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PROBABILISTIC QUARK-MODEL APPROACH

TO PROTON FRAGMENTATION

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ABSTRACT

A probabilistic approach is formulated to study the behaviour of the incident hadron quark flavours in the fragmentation process of high energy protons in low p_T reactions. Analysis of available data on pp collisions, mainly on hyperon and antibaryon multiplicities, leads to estimates of the probabilities for the different ways in which the incident valence quarks recombine into final hadrons. We find that all three incident quarks emerge in one and the same outgoing nucleon (or nucleon resonance) with probability $A_3 = 0.35 - 0.4$, that two of them emerge in one baryon and the third in another hadron (mostly a meson or meson resonance) with probability $A_2 = 0.6 - 0.5$, and that they emerge in three distinct hadrons (mostly mesons or meson resonances) with probability $A_1 = 0.05 - 0.1$. We find good support for a very simple probabilistic picture of the fragmentation process.

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1. INTRODUCTION

There is at present growing interest in studying particle production in low p_T reactions of hadrons in terms of quarks and gluons. The general problem of the quark-gluon dynamics in such collisions is not yet understood, but there is already good experimental evidence that the quark structure of hadrons reveals itself in soft collisions. This is particularly clear from the study of particle production in the fragmentation regions (large fractional longitudinal momentum x). The data indicate strongly that the quark flavours of the initial hadrons (which we shall call valence quarks in the following) tend to retain a sizable fraction of the initial hadron momentum and recombine into particles which dominate inclusive spectra in the fragmentation region. Recently, considerable amount of work has been devoted to study x distributions in specific quark recombination and fragmentation models.

In this paper we address ourselves to a more general problem. Assuming the exceptional role played by the valence quarks in low p_T collisions we attempt to estimate probabilities for the different ways in which they recombine into final hadrons.

We do not presuppose any specific recombination or fragmentation model but develop a probabilistic approach similar in spirit to the previous works by Anisovich and Shekhter ¹⁾, and Bjorken and Farrar ²⁾, but adapted to the fragmentation of the incident hadrons. We base our study mainly on the analysis of baryon multiplicities and show how the present data give already a great deal of information. Study of baryon production is particularly convenient since the central production of baryon-antibaryon pairs is small even at high energies and can be approximately estimated from the antibaryon production. Our study deals with proton-proton collisions.

The plan of the paper is as follows. The principles of our method are explained in section 2. We define in section 3 what we count as fragmentation baryons and mesons of a proton. We also are led to introduce the concept of fragmentation antibaryons. Section 4 discusses fragmentation kaons and hyperons. The special cases of Ξ and Ω hyperons are considered in section 5 which also treats the fragmentation antibaryons. It is in sections 4 and 5 that we derive

estimates of the probabilities for the various ways in which the incident valence quarks of a low- p_T pp collision recombine into final hadrons. Section 6 shows that these estimates are compatible with a very simple picture where the probability for an incident valence quark to recombine with another incident valence quark (of the same incident proton) is about 3 times the probability for its recombination with another quark or antiquark. The paper ends with a brief summary and some concluding remarks.

2. FORMULATION OF THE PROBABILISTIC APPROACH

We consider high energy inelastic pp collisions of low- p_T type. By means of simple probabilistic considerations, we try to study what happens in such collisions to the valence quarks of the incident protons. All possible fates of the valence quarks can be surveyed by imagining all possible Feynman diagrams for the collisions. These diagrams can be based on a field theory for quarks and gluons, such as Quantum Chromodynamics. They contain quark and gluon lines, but the external lines can be taken to correspond to the valence quarks or antiquarks of the incident or outgoing hadrons (the sea partons of these hadrons being represented by internal lines connected to their valence quark lines). We shall only have to consider the external lines, i.e. the valence quark lines of incident and outgoing hadrons, and we shall simply use the word "lines" to designate them. We call "incident lines" those belonging to the incident protons, and "non-incident lines" the others. In our discussion, we take all hadrons to be non-exotic (valence quark structure qqq , $q\bar{q}$ or $\bar{q}q\bar{q}$).

In all diagrams for pp collisions, all incident lines must necessarily emerge from the collision as lines of outgoing hadrons, but the latter may also contain non-incident lines. Take the incident lines q_1, q_2, q_3 of one incident proton. There are three classes of diagrams (and hence of terms in the collision amplitude) :

- A₁) those where the lines q_1, q_2, q_3 emerge as lines of three distinct outgoing hadrons;
- A₂) those where two of the lines q_1, q_2, q_3 (say q_1, q_2) emerge in one outgoing baryon, the third (q_3) emerging in another hadron;
- A₃) those where the three lines q_1, q_2, q_3 emerge in one and the same outgoing baryon.

Our assumption I is that these three classes of diagrams correspond to three distinct classes of collisions, i.e., that interference effects between the classes are negligible. We call A_1, A_2, A_3 the probabilities for a collision to belong to the corresponding class. Since they exhaust the possibilities, we have

$$A_1 + A_2 + A_3 = 1 \quad (1)$$

At very high energy, valence quark exchange processes between the forward and backward centre-of-mass hemispheres are expected to be rare. Our assumption II consists in neglecting them. Then, if the incident proton considered above is the one flying forward, the outgoing hadrons containing q_1, q_2, q_3 will all fly forward. Similarly the incident lines of the backward incident proton will emerge in outgoing hadrons flying off in the backward hemisphere. In other words, each incident line will emerge in a fragmentation product of the incident proton it belonged to. In particular, no outgoing baryon will contain incident lines originating from both incident protons. The classes and probabilities A_1, A_2, A_3 belonging to the fragmentation of one incident proton are then naturally expected to be uncorrelated to those of the other, but our considerations make no assumption to this effect.

