TOPOLOGICAL COLOUR AND DISC MATING
UNITARITY AS THE SOURCE OF ZERO TRIALITY

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A B S T R A C T

Unitarity in bootstrap quark theory is shown to preclude a general requirement that adjacent quark-antiquark pairs have matching topological colour, as is the rule for flavours. Colour matching occurs instead for certain pairs of "mated" three-quark discs that need not be mutually adjacent. Zero triality thereby becomes recognized as an ingredient in the satisfaction of unitarity.

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In Ref.(1) a bootstrap quark theory is outlined, based on S-matrix unitarity and causality as approached through a topological expansion. S-matrix elements are expanded into an infinite series of components corresponding to polyhedra whose faces belong to ingoing and outgoing particles, the triangulation pattern of each face determining the particle quantum numbers. Orientable disc faces, corresponding to hadrons, have a triangulation pattern describable in terms of "hadron-quarks"-- oriented triangular subdiscs that cover the hadron face. Baryon number and topological color reside in the orientations of the three edges forming the quark subdisc perimeter, while the interior of a hadron quark may be triangulated in patterns that correspond to flavor. It is the purpose of the present letter to prescribe, more carefully than in Ref. (1), unitarity requirements on the matching of quark color and flavor patterns in the different hadron faces that cover an orientable polyhedron. Certain proposals made in Ref. (1) will be amended, and some understanding of zero triality will emerge. In the interest of brevity it will be assumed that the reader is familiar with the contents of Ref.(1).

A central requirement, introduced in Ref.(2) and stressed heavily in Ref. (1), is the complete contractibility of each polyhedron when particle interface boundaries are ignored. Another central feature of Ref.(2), preserved in Ref.(1) as a means to ensure zero triality, is that any hadron face be buildable by starting from a disc composed of adjacent triangular 3-quark and 3-antiquark subdiscs (baryons and antibaryons) and then contracting. Ref.(1) stressed that the contractions must erase boundaries between the

3-quark subdiscs, and it was further suggested that complete contractibility should be achieved by requiring each hadron quark to be adjacent to a conjugate antiquark. We propose here to show that, although adjacent flavor patterns (flavor lunes) must indeed be conjugate, a general requirement of conjugate color patterns for adjacent quarks will violate unitarity. Instead, the appropriate demand is for certain "mated" subdisc pairs (baryon-antibaryon pairs) to have matching color patterns, regardless of whether the two pair members are mutually adjacent. The concept of "mates" identified in Ref.(2) , will be restated here; the definition implies zero triality.

Let us first see why unitarity is violated by uniform insistence on conjugate color patterns for adjacent hadron quarks. It was shown in Ref.(1) that meson faces are colorless, the color-carrying perimeter edges of the quark and antiquark becoming erased by contraction. Mesons are topologically characterized by flavor in precisely the sense proposed by Harari and Rosner ⁽³⁾ , but there is no color. At the same time mesons communicate with (appropriately-flavored) baryon-antibaryon channels-- which means that a channel disc, built from baryon and antibaryon triangular faces that share two edges, contracts to a meson disc. In order that the 2-edge interface be erasable by contraction, the adjacent flavor lunes must be conjugate. In Ref.(1) it was suggested that the corresponding color structures also should be conjugate; the quark and antiquark on the perimeter of the channel disc then carry a definite color. Suppose, that is, we label the different colors of the 3 baryon

quarks as α , β , γ and those of the antibaryon as $\bar{\alpha}$, $\bar{\beta}$, $\bar{\gamma}$. Then if we match β against $\bar{\beta}$ and γ against $\bar{\gamma}$, the remaining peripheral nonadjacent quark-antiquark pair is colored $(\alpha, \bar{\alpha})$. On the other hand if we match α against $\bar{\alpha}$ and γ against $\bar{\gamma}$, the remaining peripheral pair is colored $(\beta, \bar{\beta})$. Do these two different channels communicate on the sphere? The answer is evidently "no" if we demand color matching of adjacent peripheral quark-antiquark pairs. But if the flavors attached to colors α and β happen to be the same, both channels communicate with the same (colorless) meson poles, so unitarity on the sphere is violated if adjacent quark-antiquark pairs are uniformly required to have matching color.

