibility of a two-singularity scheme à la Harari-Freund. We start from results obtained previously at the lowest order in $\rho = N_f/N_c$ and study what can be the role of flavour-loop corrections in the real world where $\rho \sim 1$. It is argued that, in spite of the actual value of ρ , precise mechanisms exist which can weaken the role of the flavour-loop corrections in the kinematical region where "soft" dynamics dominates. We also claim that the S matrix at large N_c ($\rho \simeq 0$) incorporates already many realistic features, and we refute some objections recently raised against the Harari-Freund picture.

1. INTRODUCTION

In the topological expansion¹⁾ of unitary dual amplitudes for strong interactions at small p_T , two competing schemes exist to describe diffraction within the cylinder topology²⁾. On the one hand, conventional multi-Regge approximations allow to carry out many explicit calculations³⁾ and have been rather successful in correlating a large amount of data. Consequences of this model are the so-called P-f identity⁴⁾ and the "flavouring" of the Pomeron⁵⁾. A less well explored approach, but which seems to be favoured by Dual Field Theory (DFT)⁶⁾, string models⁷⁾ or even general analyticity considerations⁸⁾, is the two singularity scheme à la Harari-Freund⁹⁾ based on the fact that a new singularity appears in the cylinder graph of DFT¹⁰⁾. Also Quantum Chromodynamics (QCD)¹¹⁾ is expected to produce a glueball singularity at four dimensions, and bag models have seriously considered gluonium bound states¹²⁾.

In a previous paper¹³⁾ we discussed the problem of f extinction from both theoretical and phenomenological points of view going beyond multi-Regge approximations and using DFT as a guide. In particular, we analyzed the intermediate state structure of the one-loop cylinder graph and showed that glueball exchange is generated by the production in the direct channel of two massive particles which, when considered as composite or decaying objects, overlap in the rapidity plot. Daughter exchanges appeared to play a crucial role in this phenomenon and we ended up with an explicit realization of Veneziano's conjecture¹⁴⁾ that adding background terms to the twisted leading Reggeons should restore the two singularity scheme in realistic multiperipheral calculations. At this stage, one can consider the possibility that the Pomeron is mainly a promoted f and the non-leading singularity the glueball¹⁴⁾, or vice versa¹⁵⁾. The usual expectation of DFT authors is that the glueball singularity corresponds to the phenomenological Pomeron, in agreement with the conventional Harari-Freund duality. As was pointed out in I, in such a case the f and ω dipoles are both coupled to a sum of two components -- the multiperipheral one and the glueball -- which appear to have opposite signs. We then have a compensation mechanism which, in principle, could allow both Regge poles to stay close to planar intercept and ideal mixing.

However, these results were obtained with heavy zero width resonances and in practice such objects decay into many particle states. The weight of flavour loop corrections to a given zero width diagram is governed by a parameter proportional to $\rho = N_f/N_c$ ¹⁴⁾ (N_f = number of flavours, N_c = number of colours). If ρ is small, we can reasonably believe what we learn from narrow objects, but in the real world one has $\rho \sim 1$ and therefore the question arises whether including flavour loops (cylinder with holes) will not spoil the initial Harari-Freund picture.

Of course, in the lack of a precise realistic dual theory of hadrons, it is not possible to give to this question a definite numerical answer. We even do not know, in DFT, how to escape the zero mass condition α_R (Reggeon intercept) = 1 and α_G (glueball intercept) = 2 : will flavour loops provoke it spontaneously or should we look for a best suited family of Born terms, with reasonable intercepts from the beginning? But, though this question remains unanswered, we can use our present knowledge of the general qualitative features of dual amplitudes and, combining it with basic experimental information, explore the plausibility of a given scheme. For instance, assume zero width amplitudes exist with α_R and α_G away from 1 and 2, respectively, and $\alpha_G > \alpha_R$. Has this situation à la Harari-Freund a choice of surviving when flavour loops are included with the realistic coupling constant? If so, would it look compatible with data? This is one of the most crucial problems concerning the structure of the Pomeron and remains a source of controversy^{2), 13) - 20)}. We shall try to cast some light on it in the present paper.

We argue that, in spite of the actual value of ρ , DFT is able to generate some precise mechanisms which can weaken the role of flavour loop corrections in the kinematical region where "soft" dynamics dominates. We also consider the general features of data at both high and low energies, including many particle production, and compare them with what we expect from the DFT glueball. We find that the situation is compatible with a nice survival of the $P + f$ scheme and with the f and ω not too far from planar intercept and ideal mixing. The analysis clarifies several points concerning the behaviour of the glueball singularity which will be crucial for phenomenology if the Harari-Freund picture of DFT is actually the right alternative. Our goal is not to prove that this is the case, since we think that elucidating the question requires a - yet unfound - more powerful approach, but to discuss why and how such a scheme appears as a serious possibility.

Section 2 is devoted to the results of DFT in the zero width approximation, how they can be understood and generalized through an underlying constituent dynamics, and how the structure of the glueball fits with ISR information on many particle production. In Chapter 3 we discuss the general problems raised by flavour loops, as well as the properties of the dual T matrix at $N_c = \infty$ ($\rho = 0$). We argue that, apart from couplings and widths, the situation at $N_c = \infty$ has already many realistic features (multiplicities, Regge cut-offs and correlations, etc.) which do not require a drastic modification when we go to the realistic case $N_c = 3$.

In Section 4 we discuss one-loop corrections for both the planar topology and the cylinder, and show that data are compatible with flavour loops being governed at small t by an effective parameter of order 0.1. The role of planar loop divergencies is particularly emphasized to understand how such a situation would appear. Looking at the cylinder Reggeon dipole, we again find consistency with a moderate departure from ideal mixing for both f and ω trajectories.

Section 5 deals with the low energy properties of the cylinder in DFT. Contrarily to conventional expectations, glueball exchange appears not to allow a power like extrapolation to the resonance region. This has important consequences for phenomenological models of two component duality. In Chapter 5 we discuss cross-sections and flavouring. We point out that, in the scheme under consideration, the relation between the rise of cross-sections, the production of heavy flavours and the energy dependence of multiplicities is quite different from ordinary multi-Regge cluster models. As a result, the experimental situation is perfectly compatible with a Pomeron near one from the beginning, contrarily to recent claims¹⁶⁾.

2. DYNAMICAL PICTURE FOR GLUEBALL EXCHANGE

As was discussed in I (paper I, see References) and in Ref. 20), the cylinder singularity of DFT corresponds to the production of two heavy intermediate resonances in the kinematical region where

$$s_1 s_2 \gg s$$

s_1 and s_2 being the squared masses of the resonances and s the total squared energy in the direct channel centre-of-mass. In the limit $s/s_1 \rightarrow \infty$, $s/s_2 \rightarrow \infty$ and $s/s_1 s_2 \rightarrow 0$, the amplitude associated to the dual Feynman diagram of Fig. 1 (before integration over internal momentum) behaves as:

$$F(s, s_1, s_2) \sim \left(\frac{s}{s_1}\right)^{\alpha_R(0)} \left(\frac{s}{s_2}\right)^{\alpha_R(0)} \left(\frac{s_1 s_2}{s}\right)^{\alpha_G(0)} \quad (1)$$

when the volume element for internal momentum is taken to be:

$$\frac{ds_1}{s_1} \frac{ds_2}{s_2} d^{d-2} k_T$$

and k_T means transverse momentum, d being the number of space-time dimensions. $\alpha_R(t) = \alpha_t = \alpha(t)$ is the planar Regge trajectory and $\alpha_G(t)$ the glueball trajectory.

Using the techniques of I, it is also possible to discuss the dependence on transverse momentum, and the k_T distribution for each of the two produced resonances can be easily shown to be:

$$f(k_T^2) \simeq e^{2\pi^2 \alpha' \frac{k_T^2}{\ln q}} \quad (2)$$

where q is the position of the saddle point which gives the leading glueball contribution, so that one gets

$$\ln q \simeq -\ln \frac{s_1 s_2}{s}$$

Hence the average squared k_T of each resonance is

$$\langle k_T^2 \rangle \simeq \left(\frac{d}{2} - 1 \right) \frac{\ln \frac{s_1 s_2}{s}}{2\pi^2 \alpha'} \quad (3)$$

which in particular implies $\langle k_T^2 \rangle \ll s_1, s_2$.