In collision class A_1 , as defined above, the incident lines q_1, q_2 and q_3 can emerge either in a meson or in a baryon, and the same applies to q_3 in collision class A_2 . We call η the probability for such a line to emerge in a meson and $\eta' = 1 - \eta$ the probability to emerge in a baryon. Our assumption III is that η has a single value independently of the incident line concerned and of what happens to the other incident lines.

Most of the outgoing hadrons contain non-incident lines. Our assumption IV is that the probabilities for such non-incident lines to correspond to strange, up or down quarks (or antiquarks) are ξ , $\frac{1}{2}(1-\xi)$ and $\frac{1}{2}(1-\xi)$ respectively, ξ having a single value for all cases concerned.

The consequences of our assumptions for the fragmentation of the incident protons are discussed in the next section.

3. FRAGMENTATION HADRONS

Take one incident proton, say p_1 , and consider the classes and probabilities $A_1, A_2, A_3, \eta, \eta'$ defined in section 2. They lead to specific probabilities for the various numbers n_b of baryons containing $n_i = 1, 2$ or 3 incident lines of p_1 . The various possibilities are listed in Table I as cases 1 to 6, with their respective probabilities. All such baryons will be called fragmentation baryons of p_1 .

A further type of outgoing baryon should also be included in this category, corresponding to case 7 of Table I ($n_b = n_i = 0$; probability $A_1\eta^3$; the probabilities of cases 1 - 7 add up to 1). In case 7, the three incident lines of p_1 emerge in three outgoing mesons (one line in each meson). This creates a surplus of 3 valence quarks over antiquarks in the set of outgoing hadrons which contain no incident line of p_1 , with the result that one of them must be a baryon (see two examples in Fig. 1, where M = meson, B = baryon). It is natural to expect that this outgoing baryon flies off in the same hemisphere as p_1 , a property we adopt as first part of our assumption V. It leads us to consider this baryon also as a fragmentation baryon of p_1 . Note that from assumptions II and V it contains 3 non-incident lines.

The 5th column of Table I gives, with the above definitions, the number n_{fb} of fragmentation baryons of incident proton p_1 corresponding to the various cases. The mean value of n_{fb} is readily calculated from the table, with the result

$$\langle n_{fb} \rangle = A_1\eta^3 + (3A_1 + A_2)\eta' + A_2 + A_3 \quad (2)$$

That this value is not 1 as suggested at first sight by baryon number conservation is due to the fact that, in cases 4, 5 and 6 of Table I ($n_b = 2, 3$), there is a surplus of 3 or 6 valence antiquarks in those outgoing hadrons which contain no incident line of p_1 , with the result that $n_b - 1 = 1$ or 2 of them are antibaryons (see an example with $n_b = 2$ in Fig. 2, where $\bar{B}$ = antibaryon). These can be called fragmentation antibaryons. Extending our assumption V, we also assume them to be in the same hemisphere as p_1 . Their number n_{fa} is given in the last column of Table I, with the average

$$\langle n_{fa} \rangle = A_1 \eta'^2 (3 - \eta') + A_2 \eta' = \langle n_{fb} \rangle - 1 \quad (3)$$

Eq. (3) expresses baryon conservation per hemisphere.

The actual numbers of outgoing baryons and antibaryons experimentally found in the c.m. hemisphere of p_1 can differ from n_{fb} and n_{fa} on two counts. Firstly, exchange processes can take place which we have excluded through our assumptions; such effects are presumably more important at lower than at higher energies. Secondly, additional baryon-antibaryon pairs can be created in increasing number as the energy increases; such pairs are presumably concentrated in the central region of the collision (fractional longitudinal momentum x close to zero, perhaps $-0.1 \lesssim x \lesssim 0.1$). To compare with data, the best is probably to go to high collision energies and to concentrate on the fragmentation regions, say $0.1 \lesssim x \lesssim 1$ and $-1 \lesssim x \lesssim -0.1$. As last part of our assumption V, we assume that the (anti)baryons produced in these regions are fragmentation (anti)baryons in the sense we have defined, accepting that this identification can only be approximate.

We now proceed with the consideration of the fragmentation mesons. We shall call fragmentation mesons of incident proton p_1 those outgoing mesons which contain an incident line of p_1 . Examples are the top three mesons in Fig. 1 and the meson in Fig. 2. The number n_{fm} of fragmentation mesons of a given incident proton can be 0, 1, 2 or 3, with probabilities

$$\left. \begin{array}{ll} A_1 \eta'^3 + A_2 \eta' + A_3 & \text{for } n_{\text{fm}} = 0 \\ 3 A_1 \eta \eta'^2 + A_2 \eta & \text{for } n_{\text{fm}} = 1 \\ 3 A_1 \eta^2 \eta' & \text{for } n_{\text{fm}} = 2 \\ A_1 \eta^3 & \text{for } n_{\text{fm}} = 3 \end{array} \right\} \quad (4)$$

The sum of these probabilities is of course equal to one. The average number of fragmentation mesons is

$$\langle n_{\text{fm}} \rangle = (3A_1 + A_2) \eta \quad (5)$$

It is interesting to introduce at this stage, in addition to A_1 , A_2 , A_3 and $\eta = 1 - \eta'$, alternative parameters P_1 , P'_1 , P_2 and P_3 , defined as the probabilities for the various ways in which an incident line can emerge in an outgoing hadron, namely

P_1 : in an outgoing meson,

P'_1 : in an outgoing baryon containing no other incident line,

P_2 : in an outgoing baryon containing one other incident line,

P_3 : in an outgoing baryon containing two other incident lines.

