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ASSOCIATED PRODUCTION OF HEAVY FLAVOUR STATES
IN pp AND $p\bar{p}$ INTERACTIONS : SOME QCD ESTIMATES

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A B S T R A C T

A perturbative, non-scaling, QCD approach is used to calculate production cross-sections for 'naked' heavy flavour states in pp and $p\bar{p}$ collisions. The basic subprocesses considered, using lowest order perturbation theory, are : $q\bar{q} \rightarrow c\bar{c}$ and $g\bar{g} \rightarrow c\bar{c}$ (flavour creation), $q\bar{c} \rightarrow q\bar{c}$ and $g\bar{c} \rightarrow g\bar{c}$ (flavour excitation). The dependence of the calculation on various assumptions and parameters is examined. Numerical results are given over large ranges of the centre-of-mass energy and the heavy quark mass. Contrary to intuitive expectation, the flavour excitation processes may be an important, possibly the dominant, contribution. For the case of charm, the present experimental situation is reviewed, and it is found that our results are compatible with most of the data (mainly upper limits) but are rather difficult to reconcile with the 100-500 μb cross-sections inferred by two bubble chamber groups in the CERN-SPS beam dump experiments.

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1. - INTRODUCTION

Quantum chromodynamics (QCD) ¹⁾, the $SU(3)$ colour gauge field theory of quarks and gluons, possesses many attractive qualitative features. It is renormalizable, asymptotically free and provides a dynamical reason for the existence of colour degrees of freedom. In particular, the asymptotic freedom of the theory, in which the effective coupling constant becomes small at large (strictly, Euclidean) momenta, raises the hope that the surprisingly successful parton model description of processes involving hadrons and large momenta may be reproducible within a realistic and consistent quantum field theory of the strong interactions.

However, the full content of QCD is still very far from being understood. This is particularly true of the large distance behaviour of the theory and its non-perturbative aspects : notably the question of quark confinement. Moreover, even for reactions which supposedly probe the short-distance nature of the strong interaction, rigorous quantitative predictions of QCD are rather restricted.

For the latter, QCD has direct predictions only for the $e^+e^- \rightarrow$ hadrons total cross-section ^{2),3)}, and for the nature and magnitude of the breaking of Bjorken scaling in deep inelastic lepton-hadron scattering ²⁾. These predictions are in good agreement with present data.

The application of QCD to other parton-model processes, such as the hadronic production of large-mass lepton pairs (the question of the validity of the Drell-Yan mechanism ⁴⁾, for example), currently requires further assumptions. An attractive and popular ansatz ⁵⁾ is to retain the usual parton model formulae, with the following modifications.

- i) Introduce scale breaking in Q^2 , of the form predicted for deep-inelastic lepton-hadron scattering, into the structure functions and, possibly, also into the fragmentation functions. Q is now a typical large momentum of the process considered.
- ii) Where appropriate, introduce processes involving initial and/or final state gluons, as well as just quarks and antiquarks.
- iii) Use the QCD running coupling constant [see, for example, Ref. 6]

$$\alpha(Q^2) = \frac{12 \pi}{25 \log Q^2/\Lambda^2} \quad (1.1)$$

when calculating subprocess diagrams involving couplings between the elementary fields. Equation (1.1) is the asymptotic form appropriate to three colours and four flavours. The parameter Λ can be determined in principle from the scale-breaking seen in deep inelastic scattering. Present analyses ⁶⁾⁻¹¹⁾ suggest a value of the order of 500 MeV. The relative smallness of the effective coupling for moderate values of Q^2 encourages the use of lowest order perturbation theory for hadronic hard subprocesses.

Important support for this ansatz came from calculations ^{12),13)} of the first-order QCD corrections to the subprocess of the Drell-Yan mechanism. To leading logarithm approximation, these just give the required Q^2 dependence to the parton structure functions. A similar result was found for a quark-quark scattering contribution to inclusive large p_T hadron production in hadronic interactions ¹⁴⁾. In this case, a Q^2 dependence of the fragmentation function also develops. Subsequently, these results have been considerably extended and generalized ¹⁵⁾.

Thus, a wider range of processes can now be calculated in QCD using a reasonably well-defined and well-motivated procedure which, although it may not yet be fully established rigorously, possesses at least the elements of theoretical respectability. It is clearly important to pursue the phenomenological implications of this result. This paper will be concerned with its application to a particular class of hadronic reactions, which we come to below.

In purely hadronic processes, there are two main classes of reactions which one may hope to describe successfully by this means. The first is the inclusive production of hadrons at large transverse momentum. Hard scatterings, involving quarks and gluons, as the underlying mechanism have been receiving a good deal of attention within this framework, with some encouraging results ¹⁶⁾⁻¹⁸⁾. One of the lessons to be drawn from the phenomenology is that the hadronic gluon distribution can give rise directly to important effects.

The second class of reactions is the production of high mass hadronic states. This is the field of interest of the current work.

In this paper we consider the associated production of heavy flavour states in pp and $\bar{p}p$ interactions following the ansatz outlined above. The specific subprocesses we calculate are the two "flavour creation" mechanisms (Fig. 1) :

$$q \bar{q} \longrightarrow c \bar{c} , \quad g g \longrightarrow c \bar{c} \quad (1.2)$$

and the two "flavour excitation" mechanisms (Fig. 2) :

$$q c \longrightarrow q c , \quad g c \longrightarrow g c \quad (1.3)$$

In the latter, a heavy quark (c) is knocked out of the sea of one of the incoming hadrons by a hard collision with a quark (q) or a gluon (g) from the other. Intuitively, it might seem that this second process will be negligible compared with the first. As we shall see, this is not necessarily so.

