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QCD ANALYSIS OF CHARGED CURRENT STRUCTURE FUNCTIONS

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ABSTRACT

The structure functions $F_2(x, Q^2)$ and $xF_3(x, Q^2)$ measured in high energy neutrino charged current interactions on nuclei are compared with QCD predictions. Solutions to the moment equations of QCD are found which are in good agreement with the data and yield simple parametrisations of the structure functions. For the scale parameter Λ we find $\Lambda = 0.5 \pm 0.2$ GeV. The analysis also results in values for the width of the gluon distribution as function of Q^2 . We find $\langle x \rangle_{\text{gluons}} = 0.16 \pm 0.03$ for $Q^2 = 10 \text{ GeV}^2$.

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In this letter we give the results of an analysis of the structure functions $F_2(x, Q^2)$ and $xF_3(x, Q^2)$ measured in high energy neutrino interactions based on the predictions of quantum chromodynamics (QCD)¹⁾. A description of the experimental procedure and the experimental results has been submitted for publication elsewhere²⁾. The data on $F_2(x, Q^2)$ and $xF_3(x, Q^2)$ relevant for this analysis, corrected for resolution, acceptance and radiative effects are reproduced in figs. 1a and 1b. The errors given are statistical errors and do not include an overall scale error of $\pm 6\%$ for F_2 and of $\pm 8\%$ for xF_3 which is due mainly to the uncertainty in neutrino flux. Additional systematic point to point errors are estimated to be smaller or at most equal to the statistical errors.

The predictions of QCD which are tested and exploited in this analysis concern the evolution of the structure functions with Q^2 once these are measured for a fixed value Q_0^2 . These predictions are unambiguous for high Q^2 where the strong interaction constant $\alpha_s(Q^2)$ is small compared to unity and where quark and target mass effects can be neglected. The QCD predictions are expressed in terms of the Q^2 dependences of the moments of the structure functions which are given in equations (1), (2) and (3).

$$(1) M_3(n, Q^2) = M_3(n, Q_0^2) e^{-\gamma_{\psi\psi}^n \cdot s}$$

$$(2) M_2(n, Q^2) = [(1-\alpha_n) M_2(n, Q_0^2) - \beta_n M_G(n, Q_0^2)] e^{-\gamma_+^n \cdot s} \\ + [\alpha_n M_2(n, Q_0^2) + \beta_n M_G(n, Q_0^2)] e^{-\gamma_-^n \cdot s}$$

$$(3) M_G(n, Q^2) = [\alpha_n M_G(n, Q_0^2) - \epsilon_n M_2(n, Q_0^2)] e^{-\gamma_+^n \cdot s} \\ + [(1-\alpha_n) M_G(n, Q_0^2) + \epsilon_n M_2(n, Q_0^2)] e^{-\gamma_-^n \cdot s}$$

$$\text{Here } M_3(n, Q^2) = \int_0^1 x^{n-2} xF_3(x, Q^2) dx$$

$$M_2(n, Q^2) = \int_0^1 x^{n-2} F_2(x, Q^2) dx$$

$$M_G(n, Q^2) = \int_0^1 x^{n-2} G(x, Q^2) dx$$

$$s = \ln [\ln (Q^2/\Lambda^2) / \ln (Q_0^2/\Lambda^2)]$$

$G(x, Q^2)$ is the momentum distribution of the gluons inside the nucleon and $\alpha_n, \beta_n, \epsilon_n, \gamma_{\psi\psi}^n, \gamma_{\pm}^n$ are numbers predicted by QCD. We refer to ref. 3 for further details. Λ is the characteristic length of the strong interaction which is related to the effective strong interaction coupling constant

$$\alpha_s(Q^2) = \frac{12 \pi}{(33-2m_f) \ln(Q^2/\Lambda^2)}$$

where m_f is the number of excited flavours which is taken to be four.

The equations (1) - (3) for the moments of the structure functions are not easily translated into predictions for the Q^2 -dependence of the structure functions themselves. This is especially true for $F_2(x, Q^2)$ whose development with Q^2 depends not only on quark distributions but also on the gluon distribution which cannot be measured directly in lepton nucleon interactions.

