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ON THE DIVERGENCE OF PARTON DISTRIBUTIONS
FOR STATIC BAGS

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A B S T R A C T

It is shown that in $3+1$ dimensions the parton distribution for fermions is logarithmically divergent even for finite (non-confining) and smooth potentials.

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Bag models ¹⁾⁻⁴⁾, especially in the MIT version ³⁾, have found a widespread interest during the last years ⁵⁾. They seem to fit in a very plausible way with several current ideas in hadron physics : they provide a technical and predictive description of quark confinement, they seem to incorporate free field behaviour at short distances, they seem related to a non-perturbative vacuum structure, etc. The second of these presumed properties implies that it should be possible to extract reasonable deep inelastic structure functions or parton distributions from those models ⁶⁾. It has turned out, however ⁷⁾, that taking into account the "sea" contributions leads to divergent parton distributions, at least in the MIT bag model. Now it is not clear which of the idealizations or pathologies of the MIT bag model leads to this failure - the infinite height or the sharp rise of the confining walls, or its lack of translation invariance ; it is even not clear whether the structure functions should exist at all in such models which are related to asymptotically free field theories merely by wishful thinking. Nevertheless bag models represent at least an optimistic view of what the complication of QCD might imply and therefore the investigation of their structural properties deserves certainly further interest.

In this paper we consider again the question of the existence of the parton distribution. While we will not be able to give a decisive answer we can at least sharpen the statements of previous investigations : even if the parton distribution for a static fermion bag with an arbitrary finite (non-confining) and smooth potential wall is calculated, one finds a logarithmically divergence due to the transverse momentum integration. We find this result in second order perturbation theory ; we can, however, infer from the corresponding Yukawa-type field theory that higher orders are formally finite by power counting, so that the result is not restricted to a perturbative treatment.

The parton distribution in longitudinal momentum $q = M_0 X$ and transverse momentum $\vec{k}$ is defined by

$$f_s(X, \vec{k}) dX d^2k = \sum_s \langle B | \hat{b}_s^\dagger(q, \vec{k}) \hat{b}_s(q, \vec{k}) | B \rangle \frac{M_0}{(2\pi)^3} dX d^2k \quad (1)$$

Here $|B\rangle$ is the bag state, characterized by an occupied Dirac sea of negative energy levels corresponding to the external potential $f\phi(x)$ and some valence quark excitations; we omit the latter from our considerations since their contribution is finite. The operators $b_s^0(q, \vec{k})$ are defined by the null plane field expansion $[\vec{x}^3 + x^0 = 0, z = (x^3 - x^0)/2, q = k^0 + k^3]$ ⁸⁾

$$\psi^{(+)}(\vec{x}, z) = \sum_s \int_{q>0} \frac{d^2k dq}{(2\pi)^3} \left(b_s(\vec{k}, q) U_s^{(+)} e^{i\vec{k}\vec{x} + iqz} + d_s^+(\vec{k}, q) U_s^{(+)} e^{-i\vec{k}\vec{x} - iqz} \right) \quad (2)$$

Here $\psi^{(+)}$ is the "good" part of the Dirac field

$$\psi^{(+)} = \Lambda^{(+)} \psi \equiv \frac{1 + \alpha_3}{2} \psi \quad (3)$$

and correspondingly

$$U^{(+)} = \frac{1}{2} \begin{pmatrix} \chi_s \\ \sigma_z \chi_s \end{pmatrix}, \quad \chi_s^\dagger \chi_s = 1 \quad (4)$$

The operators b, b^+, d, d^+ are normalized via

$$\begin{aligned} \{ b_s(\vec{k}, q), b_{s'}^+(\vec{k}', q') \} &= \\ = \{ d_s(\vec{k}, q), d_{s'}^+(\vec{k}', q') \} &= (2\pi)^3 \delta_{ss'} \delta^2(\vec{k} - \vec{k}') \delta(q - q') \end{aligned} \quad (5)$$

In terms of the field operators the parton distribution reads

$$f(X, p_z) = \frac{M_0}{(2\pi)^3} \int d^2x dz d^2x' dz' e^{i p_z(\vec{x} - \vec{x}') + iq(z - z')} \langle B | \psi^{(+)\dagger}(\vec{x}, z) \psi^{(+)}(\vec{x}', z') | B \rangle \quad (6)$$