One of our basic assumptions is that a pair of heavy quarks thus produced will usually evolve into a pair of "naked flavour" states if $m_{c\bar{c}}$ is above the appropriate threshold, and will only seldom give a bound $c\bar{c}$ "hidden flavour" system (J/ψ or T , for example) or annihilate back into ordinary hadrons. Thus we assume

$$\sigma(pp, \bar{p}p \rightarrow \text{charmed particles} + X) \simeq \sigma(pp, \bar{p}p \rightarrow c\bar{c} + X, m_{c\bar{c}} > 2m_D) \quad (1.4)$$

in the case of charm, for example. For the flavour excitation processes of (1.3), we may consider that the threshold condition is automatically satisfied.

The calculation of the hadronic production of charmed particles is obviously the main motivation of this work, and in what follows we indiscriminately use charm terminology for heavy flavours in general. An example is the use of c to denote a (general) heavy quark. However, we shall also give predictions for heavier flavours, in particular for the flavoured states associated with the T .

The experimental situation regarding charm production is broadly the following. Direct observation of charmed particles has been reported in e^+e^- annihilation and photoproduction and they are also thought to be responsible, via their semileptonic decay modes, for a large proportion of the dilepton events seen in neutrino-hadron scattering¹⁹⁾. Dimuon events in deep-inelastic muon-hadron scattering²⁰⁾ may also be due to the semileptonic decays of charmed particles.

However, there is as yet very little experimental evidence of charm production in purely hadronic interactions. A selection²¹⁾⁻²⁷⁾ of experimental upper limits on the cross-sections, at FNAL and CERN-ISR energies, is shown in Table I. A single recent positive result from the ISR, inferred from muon-electron coincidences, is also shown²⁸⁾. These values should be taken only as a guide because of the various different assumptions on branching ratios, extrapolations in x_F , etc. used by the different groups. We refer the interested reader to the original papers²¹⁾⁻²⁸⁾ for further details. We just note here that the most stringent restriction at FNAL energies, $\sigma < 1.5 \mu\text{b}$ at 90% C.L. for pN collisions at $\sqrt{s} \simeq 24 \text{ GeV}$, is from an emulsion track search experiment²²⁾ which was sensitive to charmed particle lifetimes ~ 0 ($10^{-14} - 10^{-12}$ secs). This bound can be substantially eased if charmed particles decay faster or slower than expected.

Mention must also be made of the recent results from the CERN-SPS 400 GeV/c proton beam dump. Three experiments³⁰⁾⁻³²⁾ have reported an anomalously large neutrino production rate. If the excess arises from weak semileptonic decays of charmed particles, the two bubble chamber groups^{30),32)} estimate that the total charm production cross-section at this energy lies in the range 100-500 μb . The counter experiment³¹⁾, on the other hand, which finds a significantly smaller flux of extra neutrinos, is consistent with a value $\sim 30 \mu\text{b}$ for charm. There are large uncertainties in the extrapolation to the charm production cross-section, but these data are only compatible with the emulsion experiment if charmed particles have unexpected lifetimes or if there is some additional new source of prompt neutrinos. Moreover, the charm cross-sections inferred by the bubble chamber groups^{30),32)} are also inconsistent with other upper limits in Table I.

Finally, we remark that an analysis³³⁾ of cosmic ray data on associated production of long-lived states has suggested a charm production cross-section at the 10-100 μb level for incident cosmic ray energies of 10-20 TeV. We shall return to these results in the summary (Section 4).

On the theoretical side, various approaches ranging from conventional hadronic phenomenology to parton model mechanisms [Refs. 34)-37), for example] have yielded equally widely-varying predictions - from tens of nanobarns to several hundred microbarns. In particular, there have been a number of calculations in the framework of (1.2) : these have considered either $q\bar{q} \rightarrow c\bar{c}$ (38)-40) or $gg \rightarrow c\bar{c}$ (41) or both (42)-45) [generally using scaling parton structure functions, although the authors of Ref. 43) tested the effect of the introduction of an empirical scaling violation into their quark and gluon distributions]. We shall indicate comparisons with our results, for the flavour creation processes, below.

The rest of this paper consists as follows : in the next Section we describe the details of the calculation, discussing our choice of (non-scaling) structure functions and deriving the cross-section formulae, Section 3 gives the numerical results, and various comments and conclusions are presented in the final Section.

The main quantitative results of this paper are contained in Figs. 4, 5, 8 and 9, which should be taken together with the discussion of Section 4.

2. - DETAILS OF THE CALCULATION

As described in the Introduction, we follow the structure of the parton model in our calculations.

The total cross-section for associated production of heavy flavour states $c, \bar{c}$ is assumed to be equal to that for producing a pair of heavy quarks and to be given by an integral of the product of the sub-process cross-section with the parton probability distributions in the colliding hadrons :

$$\sigma_{c\bar{c}}(s) = \int_{\substack{x_1, x_2 \text{ such that} \\ \hat{s} > \text{threshold} \\ \hat{s} < s}} dx_1 dx_2 \frac{F_2^{1/A}(x_1, Q^2) F_2^{2/B}(x_2, Q^2)}{x_1 x_2} \hat{\sigma}(\hat{s}) \quad (2.1)$$

This expression is to be summed over the distinct $1+2 \rightarrow 3+4$ subprocesses considered. The functions $F_2^{1/A}$, $F_2^{2/B}$ are the familiar parton fractional momentum distributions, or structure functions, of the initial state hadrons A and B. As we shall see, the relation between the subprocess energy, $\hat{s}$, and (x_1, x_2, s) is a little ambiguous for the flavour excitation processes of (1.3); however, for flavour creation, (1.2), we have $\hat{s} = x_1 x_2 s$ and we set the threshold in $\hat{s}$ for these processes to correspond to the mass of the lowest-lying flavoured state, i.e., $\hat{s} > 4m_D^2$.

Equation (2.1) may be cast in the form

$$\sigma_{c\bar{c}}(s) = \int_{\text{threshold}}^s d\hat{s} \int dX_L \frac{F_2^{1/A}(x_1, Q^2) F_2^{2/B}(x_2, Q^2)}{x_1 x_2 (x_1 + x_2) s} \hat{\sigma}_{1+2 \rightarrow 3+4}(\hat{s}), \quad X_L = X_1 - X_2 \quad (2.2)$$

The differential cross-section $d\sigma_{c\bar{c}} / d\hat{s} dx_L$ is the integrand of Eq. (2.2), summed over the contributing subprocesses. Note that $\hat{s}$, x_L only refer to the pair of heavy quarks produced for the case of the flavour creation processes.

