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PRECONFINEMENT AS A PROPERTY OF PERTURBATIVE QCD

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ABSTRACT

We argue that the evolution of jets produced in hard processes can be computed perturbatively in QCD up to the appearance of a "preconfinement" stage consisting of finite mass colourless clusters of quarks and gluons. All basic properties of QCD go into the proof of this result : as a comparison, ϕ_6^3 is found to have unlimited mass clusters.

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During the last couple of years we have witnessed a growing confidence in the use of perturbative QCD techniques for predicting a wider and wider range of hard inclusive phenomena. As an example, in the process $e^+e^- \rightarrow \text{hadrons}$, techniques have been developed¹⁾ for computing how the large initial energy ($\sqrt{Q^2}$) is degraded through successive and almost collinear emission of quanta down to mass scales Q_0 such that $\alpha_s(Q_0^2)/\pi \lesssim 1$ (i.e. perhaps a few GeVs). Below Q_0^2 , perturbation theory loses any meaning and essentially non-perturbative effects -- hopefully leading to confinement -- are expected.

Let us recall that in calculating infrared regular quantities, terms like $\alpha_s(p^2)(\log p^2)^2$ cancel between real and virtual contributions²⁾. Nevertheless, powers of $\alpha_s(p^2) \log p^2$ do appear and must be resummed at all orders (leading log approximation) even when $\alpha_s(p^2) \ll 1$ ($p^2 \gg Q_0^2$). This is why final states (jets) contain a proliferation of quanta.

In these previous investigations little, if any, attention was paid to the way in which colour degrees of freedom evolve during the jet development. It is the purpose of this note to argue that quarks and gluons produced in this evolution from Q down to Q_0 , become organized in lumps (clusters) of colour singlets with finite (i.e. Q -independent) masses of order Q_0 a phenomenon which we are tempted to call "preconfinement". We can then hope that confinement [occurring in the evolution from Q_0 down to hadron masses $O(m_\pi, m_\rho)$] will convert these singlets of "small" mass into hadrons. If so, there would be no need to invoke any new and incalculable mechanism that involves a large reshuffling of momenta of coloured quanta (as in the standard picture of a parton fragmenting into hadrons).

The first important point to realize is that, in the axial gauge and at the leading log level we are working in, all relevant graphs are planar²⁾. It follows that the final quanta can be *ordered*, as shown in Fig. 1. Furthermore, there is a natural way to group them (Fig. 1) into sets C_i of adjacent partons each consisting of a quark, an antiquark and a number of gluons. These systems contain a dominant singlet component and, indeed, are pure colour singlets^{*)} in the $N \rightarrow \infty$ limit^{**)} , which incidentally, does also select planar diagrams.

It will be the mass of these colourless systems (e.g. of C_i) that we shall find to be cut off by a power law of the form $(M_{C_i}/Q_0)^{-\lambda}$. Moreover, this result will make use of all the basic properties of QCD, i.e. its non-Abelian character,

*) Another amusing consequence of this planarity is that neither OZI-violating processes, nor glueball formation is allowed in the leading log approximation.

**) The appropriate limit here is the one³⁾ in which N_c (number of colours) goes to infinity with $\alpha_s N_c$ fixed and N_f/N_c fixed (N_f is the number of quark flavours).

asymptotic freedom, its collinear singularity structure and its soft infrared divergence. As a comparison a version of six dimensional ϕ^3 theory (ϕ_6^3), which shares with QCD all the aforementioned properties except the last one, will only show a logarithmic cut off, so that average masses will always be of the order Q .

The fact that final partons can be grouped in limited mass colour singlets will not imply that collections of partons in other representations of $SU(N_c)$ must have unlimited masses. Although arbitrarily chosen systems will have average masses of order Q (see, for example, Ref. 1), many groupings of limited mass systems can be envisaged. In other words, any fundamental rôle played by singlets can only originate from the large distance ($p^2 < Q_0^2$) behaviour of QCD which is out of the scope of perturbation theory.

Going now into our technique, we shall need to extend the results of Ref. 1, which describe the evolution of a jet through emission of arbitrary quanta, to a situation in which there is a selection rule on the type of quanta emitted by certain lines in the jet calculus graph. The spectrum of C_1 needs the identification of the "first" antiquark and this can be achieved by replacing the quark propagator^{*)} with a new one (dashed circle in Fig. 2a) which forbids the emission of quark pairs.