The relations between the two sets of parameters are

$$P_1 + P'_1 = A_1 + \frac{1}{3} A_2, \quad P_2 = \frac{2}{3} A_2, \quad P_3 = A_3 \quad (6)$$

$$P_1 = \eta(P_1 + P'_1), \quad P'_1 = \eta'(P_1 + P'_1) \quad (7)$$

and one has of course

$$P_1 + P_1' + P_2 + P_3 = A_1 + A_2 + A_3 = 1 \quad (8)$$

Eqs. (2) and (5) can be rewritten as

$$\langle n_{fb} \rangle = A_1 \eta^3 + A_2 + A_3 + 3 P_1' \quad (9)$$

$$\langle n_{fm} \rangle = 3 P_1 \quad (10)$$

equations which are readily interpreted in terms of diagrams.

Experimentally, the separation of the fragmentation mesons as defined above from other outgoing mesons is more difficult than for baryons. Except at large x , perhaps $|x| \gtrsim 0.5$, outgoing mesons will often not contain any incident line. In addition, the production of meson and baryon resonances makes the counting of directly produced mesons difficult (this additional difficulty does not exist for counting baryons, since a baryon resonance has one and only one baryon among its decay products). We shall see below that the consideration of strange mesons should be helpful in this respect.

On the other hand, as shown elsewhere ³⁾, it is possible to estimate P_1 on the basis of the valence quark recombination model, with the result

$$P_1 \approx 0.2 - 0.4 \quad (11)$$

This estimate was made for pp collisions at c.m. energy $s^{\frac{1}{2}} = 45$ GeV, measured at the CERN ISR ⁴⁾. It will be independently confirmed and made more precise in section 5.

4. K MESONS AND HYPERONS FROM PROTON FRAGMENTATION

Since fragmentation mesons have been defined as outgoing mesons containing an incident line, they can only be strange through their non-incident line, which then corresponds to a strange antiquark (strangeness $S = +1$, i.e., strange fragmentation mesons are K^+ , K^0 mesons and their resonances). Calling n_{fK} the number of fragmentation kaons of a given incident proton, we have instead of (4) the following probabilities:

$$\left. \begin{array}{ll} (3 A_1 \eta \eta'^2 + A_2 \eta) \xi & \text{for } n_{fK} = n_{fm} = 1 \\ 6 A_1 \eta^2 \eta' \xi (1 - \xi) & \text{for } n_{fK} = 1, \quad n_{fm} = 2 \\ 3 A_1 \eta^3 \xi (1 - \xi)^2 & \text{for } n_{fK} = 1, \quad n_{fm} = 3 \\ 3 A_1 \eta^2 \eta' \xi^2 & \text{for } n_{fK} = n_{fm} = 2 \\ 3 A_1 \eta^3 \xi^2 (1 - \xi) & \text{for } n_{fK} = 2, \quad n_{fm} = 3 \\ A_1 \eta^3 \xi^3 & \text{for } n_{fK} = n_{fm} = 3. \end{array} \right\} \quad (12)$$

This gives for the average the very simple value

$$\langle n_{fK} \rangle = (3 A_1 + A_2) \eta \xi = 3 P_1 \xi = \langle n_{fm} \rangle \xi \quad (13)$$

For the purpose of the present paper, we limit ourselves to a determination of ξ based on the reasonable expectations that pions and kaons observed at large x are fragmentation mesons in the sense we have defined, and are directly produced mesons rather than decay products of resonances. This permits a direct determination of ξ from the K^+/π^+ ratio at large x ³⁾, with the result

$$\xi \approx 0.11 \quad (14)$$

for pp collisions at $s^{\frac{1}{2}} = 45 \text{ GeV}$ ⁴⁾.

Although we shall not pursue the idea here, we remark that an experimental estimate of $\langle n_{fK} \rangle$ may turn out to be easier than for $\langle n_{fm} \rangle$. The reason is that the resonance problem is probably less serious for kaons than for non-strange mesons. Indeed, a kaon resonance usually contains one and only one kaon among its decay products, and there are very few kaons among the decay products of other resonances.

The discussion of fragmentation hyperons is similar to that of strange mesons, but many more cases are possible. They derive from the 7 cases of Tabel I, each non-incident line of a fragmentation baryon having probability ξ to correspond to a strange quark (see assumption IV). For example, case 7 of Table I gives a $S = -1, -2$ or -3 hyperon with the respective probabilities

$$3 A_1 \eta^3 \xi (1 - \xi)^2, \quad 3 A_1 \eta^3 \xi^2 (1 - \xi), \quad A_1 \eta^3 \xi^3$$

Cases 1 to 6 of the table can be handled similarly and lead to the probabilities for producing 1, 2 or 3 fragmentation hyperons with strangeness $S = -1$ or -2 .