We now describe our choice of structure functions. On the basis of the QCD ansatz outlined in the previous section, we require the quark and gluon distributions to exhibit similar deviations from exact scaling to those expected in deep inelastic scattering. Buras and Gaemers (BG)⁸⁾ have given simple, analytic, Q^2 dependent expressions for the valence up-quark, valence down-quark, uncharmed sea, charmed sea, and gluon distributions in the nucleon. These expressions were obtained from considerations of the QCD predictions for the leading Q^2 behaviour of the moments of the structure functions neglecting mass effects. BG expect that over a wide range in Q^2 the Q^2 evolution of their distributions - for $x \leq 0.8$ in the case of the valence distributions and for $x \leq 0.3$ otherwise - is a good representation of the exact inversion of these predictions.

BG use parton distributions of the form

$$A(\bar{s}) x^{a_1(\bar{s})} (1-x)^{a_2(\bar{s})} \quad (2.3)$$

where

$$\bar{s} = \log \left[\frac{\log Q^2 / \Lambda^2}{\log Q_0^2 / \Lambda^2} \right] \quad (2.4)$$

Here Q_0^2 is the value of Q^2 at which the parton distributions are initially prescribed, and Λ is the same parameter as in the expression for $\alpha(Q^2)$ of Eq. (1.1). Data on electron and muon deep inelastic scattering up to $Q^2 \simeq 20 \text{ GeV}^2$ were successfully fitted ⁸⁾ using $\Lambda = 300 \text{ MeV}$ and the following initial proton parton momentum distributions at $Q_0^2 = 1.8 \text{ GeV}^2$:

$$\begin{aligned}
 x u_{val}(x) &= 5.5 x^{0.7} (1-x)^{2.6} - 3.1 x^{0.85} (1-x)^{3.35} \\
 x d_{val}(x) &= 3.1 x^{0.85} (1-x)^{3.35} \\
 x u_{sea}(x) &= x d_{sea}(x) = x s(x) = x \bar{u}(x) = x \bar{d}(x) \\
 &= x \bar{s}(x) = 0.018 (n_s + 1) (1-x)^{n_s} \\
 x c(x) &= x \bar{c}(x) = 0 \\
 x g(x) &= 0.40 (n_g + 1) (1-x)^{n_g} \quad \begin{matrix} n_s = 10 \\ n_g = 10 \end{matrix} \quad (2.5)
 \end{aligned}$$

We note the following with respect to the BG structure functions. For our purposes it will be necessary to know the sea and gluon distributions at rather larger x than is important for deep inelastic scattering. This may bring us outside the region of validity of the BG formulae. Therefore, with the idea of testing the large x region, we have compared data ⁴⁶⁾ on very large mass lepton pair production in 400 GeV/c pN collisions with the Drell-Yan formula ⁴⁾, using the initial distributions of (2.5) evolving according to the BG prescription ⁸⁾. We put Q^2 in (2.4) equal to the invariant mass-squared of the lepton pair, in line with our general approach and as indicated by the specific analyses of Refs. 12) and 13). The prediction falls well below the data, being up to three orders of magnitude too small at the highest measured masses (Fig. 3, solid curve). It may be argued that other processes are important at large lepton pair mass in pN interactions ⁴⁷⁾ (in which case heavy flavour production may be similarly enhanced ³⁸⁾⁻⁴⁰⁾). However, by modifying the initial gluon and sea distributions, it is possible to come closer to the data. Figure 3 shows the effect of different choices of the parameters n_s and n_g in (2.5), which modify the shape of the initial sea and glue without altering their respective momentum fractions. The choice $n_s = 5, n_g = 3$ (dash-dot curve) gives an acceptable fit to these data given the quoted systematic uncertainties ⁴⁶⁾ which are additional to the statistical errors indicated in the Figure. However, this choice may give a poorer description of deep inelastic scattering at small x , and, indeed,

it may not be possible to fit simultaneously both deep inelastic and large mass lepton pairs (via Drell-Yan) in a QCD non-scaling framework without using rather less simple parametrizations of the sea. Similar results have been obtained by other authors ^{48),49)}.

We would also stress that the BG results for the charmed sea, which they take to be generated from zero at $Q^2 = Q_0^2$, are rather speculative. Besides some arbitrariness in the initial condition, the subsequent QCD based evolution of the distribution neglects effects due to the mass of the charmed quark which, although non-leading in Q^2 , may be sufficiently important to continue to suppress its contribution for a significant range of Q^2 . At present, the main experimental evidence for a charm component of the sea is the observation of dimuon production in deep inelastic muon scattering ²⁰⁾. As a counter to our cautionary remarks, it is interesting to note a claim by Roy ⁵⁰⁾ that calculations using the BG charmed sea are compatible with characteristics of the presently rather limited dimuon data. This is encouraging since our calculation of flavour excitation is obviously strongly dependent on the heavy quark structure function.

In the numerical calculations of Section 3, we shall use the BG parton distributions based on (2.5), but we shall test the sensitivity of the predictions to the following changes :

- 1) $n_s = 5$ and $n_g = 3$, instead of $n_s = n_g = 10$, in (2.5) ;
- 2) $\Lambda = 600$ MeV, instead of $\Lambda = 300$ MeV, in (1.1) and (2.4) ;
- 3) variation of the value of Q^2 at which the heavy quark distribution starts to be generated.

We turn now to the cross-sections $\hat{\sigma}(\hat{s})$ of the various subprocesses. In the spirit of the QCD parton model ansatz, since the effective coupling (1.1) is comparatively small, we calculate these processes using lowest order QCD perturbation theory ⁵¹⁾ (Figs. 1 and 2).