If the non-diagonal propagator $\tilde{D}_{qg}$ is interpreted as selecting the first antiquark irrespective of other pair emission, then the mass p^2 of the decaying quark in Fig. 2a is larger than $m_{C_1}^2$ since it includes other clusters C_1, C_3 , etc. In this case -- identification I1 in Fig. 2b -- $\tilde{D}_{qg}$ is obviously given by 1 minus the probability that the gluon emits no pairs. If we interpret instead, $\tilde{D}_{qg}$ as the emission of one and only one pair then we find the identification I2 of Fig. 2b. In this case $p^2 \approx m_{C_1}^2$ but the diagram represents only a restrictive way of producing the system C_1 . We shall return to this question later on, but we remark that anyhow the fundamental objects to be computed are the gluon and quark modified propagators $\tilde{D}_g$ and $\tilde{D}_q$, which originate if only gluons are emitted.

We are interested in total decay rates, hence for $\tilde{D}_q$ we shall use the zero moment of the quark spectrum, whereas for $\tilde{D}_g$ in order to avoid multiple counting we use a momentum sum rule and consider the first moment of the gluon spectrum. With this definition, the analogous propagators $D_g(p^2)$ and $D_q(p^2)$, when no restriction on pair emission is made, are identically equal to one.

*) We recall that propagators in jet calculus¹⁾ describe how a quantum evolves in p^2 and x through the emission of other quanta.

From Fig. 3 we see that $\tilde{D}_g$ satisfies a non-linear equation whereas $\tilde{D}_q$ satisfies a linear one once $\tilde{D}_g$ is known. These equations represent a generalization of those of Altarelli-Parisi⁴⁾ (A.P.) which describe the arbitrary moment of propagating quanta when the emitted quanta are summed over. This means that for emitted quanta the moments are the ones we have just defined ($m = 1$ in Fig. 3) irrespective of the moment n of the propagator considered. But for unrestricted emission $D_g = D_q = 1$ and the resulting A.P. equation is therefore linear and diagonal in the moment n .

The form of the non-linearity of the integral equations represented in Fig. 3 will originate from a mismatch between real and virtual contributions. Indeed, while pair creation is excluded, by definition, from real contributions to $\tilde{D}$, it must be always included in virtual loops. This pair contribution to virtual processes acts as an uncompensated driving term and generates solutions which are damped for large Q^2 .

We understand therefore why I.R. divergences play an important role: they are usually compensated between real and virtual contributions²⁾ and will show up in a determining way as soon as this compensation is unbalanced. In order to avoid at first this extra (basic) complication, we prefer to start with a toy model which, apart from I.R. divergences, shows characteristics which resemble very much those of QCD.

This model is a slight generalization of six dimensional ϕ^3 theory (ϕ_6^3) including an internal $SU(N)$ symmetry group. Its Lagrangian is given by:

$$\mathcal{L} = -\frac{1}{2} \partial_\mu \phi_i^j \partial^\mu \phi_j^i - \frac{m^2}{2} \phi_i^j \phi_j^i - \frac{1}{3} \frac{\lambda}{\sqrt{N}} \phi_i^j \phi_j^k \phi_k^i ; \quad i, j, k = 1, 2, \dots, N \quad (1)$$

We further identify the first N_f indices as flavour and the remaining $N_c = N - N_f$ with colour. Three types of quanta then occur: "W-mesons", i.e. ϕ_a^b ($a, b = 1, 2, \dots, N_f$), "quarks" and "antiquarks" $\phi_a^\alpha, \phi_\alpha^a$ ($a = 1, \dots, N_f, \alpha = N_f + 1, \dots, N$) and "gluons" ϕ_α^β ($\alpha, \beta = N_f + 1, \dots, N$). Again, only planar diagrams like those of Fig. 1 (except for the presence of "W-mesons") contribute to the leading log approximation. The limit $N \rightarrow \infty$ with λ and $\delta \equiv N_f/N$ fixed will be taken in order to simulate the large N limit of QCD.