In our analysis we follow closely a method developed by *Buras and Gaemers* ³⁾. These authors have demonstrated that the parametrisation

$$xF_3(x, Q^2) = \frac{3}{B(\eta_1, \eta_2+1)} \cdot x^{\eta_1(s)} (1-x)^{\eta_2(s)}$$

with linear s dependence of the exponents

$$\eta_1(s) = \eta_{10} + \eta_{11} \cdot s \cdot \frac{4}{25}$$

$$\eta_2(s) = \eta_{20} + \eta_{21} \cdot s \cdot \frac{4}{25}$$

can satisfy the QCD moment relations in the present range of s for the first 10 moment equations with sufficient accuracy. The normalization factor $3/B$ where B is Euler's Beta function is imposed by the requirement of having 3 valence quarks inside the nucleon,

$$\int_0^1 F_3(x) dx = 3.$$

QCD-fits to xF_3

The fits to the valence structure function have been performed for different values of Q_0^2 giving consistent results for $\eta_1(Q^2)$, $\eta_2(Q^2)$ and Λ . The data used consist of all measurements of xF_3 as well as the measurements of F_2 for x greater than 0.4, always for $Q^2 > 3 \text{ GeV}^2/c^2$. It is possible to justify the inclusion of the F_2 -data which are more precise than xF_3 -data, by the experimental fact that the sea quark distribution is negligible in this region of large x .

All points used in the fit correspond to values of $M^2 x^2 / Q^2 < 0.03$ such that corrections due to target mass effects are expected to be small⁴⁾.

For $Q_0^2 = 20 \text{ (GeV/c}^2\text{)}$ which corresponds to the average Q^2 of our data we obtain the following results with $\chi^2/\text{DF}=56/63$

$$\eta_1 = (0.51 \pm .02) - 0.83 \cdot s \cdot \frac{4}{25}$$

$$\eta_2 = (3.03 \pm .09) + 5.0 \cdot s \cdot \frac{4}{25}$$

$$\Lambda = 0.55 \pm .15 (\pm .1 \text{ systematic}) \text{ [GeV]}$$

This simple parametrisation gives a good fit to our data as shown in figures 1a and 1b. The value of Λ , which is the most sensitive parameter, is affected by the uncertainty in the $\bar{\nu}/\nu$ flux ratio of $\pm 5 \%$, the uncertainty of $\pm 2 \%$ in the K/π ratio²⁾ and most probably by higher order effects which have been neglected in this analysis. The overall uncertainty for Λ due to systematic errors in the data is estimated to be ± 0.1 .

Common QCD fits to F_2 and xF_3

The QCD prediction for $F_2(x, Q^2)$ depends on the gluon distribution which is not measured directly except for its second moment $M_G(2, Q^2)$ which is obtained from four-momentum conservation: $M_G(2, Q^2) = 1 - M_2(2, Q^2)$. Given the limited accuracy of our data, we have chosen to follow the approximate method of ref. 3 by using the parametrisation

$$F_2(x, Q^2) = xF_3(x, Q^2) + A(s)(1-x)P(s)$$

This simple parametrisation cannot satisfy all QCD moment equations for F_2 but is reasonably good for a narrow sea-distribution⁵⁾. It has the advantage that $A(s)$ and $P(s)$ can be calculated from the second and third moment equation for F_2 with only 3 additional parameters: $A(0)$, $P(0)$ and $M_G(3, Q_0^2)$.

Common fits for $F_2(x, Q^2)$ and $xF_3(x, Q^2)$ have been obtained. The charm content of the sea has been fixed to be zero at $Q_0^2 = 1.8 \text{ GeV}^2/\text{c}^2$ following ref.3).

For $Q_0^2 = 5 \text{ GeV}^2/c^2$ we obtain with $\chi^2/DF = 101/89$

$$\eta_1 = (0.56 \pm 0.2) - 0.92 \cdot s \cdot \frac{4}{25}$$

$$\eta_2 = (2.71 \pm 0.11) + 5.08 \cdot s \cdot \frac{4}{25}$$

$$A(0) = 0.99 \pm 0.07$$

$$P(0) = 8.1 \pm 0.7$$

$$M_G(2, Q_0^2 = 5) = 0.515$$

$$M_G(3, Q_0^2 = 5) = 0.105 \pm 0.02$$

$$\Lambda = 0.47 \pm 0.11 (\pm 0.1 \text{ systematic}) [\text{GeV}]$$

Again the agreement with the data is good as shown in fig. 1a, and the parameters for the valence part and Λ are in good agreement with the values found in the fit to xF_3 only. The measurements of F_2^{eD} obtained in eD scattering experiments⁶⁾ (not used in the fit) are also shown in fig. 1a multiplied by 9/5. The QCD fits to the neutrino data describe this eD data equally well within the relative normalisation errors of the two experiments.