A question which arises is what should be chosen for Q^2 - the large momentum-squared of the subprocesses which appears in the running coupling constant and the scale-breaking of the structure functions. For the $q\bar{q} \rightarrow c\bar{c}$ mechanism of Fig. 1a, and the $qc \rightarrow qc$ mechanism of Fig. 2a, there is a single obvious candidate in each case : $Q^2 = \hat{s}$, $-\hat{t}$, respectively. (Here and below we use "hats" to denote variables associated with the subprocesses, although these may be suppressed when there is no ambiguity.)

However, in the case of the gluon-initiated mechanisms of Figs. 1b and 2b, there are several possibilities. If one chooses a different Q^2 for each of the lowest order diagrams, corresponding to the momentum of the internal propagator, then each diagram has a different effective coupling constant and exact gauge invariance of the sum is lost. This may not have any bad consequences for our numerical results, but it complicates our calculations and in the following we shall only consider different choices for the common Q^2 of any particular subprocess. We also note that in the $\hat{t}$ and $\hat{u}$ channel diagrams for $gg \rightarrow c\bar{c}$ (Fig. 1b) the momentum-squared of the exchanged heavy quark is kinematically allowed to approach zero as $\hat{s}$ increases. Accordingly, we have assumed, with little theoretical justification, that the relevant quantity is the displacement off-shell of the internal lines ($m_c^2 - \hat{t}$ and $m_c^2 - \hat{u}$ in the above example), and we will be guided by this in our choices of Q^2 .

The effective mass of the heavy quark, m_c , is also uncertain. This enters our calculations of the $\hat{\sigma}(\hat{s})$. For simplicity, and to give a reasonable threshold behaviour of $d\sigma_{c\bar{c}} / d\hat{s}$ for the flavour creation mechanisms, we have taken $m_c = m_D$, the mass of the lowest-lying flavoured state.

We now derive the $\hat{\sigma}(\hat{s})$ of the flavour creation subprocesses.

$q\bar{q} \rightarrow c\bar{c}$ (Fig. 1a)

The matrix element is given by

$$\frac{i g^2}{4 \hat{s}} (\gamma^\mu)_{\beta\alpha} (\gamma_\mu)_{\gamma\delta} (\lambda^e)_{ba} (\lambda^e)_{cd} \quad (2.6)$$

where the latin (greek) subscripts denote colour (spin) indices assigned as in Fig. 1a. The λ 's are the well-known Gell-Mann SU(3) matrices. Squaring the matrix element and summing (averaging) over final (initial) colours and spins gives

$$\sum |M|^2 = \frac{64 \pi^2 \alpha^2(Q^2)}{9} \frac{(M^2 - t)^2 + (M^2 - u)^2 + 2 M^2 s}{s^2} \quad (2.7)$$

for the subprocess, where we have neglected all masses except $M = m_c (=m_D)$. We will choose $Q^2 = \hat{s}$ for this mechanism. The cross-section $\hat{\sigma}(\hat{s})$ is obtained from (2.7) by integrating over $\hat{t}$. Explicitly,

$$\hat{\sigma} = \frac{1}{16\pi s^2} \int_{M^2 - \frac{1}{2}s(1-\sqrt{1-4M^2/s})}^{M^2 - \frac{1}{2}s(1+\sqrt{1-4M^2/s})} dt \sum |m|^2 \quad (2.8)$$

This gives

$$\hat{\sigma}_{q\bar{q} \rightarrow c\bar{c}}(\hat{s}) = \frac{8\pi\alpha^2(Q^2)}{27\hat{s}^2} (\hat{s} + 2M^2) \sqrt{1 - \frac{4M^2}{\hat{s}}} \quad , \quad Q^2 = \hat{s} \quad (2.9)$$

in agreement with Refs. 52), 42), 43) and 45) [but not, apparently, with Ref. 44].

$gg \rightarrow c\bar{c}$ (Fig. 1b)

The matrix element is given by

$$\begin{aligned} & \frac{ig^2}{4} \left\{ \frac{1}{s} [\lambda^a, \lambda^b]_{cd} \left(g^{\alpha\beta} (\not{p}_2 - \not{p}_1) + 2\gamma_{\not{p}_1}^{\alpha\beta} - 2\gamma_{\not{p}_2}^{\beta\alpha} \right)_{\gamma\delta} \right. \\ & \quad + \frac{1}{M^2 - \hat{t}} (\lambda^a \lambda^b)_{cd} \left(\gamma^{\alpha} (\not{p}_3 - \not{p}_1 + M) \gamma^{\beta} \right)_{\gamma\delta} \\ & \quad \left. + \frac{1}{M^2 - \hat{u}} (\lambda^b \lambda^a)_{cd} \left(\gamma^{\beta} (\not{p}_3 - \not{p}_2 + M) \gamma^{\alpha} \right)_{\gamma\delta} \right\} \quad (2.10) \end{aligned}$$

where the momenta and the colour and spin indices are as indicated in Fig. 1b. Here and elsewhere $\hat{t} = (p_1 - p_3)^2$, $\hat{u} = (p_1 - p_4)^2$. In (2.10) we have omitted the term

$$\frac{ig^2}{4s} [\lambda^a, \lambda^b]_{cd} \left(\gamma_{\not{p}_2}^{\alpha\beta} - \gamma_{\not{p}_1}^{\beta\alpha} \right)_{\gamma\delta} \quad (2.11)$$

which vanishes when contracted with two physical gluon polarisation vectors. Without this term, the expression (2.10) itself then vanishes identically when contracted with physical on-shell spinors and either $p_{1\alpha}$ or $p_{2\beta}$. This then allows us to replace the sum over the two transverse physical gluon polarisations in the square of the matrix element by $(-g_{\mu\nu})$ just as in QED. This trick is also used by the authors of Ref. 45).