Defining $\alpha = \lambda^2/(4\pi)^3$ this theory behaves like the usual ϕ_6^3 theory with a running coupling constant $\bar{\alpha}(p^2)$ given, at the single loop level by⁵⁾

$$\bar{\alpha}(p^2) = \alpha \left(1 + \frac{3}{4} \alpha \log P^2/m^2 \right)^{-1} \quad (2)$$

and an A.P. probability function¹⁾

$$P(x) = P_V(x) + P_R(x) ; P_V(x) = -\frac{1}{12} \delta(x-1) , P_R(x) = x(1-x) \quad (3)$$

The non-linear equation described by Fig. 3a is

$$\tilde{D}_g(p^2) = 1 + \frac{4}{3} \int_{\gamma_0}^{\gamma} \frac{d\gamma'}{\gamma'} (A_V + A_R \tilde{D}_g(p'^2)) \tilde{D}_g(p'^2) \quad (4)$$

where $\gamma = \log p^2/m^2$, $\gamma_0 = \log Q_0^2/m^2$ and the integration measure $\frac{1}{3} d\gamma/\gamma$ is nothing else but dy of Ref. 1. Furthermore

$$A_V = \int_0^1 dx \, x P_V(x) = -\frac{1}{12} , A_R = (1-\delta) \int_0^1 dx \, x P_R(x) = \frac{1-\delta}{12} \quad (5)$$

where the factor $1 - \delta$ in A_R takes into account the fact that no *real* process $g \rightarrow q\bar{q}$ is allowed. Eq. (4) is easily reduced to a non-linear differential equation whose solution is:

$$\tilde{D}_g(p^2) = \left(\frac{\alpha}{\bar{\alpha}(p^2)} \right)^{-1/3} \left[\delta + (1-\delta) \left(\frac{\alpha}{\bar{\alpha}(p^2)} \right)^{-1/3} \right]^{-1} \xrightarrow{p^2 \rightarrow \infty} \frac{1}{\delta} \left(\log p^2/m^2 \right)^{-1/3} \quad (6)$$

The equation for $D_q(p^2)$ shown in Fig. 3b is

$$\tilde{D}_q(p^2) = 1 + \frac{4}{3} \int_{\gamma_0}^{\gamma} \frac{d\gamma'}{\gamma'} (A_V^q + A_R^{q \rightarrow gq} \tilde{D}_g(p'^2)) \tilde{D}_q(p'^2) \quad (7)$$

Since $A_V^q = A_V$, $A_R^{q \rightarrow gq} = A_R$, the solution to Eq. (7) is simply $\tilde{D}_q(p^2) = \tilde{D}_g(p^2)$.

For $\delta = 0$ (no restriction on "pair creation") $A_V = -A_R$ and $\tilde{D}_g(p^2) = \tilde{D}_q(p^2) \equiv 1$ as expected. We notice also that the asymptotic form of the solutions has a $1/\delta$ singular coefficient that reflects the fact that, for $\delta = 0$, the actual solution has a different asymptotic behaviour.

Using the above results for $\tilde{D}_g$ and $\tilde{D}_q$, and identification 1 of Fig. 2 for $\tilde{D}_{gq}$, one gets after some algebra^{*)}

*) The result [Eq. (8)] contains also the contribution of two other diagrams typical of the presence of "W-mesons" in ϕ_s^3 .

$$\frac{1}{\sigma} \frac{d\sigma}{dp^2} = \frac{\delta [\alpha/\bar{\alpha}(p^2)]^{-1/3}}{9 p^2 \log p^2/m^2} \left\{ 1 + \frac{(1-\delta)}{(\delta + (1-\delta)(\alpha/\bar{\alpha}(p^2))^{-1/3})^2} \right\} \quad (8)$$

We see therefore the appearance of an *extra damping* factor {going asymptotically as $1/\delta [\alpha/\bar{\alpha}(p^2)]^{-1/3}$ } besides the naive $\bar{\alpha}(p^2)/p^2$ behaviour. This is to be contrasted with the mass spectrum of an arbitrary pair of final quanta which, by the rules of Ref. 1, is found to be:

$$\frac{1}{\sigma} \frac{d\sigma}{dp^2} = \frac{2}{9} \frac{1}{p^2 \log p^2/m^2} \left[\left(\alpha/\bar{\alpha}(p^2) \right) \cdot \left(\alpha/\bar{\alpha}(Q^2) \right) \right]^{1/3} \quad (9)$$

i.e. *broader* than the naive $\bar{\alpha}(p^2)/p^2$ spectrum. It is easy to check that the "first cluster" spectrum [Eq. (8)] is correctly normalized to one whereas the inclusive spectrum [Eq. (9)] is normalized to $\langle n(n-1) \rangle \sim 2\langle n \rangle^2$. The extra damping in Eq. (8) is only logarithmic in p^2 and is essentially due to $\tilde{D}_g(p^2)$.