The fact that formulae (1), (2) and (3) involve zero width objects without flavour loops, immediately leads us to formulate a constituent picture as the most natural way of understanding such a splitting and rearranging of the internal structure of resonances before decay. It is an old suggestion^{21),22)} that dual amplitudes correspond to highorder Feynman diagrams of a given topology in some underlying constituent field theory. Then the planar Born term of Fig. 2a corresponds to the fishnet diagram of Fig. 2b. If we consider the s channel intermediate state in terms of constituents, we can roughly assume that quarks and gluons have limited transverse momenta and are ordered in rapidity in the longitudinal plot following the vertical axis of Fig. 2c, where $y \simeq \ln s$ and $y/2$ ($-y/2$) is the rapidity of the quark (antiquark) in the centre-of-mass system. In terms of this picture, we can consider two kinds of "rapidity ordered" Feynman diagrams for the cylinder amplitude. The one of Fig. 3a corresponds to the situation where the produced resonances are separated by some rapidity gap, typically $s > s_1 s_2$ (in the figure: $Y_1 = \ln s_1$, $Y_2 = \ln s_2$, $Y_0 = y - Y_1 - Y_2$). The opposite situation is the one where the intermediate state is made of two overlapping resonances (Fig. 3b now with $y_1 = y - Y_2$, $y_2 = y - Y_1$, $y_0 = Y_1 + Y_2 - y = -Y_0$). As was discussed in I, both diagrams give separate contributions to the cylinder amplitude. In the limit $Y_0, Y_1, Y_2 \rightarrow \infty$ the result of DFT for Fig. 3a is:

$$F(y, y_1, y_2) \sim e^{y_1 \alpha_R(0) + y_2 \alpha_R(0) + y_0 \alpha_C(0)} \quad (4a)$$

up to logarithmic factors for the cut amplitude (α_C = cut trajectory) whereas for Fig. 3b we have formula (1) which in terms of rapidity variables becomes

$$F(y, y_1, y_2) \sim e^{y_1 \alpha_R(0) + y_2 \alpha_R(0) + y_0 \alpha_G(0)} \quad (4b)$$

and expression (3) becomes

$$\langle k_T^2 \rangle \simeq \lambda y_0 \quad (4c)$$

with

$$\lambda = \frac{\frac{d}{2} - 1}{2\pi^2 \alpha'}$$

From the t-channel point of view, the diagram of Fig.3b contains the so-called "gluonium" propagating in the rapidity range y_0 . It corresponds to a cylinder whose surface is a net of gluons. We then can easily understand the meaning of formulae (1), (2), and (3). In the s-channel intermediate state we have two surfaces of gluons with quark lines on their borders. The kinematical regions where there is only one surface are governed by the usual Reggeon sum rule, whereas the overlap region gives rise to the glueball of DFT. Crucial for the consistency of this picture is that k_T^2 depends on y_0 instead of y_1 or y_2 . In the region with only one net of gluons, local compensation requires that the transverse momentum of each gluon is compensated by gluons belonging to the same resonance. On the contrary, the overlap region allows for compensation with gluons from the other intermediate particle without spoiling short-range ordering. Then the transverse momenta of gluons belonging to the same particle can be relatively uncorrelated and generate a sizeable total k_T in the overlap region. From statistical considerations, the most natural result is a $\langle k_T^2 \rangle$ proportional to the number of gluons, k_T following a Gaussian distribution : this is precisely what we get from DFT through formulae (2) and (4c) since the average number of gluons in the overlap region is expected to be proportional to y_0 .

When the intermediate overlapping resonances of Fig.3b decay, we have to deal with events like the one of Fig. 4, where 0 is for the overlap region of rapidity size y_0 . It is interesting to compare the structure of the overlap region with the typical events of ISR²³⁾, which represent 80% of the total cross-section. Standard fits to correlation data suggest that an ISR cluster is made of three or four particles and spans roughly two rapidity units. Taking the total multiplicity of two jet events to be $2.5 \ln s$, a density of 3.5 particles per cluster means 0.7 clusters per rapidity unit, i.e. 1.4 clusters for two rapidity units, which implies strong overlap effects. More precisely, the average gap inside each cluster is about 0.8 and an equal spacing picture leads to a total multiplicity of $1.25 \ln s$ in the whole available rapidity range. This is just half the above-mentioned two-jet multiplicity and therefore the situation is consistent with two overlapping chains spanning over most of the longitudinal space, i.e. $y_0 \approx y$.

We can also attempt to compare with experiment the average k_T of overlapping clusters. Putting $d=4$, expression (3) becomes :

$$\langle k_T^2 \rangle \simeq 0.066 y_0 \simeq 0.066 \ln s \left(\frac{\text{GeV}}{c} \right)^2 \quad (5a)$$

whereas a statistical model of clusters with $\langle p_T^2 \rangle \simeq 0.55$ for each ISR cluster²⁴⁾ gives :

$$\langle k_T^2 \rangle \simeq 0.55 \frac{1.25 \ln s}{3.5} \simeq 0.2 \ln s \quad (5b)$$

so that there is a factor of 3 between both results which means a factor of 1.7 between the $\langle k_T \rangle$'s. Trying to improve the fit further would be dubious, since the B function we used cannot be considered as the ultimate hadronic amplitude. For instance, it is only at five or six dimensions that the precise value of the Hagedorn temperature agrees with experiment²⁵⁾. We therefore prefer to stay at a qualitative level and just conclude that the overall picture looks quite consistent with our experimental knowledge of high multiplicity reactions.

To have an idea of the validity of multi-Regge formulae for typical events, we can consider the kinematical structure of Fig. 5 where we have selected two overlapping sets of particles, one on each chain. Assuming that all particles are pions with the mean transverse mass ($m_T^2 = m^2 + p_T^2 \simeq 0.17$), we can write :

$$M^2(y_1, y_2, \dots, y_n) \simeq m_T^2 (1 + e^{y_2 - y_1} + \dots + e^{y_n - y_1}) (1 + e^{y_1 - y_2} + \dots + e^{y_1 - y_n}) - \langle |\sum_{i=1}^n p_{Ti}|^2 \rangle \quad (6)$$

where M^2 is the squared centre-of-mass energy of the n pion system under consideration. If we call s_1 and s_2 the squared masses of both planar clusters and s_{12} the squared mass of the whole system, we have :

$$s_1(y_1, y_2, y_3, y_4) = s_2(y_5, y_6, y_7, y_8) \simeq 4.2 \quad (7a)$$

which corresponds to a cluster mass of 2.05 to be compared with the value of 1.9 from standard fits²⁴⁾. The total squared mass s_{12} turns out to be :

$$s_{12}(y_1, y_2, \dots, y_8) \simeq 22 \quad (7b)$$

which gives $\frac{s_{12}}{s_1 s_2} \simeq 1.3$ so that we expect twisted non-leading Regge exchanges between clusters to be important and the use of asymptotic Regge amplitudes is not justified. We then are in the situation first described by Veneziano¹⁴⁾ where some background has to be added to the leading Regge exchanges and this reopens the possibility of a two singularity scheme in realistic multiperipheral cluster models. More precisely, DFT tells us that leading Regge exchanges generate one component of the f renormalization (which shifts it upwards à la Chew-Rosenzweig), whereas the background generates the glueball singularity plus another component of the f dipole (which carries repulsion between the f and the glueball). In the J -plane, pairs of non-leading Reggeons are coupled as t -channel intermediate states and therefore the unitarity equation is not a factorized one-dimensional one as it is usually assumed in conventional multi-Regge approaches.