We eventually obtain for the squared amplitude, summed (averaged) over final (initial) colours and spins,

$$\sum |\mathcal{M}|^2 = \pi^2 \alpha^2(Q^2) \left[\frac{12}{s^2} (M^2-t)(M^2-u) + \frac{8}{3} \frac{(M^2-t)(M^2-u) - 2M^2(M^2+t)}{(M^2-t)^2} \right. \\ \left. + \frac{8}{3} \frac{(M^2-t)(M^2-u) - 2M^2(M^2+u)}{(M^2-u)^2} - \frac{2M^2(s-4M^2)}{3(M^2-t)(M^2-u)} \right. \\ \left. - 6 \frac{(M^2-t)(M^2-u) + M^2(u-t)}{s(M^2-t)} - 6 \frac{(M^2-t)(M^2-u) + M^2(t-u)}{s(M^2-u)} \right] \quad (2.12)$$

Equation (2.12) explicitly shows the six terms corresponding to the three diagrams of Fig. 1b and their interference. Some small simplifications are possible. To obtain $\hat{\sigma}(\hat{s})$, Eq. (2.12) will be integrated over $\hat{t}$ as in Eq. (2.8). Following our earlier discussion, we choose $Q^2 = \frac{1}{2}[(M^2-\hat{t}) + (M^2-\hat{u})] = \frac{1}{2}\hat{s}$, as the $\hat{t}$ and $\hat{u}$ channel exchange graphs may be expected to dominate this subprocess. However, we will also use $Q^2 = \hat{s}$ to check our sensitivity to this choice. Both forms have the merit that they are independent of $\hat{t}$ (at fixed $\hat{s}$) and thus, as in the case of $q\bar{q} \rightarrow c\bar{c}$, the $\hat{t}$ integration of the matrix element will not involve either $\alpha(Q^2)$ or the non-scaling structure functions, and can be performed explicitly. The result is

$$\hat{\sigma}_{g\bar{g} \rightarrow c\bar{c}}(\hat{s}) = \frac{\pi \alpha^2(Q^2)}{3 \hat{s}} \left[-\left(7 + \frac{31M^2}{\hat{s}}\right) \frac{\chi}{4} + \left(1 + \frac{4M^2}{\hat{s}} + \frac{M^4}{\hat{s}^2}\right) \log \frac{1+\chi}{1-\chi} \right] \\ \chi = \sqrt{1 - \frac{4M^2}{\hat{s}}} \quad , \quad Q^2 = \frac{1}{2} \hat{s} \quad (Q^2 = \hat{s}) \quad (2.13)$$

in agreement with Refs. 52), 42), 43) and 44) [but not, apparently, with Ref. 45)].

Turning now to the flavour excitation processes of Fig. 2, these amplitudes may either be calculated independently, or just obtained from Eqs. (2.7) and (2.12) by appropriate crossing, including any extra numerical factor arising from a difference in the averaging and summing over spins and colours. Explicitly :

qc → qc (Fig. 2a)

$$\sum |m|^2 = \frac{64 \pi^2 \alpha^2(Q^2)}{9} \frac{(M^2 - u)^2 + (s - M^2)^2 + 2 M^2 t}{t^2} \quad (2.14)$$

gc → gc (Fig. 2b)

$$\begin{aligned} \sum |m|^2 = \pi^2 \alpha^2(Q^2) & \left[\frac{32 (s - M^2)(M^2 - u)}{t^2} + \frac{64 (s - M^2)(M^2 - u) + 2 M^2 (s + M^2)}{9 (s - M^2)^2} \right. \\ & + \frac{64 (s - M^2)(M^2 - u) + 2 M^2 (M^2 + u)}{9 (M^2 - u)^2} + \frac{16 M^2 (4 M^2 - t)}{9 (s - M^2)(M^2 - u)} \\ & \left. + 16 \frac{(s - M^2)(M^2 - u) + M^2 (s - u)}{t (s - M^2)} - 16 \frac{(s - M^2)(M^2 - u) - M^2 (s - u)}{t (M^2 - u)} \right] \end{aligned} \quad (2.15)$$

For qc → qc, we do not make the obvious choice $Q^2 = -\hat{t}$ noted earlier. Instead, so that the $\hat{t}$ integration can be performed analytically, we take $Q^2 = -\frac{1}{2}[\hat{t}_{\min} + \hat{t}_{\max}]$. For gc → gc we choose $Q^2 = \frac{1}{2}[-\hat{t} + (M^2 - \hat{u})] = \frac{1}{2}(\hat{s} - M^2)$. In both cases we examine the effect on our results of taking $Q^2 = \hat{s}$.

The cross-sections of the subprocesses are formally given by

$$\hat{\sigma} = \frac{1}{16 \pi (s - M^2)^2} \int_{-\frac{s}{s}}^0 dt \sum |m|^2 \quad (2.16)$$

However, in both cases the resulting $\hat{\sigma}(\hat{s})$ diverge, because of poles at $\hat{t} = 0$ in the expressions (2.14) and (2.15) which correspond to soft qc and gc interactions in the graphs of Fig. 2. This is an artificial effect (because the charmed sea is supposedly negligible in soft processes, only starting to participate when $Q^2 > Q_0^2$) which arises because we have taken Q^2 independent of $\hat{t}$ in order to simplify the $\hat{t}$ integration. Accordingly, we reduce the upper limit of integration in Eq. (2.16) to $\hat{t} = -Q_0^2$ and, in the case of gc → gc only, require that also $\hat{u} - M^2 < -Q_0^2$ so that the latter is kept sufficiently far from its $\hat{u}$ channel pole.