For identification I2 of Fig. 2, we obtain still a logarithmic cut off but with twice the power of Eq. (8). In any event the average cluster mass remains of the order of Q (up to logarithms) and is *not* finite as $Q \rightarrow \infty$. The fact that QCD will imply a stronger cut off stems from the observation that the exponent of $\alpha/\bar{\alpha}(p^2)$ in Eqs. (6) and (8) is directly related to the value of A_V which in QCD is infrared divergent. We shall show that this infinity amounts to change the logarithmic cut off into a power cut off rendering the average cluster mass finite as $Q^2 \rightarrow \infty$.

Using the explicit form of the A.P. probability functions⁴⁾ we find:

$$\begin{aligned} A_V^g &= -A_R^g - N_c/3 \quad ; \quad A_V^g = -A_R^g \\ A_R^g &= 2C_A \int_0^{1-\epsilon} dx \cdot x \left[\frac{x}{1-x} + \frac{1-x}{x} + x(1-x) \right] = 2N_c \left\{ \log 1/\epsilon - 11/2 \right\} \\ A_R^g &= C_F \int_0^{1-\epsilon} dx \frac{1+x^2}{1-x} = (N_c - 1/2N_c) \left\{ \log 1/\epsilon - 3/4 \right\} \\ C_A &= N_c \quad ; \quad C_F = \frac{1}{2N_c} (N_c^2 - 1) \end{aligned} \quad (10)$$

These expressions diverge if we let the cut off $\epsilon \rightarrow 0$. In the A.P.-type evolution equations the limit $\epsilon \rightarrow 0$ can be safely taken²⁾ due to the cancellation of soft

divergences between A_V and A_R . Here we must be more careful and exhibit the explicit dependence of ε on the kinematical variables.

It is easy to see that, in the kinematical configuration shown in Fig. 4a, the fraction of momentum x carried by the gluon of momentum p'' is limited near $x = 1$ by ⁶⁾ $1 - x > p'^2/p^2$, $(1 - x) > p^2/Q^2$. The second constraint, however, will be automatically satisfied in the damped solutions we will obtain. Inserting $\varepsilon \approx p'^2/p^2$ in Eqs. (10) we then get

$$A_R^g \approx C_A/C_F A_R^g \approx 2N_c \log(p'^2/p^2) \quad (11)$$

It then follows that, in Fig. 3a, one can integrate as usual in p''^2 up to p^2 , while the integration over p'^2 must take into account the p'^2 dependence of the vertex (A_V, A_R). As a result, a recursive equation can only be obtained* for the unintegrated (differential) distribution Δ defined by

$$\tilde{D}_g(x, x_0) = 1 + \int_{x_0}^x \frac{dx'}{x'} \Delta(x', x_0) \quad ; \quad \Delta(x, x_0) = x \frac{d}{dx} \tilde{D}_g(x, x_0) \quad (12)$$

The equation for Δ reads

$$\begin{aligned} \Delta(x, x_0) &= \tilde{D}_g(x, x_0) \left[-\delta + \int_{x_0}^x \frac{dx'}{x'} (x - x') \Delta(x', x_0) \right] = \\ &= \tilde{D}_g(x, x_0) \left[-\delta + \int_{x_0}^x dx' (\tilde{D}_g(x', x_0) - 1) \right] \end{aligned} \quad (13)$$

where we have defined**):

$$x = \frac{C_A}{\pi b} \log \frac{p^2}{\Lambda^2}, \quad \delta = \frac{N_f}{6\pi b}, \quad b = \frac{1}{12\pi} (11N_c - 2N_f) > 0 \quad (14)$$

and we have integrated once by parts using Eq. (12). The term $-\delta$ in Eq. (13) represents $(A_V + A_R)/2\pi b$ and gives the driving term ($\Delta \equiv 0$, $\tilde{D} \equiv 1$ for $N_f = \delta = 0$), whereas the second term is the combined result of the common singular terms in A_V and A_R . Equation (13) can be written as a second order non-linear differential equation for the primitive of $\tilde{D}_g$. Indeed, defining

*) We wish to thank A. Bassetto, M. Ciafaloni and G. Marchesini for discussions regarding this point.

