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IMPROVED STERMAN-WEINBERG JET FORMULA

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A B S T R A C T

The fraction of non-two-jet events in e^+e^- annihilation and deep inelastic scattering is calculated exactly to the order $\alpha_s(Q^2)$.

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In this letter we present an exact calculation of Stermann-Weinberg (SW) jet formula¹⁾ for e^+e^- annihilation and deep inelastic scattering. In the original paper it was shown that the jet structure follows from a perturbative calculation in QCD. The authors calculated the fraction of two-jet events $f(\epsilon, \delta)$ in which a fraction of the total energy, smaller than ϵ , is emitted outside two opposite cones of aperture angle 2δ . This definition is independent of the choice of any jet axis and is therefore unbiased. In e^+e^- annihilation the fraction of non-two-jet events is then

$$1 - f = \frac{4}{3\pi} \alpha_s(Q^2) \left[4 \ln \delta \left(\ln 2\epsilon + \frac{3}{4} \right) + \frac{\pi^2}{3} - \frac{7}{4} \right], \quad (1)$$

where

$$\alpha_s(Q^2) = 4 \frac{\pi}{9} \frac{1}{\ln Q^2/\Lambda^2}$$

is the running coupling constant. This formula exhibits infrared singularities due to the collinear gluon emission ($\ln \delta$) and mass singularities ($\ln \epsilon$). The range of applicability of (1) is narrow because for small enough ϵ or δ the effective coupling becomes large, making perturbation theory unreliable, and for sizeable ϵ and δ higher correction terms of order ϵ and δ have been neglected. These corrections have now been calculated²⁾ but no exact treatment has so far been proposed for the entire range in δ and ϵ .

To calculate the non-two-jet contribution in this context, one has to take into account the kinematical conditions which define a true three-jet configuration.

- a) All x_i satisfy $x_i > 2\epsilon$, where $x_i = 2E_i/E_{\text{tot}}$ is the fraction of the total energy carried by the i^{th} parton in the hadron rest frame (Q for e^+e^- and W for leptonproduction).
- b) The angle between any two of the three particles is larger than 2δ .

This gives a partition of the (ϵ, δ) plane into different regions (Fig. 1). Region I is the region where (1) was originally derived; that is, where (a) and (b) are independently imposed. In region III, δ is large enough so that (b) remains the only relevant constraint, (a) being automatically satisfied; the contribution

of this region is then ϵ independent. The same is true for the region IV but the roles of ϵ and δ are exchanged; the contribution of region IV is δ independent.

As noted earlier, (a) and (b) do not exhaust all possible configurations; it may happen that the angle between two particles is greater than $\pi-2\delta$, a case that corresponds to a two-jet event. This complication gives rise to three additional distinct regions in the phase space II, V, and VI, for moderate δ values. As we shall see later, the behaviour of $(1-f)$ exactly calculated in these regions is very different from that of the naive extrapolation of (1).

Let us now turn to the calculation of $(1-f)$ in e^+e^- annihilation. The matrix element is Q^2 independent when normalized to σ_{tot} and is just $(x_1^2 + x_3^2)/(1-x_1)(1-x_3)$, therefore if $R(\epsilon, \delta)$ denotes the relevant phase-space region :

$$1 - f(\epsilon, \delta) = \frac{2\alpha_s(Q^2)}{3\pi} \int_{R(\epsilon, \delta)} dx_1 dx_3 \frac{x_1^2 + x_3^2}{(1-x_1)(1-x_3)} \equiv \frac{2\alpha_s}{3\pi} Q(\epsilon, \delta) . \quad (2)$$

The results are presented in Fig. 2 for various values of ϵ in full lines; for comparison we also include the SW calculations as given by (1), which are plotted in dashed lines. Clearly the two calculations are only similar for small ϵ and δ , but differ elsewhere. In particular for small δ values and $\epsilon > 0.1$ we are in region II, and the contributions show divergent behaviour sooner than expected by simply extrapolating (1). In Fig. 2, once the boundary between regions I and II is crossed $(1-f)$ changes rapidly, and perturbation theory no longer makes sense. As δ gets larger our estimates are much smaller than the SW calculations, which are anyway unreliable because of the absence of correction terms in δ .

In the application to deep inelastic scattering we consider the explicit case of νp scattering. All the calculations can be repeated from e^+e^- , apart from the matrix element which is more complicated³⁾. From our analysis it emerges that $(1-f)_{\nu p}$ is scarcely dependent on Q^2 and W^2 , so the curves in Figs. 3 are integrated over W^2 for $Q^2 = 10$ and 40 GeV^2 . It is clear that in the two cases they are very similar. Even the $\alpha_s(Q^2)$ dependence is washed out; this is very different

from e^+e^- and contradicts available experimental results where $(1 - f)$ seemed to decrease with $\ln Q^2$ ⁴⁾. When comparing the absolute size of $(1 - f)_{\nu p}$ and $(1 - f)_{e^+e^-}$, one sees that in e^+e^- annihilation the fraction of non-two-jet events is expected to be two or three times that in neutrino scattering. This is consistent with previous calculations⁵⁾ of average values of thrust and sphericity in deep inelastic scattering where the QCD predictions are twice as small as in e^+e^- . The results for $\bar{\nu}p$ scattering are the same as those for νp within 10%; we therefore expect our calculations to be valid for ep and μp when Q^2 is fixed.

We did not take into account the non-perturbative effects, which could turn out to be large at present energies⁵⁾. However, in regions III and IV as $(1 - f)$ is independent of ϵ or δ , the hadronization should become irrelevant; the corresponding fraction of events is small and such a measurement would require high statistics experiment.

Our calculation extends the SW formula in the whole domain of ϵ and δ and makes it correct for large values of these variables, where QCD perturbation theory is reliable^{*)}. It appears also that, except for small δ , the results are stable and vary smoothly with ϵ and δ . From the experimental point of view it appears that e^+e^- annihilation is a good place to test such effects due to the $\alpha_s(Q^2)$ explicit dependence, whereas in leptonproduction Q^2 and W^2 dependences interplay in a complicated manner and make the analysis difficult.

*) After completion of this work we received a preprint⁶⁾ in which the exact calculation is performed for e^+e^- in region I.

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Figure captions

Fig. 1 : Different regions of integration in (ϵ, δ) plane:

$$m(\delta) = \frac{1}{2 \cos^2 \frac{\delta}{2}}, \quad g(\delta) = \frac{2 \sin \delta}{1 + \sin \delta}, \quad h(\delta) = \frac{1}{2 \cos^2 \delta}, \quad \ell(\delta) = 2 - \frac{1}{\sin^2 \delta}.$$

Fig. 2 : $Q(\epsilon, \delta)$ and $1 - f = [4\alpha_s(Q^2)/(3\pi)] Q(\epsilon, \delta)$ at $Q^2 = 40 \text{ GeV}^2$ as a function of δ , at fixed ϵ in e^+e^- annihilation. The dashed line is the behaviour of the original Sterman-Weinberg formula extrapolated to finite ϵ and δ . The shaded boundary limits the domain where region II contributes.

Fig. 3 : $1 - f$ as a function of δ , at fixed ϵ and Q^2 in neutrino production:
a) $Q^2 = 10 \text{ GeV}^2$, b) $Q^2 = 40 \text{ GeV}^2$.

Fig. 1

Fig. 2

Fig. 3