Performing these truncated integrations, we finally obtain

$$\hat{\sigma}_{qc \rightarrow qc}(\hat{s}) = \frac{4\pi\alpha^2(Q^2)}{9(\hat{s}-M^2)^2} \left[\left(1 + \frac{2\hat{s}}{Q_o^2}\right) \left(\frac{\{\hat{s}-M^2\}^2}{\hat{s}} - Q_o^2\right) - 2\hat{s} \log \frac{(\hat{s}-M^2)^2}{Q_o^2 \hat{s}} \right]$$

$$Q^2 = -\frac{1}{2} [t_{min} + t_{max}] \quad (Q^2 = \hat{s})$$

$$= \frac{1}{2} \left(Q_o^2 + \frac{\{\hat{s}-M^2\}^2}{\hat{s}} \right) \quad (2.17)$$

and

$$\hat{\sigma}_{gc \rightarrow gc}(\hat{s}) = \frac{\pi\alpha^2(Q^2)}{(\hat{s}-M^2)^2} \left[\left(1 + \frac{4\{\hat{s}+M^2\}^2}{9\{\hat{s}-M^2\}}\right) (L - Q_o^2) + \frac{2(Q_o^4 - L^2)}{9(\hat{s}-M^2)} \right.$$

$$+ 2(\hat{s}+M^2) \log \frac{Q_o^2}{L} + \frac{4}{9} \frac{\hat{s}^2 - 6M^2\hat{s} + 6M^4}{\hat{s}-M^2} \log \frac{\hat{s}-M^2-Q_o^2}{\hat{s}-M^2-L}$$

$$\left. + 2(\hat{s}-M^2)^2 \left(\frac{1}{Q_o^2} - \frac{1}{L}\right) + \frac{16M^4}{9} \left(\frac{1}{\hat{s}-M^2-L} - \frac{1}{\hat{s}-M^2-Q_o^2}\right) \right]$$

$$L = \min\left(\hat{s}-M^2-Q_o^2, \frac{(\hat{s}-M^2)^2}{\hat{s}}\right), \quad Q^2 = \frac{1}{2}(\hat{s}-M^2) \quad (Q^2 = \hat{s}) \quad (2.18)$$

The cuts on the integration range also have the effect of raising the threshold from $\hat{s} > M^2$ to :

$$\hat{s} > M^2 + \frac{1}{2} Q_o^2 + (M^2 Q_o^2 + \frac{1}{4} Q_o^4)^{1/2} \quad \text{for all } qc \rightarrow qc, \text{ and}$$

$$\quad \text{for } gc \rightarrow gc \text{ if } Q_o^2 < \frac{1}{2} M^2.$$

$$\hat{s} > M^2 + 2Q_o^2 \quad \text{for } gc \rightarrow gc \text{ if } Q_o^2 > \frac{1}{2} M^2. \quad (2.19)$$

It is appropriate to discuss here the relation between $\hat{s}$ and x_1, x_2 in Eqs. (2.1) and (2.2). For the flavour creation processes in which the interacting constituents have zero or negligible masses we naturally use $\hat{s} = x_1 x_2 s$. However, the situation is not so clear for flavour excitation where one of the interacting constituents is massive. Also, one may suspect that a heavy quark from the sea of a nucleon will be far off mass-shell whereas our above calculations of the $\hat{\sigma}(\hat{s})$ have been made for on-shell partons. In spite of these points, however, we have decided to continue to calculate with $\hat{s} = x_1 x_2 s$, together with the on-shell $\hat{\sigma}(\hat{s})$, but we will partially test this assumption using another possibility for the $x_1 - x_2 - \hat{s}$ relation: $\hat{s} = m_c^2 + x_1 x_2 s$. Note that the former relation is the more conservative in that it sets higher thresholds for x_1, x_2 .

Finally, we note that, for flavour excitation, the upper limits of integration in Eqs. (2.1) and (2.2) probably should be modified to allow room for the heavy flavoured state left in the background soft hadronic interaction. We have been similarly careless of other such corrections, notably that due to baryon number conservation in pp collisions. These may give non-negligible effects at the lower values of $\sqrt{s}/m_c$ where the excitation curves are steep. The interested reader may wish to consider appropriate shifts or rescalings of the horizontal ($\sqrt{s}$ -) axes in Figs. 4 and 8 below.

3. - NUMERICAL RESULTS

In the following, the standard choice of parameters and assumptions, described in the previous section, will be used unless stated otherwise.

The predictions divide rather naturally into those for flavour creation (Fig. 1) and those for flavour excitation (Fig. 2). We consider the former first.

Figures 4 and 5 show the predicted "creation" total cross-sections for associated production of heavy flavoured states in pp and $\bar{p}p$ interactions, plotted as a function of the centre-of-mass energy (Fig. 4) and as a function of the heavy quark mass (Fig. 5). The two sets of curves in Fig. 4 correspond to charm (putting $m_c = m_D = 1.87$ GeV), and to the new

flavour (beauty ?) presumably associated with the T states, where our choice of the beautiful quark mass ($m_c = m_D = 5.4$ GeV) has been guided by the apparent absence of T states in the $\mu^+\mu^-$ spectrum above ~ 10.6 GeV⁵³⁾. In Table II we examine the sensitivity of the predictions to our particular inputs, at two values of the heavy quark mass and three different energies. We show results of changing one of (or one set of) the parameters, while holding the others fixed at their standard values, expressed as a ratio of the original prediction. Table II also compares results of an exact scaling model, obtained by putting $Q^2 = Q_0^2$ in (2.4) (i.e., the structure functions never evolve from their initial values at $Q^2 = Q_0^2 = 1.8$ GeV²) and using either $\alpha = \alpha(Q^2 = 1.8) \simeq 0.5$ or $\alpha = 0.25$.

The Feynman x distributions of charmed quark-antiquark pairs produced via the flavour creation mechanisms in pp collisions at three different energies are shown in Fig. 6. $\bar{x}_F$ is related to x_L by $x_F = (p_{||}^c + p_{||}^{\bar{c}})^{cm} / (p_{||}^c + p_{||}^{\bar{c}})_{max}^{cm} = x_L / (1 - \hat{s}/s)$. The corresponding curves for $\bar{p}p$ interactions have rather similar shapes. Similar trends are apparent when $m_c = 5.4$ GeV. If we take flatter initial sea and glue distributions by choosing $n_s = 5, n_g = 3$ then most of the distributions shown in Fig. 6 broaden but they still retain many of their qualitative features; in particular, the emergence with increasing energy of the peak at $x_F = 0$, dominated by the gluon process, still occurs although the magnitude of the effect is significantly reduced.