We concentrate here on an average of the number n_{fh} of fragmentation hyperons resulting from a given incident proton. Its value turns out to have a simple form if each hyperon is counted with a weight equal to minus its strangeness (i.e. each Ξ is counted for 2 and each Ω^- for 3). One then finds for this strangeness-weighted average of the number n_{fh}

$$\langle n_{fh} \rangle_S = b\xi, \quad b = 3 A_1 \eta^3 + A_2 + 6P_1' \quad (15)$$

To order ξ , this is also the average number of $S = -1$ hyperons. The average numbers of Ξ 's and Ω 's, which are of order ξ^2 and ξ^3 respectively, will be discussed in the next section. The terms in eq. (15) have each a simple physical meaning. The terms A_2 , $6P_1'$ and $3 A_1 \eta^3$ refer respectively to the fragmentation hyperons containing 2, 1 and 0 incident lines.

The usefulness of eq. (15) stems from two circumstances. Firstly, a reasonably reliable experimental determination of b seems possible. This is because, at least for the lighter ones, hyperon resonances decay into one and

only one hyperon, which has a value of x close to the x -value of the resonance. A reasonable approximation to $\langle n_{fh} \rangle_S$ and b can therefore be obtained by counting the metastable hyperons (Λ , Σ , Ξ , Ω) produced in the fragmentation regions (say with $x \geq 0.1$ or $x \leq -0.1$), irrespective of whether they are produced directly or are decay products of resonances. More detailed work distinguishing Λ 's and Σ 's would require an experimental separation of the resonances.

Secondly, once b is known, eq. (15) imposes interesting constraints on our probabilities. The simplest case obtains for $P_1' = 0$, i.e. $\eta = 1$. We then have

$$b = 3 P_1 = 3 A_1 + A_2 \quad (16)$$

and the remaining freedom of the A_i 's, resulting from $0 \leq A_i \leq 1$, is

$$0 \leq A_1 \leq b/3, \quad b \geq A_2 \geq 0, \quad 1 - b \leq A_3 = 1 - A_1 - A_2 \leq 1 - b/3 \quad (17)$$

where the first bounds go together and so do the second bounds.

In the more general case where $P_1' > 0$, i.e. $\eta < 1$, one finds that for fixed P_1, P_1' the quantity b is no longer fixed but ranges over a rather narrow interval illustrated in Fig. 3. The limits are

$$\left. \begin{aligned} b_{\min} &= 3 P_1 (\eta^3 - 2\eta + 2)/\eta \quad \text{for } P_1/\eta = P_1 + P_1' \leq 1 \\ b_{\max} &= \begin{cases} 3 P_1 (3 - 2\eta)/\eta & \text{for } P_1/\eta \leq \frac{1}{3} \\ [3 P_1 (\frac{3}{2} - 2\eta + \frac{3}{2} \eta^3)/\eta] + \frac{3}{2} (1 - \eta^3) & \text{for } \frac{1}{3} \leq P_1/\eta \leq 1 \end{cases} \end{aligned} \right\} \quad (18)$$

We have plotted the function $b_{\min}/(3 P_1)$ in Fig. 4. Note that this function has a minimum of 1, reached at $\eta = 1$. For $P_1' > 0$, the A_i are functions of $P_1 + P_1'$ and b :

$$\left. \begin{aligned} A_1 &= P_1 + P_1' - \frac{1}{3} (b - b_{\min}) / (1 - \eta^3) \\ A_2 &= (b - b_{\min}) / (1 - \eta^3) \\ A_3 &= 1 - (P_1 + P_1') - \frac{2}{3} (b - b_{\min}) / (1 - \eta^3) \end{aligned} \right\} \quad (19)$$

One sees that $b = b_{\min}$ corresponds to $A_2 = 0$, whereas $b = b_{\max}$ corresponds to $A_1 = 0$ for $P_1/\eta \leq \frac{1}{3}$ and to $A_3 = 0$ for $P_1/\eta \geq \frac{1}{3}$.

A reliable determination of b at high energy is so far not possible. A rather complete measurement of hyperon production has been made for pp collisions at $p_{\text{lab}} = 24$ GeV by Blobel et al.⁵⁾ If we assume all hyperons to be of fragmentation type it gives $\langle n_{fh} \rangle_S \approx 0.05$. Fermilab and ISR data⁶⁾⁻⁹⁾ show a slow increase of Λ production, from $\langle n_{\Lambda} \rangle - \langle n_{\Lambda}^- \rangle \approx \langle n_{\Lambda} \rangle \approx 0.06$ at $p_{\text{lab}} = 24$ GeV⁵⁾ to $\langle n_{\Lambda} \rangle - \langle n_{\Lambda}^- \rangle \approx 0.1$ at $p_{\text{lab}} \gtrsim 200$ GeV ($n_{\Lambda}, n_{\Lambda}^-$ total multiplicities in pp collisions), with Λ production in the proton fragmentation region appearing to flatten off above $p_{\text{lab}} \approx 200$ GeV. Assuming the same increase for the other hyperons, we estimate $\langle n_{fh} \rangle_S \approx 0.05 \times (0.1/0.06) = 0.08$. With (14) we therefore adopt the estimate

$$b \approx 0.75 \quad (20)$$

We mentioned above that $b_{\min} \geq 3 P_1$. Hence we conclude from (20) that $P_1 \leq 0.25$. If we combine this with (11) we obtain

$$P_1 \approx 0.2 - 0.25 \quad (21)$$

Fig. 4 then shows that $\eta \approx 1$, e.g.

$$\eta \geq 0.74 \quad \text{for } P_1 = 0.2, \quad \eta = 1 \quad \text{for } P_1 = 0.25 \quad (22)$$

Independent evidence for $\eta \approx 1$ will be obtained in section 5, and we can then conclude $P_1 \approx 0.25$ from (18) without using (11).