Now it is convenient to expand $\psi^{(+)}$ in terms of the solutions of the Dirac equation in the external potential, since the state $|B\rangle$ is annihilated by the corresponding operators $b_s(\vec{k}, q)$ and $d_s(\vec{k}, q)$. So we write

$$\psi^{(+)}(\vec{x}, z) = \sum_s \int_{q>0} \frac{d^2k dq}{(2\pi)^3} \left(b_s(\vec{k}, q) U_{s, \vec{k}, q}^{(+)}(\vec{x}, z) + d_s^+(\vec{k}, q) U_{s, -\vec{k}, q}^{(+)}(\vec{x}, z) \right) \quad (7)$$

The spinors $U_{s,k,q}^{(+)}$ satisfy

$$(H_0 + H_1) U^{(+)} = \omega U^{(+)} \quad (8)$$

where ⁸⁾

$$(H_0 u)(\vec{x}, z) = -\frac{i}{2} \frac{\partial u(\vec{x}, z)}{\partial z} + \frac{1}{2} (\vec{\nabla}_\perp^2 + M^2) i \int_{-\infty}^z d\bar{z} u(\vec{x}, \bar{z}) \quad (9)$$

$$\begin{aligned} (H_1 u)(\vec{x}, \bar{z}) = & \frac{1}{2} (-i \vec{\alpha}_\perp \vec{\nabla}_\perp + \beta M) i \int_{-\infty}^z d\bar{z} \beta f \phi(\vec{x}, \bar{z}) u(\vec{x}, \bar{z}) \\ & + \frac{1}{2} \beta f \phi(\vec{x}, z) i \int_{-\infty}^z d\bar{z} (-i \vec{\alpha}_\perp \vec{\nabla}_\perp + \beta M) u(\vec{x}, \bar{z}) \\ & + \frac{1}{2} f \phi(\vec{x}, z) \int_{-\infty}^z d\bar{z} f \phi(\vec{x}, \bar{z}) u(\vec{x}, \bar{z}) \equiv \end{aligned} \quad (10a)$$

$$\equiv \int d\bar{z} H_1(z, \bar{z}, \vec{x}) u(\vec{x}, \bar{z}) \quad (10b)$$

We adopt the convention that M is the heavy (vacuum) quark mass ; so the confining potential is negative and $-M$ inside the bag. We omit the bound states from our consideration in the sense of a perturbative treatment. We have then

$$U_{s,k,q}^{(+)}(\vec{x}, z) \xrightarrow[\vec{x}, z \rightarrow \infty]{} U_s^{(+)} e^{i(\vec{k}\vec{x} + iqz} \equiv U_{s,k,q}^{(+)}(\vec{x}, z) \quad (11)$$

Therefore

$$\omega = \frac{1}{2} \left(q + \frac{\vec{k}^2 + M^2}{q} \right) \quad (12)$$

and positive and negative energy levels are characterized by the sign of q . Our parton distribution can now be expressed in terms of the negative q solutions as

$$\begin{aligned} f(X, \vec{k}) = & \frac{M_0}{(2\pi)^3} \int d^3x dz d^3x' dz' e^{i\vec{k}(\vec{x} - \vec{x}') + iq(z - z')} \\ & \times \int_{q' > 0} \frac{d^2k' dq'}{(2\pi)^3} \sum_s U_{s, -\vec{k}', -q'}^{(+)*}(\vec{x}, z) U_{s, -\vec{k}', -q'}^{(+)}(\vec{x}', z') \end{aligned} \quad (13)$$

We now expand perturbatively

$$U^{(+)} = U^{(+)}_0 + U^{(+)}_1 + \dots \quad (14)$$

The $U^{(+)}_0$ part of $U^{(+)}$ cannot contribute in (13) since it leads to a $\delta(q' + M_0 X)$ which vanishes because $q' > 0$. The first non-vanishing contribution comes therefore from the term involving $U^{(+)}_1(\vec{x}, z) U^{(+)}_1(\vec{x}', z')$.