3. THE ROLE OF FLAVOUR LOOPS

In Section 2 we have considered DFT Feynman diagrams without flavour loops so that our intermediate states were heavy narrow resonances. But, when comparing with experiment, we had to consider realistic many-particle production. Resonances decay into many particle states through diagrams involving many flavour loops, such as those considered in Fig. 6. In this way, direct channel iteration shifts the

resonance poles away from the real axis. The total contribution of a given topology to the transition amplitude is given by the sum over all flavour loops. Therefore the results from zero width diagrams are significant only if they can survive to flavour-loop corrections. In particular, even starting from a nice Harari-Freund picture at the zero width approximation, the scheme can be strongly deformed by the remaining contributions and the physical result might be very different from the initial situation. In the real world we have $\rho \sim 1$ so that we a priori expect that for any n , the contribution of the diagram with n flavour loops associated to a given topology will be of the same order as the zero width term. For this reason, a perturbative treatment of flavour loops should in principle be excluded^{14),26)}. However, in the next section we shall argue that, in spite of the effective coupling which governs strong interactions, dual loops are drastically different from many cluster amplitudes and other elements come into play which can weaken the weight of flavour loops in the small t region. As a consequence, the dual perturbation series inside each topology could still mean something, as far as soft dynamics is concerned, in which case zero width results would keep a reasonable contact with reality.

In what follows we forget about the well-known intercept problems of DFT and assume that a Born term exists with $\alpha_R \neq 1$ which produces a glueball at $\alpha_G \neq 2$. In particular, we shall implicitly have in mind the possibility that $\alpha_R \simeq \frac{1}{2}$ and $\alpha_G \simeq 1$. The fact that until now nobody has been able to construct such an amplitude because of the role of Virasoro gauges⁶⁾ has led some authors^{27),28)} to conjecture that perhaps dual loops will spontaneously generate some kind of Higgs mechanism after a suitable treatment of the flavour loop divergencies. Another alternative, that we adopt here, is to think of a situation where gauge identities are not explicit because the required Higgs mechanism is at work from the beginning.

Since we are going to discuss the role of planar-loop corrections, we can start asking the question : what are exactly the unphysical features of the narrow width approximation? To what extent the realistic $N_c = 3$ theory needs to be fundamentally different from that at $N_c = \infty$? As it was pointed out long time ago²⁹⁾, the planar narrow resonance amplitude is not only the Born term of DFT but also, with suitable normalization, the limit of the whole unitary T-matrix when the coupling constant tends to zero. This can be seen introducing explicitly the widths of resonances in the planar unitary equation as should always be done when one works with unstable particles. Then, if we decompose the two-body amplitude as in the topological expansion :

$$T = T_{\text{planar}} + T_{\text{cylinder}} + \dots \text{ (other topologies)} \quad (8)$$

and analogously for the n particle production cross-section $\sigma_{2 \rightarrow n}$

$$\sigma_{2 \rightarrow n} = \sigma_{2 \rightarrow n}^{pl} + \sigma_{2 \rightarrow n}^c + \dots \text{ (other topologies)} \quad (9)$$

with pl standing for planar and c for cylinder, it turns out²⁹⁾ that the widths of resonances invalidate naïve power counting and the quantities:

$$\tilde{T}_{pl} = \lim_{N_c \rightarrow \infty} N_c T_{\text{planar}} \quad (10a)$$

$$\tilde{\sigma}_{2 \rightarrow n}^{pl} = \lim_{N_c \rightarrow \infty} N_c \sigma_{2 \rightarrow n}^{pl} \quad (10b)$$

$$\langle n \rangle_{1 \text{ jet}} (N_c = \infty) = \lim_{N_c \rightarrow \infty} \frac{\sum_n n \sigma_{2 \rightarrow n}^{pl}}{\sum_n \sigma_{2 \rightarrow n}^{pl}} \quad (10c)$$

$$\tilde{T}_c = \lim_{N_c \rightarrow \infty} N_c^2 T_{\text{cylinder}} \quad (11a)$$

$$\tilde{\sigma}_{2 \rightarrow n}^c = \lim_{N_c \rightarrow \infty} N_c^2 \sigma_{2 \rightarrow n}^c \quad (11b)$$

$$\langle n \rangle_{2 \text{ jets}} (N_c = \infty) = \lim_{N_c \rightarrow \infty} \frac{\sum_n n \sigma_{2 \rightarrow n}^c}{\sum_n \sigma_{2 \rightarrow n}^c} \quad (11c)$$

are all finite, non-zero functions of the energy variables. In other words, the limit $N_c \rightarrow \infty$ can be formulated in terms of the usual S matrix states involving only stable particles and not heavy resonances, even if the widths of the latter tend to zero. The total cross-section is always given by the sum of $2 \rightarrow n$ cross-sections for systems of n stable particles: $\sigma_{\text{Tot}} = \sum_n \sigma_{2 \rightarrow n}$

In particular, we have $\langle n \rangle_{1 \text{ jet}} (N_c = \infty) > 2$ and $\langle n \rangle_{2 \text{ jets}} (N_c = \infty) > 4$. More precisely, Regge cut-offs will as usual kill large gaps and this forces the mean multiplicities to grow logarithmically with energy as was already found by Green and Veneziano²⁹⁾ in a cascade model.

Another interesting point is two-body fragmentation of heavy resonances. The two-body discontinuity of the one-loop planar amplitude was studied by Chan et al.³⁰⁾ in the large energy region and, with the notation of Figs. 1 and 7a, the result was

$$F(s, s_1, s_2) \sim \delta(s_1, t_1, t'_1, t) P\left(\frac{s}{s_1 s_2}, t_1, t'_1\right) \delta(s_2, t_1, t'_1, t) \quad (12a)$$

where in the limit $s_i \rightarrow \infty$:

$$\delta(s_i, t_1, t'_1, t) \sim \frac{s_i^{\alpha(t)} \alpha(t)}{\Gamma(1 + \alpha_t - \alpha_{t_1} - \alpha_{t'_1})} \quad (12b)$$

So that we have to deal with very simple factorized expressions. In particular, we see that when both s_1 and s_2 are large the decay probability depends only on the size of the gap ($Y_0 = \ln \frac{s}{s_1 s_2}$) and not on its precise location in the longitudinal space (sizes of clusters 1 and 2). This is quite characteristic of short range correlations and in fact can be used to build a heuristic mathematical argument.

Since large gaps (Fig. 7b) carry Regge cut-offs, we can use a finite average gap $\bar{Y}_0$ in order to simplify mathematics. Then, let us consider what happens to a heavy resonance of squared mass $M^2 = s$ when it decays into a many particle state. This will take place through diagrams like those of Figs. 7c and 7d, where the intermediate resonances fragment into lighter objects until stable particles are produced^{*)}. In the sense of duality, i.e., after summing over all possible masses of the intermediate objects, diagrams 7c and 7d are equivalent and including both would involve double coupling. But this is not the case, in the rapidity plot, for diagrams 7e and 7f which correspond to individual events of 7c and 7d respectively with all resonances having precise masses. Moreover, even if the final state is the same, the mixing between the intermediate states of 7e and 7f is proportional to g^2 ($\sim 1/N_c$)^{**)} and, at large N_c , we can neglect interference term contributions

*) At the leading order in $1/N_c$, we take all renormalized tree diagrams and we restrict ourselves to physical values of all direct channel variables (resonance energies) which is the relevant kinematical region for the argument of Ref. 29). Then we lose duality between different tree diagrams (like 7c and 7d), and we have the sum over all of them.

**) In the argument of Ref. 29 each power of g^2 is compensated by the modulus squared of a propagator and this cannot occur for interference terms.

to the total probabilities. In other words, in such a limit the total probability associated to $7e$ plus $7f$ is the sum of the squares of both amplitudes, and we do not have any leading order contribution from the interference term described in $7g$.

Being aware of this, we can start discussing the scope of correlations. The following simple example can sketch the general argument. Assume we want to know whether there is any correlation between final particles produced at the two ends of the longitudinal axis ($y = \pm \frac{Y}{2}$, $Y = \ln s$). The particles under consideration will be produced after a certain number of decays which depend on the intermediate process. But already for the first decay we know that the hole has equal probability of being produced at any value of y so that the probability that it appears near the end of the diagram is proportional to $\frac{1}{Y}$ which tends to zero for $s \rightarrow \infty$. Then, in most of events, the first decay will produce a hole at large values of both Y_1 and Y_2 , so that we can use the factorized expressions of formulae (12) and Ref. 30. Since afterwards each produced resonance will decay independently^{*)}, we conclude that there cannot be any long range correlation (apart from trivial energy-momentum conservation) between both ends of the chain.