The way out of the dilemma was indicated already in Ref.(2) through the identification of "mated" pairs of the 3-quark or 3-antiquark subdiscs out of which hadron faces must be built. Before contraction the allowable sphere-covering patterns of these triangular subdiscs is always, according to Ref.(2), such that each 3-quark disc perimeter shares all three vertices with exactly one 3-antiquark disc perimeter*. The two members of every such mated pair need not have perimeter edges in common(they need not be adjacent), but some mates will be adjacent and their contraction brings further mates into mutual adjacency. The contraction scheme of Ref.(2) consists entirely of a succession of mate-pair annihilations. Now Ref.(2) did not consider flavor, but if in the scheme of Ref.(1)

* The discussion of Ref.(2) employs graphs dual to the polyhedra.

In place of a triangular disc there is a 3-vertex. Vertices of polyhedron faces become faces of the dual graphs. The two members of a mated pair of 3-vertices then lie on the boundaries of the same 3 faces.

we begin the contraction by annihilating all pairs of adjacent flavor lunes, we arrive at a collection of triangular discs with symmetrical perimeters, no longer flavored but still colored (a different color for each of the 3 quarks) that stand in one-to-one correspondence with the triangular discs of Ref.(2). We may thus continue to identify mate pairs. Insisting only that the two members of a pair have matching colors*, complete contractibility is ensured by the reasoning of Ref.(2).

Let the reader now review the example given above that led to unitarity violation for a 4-face sphere, covered by two baryon and two antibaryon faces. It will be recognized that the baryon-antibaryon pairs sharing two perimeter edges are mates since they also share 3 vertices; the baryon-antibaryon pairs that share only one edge are not mates since they have only two vertices in common. Complete contractibility for the sphere demands color matching for the former pairs but not for the latter. Figure 1 shows an allowed pattern where the nonmated BB pairs have nonmatching color for the adjacent quark and antiquark.

Although in this simple example both mated pairs share two perimeter edges as well as having three vertices in common, the latter feature is the generally essential one in determining where color matching is required to assure complete contractibility for the closed surface.

The requirement of Ref.(2) (restated in Ref.(1)) that all three

* All 3 colors must match, the 3 shared vertices determining the order of matching.

of the perimeter vertices of a 3-quark or 3-antiquark subdisc building a hadron face should be located on the face perimeter, means that the two members of any mated disc pair necessarily lie in different faces. Requiring color-matching (cancellation) only for mates then invalidates another rule proposed as general in Ref.(1): That any n-edge hadron face consists of n "peripheral" hadron quarks, each of which contributes one edge to the face perimeter. If in building the face from a combination of 3-quark and 3-antiquark discs, adjacent quark-antiquark pairs do not happen all to be color matched, then it will not be possible to annihilate internally all the nonperipheral quarks within the face in question. There necessarily then exist n-edge hadron faces that (after all possible contractions) contain more than n hadron quarks. This phenomenon does not, of course, occur for $n=2$ (mesons) or $n=3$ (baryons). Note also that when nonperipheral quarks survive, their flavor lunes are all internally matched and therefore cancelled. Flavor resides entirely on the perimeter of a hadron face.

We have achieved through the foregoing arguments a partial understanding of zero triality-- the requirement that hadron faces be built from 3-quark, 3-antiquark combinations. The concept of mates depends on this requirement-- the central point of Ref.(2) ; the mate concept then provides a means for achieving unitarity.

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FIGURE CAPTION

1. A sphere covered by two baryon and two anti-baryon faces, each face being composed of 3 quarks or 3 antiquarks. The two edges marked "a" are to be identified with each other. It is assumed that flavor lunes are matched and so have been annihilated; only color remains. Color matches within each baryon-antibaryon mate pair but fails to match for the nonmated pairs.

Figure 1.