Figure 7 gives the differential charm "creation" cross-sections with respect to $\sqrt{\hat{s}}$, the invariant mass of the $c\bar{c}$ pair produced, in pp interactions at $\sqrt{s} = 19.4, 62.4$ and 800 GeV. As can be readily seen, the charmed quark-antiquark pairs are predominantly produced at low invariant masses not far from threshold. For the above energies, $\langle \sqrt{\hat{s}} \rangle = 4.4, 5.0$ and 7.4 GeV, respectively. For $\bar{p}p$ interactions, the curves of Fig. 7 tend to be somewhat broader, particularly at lower energies; the corresponding values of $\langle \sqrt{\hat{s}} \rangle$ are $4.8, 5.1$ and 7.4 GeV. Using the flatter $n_s = 5, n_g = 3$ sea and glue distributions gives (for pp) $\langle \sqrt{\hat{s}} \rangle = 4.5, 5.5$ and 8.4 GeV at the three energies, while with $m_c = 5.4$ GeV (and for pp , with $n_s = n_g = 10$) $\langle \sqrt{\hat{s}} \rangle = 11.3, 12.7$ and 18 GeV, respectively.

We turn now to the predictions for the flavour excitation mechanisms. These will be the same for pp and $\bar{p}p$.

Figures 8 and 9 show the predicted "excitation" total cross-sections for associated production of heavy flavoured states, plotted as a function of the centre-of-mass energy (Fig. 8) and as a function of the heavy quark mass (Fig. 9). For the case of a general heavy quark, we assumed Q_0^2 - the value of Q^2 at which the heavy quark sea starts to be generated from zero - to be proportional to m_c^2 , and hence we took

$$Q_0^2 = 1.8 \left(\frac{m_c}{1.87} \right)^2 \simeq 0.51 m_c^2 .$$

For calculational convenience, we assumed that the initial values of the other parton distributions at these new values of Q_0^2 were still given by Eq. (2.5), thus neglecting the scaling violation in these distributions in going between $Q^2 = 1.8 \text{ GeV}^2$ and $Q^2 = Q_0^2$ *). For quark masses heavier than charm, this means that these distributions were not sufficiently soft. The third set of entries in Table III below indicates that this may give some overestimate of the cross-sections at FNAL and ISR energies. Also, here and in the other calculations, we treat the general heavy quark just by varying m_c , i.e., we never assume other than four flavours for the formulae used.

The cross-sections for flavour excitation shown in Figs. 8 and 9 tend to be rather larger, typically by an order of magnitude, than the flavour creation cross-sections of Figs. 4 and 5. This perhaps surprising result is due partly to the fact that the charmed sea, although starting from zero at $Q^2 = Q_0^2$, is generated quite fast in QCD [see the numerical results of Ref. 8], if mass effects are not important. However, the main cause is that the subprocess cross-sections for flavour excitation are substantially larger than those for flavour creation and they have lower thresholds (in $\hat{s}$). This is illustrated in Fig. 10, where $(\hat{s}/\alpha^2(Q^2))\hat{\sigma}$ is plotted for the four subprocesses. [This dimensionless quantity depends just on $\hat{s}^{1/2}/m_c$, and, for the excitation processes, Q_0^2/m_c^2]. Figure 10 also shows the excitation subprocess cross-sections in the case $Q_0^2 = m_c^2$.

In Table III we examine the dependence of the predictions for flavour excitation on various parameters and assumptions. As in Table II, the values shown are the results of making the indicated change, while holding all the other quantities fixed at their standard values, expressed as a ratio of the original prediction. Table III also indicates the effect

*) A similar remark applies to the fourth set of entries in Table III.

of putting the heavy quark sea equal to the ordinary sea while keeping the Q_0^2 cut-offs ; the results show that the relative suppression of the heavy quark sea is not overwhelmingly large, in agreement with the remark on its generation with Q^2 above. (Note that in Tables II and III the effects of making two or more of the indicated changes simultaneously are not necessarily well-estimated by multiplication, or other simple combination, of the relevant entries.)

4. - DISCUSSION

In this final section we shall (i) discuss the theoretical status of our approach and the reliability of our numerical results, (ii) discuss the results themselves, noting how our flavour creation cross-sections compare with those of other authors, (iii) review the experimental situation indicated in the Introduction in the light of our results, and (iv) conclude with some assorted comments. Many of the observations and conclusions in this section - particularly regarding the flavour creation processes - have been noted by previous authors.

(i) Roughly speaking, the present theoretical justification, for calculating "naked" heavy flavour production via the lowest order diagrams of Figs. 1 and 2, is at a similar level to that for the QCD approaches to hadronic interactions at large p_T described in Refs. 16)-18). There are certain differences, of course. On the one hand, our results are presumably less sensitive to the details of the parton transverse momentum distributions, which play a (suspiciously) crucial rôle in the more successful of the QCD descriptions ^{16),17)} for the large p_T single-particle data. On the other hand, since the flavour creation processes occur predominantly close to threshold (Fig. 7) with values of $\sqrt{s}$ typically in the region where structure is still observed in the e^+e^- annihilation total cross-section, there should be bound state effects, etc. which one must assume are either unimportant or have been effectively averaged over in the calculation. There may also be an important danger in our approach of double-counting by considering the flavour creation and flavour excitation processes separately. This is illustrated by the diagram of Fig. 11a which is part of both a flavour creation amplitude (Fig. 11b) and a flavour excitation amplitude (Fig. 11c).

The point of view taken in this paper is that the final states are sufficiently different that all four of our basic subprocesses can be considered essentially distinct.

We have ignored the possibility of charmed particles being formed in the fragmentation of a gluon or an ordinary quark which has undergone a hard scattering. It has been suggested that such a mechanism can be important for charm production in deep inelastic scattering ⁵⁴⁾.