The argument can be readily refined and made stronger in order to show that the range of correlations grows slower than any increasing function of Y , $f(Y)$ (and therefore stays constant). For this we take $f(Y) \ll Y$ at large Y and, for simplicity, consider a region of size $\sqrt{f(Y)}$ starting from $y = -\frac{Y}{2}$. Then we draw $g(Y)$ holes with $\sqrt[4]{f(Y)} g(Y) \ll Y$ and $\sqrt{f(y)} g(Y) \gg Y$. The probability that one of these holes lies between $-\frac{Y}{2}$ and $-\frac{Y}{2} + \sqrt[4]{f(Y)}$ will be :

$$P_1 \approx 1 - \left[1 - \frac{\sqrt[4]{f(Y)}}{Y} \right]^{g(Y)} \approx 1 - e^{-\frac{\sqrt[4]{f(Y)} g(Y)}{Y}} \quad (12c)$$

which tends to zero at large Y .

*) The situation at $N_c = \infty$ is simple, since recombination effects are suppressed by powers of g by a mechanism similar to the one which suppresses interference terms.

On the contrary, the probability of having a gap between $-\frac{Y}{2}$ and $-\frac{Y}{2} + \sqrt{f(Y)}$ will be :

$$P_2 \simeq 1 - \left[1 - \frac{\sqrt{f(Y)}}{Y} \right]^{g(Y)} \simeq 1 - e^{-\frac{\sqrt{f(Y)} g(Y)}{Y}} \quad (12d)$$

which tends to 1 in the limit $Y \rightarrow \infty$, so that the lowest hole (in most events) will lie somewhere between $-\frac{Y}{2} + \sqrt{f(Y)}$ and $-\frac{Y}{2} + \sqrt{f(Y)}$. Using again the factorization argument from (12a) and (12b), we conclude that the decay dynamics near $y = -\frac{Y}{2}$ is uncorrelated with what happens at $y > -\frac{Y}{2} + \sqrt{f(Y)}$, for any $f(Y)$ growing with Y . Therefore we get multiperipheral short-range order for the planar term already at $N_c = \infty$, without using the conventional Reggeized formulae for many particle final states : the constituent structure of heavy resonances before decay appears to contain already the chief features of correlations.

A similar reasoning can be used for the cylinder topology provided that the Pomeron is a pole, considering two-jet events as the production of two heavy resonances which decay in a way similar to the planar case. Then, to the diagram of Fig.7h one has to associate, in the limit where all y_i are large, and neglecting transverse dimensions, an expression of the kind :

$$F(y_1, y_2, y_3, y_4, y_5) \sim e^{\gamma_1 \alpha_R + \gamma_2 \alpha_G + \gamma_3 \alpha_R + \gamma_4 \alpha_G + \gamma_5 \alpha_R} \quad (12e)$$

which is the natural extension of (4b). Now, with $\alpha_G > \alpha_R$, the cut-off for the gap y_3 is proportional to $\alpha_G - \alpha_R$ and we can proceed as in the planar case.

In conclusion we can say that the large N_c S matrix has already quite reasonable features apart from widths and coupling constants. Planar-loop corrections to each topology become important when $\frac{N_f}{N_c}$ increases, but it is not clear that they play the role of restoring anything fundamental, apart from resonance widths.

4. FLAVOUR LOOP CORRECTIONS TO THE PLANAR AND CYLINDER TOPOLOGIES

After flavour loops have been included, we can essentially expect two possibilities : one with Pomeron - f identity and a lower singularity associated to the DFT glueball. This preserves phenomenology based on direct connection between

Pomeron and f couplings, but the second singularity has no reason to behave like the Harari-Freund Reggeon. In its spirit, such an alternative has something in common with one-singularity schemes. The second possibility is the converse one, where we keep the $P + f$ picture but there is no direct relation between P and f couplings.

The first alternative would be obtained assuming that flavour-loop corrections shift the planar Reggeon from 1 to near $\frac{1}{2}$ and the glueball from 2 to less than the intercept of the renormalized f ^{*)}. This may be reasonable considering that $\rho \sim 1$. On the contrary, glueball-Pomeron identity would prefer considering the identity $\alpha_G = 2\alpha_R$ as something deeper than an accident at 26 dimensions ³¹⁾. In particular, one can think of $\alpha_R \simeq \frac{1}{2}$, $\alpha_G \simeq 1$ and weak flavour-loop corrections. But we then have to understand how this could happen in spite of the fact that the effective coupling constant is of order 1.

The key point is that at $|t|$ not too large, the ρ , f , ω and A_2 trajectories show only small departures from linearity. Then the only renormalization term which is really unavoidable is the one which comes from resonance widths through a dispersion relation and it is of order 0.1 rather than 0.5 or 1. Therefore one is tempted to conjecture that perhaps realistic dual field theory can be formulated - using eventually the renormalization group - in such a way that the Born term is already close to the real world for planar Reggeons and produces a quite realistic gluonic Pomeron from the beginning, the flavour-loop corrections being governed by a reasonably small parameter.

In what follows we shall attempt a further discussion of this possibility, starting with the planar topology in order to fix the general ideas and then extending the argument to the cylinder topology.

i) The planar term at finite N_c

In the real world it appears that the $\rho\rho$ threshold lies quite high. This weakens its contribution to the non-linear part of the ρ and f trajectories at low mass, contrary to what happens in the planar bootstrap ²⁶⁾ where the 3ρ coupling is the parameter which governs many cluster cross-sections. If we consider the one-loop planar amplitude for two-body collisions, at energy large enough and fixed t we can use the Regge approximation and write ³²⁾:

*) One of us (L.G.) thanks G. Veneziano for a discussion on this point.

$$A_{pe}^{(1)}(s,t) \simeq g^4 N_f \Gamma(-\alpha_t) \Sigma(t) s^{\alpha_t} \ln s + \\ + g^4 N_f [\Gamma(-\alpha_t) \beta(t) - \Gamma'(-\alpha_t) \Sigma(t)] s^{\alpha_t} \quad (13a)$$

which is to be compared with the asymptotic behaviour of the Born term :

$$A_{pe}^{(0)}(s,t) \simeq g^2 \Gamma(-\alpha_t) s^{\alpha_t} \quad (13b)$$

so that the relative weight between the dipole component and the zero width amplitude is governed by the trajectory shift $g^2 N_f \Sigma(t)$ where $g^2 N_f$ is of order one and $\Sigma(t)$ is independent of N_f and N_c . But we have not finished yet since in fact $\Sigma(t)$ is divergent and requires renormalization. The usual procedure amounts to a subtraction at the mass of the ground state so that $\Sigma(t)$ turns into $\bar{\Sigma}(t) = \Sigma(t) - \Sigma(m^2)$ with $\alpha(m^2) = 0$. In fact, the counterterm is in general not unique³³⁾ and the details of the renormalization can change provided the key features (finite result, duality and Reggeization, absence of two-body discontinuities in the counterterm) are respected.

An important feature is that $\Sigma(t)$ behaves asymptotically as a power less than linear ($t^{\frac{\alpha_G}{2}}$ in the specific case we are considering) and therefore $\bar{\Sigma}(t)$ satisfies a dispersion relation with only one subtraction. This allows to relate its slope at small t to the discontinuity on the positive real axis, i.e. to resonance widths. We can write :

$$\frac{\bar{\Sigma}(t) - \bar{\Sigma}(\mu^2)}{t - \mu^2} = \frac{1}{\pi} \int_{t_0}^{\infty} \frac{dt' \text{Im} \bar{\Sigma}(t')}{(t' - \mu^2)(t' - t)} \quad (14)$$

where t_0 is the lowest threshold and μ^2 an arbitrary value of t where we have performed the required subtraction. Since $\bar{\Sigma}(t)$ vanishes at $t = m^2$, we end up with the very pleasant result that $\Lambda(t) = g^2 N_f \bar{\Sigma}(t)$ will be of the same order as the experimental widths of low mass resonances. For instance, if $\text{Im} \Lambda(t) \simeq \gamma \sqrt{t - t_0}$, we obtain $\Lambda(0) \simeq \gamma(\sqrt{t_0} - \sqrt{t_0 - m^2})$.