We illustrate the consequences following so far for the A_i by considering two cases. Firstly, for $\eta = 1$, i.e. $P_1' = 0$, we apply eqs. (16) and (17) to find

$$\underline{P_1 = 0.25}, \quad \underline{\eta = 0} \quad : \quad 0 \leq A_1 \leq 0.25, \quad A_2 = 0.75 - 3A_1, \quad A_3 = 0.25 + 2A_1 \quad (23)$$

We take $P_1 = 0.2$ as second example. We then have

$$b_{\min} \leq 0.75 \leq b_{\max} \quad \text{for} \quad 0.74 \leq \eta \leq 0.95$$

(with $\eta = 0.74$ corresponding to $b_{\min} = 0.75$). This gives as extreme cases

$$\underline{P_1 = 0.20}, \quad \underline{\eta = 0.74} : \quad A_1 = P_1 + P_1' = 0.28, \quad A_2 = 0, \quad A_3 = 0.72 \quad (24)$$

$$\underline{P_1 = 0.20}, \quad \underline{\eta = 0.95} : \quad A_1 = 0, \quad A_2 = 3(P_1 + P_1') = 0.63, \quad A_3 = 0.37$$

Great variations are seen to remain. In the next section we shall obtain more precise estimates, not relying on eq. (11).

An interesting question is whether A_3 can vanish (one has $A_3 = 0$ in dual unitarization models neglecting the triple-Pomeron coupling). As mentioned after eq. (19), $A_3 = 0$ corresponds to

$$b = b_{\max}, \quad \frac{1}{3} \leq P_1/\eta \leq 1, \quad A_1 = 1 - A_2 = \frac{1}{2} [(3P_1/\eta) - 1] \quad (25)$$

For given η , the smallest b value for which this can happen is $b = 3 - 2\eta$, corresponding to $P_1 = \eta/3$. This b -value itself reaches its minimum, $b = 1$, for $\eta = 1$. At this minimum, $P_1' = A_1 = A_3 = 0$ and $A_2 = 3P_1 = 1$. The case $A_3 = 0$ seems very difficult to reconcile with existing data. We therefore expect that gluon exchange processes must make an important contribution to high energy hadronic collisions (gluon exchange is compatible with any value of the A_i 's).

5. Ξ , Ω HYPERONS AND ANTIBARYONS FROM PROTON FRAGMENTATION

The considerations of section 4 concentrated on the total number of fragmentation hyperons, i.e. mainly the $S = -1$ hyperons. We now discuss more specifically the Ξ and Ω hyperons, as well as the fragmentation antibaryons in the sense defined in section 3 (see also Fig. 2).

The Ξ and Ω being quite heavy, it is reasonable to assume that, in most cases where such particles are observed, they are directly produced particles and not decay products of heavier resonances. From the fragmentation cases listed in Table I, Ξ 's can be produced in all cases except 2 and 3, and Ω 's in case 7. With the probabilities in Table I and with eqs. (6), an easy calculation gives the following average numbers for the fragmentation of one proton (normal counting is used, every Ξ or Ω counting for 1) :

$$\langle n_{f\Xi^0} \rangle = \xi^2 b_0, \quad b_0 = 2 P_1' + \frac{3}{2} (1 - \xi) A_1 \eta^3 \quad (26)$$

$$\langle n_{f\Xi^-} \rangle = \xi^2 b_-, \quad b_- = P_1' + \frac{3}{2} (1 - \xi) A_1 \eta^3 \quad (27)$$

$$\langle n_{f\Omega} \rangle = \xi^3 b_\Omega, \quad b_\Omega = A_1 \eta^3 \quad (28)$$

The terms in P_1' refer to Ξ 's containing one incident line, those in $A_1 \eta^3$ to hyperons without incident line. As to the number of fragmentation antibaryons per incident proton (cases 4, 5, 6 of Table I), its average was already given in eq. (3).

On the basis of a recent experiment in the hyperon beam at the CERN SPS, Bourquin et al.¹⁰⁾ have published inclusive data on the small angle production of various hyperons and antibaryons by protons on a nuclear target, at incident momenta $p_{lab} = 200$ and 210 GeV/c ($s^{\frac{1}{2}} \approx 20$ GeV). The measurements are made around $x = 0.5$ ($x = 0.48$ for all particles measured, $0.35 \leq x \leq 0.66$ for some), at a production angle of 6 mrad for positive secondaries, and at an average angle of 2 mrad for negatives. The data are mostly given as particle

ratios at given x . Combined with data of earlier experiments, they also indicate the trend of the x -dependence of inclusive cross sections for $0.4 \lesssim x \lesssim 0.8$, which is found to be compatible with a parametrization

$$E \frac{d^3\sigma}{dp^3} \propto (1-x)^n \quad (29)$$

For meson production the data are compatible with the ISR data of Singh et al.⁴⁾. They show no significant variation with $s^{\frac{1}{2}}$ from 20 GeV to 45 GeV.