**) In terms of γ the integration measure dy of Ref. 1 reads $1/2\pi b \, d\gamma/\gamma = dp^2/p^2 \times \alpha_s(p^2)/2\pi$.

$$H(\gamma) = \int_{\gamma_0}^{\gamma} \tilde{D}_g(\gamma', \gamma_0) d\gamma' - \delta + \gamma_0 \quad (15)$$

one easily gets the equation ($H' = d/d\gamma H$, etc.)

$$H'' = H' \left[\frac{H}{\gamma} - 1 \right] ; H(\gamma_0) = \gamma_0 - \delta ; H'(\gamma_0) = 1 \quad (16)$$

We have been unable to solve completely Eq. (16). We have obtained, however, its asymptotic ($\gamma \rightarrow \infty$) behaviour in the form:

$$H \underset{\gamma \rightarrow \infty}{\sim} a - \alpha \int_{\gamma}^{\infty} d\gamma' e^{-(\gamma' - \gamma_0)} (\gamma'/\gamma_0)^a \quad (17)$$

The constants a and α cannot be determined from the asymptotic analysis of Eq. (16) but a qualitative inspection allows to deduce that a increases when δ decreases. From Eqs. (15) and (17) one finds

$$\tilde{D}_g(p^2) \underset{\gamma \rightarrow \infty}{\sim} e^{-(\gamma - \gamma_0)} (\gamma/\gamma_0)^a = \left(p^2/Q_0^2 \right)^{-\frac{N_c}{\pi b}} (\log p^2)^a \quad (18)$$

showing the anticipated power damping. Again, $\tilde{D}_q$ can be obtained in terms of $\tilde{D}_g$ by quadratures through the equation depicted in Fig. 3b. After some straightforward calculations we get:

$$\begin{aligned} \tilde{D}_q &= \exp \left[\frac{C_F}{C_A} \int_{\gamma_0}^{\gamma} \frac{d\gamma'}{\gamma'} \int_{\gamma_0}^{\gamma'} d\gamma'' (\tilde{D}_g(\gamma'') - 1) \right] \simeq \\ &\simeq \left[\exp(-\gamma) (\gamma/\gamma_0)^{\delta+a} \right]^{C_F/C_A} \simeq \left[(\log p^2)^{\delta} \tilde{D}_g(p^2) \right]^{C_F/C_A} \end{aligned} \quad (19)$$

Introducing the quantity $\Delta_{qg} = \gamma d/d\gamma \tilde{D}_{qg}$, the p^2 spectrum of diagram 2a will be given by

$$\frac{1}{\sigma} \frac{d\sigma}{dp^2} = \frac{C_F}{C_A} \frac{\tilde{D}_g(\gamma)}{p^2 \log p^2} \int_{\gamma_0}^{\gamma} \frac{d\gamma'}{\gamma'} (\gamma - \gamma') \Delta_{qg}(\gamma') \xrightarrow{\gamma \rightarrow \infty} \begin{cases} \frac{\alpha_s(p^2)}{p^2} \left(p^2/Q_0^2 \right)^{-C_F/\pi b}, & \text{for I1} \\ \frac{\alpha_s(p^2)}{p^2} \left(p^2/Q_0^2 \right)^{-\frac{C_F+C_A}{\pi b}}, & \text{for I2} \end{cases} \quad (20)$$

As discussed before, for II we expect a normalization condition:

$$\int_{Q_0^2}^{Q^2} dq^2 \frac{1}{\sigma} \frac{d\sigma}{dq^2} = 1 - \tilde{D}_g(Q^2) \quad (21)$$

The fact that Eq. (21) can be proven to hold even without knowing the explicit form of $\tilde{D}_g(p^2)$ and $\tilde{D}_q(p^2)$ represents a non-trivial test of our method.

As stated before, we consider the behaviour for II an underestimate of the damping. Indeed, in this case, the mixed propagator $\tilde{D}_{qg}$ provides no damping at all because the gluon emits an increasing number of $q\bar{q}$ pairs as p^2 increases and hence p^2 includes an increasing number of clusters. We conjecture that the actual damping of $m_{C_1}^2$ is somewhere between the two shown in Eq. (20) and probably close to the $(p^2/Q_0^2)^{-N_c/\pi b}$ form of $\tilde{D}_g$ or $\tilde{D}_q^2$. If this conjecture is confirmed, the same damping would follow for the mass of any final cluster C_i of Fig. 1. We notice that $\tilde{D}_q^2(Q^2) \propto (Q^2/Q_0^2)^{-(N_c^2-1)/\pi b N_c}$ is the behaviour we predict for

$$\frac{\sigma(e^+e^- \rightarrow q\bar{q} + \text{gluons})}{\sigma(e^+e^- \rightarrow \text{all})}$$

In order to clarify this point we are now envisaging a more refined study of how transverse momentum -- and therefore mass -- gets distributed in QCD jet evolutions.