At this stage we can compare with the real world and we see that for the ρ trajectory one has $\gamma \simeq 0.17$. We then need to have in the Born term an intercept already close to the observed one, and analogously for the slope. Since the lowest

resonance lies above the $\pi\pi$ threshold we cannot subtract $\Sigma(m^2)$ at the ρ mass due to the fact that the imaginary part of Σ must remain unaltered. If the subtraction is done at $\alpha_\rho = 0$, then the result is :

$$\Lambda_\rho(0) \simeq -\delta \left(\sqrt{4m_\pi^2 + \frac{\alpha_\rho(0)}{\alpha'}} - 2m_\pi \right) \simeq -0.1 \quad (15)$$

so that $|\Lambda_\rho(t)| \leq 0.1$ for $-1 \leq t \leq 0$. Assuming that $\beta(t)$ leads after renormalization to a result of the same order as $\Sigma(t)$, we conclude that in the kinematical region under consideration the one-loop planar amplitude just gives a weak correction to the dual Born term. Of course, the factor $\ln s$ of the dipole term will at the end dominate, but at $s = 100 \text{ GeV}$ one has $0.1 \ln s \simeq 0.46$ so that the main contribution still comes from the Born term. This leaves the possibility open that even at $N_c = 3$, flavour loops admit a perturbative treatment with an effective parameter of order 0.1. In this case we do not expect the expansion to be a good one in all kinematical regions, but it will be so in the region where soft dynamics dominates. There the dual Born term will be seen in a similar way to the QCD tree diagram at large p_T . The role of planar loops will then be to give finite widths to resonances without drastically changing in average the original zero width amplitude. This is possible essentially because: a) The two-body discontinuity is not the only contribution to the imaginary part of the loop amplitude and the remaining terms involve the regularization of a divergency. b) Finite energy kinematical effects (mass of the ρ , etc..) are such that the imaginary part of realistic Regge trajectories is governed by a parameter of order 0.1. Since we have seen that the $N_c = \infty$ limit defines a not too crazy S matrix structure, the semilocal average of planar loops can be comparatively small without going in principle against anything fundamental.

ii) The cylinder topology

The removal of planar loop divergencies in the cylinder topology has never been carried out explicitly, but the graph of Fig. 8a is known³⁴⁾ to lead to a divergent glueball self-energy. Then regularization will imply a redefinition of the corresponding dipole term, related to the cure of the planar divergency in the diagram of Fig. 8b. Again, we cannot exclude the possibility that the resulting finite glueball self-energy is small and slowly varying in the small t region. If it appears to be so, then flavour-loop corrections will not drastically change the structure of the glueball and we shall really be seeing it in the small t region. This has nothing to do with having heavy narrow resonances since they

decay with realistic widths, nor with any apparent violation of short range order (as it has been claimed ¹⁶⁾) since already the $N_c = \infty$ theory looks rather reasonable from this point of view. It would only mean that the glueball singularity mixes weakly with Reggeons in DFT.

The imaginary part of the glueball self-energy is a sum of δ functions associated to the Reggeon resonance poles (leading and daughters). When resonance widths are taken into account, it turns into a continuum dominated by these poles. Unfortunately, we do not possess here as clean experimental information as for the planar Reggeon dipole. We can attempt a rough numerical estimate writing $\Lambda_G \sim g_{RG}^2$ where Λ_G is the glueball self energy and g_{RG} is the direct coupling between gluonium and tensor resonances. Then the decay of the $\lambda\bar{\lambda}$ tensor state $f'(1514)$ will include the diagram of Fig. 9, so that we can write roughly:

$$\frac{\Gamma(f' \rightarrow 2\pi) \text{ ph. s. } (f' \rightarrow K\bar{K})}{\Gamma(f' \rightarrow K\bar{K}) \text{ ph. s. } (f' \rightarrow 2\pi)} \sim g_{RG}^4 \quad (16)$$

where ph. s. stands for phase space factor.

Experimentally, $K\bar{K}$ is the dominant decay mode of the f' but the data are not precise enough to give the required information on partial branching ratios. The upper bound on the ratio $\Gamma(f' \rightarrow 2\pi) / \Gamma(f' \rightarrow K\bar{K})$ is less than 0.06 ³⁵⁾. The phase space factor is $k^{2J+1} = k^5$, where k is the centre-of-mass momentum. Then one gets the upper-bound :

$$g_{RG}^2 < 0.13$$

which can be compared with the value used in the Appendix of I ($g_{RG}^2 \approx 0.05$ after adequate normalization).

iii) Renormalization of $I = 0$ Regge trajectories

Data are consistent with a small exchange degeneracy breaking between α_p and α_f in the resonance region. In Pomeron-f identity schemes, this is attributed to asymptotic planarity ²⁾. But it has recently been shown by Pennington and

B. and F. Schrempp ⁸⁾ that standard dispersion relations lead to inconsistency of Pomeron-f identity with asymptotic planarity and therefore one-singularity schemes can be in trouble with conventional unitarity and analyticity constraints. The main problem comes from the slope of the trajectory at negative t . On the contrary, in a two-singularity scheme with glueball-Pomeron identity we only need a weak effect in order to shift the glueball slope from $\frac{\alpha'_G}{2}$ to 0.3 which is the usual value for the bare Pomeron in Reggeon calculus ³⁶⁾. Then asymptotic planarity is easy to satisfy escaping naive analyticity constraints. For instance in the simplified G-f mixing model proposed by Veneziano ¹⁵⁾ one has :

$$\alpha_{\pm} = \frac{1}{2} (\alpha_R + \alpha_G) \pm \frac{1}{2} \sqrt{(\alpha_G - \alpha_R)^2 + 4g_{RG}^2} \quad (17)$$

α_+ being the output bare Pomeron. We immediately see that α_+ and α_- separately do not satisfy Hermitic analyticity in the t variable, because of the square root singularities, although by construction the total amplitude still does.

At this stage an important point arises. We know that under standard hypothesis one singularity is not enough and we have to add something else. But the properties of this second object are not completely arbitrary. For instance, assume we add a non-leading glueball to the shifted f in such a way that they do not mix. Then both trajectories remain analytic, separate dispersion relations are still valid and we have not made any progress. Therefore an important departure of the leading singularity from Hermitian analyticity can only be obtained with an important glueball component.

The situation does not change when we modify Veneziano's proposal adding a multi-Regge component to the non-planar f self energy. Then the J -plane propagator becomes ¹³⁾ :

$$A(t, J) \sim \frac{1}{J - \alpha_R(t) - \lambda_R(t) - \frac{g_{RG}^2}{J - \alpha_G(t)}} \quad (18)$$

which leads to a formula similar to (17) but with $\alpha_R + \lambda_R$ instead of α_R (we are neglecting also SU(3) complications). Before mixing, both $\alpha_R(t) + \lambda_R(t)$ and $\alpha_G(t)$ satisfy Hermitian analyticity, and again either it is α_G which is close to the Pomeron, or a strong mixing will be necessary in order to reach agreement with data. For the f trajectory in the resonance region, we can write :

$$\alpha_f(t) = \bar{\alpha}_R(t) + \mu(t) \quad (19a)$$

where μ is the component which violates Hermitic analyticity, and we have put $\bar{\alpha}_R(t) = \alpha_R(t) + \lambda_R(t)$. Then, from (18) we have:

$$\mu(t) = \frac{1}{2} (\bar{\alpha}_R(t) - \alpha_f(t)) \left(\sqrt{1 + \frac{4g_{RG}^2(t)}{(\bar{\alpha}_R(t) - \alpha_f(t))^2}} - 1 \right) \quad (19b)$$

From expression (19b) we learn that asymptotic planarity is satisfied if $\lambda_R(t) \rightarrow 0$ and $g_{RG}^2(t)/t \rightarrow 0$ in the limit $|t| \rightarrow \infty$. This is compatible with a constant g_{RG}^2 and does not by itself require a fast t dependence of λ_R in a given interval. Therefore the parameters which govern f renormalization can be as small at negative t as data suggest them to be in the resonance region (typically of order 0.1 or less at $t \simeq 1.5$). This has to be combined with the fact that, if $\alpha_G > \alpha_R$, the two components of the Reggeon dipole produce opposite effects.