While the data of Ref. 10 are insufficient for determining quantitatively the numbers (26) - (28) and the number $\langle n_{fa} \rangle$, they show interesting qualitative trends. The $S = -1$ and $S = -2$ hyperons show a leading particle effect, i.e. a relatively low power n in eq. (29), $n \approx 1.5$ for Σ^- and $n \approx 3$ for Ξ^- . It is of course natural to relate this to the fact that a $S = -1$ hyperon can contain two incident lines (A_2 term in expression (15) for b), whereas a Ξ contains at most one (P_1' terms in b_0, b_-). As expected, the antiproton x -dependence is very steep; the exponent is found to be $n \approx 7$. For the Ω^- , the measurements are limited to $x = 0.48$ and 0.55 , so that the x -dependence remains unknown.

Another interesting result of Ref. 10 is that the antibaryon to baryon ratio at $x = 0.48$ increases rapidly with strangeness, from $\bar{p}/p \approx 7.4 \times 10^{-4}$ to $\bar{\Xi}^-/\Xi^- \approx 2.8 \times 10^{-2}$, the $S = \pm 1$ cases being intermediate. Here again qualitative understanding is easy in terms of the number of incident lines which can be contained in the baryon (up to 3 for protons, up to 2 for Λ and Σ , up to 1 for Ξ).

If total numbers of baryons and antibaryons are known in the fragmentation regions, direct use can be made of our eqs. (2), (3), (15) and (26) - (28). For example, assuming

$$\langle n_{fp} \rangle / \langle n_{fa} \rangle \approx \langle n_{fp} \rangle / \langle n_{fb} \rangle \approx \frac{1}{2} \quad (30)$$

one finds

$$\langle n_{fp}^- \rangle / \langle n_{fp} \rangle \approx \langle n_{fa} \rangle / \langle n_{fb} \rangle = \eta' [A_2 + A_1 \eta'^2 (3 - \eta')] / [A_2 + A_3 + A_1 \eta'^3 + (3A_1 + A_2) \eta']$$

which for small η' reduces to

$$\langle n_{fp}^- \rangle / \langle n_{fp} \rangle \approx \eta' A_2 / (A_1 + A_2 + A_3) = \eta' A_2 \quad (31)$$

For the Ξ 's one has to leading order in ξ

$$\langle n_{f\Xi^-}^- \rangle / \langle n_{f\Xi^-} \rangle = \frac{3}{2} \langle n_{fa} \rangle / b_-$$

hence for small η'

$$\langle n_{f\Xi^-}^- \rangle / \langle n_{f\Xi^-} \rangle \approx \eta' A_2 / A_1 \quad (32)$$

Similarly, for the $S = \pm 1$ hyperons (sum of Λ 's and Σ 's), one has to leading order in ξ and η'

$$\langle n_{fh}^- \rangle / \langle n_{fh} \rangle \approx 3 \langle n_{fa} \rangle / b \approx 3 \eta' A_2 / (A_2 + 3A_1) \quad (33)$$

Estimates can be attempted for (31) and (32) on the basis of the data of Bourquin et al. ¹⁰⁾. The $\bar{p}/p$ ratio is 7.4×10^{-3} at $x = 0.48$, the distribution (29) falls as $(1-x)^7$ for $\bar{p}$ and is approximately flat for p . By integrating over $0 \leq x \leq 1$ one then obtains the estimate

$$\langle n_{fp}^- \rangle / \langle n_{fp} \rangle \approx \eta' A_2 \approx 0.9 \times 10^{-2} \quad (34)$$

For Ξ^-/Ξ , the ratio is 2.8×10^{-2} at $x = 0.48$, the x -distribution falls as $(1-x)^3$ for Ξ^- , and it is reasonable to assume the same $(1-x)^7$ for Ξ as for p . This gives

$$\langle n_{f\Xi^-}^- \rangle / \langle n_{f\Xi^-} \rangle \approx \eta' A_2 / A_1 \approx 0.19 \quad (35)$$

From the ratio of (34) and (35) we deduce $A_1 \approx 0.05$. To find A_2 and A_3 we use our estimate (20) for b . In the small η' limit it gives

$$A_2 \approx b - 3A_1 \approx 0.6, \quad A_3 = 1 - A_1 - A_2 \approx 0.35 \quad (36)$$

and we obtain an independent estimate of P_1

$$P_1 \approx A_1 + \frac{1}{3} A_2 \approx 0.25 \quad (37)$$

remarkably close to (21). From (34) and (36) now follows

$$\eta' = 1.5 \times 10^{-2}, \quad P_1' \approx 3.8 \times 10^{-3} \quad (38)$$

which justifies a posteriori the small η' approximation. Note that (33) now gives $\langle n_{fh}^- \rangle / \langle n_{fh} \rangle \approx 3.6 \times 10^{-2}$, indeed intermediate between (34) and (35).