Q^2 -dependence of gluon moments

The Q^2 -dependence of the second and third moments for all nucleon constituents can be calculated from their fitted values for $Q_0^2 = 5 \text{ GeV}^2/c^2$ given above using the moment equations (1), (2) and (3). It should be noted, however, that the QCD fit to F_2 and xF_3 makes no use of equation (3) for the moments of the gluon distribution except for $M_G(2, Q^2)$ which is automatically satisfied by four-momentum conservation. $M_G(3, Q_0^2)$ is obtained in the fit as a free parameter which is relevant for the Q^2 -dependence of the sea-quarks. This allows an important quantitative test of the QCD prediction for the third gluon moment $M_G(3, Q^2)$.

This is done by repeating the fit for $F_2(x, Q^2)$ for different values of Q_0^2 , keeping the parameters for the valence distribution and Λ fixed to the values obtained at $Q_0^2 = 5 \text{ GeV}^2/c^2$. The only free parameters left are then the second and third moment of the sea-distribution and the third moment of the gluon distribution at $Q^2 = Q_0^2$. The results of these fits are summarized in fig.2, where the total fractional momentum of all constituents and their average value $\langle x \rangle$ are shown for the Q^2 range $3 \leq Q^2 \leq 20 \text{ GeV}^2/c^2$ for which this experiment has relevant data in the sea region.

These QCD-fits give a definite increase of the total momentum carried by the sea quarks, a decrease of the total momentum of quarks plus antiquarks and a corresponding increase of the momentum carried by the gluons. The important new result, which goes beyond the results of the last section, is however the Q^2 -dependence of $\langle x \rangle_G = M_G(3, Q_0^2) / M_G(2, Q_0^2)$ obtained in these fits (points with error bars in fig.2). The average value $\langle x \rangle_G$ is found comparable to that of the total quark distribution at $Q^2 = 5 \text{ GeV}^2/c^2$ and shows a substantial shrinking with increasing Q^2 . The dotted line gives the QCD prediction for $\langle x \rangle_G$ which is calculated from the moment equation (3) (which was not used in the fits) starting with the measured values $M_2(3, Q_0^2 = 5 \text{ GeV}^2/c^2) = 0.127$ and $M_G(3, Q_0^2 = 5 \text{ GeV}^2/c^2) = 0.105$. The agreement with the fitted values is very good. It should be noted that in the present fit method, every point shown in fig.2 represents the same data and carries therefore the full statistical weight of all data, such that there are no statistical point to point errors. The good agreement between the fitted values of $\langle x \rangle_G$ and the calculated QCD curve constitutes thus another successful test of QCD. Since the gluon distribution can not be directly measured, the results of this analysis for $\langle x \rangle_G$ as

function of Q^2 is of interest. At e.g. $Q^2 = 10 \text{ GeV}$ we find $\langle x \rangle_G = 0.16 \pm 0.03$, which can be compared with $\langle x \rangle_{q-\bar{q}} = 0.28 \pm 0.01$ and $\langle x \rangle_{\bar{q}} = 0.10 \pm 0.01$.

The points for the total momentum of the constituents in fig. 2 have a common scale error of $\pm 6\%$ and all errors given are relative errors with respect to the valence quarks $q - \bar{q}$. The measurements of the average x of the gluon distribution show systematic variations with different cuts in Q^2 to the data. This is probably due to the approximate nature of the fitting method. They are also slightly dependent on the assumptions about the charmed sea and variations of Λ and the valence structure function within errors. These effects can lead to an overall shift of all points by about one standard deviation but do not change the Q^2 -behaviour.

In summary, parametrized solutions of the *Buras - Gaemers* type to the QCD moment equations can be found which are in good agreement with the neutrino and electron deep inelastic scattering data. This constitutes an important quantitative confirmation of the validity of QCD. For the gluon constituents of the nucleon this fit gives the Q^2 dependence of both the total fractional momentum and the average value of the gluon momentum distribution.

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Figure captions:

Fig.1a: F_2 in different x bins as a function of $\ln Q^2$ (solid symbols). The values of F_2 measured in electron deuteron scattering are also shown as open symbols. The solid lines represent the QCD fit to the neutrino data.

Fig.1b: same as 1a) for the structure function $xF_3(x, Q^2)$

Fig.2: The total fractional momentum of all nucleon constituents and the average value of their momentum distributions as a function of Q^2 as obtained from QCD-fits to $xF_3 = q - \bar{q}$ and $F_2 = q + \bar{q}$. The dotted line gives the QCD prediction for the third gluon moment which was not used in the fit.

References:

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Fig. 1a

Fig. 1b

Fig. 2