Something similar can be said for the ω trajectory, including a $C = -$ glueball daughter as was proposed in I. Note, in any case, that due to the effect of higher topologies, when the mass difference between the π and the η is taken into account, the ρ - f and ω - A_2 trajectories appear to be quite different in threshold structure and this will produce an exchange degeneracy breaking between the f and the ω which is not due to the cylinder term.

5. STRUCTURE OF THE TWO JET STATES AND LOW ENERGY BEHAVIOUR

As we discussed in I, in terms of the standard integration variables (Fig. 10), the glueball of DFT is generated by a series of powers of $\omega = x_1 x_2 x_3 x_4$ which diverges at $\omega = 1$. In terms of the dual model harmonic oscillators, this is associated to one or more oscillators going all around the loop without being contracted with any of the external momenta (closed string). Then the s channel glueball intermediate states are characterized by the fact that both resonances carry in common a certain number of oscillators with spin-spin coupling, and that the same oscillators are carried by the exchanged twisted Reggeon (which in general belongs to the daughter's level). These details, which can only be obtained from DFT, will be used in what follows to discuss the low-energy behaviour of the cylinder amplitude.

It is commonly admitted that the Harari-Freund picture agrees with low-energy data if the Pomeron is flat³⁷⁾. But, since cross-sections have been found to grow, one can ask what is the correct extrapolation of the Pomeron to the resonance region and what happens to the usual two component version of duality^{38),16)}. We shall discuss this question neglecting absorption complications, although we know from DFT diagrams and from dual absorption models^{39),40)} that actually poles and cuts mix in the duality scheme.

In the planar amplitude we have resonances spaced roughly by one unit of s , starting near threshold and turning into a continuum at $s \approx 6 \text{ GeV}^2$. For instance, in πN scattering, the first SU(6) multiplet of resonances involves the N (bound state) and the Δ , which are both close to threshold and separated by 0.5 units of s . This is typically the situation where the smooth power-like extrapolation provides a good description of the resonance region in the sense of a semi-local average over one unit of s . The Pomeron amplitude has intermediate states made of two jets. In the narrow resonance approximation, this corresponds to the two particle thresholds formed by pairs of zero width resonances from the dual model spectrum. The usual assumption within the Harari-Freund picture is that the Pomeron is smooth in the resonance region and can be described by a power-like extrapolation of the high-energy behaviour. These are precisely the points we want to discuss from the point of view of DFT.

i) How smooth is the Pomeron at low energy?

As far as the masses of both intermediate jets lie in the narrow resonance region, we can obviously use the one-loop cylinder graph of DFT, i.e., the cylinder without holes, as the basis for our discussion. The first SU(6) multiplet for two-resonance continua contains the channels πN , $\pi\Delta$, ρN and $\rho\Delta$, to consider non-strange particles only. In terms of the standard variable $v = s-u/2M$ (M = nucleon mass), the thresholds lie between $v = 0$ (πN) and $v \approx 1.5$ ($\rho\Delta$), spaced roughly by one unit of s . Each two-particle continuum starts from zero at threshold (in imaginary part), grows and spans over some energy interval before it falls asymptotically like a Regge-Regge cut. Many-particle thresholds in this energy region appear only through the former ones. For instance, the $\pi\pi N$ channel is dominated by $\pi\Delta$ and ρN . In this situation, the Pomeron has no reason to be completely smooth at low energy, since it contains precise individual threshold effects. It is worth noticing that already the first phenomenological discussion of the Harari-Freund picture³⁷⁾ was confronted with the occurrence of some structure in the low-energy background, although this was thought to be due to the practical problems of precise phase shift analysis. In fact, these threshold effects are similar for both Pomeron-f identity and two-singularity schemes.

ii) Low-energy behaviour and power-like extrapolation

Using the notation of I, the cylinder amplitude can be written as:

$$A_c^{(0)}(s,t) = \int dx_1 dx_2 dx_3 dx_4 F(s,t,x_i) f(\omega)^{-d} \quad (20)$$

where $f(\omega)$ is the usual partition function and F is a function we are not interested in. Writing:

$$f(\omega)^{-d} = \sum_n a_n^{(d)} \omega^n \quad (21)$$

we get:

$$A_c^{(0)}(s,t) = \sum_n a_n^{(d)} A^{(n)} \quad (22)$$

with

$$A^{(n)} = \int dx_i F(s,t,x_i) \omega^n \quad (23)$$

$A^{(n)}$ has two-resonance thresholds at all possible combinations of masses above

$$s_n = 4 \frac{n - \alpha_0}{\alpha'}$$

in the conventional B_N model. The lowest threshold s_n is associated to the case where the intermediate state carries only closed string oscillators. Each $A^{(n)}$ behaves asymptotically like a constant, up to logarithmic factors, at $t = 0$. When the energy increases, we get contributions from more and more different $A^{(n)}$ and this shifts the energy dependence from zero intercept to α_0 .

In the realistic case of πN scattering the threshold s_0 corresponds to the above discussed $SU(6)$ multiplet involving $L = 0$ states without radial excitations. The threshold s_1 has to be associated with pairs of $L = 1$ states ($H, A_1, f, A_2 - N(1520), N(1670), \Delta(1670)$) which give thresholds between $s \approx 6.3$ and $s \approx 8.5$. The threshold s_2 corresponds to $L = 2$ states ($A_3(1640), g, \omega(1675) - N(1688), \Delta(1890), \Delta(1950)$) with thresholds between $s \approx 11$ and $s \approx 13$ and to pairs of $L = 0$ first radial excitations ($\rho'(1600) - N(1780)$)*) whose thresholds lie at similar energies. For higher powers of ω , pairs of resonances lying on daughter trajectories become the main ingredient of $A^{(n)}$.

*) We can also consider the $N(1470)$, but its mass is substantially different from that of $L = 2$ states. Perhaps it should be associated with the $\rho'(1235)$. In any case, our argument does not depend on this particular choice.

The crucial point is that $\mathcal{A}^{(0)}$ does not depend at all on the glueball-intercept, and it provides the only contribution to the cylinder amplitude below $s \approx 6.3$ ($\nu \approx 2.7$). Therefore the power-like extrapolation cannot provide a suitable description of the Pomeron amplitude in the resonance region, even in a semi-local sense. It is only looking at the whole interval $1 \leq s \leq \tilde{s}$ with $s \approx 15$ GeV that we can get some information on α_G . In particular, the energy dependence of the background between the πN threshold and $s \approx 6$ will not be governed by the power s^{α_G} .

iii) Is the Pomeron monotonic?

The series $f(\omega)$ near $\omega = 1$ reaches its limit rather slowly and it is not obvious that the asymptotic behaviour of $A(s,t)$ can be reproduced with only a small number of $\mathcal{A}^{(n)}$. In the realistic case of πN scattering, at $s = 20$ only $\mathcal{A}^{(0)}, \mathcal{A}^{(1)}, \mathcal{A}^{(2)}$ and $\mathcal{A}^{(3)}$ contribute to the imaginary part of the amplitude. This is to be compared with the result of Roberts, Gavay and Roy¹⁷⁾, who re-analyzed meson-nucleon scattering data in order to check the consistency of two component duality with ideal mixing and exchange degeneracy for the $f-\omega$ system. They were led to consider a Pomeron described by non-monotonic amplitudes and whose low-energy structure does not correspond to a naïve extrapolation of the high-energy behaviour. With the Pomeron of Fig. 11 they got reasonable fits for the Harari-Freund picture (note, however, our discussion of Section 4 on $f-\omega$ exchange degeneracy). We see that it is only at $s > 20$ that the Pomeron develops its asymptotic behaviour. At first sight, such a Pomeron amplitude looks rather unorthodox, but in the light of the above analysis we can say that actually there is nothing strange in it since we have seen that: i) the power-like extrapolation does not govern, even in semi-local average, the energy dependence of the glueball in the resonance region, and ii) threshold effects can strongly delay the appearance of the asymptotic behaviour s^{α_G} . The question requires investigation within a more powerful theoretical framework, that we do not have at our disposal. But in any case we learn that one must be careful with power-like extrapolations of the Pomeron at low energy, and that their eventual failure does not provide any serious argument against the Harari-Freund picture.