We are led to values of P_1, P_1' and the A_i 's which are fully compatible with the limits obtained in section 4, eqs. (21) - (24). While the actual numbers are of course no more than estimates, this compatibility is noteworthy and tests the validity of our approach.

We conclude this section with a remark on Ω^- production. To leading order in ξ and neglecting η' we derive from (27) and (28)

$$\langle n_{f\Omega}^- \rangle / \langle n_{fE^-} \rangle \approx \frac{2}{3} \xi \approx 0.07 \quad (39)$$

At $x = 0.48$ the experimental Ω^-/E^- ratio is ~ 0.02 with a large error due mainly to the uncertainty of the Ω^- lifetime. We cannot explain the discrepancy with (39) by assuming a difference in x -dependence between Ω and E , since for small η' these particles originate mainly from the same mechanism ($A_1\eta^3$ term). Indeed, the P_1' term of eq. (27) for E^- production turns out to be some 6% only of the term in $A_1\eta^3$. Hence we expect the E 's and Ω 's to have essentially the same x -dependence.

6. A TWO-PARAMETER SCHEME

Our scheme depends so far on three parameters (P_1 , P_1' , P_2 , or equivalently A_1 , A_2 , η). We now show that a further simple probabilistic consideration leads to one more relation, restricting the number of independent parameters to two, with a very weak dependence on one of them. It is best expressed in terms of P_1 , P_1' , P_2 and P_3 .

The basic idea is inspired by quark recombination models, but is of more general nature in many respects. As explained in section 2, the outgoing hadrons are combinations of (quark and/or antiquark) lines, which are either incident lines (i.e. belonging to an incident proton), or non-incident lines. Considering the lines of the outgoing hadrons flying out in a given c.m. hemisphere, we assume this combination process to obey the following simple probabilistic rule.

Consider an incident line. We call Q the probability that it emerges combined with another incident line in the same c.m. hemisphere, and Q' the probability that it emerges combined with a non-incident line of the same c.m. hemisphere. Our new assumption VI is that these probabilities have unique values for the various possible combinations and simply multiply to give the probabilities for combinations with two lines, in equations :

$$P_1 = NQ', \quad P_1' = \frac{1}{2} N(N-1)Q'^2, \quad P_2 = 2NQQ', \quad P_3 = Q^2 \quad (40)$$

N is here the effective number of non-incident lines available for combination with a given incident line. Note that assumption VI is compatible with quark confinement as long as eq. (8) holds.

Eqs. (40) lead to

$$P_2^2 = 4 P_1^2 P_3 = 4 P_1^2 (1 - P_1 - P_1' - P_2) \quad (41)$$

$$P_1' = c P_1^2 \quad (42)$$

$$c = \frac{1}{2} (N - 1)/N, \quad 0 \leq c \leq \frac{1}{2} \quad (43)$$

where the last inequalities result from $N \geq 1$, as required by (40). Eq. (41) can now be solved for P_2

$$P_2 = 2 P_1 [1 - P_1 + (1 - c) P_1^2]^{\frac{1}{2}} - 2 P_1^2 \quad (44)$$

and we are left with two free parameters only, P_1 and c . Fig. 5 shows the resulting dependence of b and the A_i 's on P_1 for the extreme cases $c = 0$, $c = 0.5$, and for $0 \leq P_1 \leq 0.5$ (the allowed interval of P_1 is given by $0 \leq P_1 + c P_1^2 \leq 1$). One notes the weak dependence on c , especially at low P_1 .

Our estimates of section 5 can now be compared with this more restrictive scheme. Eqs. (37), (38) give

$$P_1 = 0.25, \quad c = 6.1 \times 10^{-2}$$

With eq. (44) one then calculates

$$A_1 = 0.09, \quad A_2 = 0.49, \quad A_3 = 0.42, \quad b = 0.77 \quad (45)$$

These values correspond to

$$Q = 0.65, \quad Q' = 0.22, \quad N = 1.14 \quad (46)$$

The results (45) must be compared with our estimates of section 5 :

$$A_1 = 0.05, \quad A_2 = 0.6, \quad A_3 = 0.35, \quad b = 0.75 \quad (47)$$

In view of the uncertainties affecting the estimates of section 5, especially A_1 , which was obtained by dividing the small and poorly known numbers (34) and (35), we conclude that (45) and (47) compare quite well, thereby giving support to our probabilistic scheme including assumption VI. It is clear, however, that more complete data are needed to perform quantitative tests of our approach.

7. CONCLUDING REMARKS

We first briefly summarize the contents of this paper. The outgoing hadrons of a pp collision contain valence quarks of two types, those which entered the collision as valence quarks of the incident protons, and those which did not. At high energy and low p_T , assuming that the three valence quarks of either incident proton emerge in hadrons flying off in the centre-of-mass hemisphere which contained that proton, we have called A_3 the probability that all three emerge in one and the same outgoing nucleon (or nucleon resonance), A_1 the probability that they emerge in three distinct hadrons (or hadron resonances), and A_2 the probability that two of them emerge in one baryon (or baryon resonance) and the third in another hadron (or hadron resonance). Using simple probabilistic arguments, and using data on meson, hyperon and antibaryon production, we have estimated the following values for the probabilities A_i , see eqs. (45) and (47) :

$$A_1 = 0.05 - 0.1, \quad A_2 = 0.6 - 0.5, \quad A_3 = 0.35 - 0.4 \quad (48)$$

where the first values belong together, as do the second ones. We have found in addition that when an outgoing hadron contains one and only one incident valence quark, it is most of the time a meson or meson resonance, and only in 1-2 % of cases a baryon or baryon resonance.