Using

$$U^{(+)}_1 = -H_0^{-1} H_1 U^{(+)}_0 \quad (15)$$

we obtain

$$\begin{aligned} f\left(\frac{q}{M_0}, \vec{k}\right) &= \frac{M_0}{(2\pi)^3} \int \frac{d^2 k' dq'}{(2\pi)^3} \frac{1}{4} \int d^2 x dz d^2 x' dz' e^{i\vec{k}(\vec{x}-\vec{x}') + iq(z-z')} \\ &\quad \frac{1}{(\omega + \omega')^2} \text{Tr} \{ H_1(\vec{x}, z, \bar{z}) \Lambda^{(+)} H_1(\vec{x}', z', \bar{z}') \Lambda^{(+)} \} e^{i\vec{k}'(\vec{x}-\vec{x}') + iq'(z-z')} \\ &\quad + \text{higher order contributions} \end{aligned} \quad (16)$$

Inserting the explicit expression (10a) for $H_1(\vec{x}, z, \bar{z})$ and performing the trace leads to

$$\begin{aligned} f\left(\frac{q}{M_0}, \vec{k}\right) &= \frac{M_0}{(2\pi)^3} \int_{q>0} \frac{d^2 k' dq'}{(2\pi)^3} \frac{(\frac{\vec{k}'}{q'} - \frac{\vec{k}}{q})^2 + M^2 (\frac{1}{q'} - \frac{1}{q})^2}{[q+q' + \frac{\vec{k}^2 + M^2}{q} + \frac{\vec{k}'^2 + M^2}{q'}]^2} \\ &\quad \times \left| \int e^{i\vec{k}_+ \vec{x} + i(q+q')z} f \Phi(\vec{x}, z) d^2 x dz \right|^2 \\ &\quad + \text{higher order contributions.} \end{aligned} \quad (17)$$

where $\vec{k}_\pm \equiv (\vec{k} \pm \vec{k}')$. The first factor in the integrand behaves well at large q' ($\sim q'^{-4}$) and at small q' ($\rightarrow 1$) for all values of $\vec{k}$, $\vec{k}'$ and q . As a function of $\vec{k}'$ it goes like k'^{-2} for large k' , which would imply a logarithmic divergence of the $d^2 k'$ integration if the Fourier transform in the second factor would not provide an additional damping. However, if we ask for the parton distribution integrated over transverse momentum

(as it appears in deep inelastic structure functions) then the Fourier transform damps only the k_+ integration while the integration over d^2k_- remains divergent. Evaluation of the divergent part leads to

$$F(X) \equiv \int f(x, \vec{k}) d^2k \approx \frac{M_0}{(4\pi)^2} \int d^2\vec{x} \left| \int dz f \phi(\vec{x}, z) e^{i(q+q')z} \right|^2 \times \int \frac{dk_-}{k_-} \quad (18)$$

So we have found that the parton distribution $F(X)$ diverges for all X even for completely harmless potentials. It is interesting to consider somewhat closer the structure of the divergent term. This is facilitated if one thinks of $\phi(x)$ as a scalar field in a Yukawa-type field theory. Then the divergent part (18) of $F(X)$ is related to the fact that the scalar confining field can split up into parton-antiparton pairs. Indeed this process gives rise to a logarithmically divergent wave function renormalization of the field ϕ . To see this directly we integrate (18) over $X dX$ to obtain the expectation value of the parton contribution to the null plane momentum P_+ . This yields

$$\langle P^+ \rangle_{\text{partons}} \propto \int \left(\frac{\partial \phi}{\partial z} \right)^2 d^2\vec{x} dz \int \frac{dk_-}{k_-} \quad (19)$$

which has precisely the form of a wave function renormalization. The difficulty seems therefore related to the fact that the underlying field theory is not of a type (free or super-renormalizable) for which the concept of a naïve parton distribution can be expected to make sense.

The case is not closed nevertheless ; it remains to be seen whether the use of a momentum eigenstate - instead of our localized state $|B\rangle$ - affects our results ; a calculation of a parton distribution of a soliton in 1+1 dimensions ⁹⁾ suggests, however, that the finiteness of the parton distribution is not affected by taking translation invariance into account. The corresponding investigation in 3+1 dimensions requires unfortunately a formal framework going beyond the existing formulations ^{1), 3) 10)} of "bag field theory" ; this remains to be done.

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