6. FLAVOURING AND HIGH-ENERGY THRESHOLDS

The relation between the production of heavy pairs ($K\bar{K}$, $N\bar{N}$, etc.) and the observed rise of cross-sections has been for many years the object of controversy⁴¹⁾. In the framework of factorized multi-Regge models one can attempt to establish a precise quantitative relation between heavy pair production and the shift of the effective Pomeron trajectory from one region to another. This has

been used not only to reconcile the Chew-Rosenzweig Pomeron with high-energy data⁵⁾, but also to argue that a Harari-Freund Pomeron around one would get too high an intercept when pair production is included¹⁶⁾.

It is instructive to start discussing the case $N_c = \infty$. Assume, for instance, an $SU(3)$ meson theory where the symmetry is broken by the mass of the strange quark. Then, in $\pi\pi$ scattering, planar loops will contain thresholds associated to the production of $K\bar{K}$ pairs. But, from an s channel point of view, the resonance spectrum coupled to $\pi\pi$ is the same as in $SU(2)$ without the λ quark. In the s channel unitarity equation, the only change is that now $\pi\pi$ resonances have got some extra width and their decay modes incorporate $K\bar{K}$ pairs. We then get non-zero values of $\tilde{\sigma}_{p1}^{K\bar{K}}$, $\tilde{\sigma}_c^{K\bar{K}}$, etc., for the production of $K\bar{K}$ pairs, but the amplitudes $\tilde{A}_{p1}$, $\tilde{A}_c$, etc. are governed by the zero width approximation which, for $\pi\pi$ scattering, does not involve the λ quark. This provides a precise example where heavy pairs are produced without originating any shift in the effective intercept of total cross-sections, even if the multiplicity of such pairs grows logarithmically with s and the corresponding cross-section is of leading order in $1/N_c$.

Furthermore, using the argument of Ref. 29) for renormalized tree diagrams, we find that $\sigma_{2 \rightarrow n}^{p1}$ is proportional to g^2 and not to $(g^2 N_f)^{n-1}$. Similar results are obtained for $\sigma_{2 \rightarrow n}^c$ and for all other topologies. Therefore, all $\langle n \rangle_m$ jets are independent of N_f at $N_c = \infty$. In this case, multi-Regge flavouring for the cylinder topology would be a misleading procedure, even if short-range order holds as we suggested in Section 3.

At finite N_c , σ_{tot} remains unflavoured provided that the semi-local average of quark-loop corrections is numerically small. High-energy thresholds depend on the weight of diagrams with holes inside each topology. For instance, in the planar term the shift of the leading trajectory will be of the order of the partial widths of ideally mixed resonances into $K\bar{K}$, which are known to be rather small. In the cylinder topology, the λ quark contribution to the glueball self-energy will involve a coupling of order g^2 that we have proposed to relate to the partial width $f' \rightarrow 2\pi$. The situation is in any case different from the Chew-Rosenzweig model since there we need an important renormalization effect to shift the f to 0.85 and later to 1. If the Pomeron is near one from the beginning, we expect renormalization to be much weaker and the effect of the λ quark to be proportional to the parameters which govern planar-loop corrections to the cylinder topology. Threshold effects in σ_{tot} are proportional to diagrams which involve the subtraction of planar-loop divergencies and do not depend in a simple way on the observed multiplicity of heavy pairs.

The situation is different for multiplicities. At finite N_c , even if flavour loop amplitudes remain numerically weak, individual resonance widths are no longer small. This kills the above argument which prevented $\sigma_{2 \rightarrow n}^{p1}$ from carrying a factor $(N_f)^{n-1}$, and analogously for the $\sigma_{2 \rightarrow n}^c$ and all other topologies. Therefore it seems that the mean multiplicities can present flavour thresholds even if σ_{tot} does not, so that a significant difference appears between $N_c = \infty$ and N_c finite. Yet it is not clear whether such an effect can be related to conventional multi-Regge parameters.

In any case, heavy pair multiplicity can be associated to rather simple - and presumably model independent - effects. Assume that we have a pair of overlapping heavy resonances which span over the whole available rapidity interval, $y \simeq \ln s$, and that they fragment into smaller objects whose size is typically of two rapidity units, with small gaps (Fig. 12). In this kinematical configuration we can neglect SU(3) breaking for the masses whereas the effect through Reggeon intercepts is not enormous (less than 30% for an average gap of 0.7). We can then estimate to roughly 0.25 the probability of producing a strange quark in a given flavour loop, so that the average number of λ quark loops in Fig. 12 will be:

$$\langle n_\lambda \rangle \simeq 2 \frac{0.25 \ln s}{\langle Y_c + Y_g \rangle} \simeq 0.2 \ln s \quad (24)$$

where Y_c and Y_g are respectively for cluster and gap size. For each individual strange particle, this guarantees a multiplicity of $0.1 \ln s$, which is roughly the observed value. Notice that (24) has been obtained independently of the value of the coupling constant and of the weight of DFT diagrams involving quark loops.

We conclude that the observed production of heavy pairs cannot be related in a trivial way to a precise shift of the Pomeron trajectory at high energy^{*)}, and that in principle it does not originate any difficulty to a Harari-Freund picture with a Pomeron near one from the beginning. The effect can certainly exist, but there is no reason on theoretical grounds to estimate it to 0.2 for the Pomeron intercept, as has been recently claimed¹⁶⁾. Also it is not obvious how to relate the energy dependence of the total multiplicity with the one of σ_{tot} and with heavy pair productions. Again, a deeper knowledge of dual hadron dynamics is needed before these questions can be elucidated.

*) Dash et al.⁵⁾ suggested that there is some similarity between the flavour thresholds of the Pomeron and those of $R = \sigma(e^+e^- \rightarrow \text{hadrons})/\sigma(e^+e^- \rightarrow \mu^+\mu^-)$. But, at $N_c = \infty$, the thresholds of $\sigma(e^+e^- \rightarrow \text{hadrons})$ remain whereas the Pomeron amplitude is independent of N_f and presents no flavour thresholds.

7. CONCLUSIONS AND COMMENTS

Using DFT as a guide, we have discussed some general features of planar-loop corrections and of the cylinder structure which can be relevant to understand the dynamical origin of the Pomeron in the real world.

We have pointed out that, in spite of the fact that in the real world $N_f/N_c \sim 1$, flavour-loop corrections to the narrow width amplitude associated to a given topology could actually be governed in semi-local average by an effective parameter of order 0.1, related to low-energy resonance widths, OZI violations, etc. If this is so, we do not need a small coupling to get close to the conventional $P + f$ picture à la Harari-Freund. Crucial to this are the details of planar renormalization for the Pomeron and for the planar Reggeon, which both involve the same kind of divergency in DFT.

Then the situation would be roughly the same as at large N_c , up to couplings and widths, and it would be possible to observe experimentally gluonic fish-net diagrams weakly perturbed by quark loops. As we have discussed, the T matrix at $N_c = \infty$ has actually rather reasonable features: factorized expressions for resonance fragmentation (which allow to argue for short-range order), reasonable transverse momenta for two jet events, logarithmically growing multiplicities, etc. None of them requires a drastic modification in going from $N_c = \infty$ to $N_c = 3$.

Even if flavour-loop corrections are not that small, it remains that standard analyticity constraints seem to require more than one J -plane singularity in order that the leading $I = 0$ exchange agrees with the real world. Then, in simple $f + G$ schemes, gluonium must be directly responsible for the structure of the Pomeron at $t \leq 0$, eventually together with the promoted f component.

From DFT we also get non-trivial information on the low-energy behaviour of the cylinder graph. The situation appears to be quite consistent with non-monotonic Pomeron amplitudes in the standard $P + f$ picture. This is important for phenomenology since recent fits to meson-baryon total cross-sections¹⁷⁾ seem to require such a kind of amplitude. Finally, if the Pomeron is near one from the beginning, renormalization effects should be much weaker than in one singularity schemes and we therefore expect heavy pair production not to originate a strong shift of the effective Pomeron intercept. Whether such pairs are the origin of the observed rise of cross-sections remains an open question within the Harari-Freund picture.