We end with a few remarks. Our study concerns the bulk of the in-elastic collisions, which are non-diffractive and of low- p_T type. The latter restriction is of no importance for the probabilities we have considered, simply because high- p_T collisions are very rare and, whatever their characteristics, their inclusion would not change the above estimates appreciably. We discuss briefly the question of inclusion or exclusion of diffractive collisions.

Our estimates use data on certain mean multiplicities of outgoing hadrons in the fragmentation region of an incident proton. Each of these is obtained as the appropriate inclusive cross section divided by the appropriate total cross section. Both cross sections are usually taken to be for all in-elastic processes including diffraction dissociation, but could also be taken excluding diffraction dissociation. Since the outgoing hadrons of interest are

those in the fragmentation regions, the inclusive to total cross section ratios will not be very different whether we include or exclude collisions where the proton fragments by diffraction dissociation, and we expect our estimates to be about the same in both cases. Differences would undoubtedly occur at a higher level of precision, but then one might trespass the limits of validity of our assumptions. On the other hand, it is clear that collisions where the proton does not fragment should not be included.

We saw that the probability A_2 is quite considerable, and that it mostly corresponds, in the c.m. hemisphere considered, to one outgoing baryon B containing two incident valence quarks and one outgoing meson M containing the third incident valence quark (B and M could also be resonances). An obvious question concerns the flavour correlation between the third valence quark of B and the valence antiquark of M . They could have all their flavour quantum numbers opposite, as would be the case if they originate by single pair creation in the gluon field, but they could also have weaker or vanishing flavour correlation. This could be studied experimentally by taking for B the baryon (or clearly separated baryon resonance) of largest x , and similarly for M the meson (or clearly separated meson resonance) of largest x , choosing such configurations where B and M are likely not to form a resonance together (thereby reducing the probability to be in the A_3 case).

The extension of our approach to meson-nucleon and antinucleon-nucleon collisions is straightforward. It should be noted, however, that exchange processes involving the incident valence partons will be more abundant in all cases where the incident s-channel is not exotic, since a valence antiquark of one incident hadron can then annihilate with a valence quark of the other. Such effects appear to remain important up to quite high energies ^{*)}. Except for these $q\bar{q}$ annihilation processes, the fragmentation of antinucleons should be very similar to the one of protons, and the fragmentation of mesons should be dominantly of the two following types :

*) Private communication from B. Buschbeck, Vienna.

- i) the incident valence quark and antiquark emerge in the same outgoing meson or meson resonance;
- ii) they emerge in two distinct mesons or meson resonances.

Extending to mesons the considerations of section 6, and assuming Q/Q' to have the same value (~ 3) there found for protons, we predict that type i) processes should be ~ 3 times more frequent than type ii).

As a last remark we note that we have not discussed the x -distributions of the outgoing hadrons, which is the main subject of study of the quark recombination and fragmentation models which recently received a great deal of attention. The applications of such models are usually restricted to inclusive x -distributions and their relation to incident quarks. We have addressed the problem of the simultaneous fate of all incident valence quarks and have proposed an answer to the most immediate question, which is to estimate probabilities for the various ways in which they re-emerge from the collision as valence quarks of outgoing hadrons. A next step might be to combine the two approaches by considering also the momentum distributions and correlations of all fragmentation products of the incident hadrons.

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Table I. Baryons and antibaryons from proton fragmentation

Case	n_b	n_i	Probability	n_{fb}	n_{fa}
1.		1	$3 A_1 n^2 n'$		
2.	1	2	$A_2 n$	1	0
3.		3	A_3		
4.	2	1, 1	$3 A_1 n n'^2$	2	1
5.		2, 1	$A_2 n'$		
6.	3	1, 1, 1	$A_1 n'^3$	3	2
7.	0	0	$A_1 n^3$	1	0

FIGURE CAPTIONS

Figure 1 Examples of diagrams for proton fragmentation with emission of a baryon B when the valence quarks of the proton all emerge in mesons M .

Figure 2 Example of a diagram for proton fragmentation with emission of two baryons B , an antibaryon $\bar{B}$ and a meson M .

Figure 3 Allowed domain of b as function of P_1, P_1' . The domain is bounded by the surfaces OAB and $AA''B''B$ for $b_{\max}$, and $OA''B''$ for $b_{\min}$. The straight lines OL, LL'' and OL'' show how these surfaces are constructed. The curve $A''B''$ has the equations

$$P_1 + P_1' = 1, \quad b = 3(P_1^3 - 2P_1 + 2)$$

Its minimum is in the point N with $P_1 = (2/3)^{1/2} \approx 0.82$. The table gives the co-ordinates of various points of the figure.

Figure 4 The shape of the function $b_{\min}/3P_1$ as given by the first eq. (18).

Figure 5 The P_1 -dependence of A_1, A_2, A_3 and b resulting from eqs.(41 - 44) with eqs. (6) and (15), for the two extreme values of the parameter c .

FIG.1

FIG.2

FIG.3

FIG.4

FIG.5