We want to mention here that, if we simplify Eq. (13) by neglecting γ' in $(\gamma - \gamma')$ on the right-hand side, the corresponding differential equation becomes of the first order (as is ϕ_g^3) and its exact solution can be found to be:

$$\begin{aligned} \tilde{D}_g(x) &= (x/x_0)^{-\delta} e^{-(x-x_0)} \left[1 - \int_{x_0}^x dx' e^{-(x'-x_0)} (x'/x_0)^{-\delta} \right]^{-1} \simeq \frac{(\log p^2)(p/Q_0^2)^{-\frac{C_A}{\pi b}}}{\delta} \\ \tilde{D}_q(x) &= \left[(x/x_0)^{\delta} \tilde{D}_g(x) \right]^{C_F/C_A} \end{aligned} \quad (22)$$

Notice that these solutions have exactly the asymptotic power damping and the relation between $\tilde{D}_q$ and $\tilde{D}_g$ found from the exact equation. Furthermore, they exhibit the expected singular behaviour as $N_f \rightarrow 0$ and obey $\tilde{D}_g = \tilde{D}_q \equiv 1$ for $\delta = 0$.

In order to have an idea of the quantitative implications of our results we notice that the damping power in $\tilde{D}_g$ has an exponent

$$\frac{N_c}{\pi b} = \frac{12}{11 - 2N_f/N_c} \gtrsim 1 \quad (4/3 \text{ for } N_f = N_c = 3) \quad (23)$$

Our expectation for the cluster mass distribution is therefore of the type $\sim (M_C/Q_0)^{-4}$ so that the average cluster mass is finite and of the order Q_0 , where

we recall that Q_0 was defined as the lower end of applicability of perturbative QCD. Furthermore, since $\delta = O(N_f/N_c)$ acts as a driving term and the damping increases with δ (in particular we have seen that a in Eq. (18) decreases as δ increases), the average cluster mass will turn out to be a decreasing function of N_f/N_c , a result expected on rather general grounds³⁾.

In conclusion, from our arguments, an appealing and consistent picture emerges for QCD jets. The perturbative (short time scale) evolution from Q^2 to Q_0^2 already prepares colour singlet states with masses of order Q_0 . These can then be converted into hadrons by the non-perturbative (confining) dynamics which can involve only momentum transfers smaller than Q_0 (i.e. a large time scale). The resulting picture looks more like a "fusion" of constituent quanta into hadrons than the commonly adopted fragmentation of single quanta into hadrons. Even if the two pictures need not be incompatible, the first one stems directly from QCD, as we have shown, and implies that hadronic clusters (of mass $\geq Q_0$) produced in hard processes are in principle calculable from perturbative QCD. Furthermore, the simplicity of our approach and the physically intuitive nature of the graphs to be computed, lend us some hope that a simple algorithm can eventually emerge for computing hadronic level jet quantities without having to tackle the full complexity of the confinement problem.

We are happy to thank A. Bassetto, M. Ciafaloni, K. Konishi and G. Marchesini for helpful discussions and most constructive criticism and advice.

Note added

Recently, by introducing a generating functional, Bassetto, Ciafaloni and Marchesini were able to extend our method to arbitrary moments, i.e. to x distributions. This allows them to confirm the conjecture made in this paper that $\tilde{D}_{qg}$ provides an extra damping factor of order $\tilde{D}_q \approx \tilde{D}_g^{1/2}$.

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Figure captions

- Fig. 1 : Planar diagrams contributing to the leading log evolution of e^+e^- jets in the axial gauge. Final partons are $O(Q_0^2)$ off-shell and naturally group into ordered colour singlets C_i .
- Fig. 2 : a) Modified jet calculus graph for the spectrum of C_1 .
b) Identifications I1 and I2 for the modified off-diagonal propagator $\tilde{D}_{qg}$.
- Fig. 3 : Self-consistency equations for $\tilde{D}_g$ and $\tilde{D}_q$.

Fig. 1

Fig. 2

Fig. 3