ACKNOWLEDGEMENTS

We are grateful to X. Artru, A. Capella, J. Dash, R. Hong Tuan, K. Konishi and Alex Martin for discussions, and to V. Alessandrini, F. Schrempp and G. Veneziano for discussions and for reading the manuscript.

NOTE ADDED

After the typewriting of this paper we became aware of recent work by J. Dash and C.I. Tan (Brown University preprint HET-374, September 1978) where it is pointed out that flavouring allows to escape standard analyticity constraints for the leading trajectory within the Pomeron-f identity scheme.

REFERENCES

- 1) G. Veneziano, Phys. Letters 52B (1974) 220 and Nuclear Phys. B74 (1974) 365.
- 2) See, for instance, G.F. Chew and C. Rosenzweig, Phys. Reports 41C (1978) 263.
- 3) See Ref. 2) and H.M. Chan and S.T. Tsou, Bielefeld lectures, Rutherford preprint RL-76-080 (1976).
- 4) C. Rosenzweig and G.F. Chew, Phys. Letters 58B (1975) 93.
P.R. Stevens, G.F. Chew and C. Rosenzweig, Nuclear Phys. B110 (1976) 355.
H.M. Chan, J.E. Paton, T.S. Tsun and S.W. Ng, Nuclear Phys. B92 (1975) 13.
T.S. Tsun, Phys. Letters 65B (1976) 81; Phys. Rev. D16 (1977) 2353.
- 5) C. Rosenzweig and G.F. Chew, Ref. 4).
J.W. Dash, E. Manesis and S.T. Jones, Phys. Rev. D18 (1978) 303.
- 6) For a comprehensive review, see, M. Jacob, Dual Theory, North Holland/
American Elsevier, Amsterdam (1974).
- 7) See, for instance, C. Rebbi and S. Mandelstam in Ref. 6).
- 8) M.R. Pennington, B. Schrempp and F. Schrempp, CERN preprint TH.2476 (1978).
- 9) H. Harari, Phys. Rev. Letters 20 (1968) 1395.
P.G.O. Freund, Phys. Rev. Letters 20 (1968) 235.
- 10) D.J. Gross, A. Neveu, J. Scherk and J.H. Schwarz, Phys. Rev. D2 (1970) 697.
D. Olive, in Ref. 6).
- 11) For a review, see, W. Marciano and H. Pagels, Physics Reports 36 (1978) 137.
- 12) See, for instance, P. Hasenfratz and J. Kuti, Phys. Reports 40C (1978) 75.
- 13) P. Aurenche and L. González Mestres, preprint Ecole Polytechnique No. A 270.077
and Orsay LPTHE 77/33, to be published in Phys. Rev. D. Hereafter referred
to as I.
- 14) G. Veneziano, Nuclear Phys. B117 (1976) 519.
- 15) G. Veneziano, Nuclear Phys. B108 (1976) 285.
- 16) J.W. Dash and A. Martin, Marseille preprint 77/P.964 and revised version
(1978). See also J.W. Dash, Talk given at the Moriond Conference (1978).
- 17) R.G. Roberts, R.V. Gavay and D. Roy, Nuclear Phys. B133 (1978) 285.
- 18) S.S. Pinsky and D.R. Snider, Phys. Rev. D13 (1976) 1470.
- 19) R. Hong Tuan, Orsay preprints LPTHE 77/4 and 77/28 (1977).
- 20) C.B. Chiu and S. Matsuda, Nuclear Phys. B134 (1978) 463.
- 21) H.B. Nielsen and P. Olesen, Phys. Rev. B32 (1970) 203.
B. Sakita and M.A. Virasoro, Phys. Rev. Letters 24 (1970) 1146.
P. Olesen, Nuclear Phys. B29 (1971) 77.
- 22) G. 't Hooft, Nuclear Phys. B75 (1974) 461.

- 22) See also, G. Tiktopoulos, Talk given at the Moriond Conference (1977) in connection with hadronic many gluon structures.
- 23) See, for instance, M. Jacob, ISR workshop 2-1 (1977).
- 24) See, for instance, M. Le Bellac, Budapest talk, preprint CERN TH.2361 (1977).
- 25) See, for instance, G. Veneziano, in Ref. 6).
- 26) C. Rosenzweig and G. Veneziano, Phys. Letters 52B (1974) 335.
- 27) G. Veneziano, Ref. 1) Nuclear Phys. B74 (1974) 365.
- 28) E. Cremmer and J. Scherk, Nuclear Phys. B72 (1974) 117.
- 29) A. Di Giacomo, S. Fubini, L. Sertorio and G. Veneziano, Phys. Letters 33B (1970) 171.
M. Green and G. Veneziano, Phys. Letters 36B (1971) 477.
- 30) H.M.Chan, J.E. Paton and T.S. Tsun, Nuclear Phys. B86 (1975) 479.
- 31) See, for instance, J.H. Schwarz, in Ref. 6).
- 32) A. Neveu and J. Scherk, Phys. Rev. D1 (1970) 2357.
- 33) D.J. Gross et al., Ref. 10).
- 34) H. Dorn and D. Ebert, Nuclear Phys. B72 (1974) 228.
- 35) See Particle Data Group, Review of Particle Properties (1978) and references therein.
- 36) See, for instance, M. Moshe, Phys. Reports 27C (1978) 256.
- 37) H. Harari and Y. Zarmi, Phys. Rev. 187 (1969) 2230.
- 38) C. Quigg and E. Rabinovici, Phys. Rev. D13 (1976) 2525.
- 39) B. Schrempp and F. Schrempp, Nuclear Phys. B54 (1973) 525; *ibid.* B60 (1973) 110; *ibid.* B77 (1974) 453.
- 40) J.P. Ader, R. Peschanski, R. Lacaze, G. Cohen-Tannoudji and C. Gilain, Nuovo Cimento, 27A (1975) 385.
L. González Mestres and R. Hong Tuan, Nuovo Cimento 40A (1977) 217.
- 41) M.B. Einhorn and S. Nussinov, Phys. Rev. D9 (1974) 3032.
C.I. Tan, Talk at the Moriond Conference (1974).

Figure captions

- Fig. 1 : The dual crossed loop diagram. The same kinematics will be used for the planar box diagram.
- Fig. 2 : a) The dual zero-width planar amplitude.
b) Its representation in fishnet diagrams (solid lines = quarks, wavy lines = gluons).
c) s-channel content of the resonant intermediate state in terms of rapidity ordered quarks and gluons.
- Fig. 3 : Rapidity ordered fishnet representation of the two components of the dual crossed loop: a) conventional multiperipheral Reggeon dipole, b) f-dominated glueball.
- Fig. 4 : Decay products of the glueball s-channel intermediate state.
- Fig. 5 : Example of typical configuration in many particle final states: the secondaries originate in overlapping clusters.
- Fig. 6 : Flavour-loop corrections to: a) the planar topology and b) the cylinder topology.
- Fig. 7 : Decay of a heavy resonance: a)-b) two-body fragmentation, c)-d)-e)-f) decay into many stable particles, g) mixed contribution to the modulus-squared total amplitude, h) fragmentation in the cylinder topology. The dotted lines in g) and h) indicate the s-channel unitarity cut.
- Fig. 8 : Planar loop renormalization of the cylinder diagram: a) rapidity representation of the glueball dipole; b) dual diagram exhibiting the planar loop explicitly.
- Fig. 9 : The f'-f mixing diagram via gluonium.
- Fig. 10 : Dual crossed loop definition of the integration variables.
- Fig. 11 : Low-energy extrapolation of the Pomeron cross-section with

$$\sigma^P = 12.84 \left[1 + 0.0152 \left(\ln \frac{PL}{2.69} \right)^2 \right]$$
 according to Roberts, Gavay and Roy, see Ref. 17).
- Fig. 12 : Intermediate kinematical configuration in s-channel two-jet decay.

FIG.1

=

(a)

(b)

(c)

FIG.2

FIG. 3

FIG. 4

FIG. 5

FIG. 6

FIG.7

(a)

(b)

FIG. 8

FIG. 9

FIG.10

FIG.12