Given the theoretical basis for our calculations, one may wonder how sensitive the results are to the shapes of the little-known sea and glue distributions and to the various other phenomenological choices which have had to be made. The encouraging indication from Tables II and III is that, when the predicted cross-sections are sufficiently large, they are surprisingly stable - even against, in some cases, quite major changes in these parameters. We remark in passing that the entries of Tables II and III (note particularly the $\sqrt{s}$ dependence in certain cases) generally support one's qualitative expectations.

One possibility we have not so far discussed is that heavy quarks formed in hadronic collisions may give rise to heavy hadrons not by fragmentation but by combining with other quarks present after the initial interaction. This is in the spirit of the recombination models used to describe aspects of ordinary hadronic interactions ⁵⁵⁾. This would change the way in which we implement the heavy particle threshold if it were the dominant mechanism. In particular, if $m_c < m_D$, this allows naked charm production via flavour creation for $4m_c^2 < \hat{s} < 4m_D^2$. Some indication of the possibly large resulting enhancement, for reasonable values of m_c , can be seen in Fig. 5.

(ii) As already noted in Section 3, perhaps the most surprising feature of our results is the size of the flavour excitation cross-sections. Unfortunately, since their calculation is crucially dependent on the charmed sea structure function, there is more uncertainty in these results than in those for flavour creation. Nevertheless, if Figs. 8 and 9 together with Table III are any guide, flavour excitation is an important, and possibly the dominant, mechanism for hadronic production of heavy flavoured states.

One consequence of this is that it further dilutes the advantage of using a $\bar{p}$ beam instead of a proton beam. Already, for flavour creation alone, the potential gain is not particularly dramatic (Figs. 4 and 5) except at comparatively low $\sqrt{s}/m_c$ where the cross-section is smaller. For a pion beam, detailed predictions require extra assumptions. A useful estimate may be :

$$\sigma(\pi p \rightarrow c \bar{c} + X; s) = \frac{1}{3} \left[\sigma_{pp}(\frac{3}{2}s) + \sigma_{\bar{p}p}(\frac{3}{2}s) \right] \quad (4.1)$$

This simple relation, based on the pion containing fewer but faster partons, was suggested by the successful use of a similar expression in large p_T π^0 production⁵⁶⁾.

As with the QCD-based descriptions of large p_T ¹⁶⁾⁻¹⁸⁾, processes involving gluons can be very important. For both flavour creation and flavour excitation, separately, the respective gluon mechanism may give the major contribution (Figs. 4, 5, 8 and 9).

It is amusing to note that Figs. 5 and 9 imply non-negligible cross-sections ($\sim 10^{-35} - 10^{-34} \text{ cm}^2$) at ISABELLE energies for associated production of heavy flavoured states with masses in the 60-100 GeV range. However, since the experimental search for charm presently has difficulty reaching a sensitivity of $1 \mu\text{b}$, it seems likely that such states would be discovered only if they had sufficiently favourable decay modes or were rather long-lived. At the other end of the scale, Fig. 5 shows flavour creation cross-sections for strangeness - $m_c = 500 \text{ MeV}$. (Figure 9 only goes down to $m_c = 1 \text{ GeV}$ below which Q_0^2 becomes unacceptably small.) Our results must be regarded as less reliable at such a value of m_c ; we would just note that at least the values obtained are not completely ridiculous.

As mentioned in the Introduction, the flavour creation processes have been considered by a number of authors³⁸⁾⁻⁴⁵⁾. References 42)-45) are quite similar to our approach except that they use exact scaling parton structure functions [apart from Ref. 43), briefly]. In addition to these calculations, there is a recent paper by Hagiwara and Yoshino⁵⁷⁾ who have also used the BG⁸⁾ non-scaling structure functions (with $n_s = 10$, $n_g = 5$). Our treatment of flavour creation is close to theirs in certain respects. We find that our results for flavour creation are generally comparable with those of Refs. 42)-45) and 57), except that we tend to somewhat smaller

cross-sections at lower energies (in the FNAL, CERN-SPS range, say). Our implementation of the heavy particle threshold (" $m_c = m_D$ "), and the very steep glue and sea that we use for most of the calculations, may be the main origins of this effect.

(iii) Our results are easily compatible with most of the experimental upper limits on charm production in Table I. The ISR result of $70 \mu\text{b}$ ²⁸⁾ is in line with our expectations and, in our calculation, indicates a substantial contribution from flavour excitation. However, the large quoted error on this result ($\pm 36 \mu\text{b}$) ²⁸⁾ should be noted, as should also the upper limit of $76 \mu\text{b}$ (95% C.L.) reported from the other ISR experiment ²⁷⁾ in Table I. Going to higher energies, there is a tendency for our charm production cross-section, when flavour excitation is included, to be somewhat above the cosmic ray estimate ³³⁾ mentioned in the Introduction.

The experimental upper limit which squeezes our calculation most is the $1.5 \mu\text{b}$ bound (90% C.L.) at $\sqrt{s} = 23.8 \text{ GeV}$ of the FNAL emulsion experiment ²²⁾. At this energy, with our standard choice of parameters, we find $0.08 \mu\text{b}$, $0.06 \mu\text{b}$, $3.4 \mu\text{b}$ and $1.0 \mu\text{b}$ for the $q\bar{q}$, gg , qc and gc mechanisms, respectively, giving a total of $4.6 \mu\text{b}$. A similarly restrictive upper limit has been inferred by Lauterbach ⁵⁸⁾ from the measured small (consistent with zero) polarization of prompt muons produced in pN interactions at $\sqrt{s} = 27.4 \text{ GeV}$.

Both these two upper limits and our results lie well below the charm interpretations ³⁰⁾⁻³²⁾ of the CERN-SPS beam dump prompt neutrino cross-sections noted in the Introduction. In view of the possible importance of this discrepancy, we extend our earlier discussion by remarking the following points.

A recent analysis ⁵⁹⁾ suggests that the null result of the emulsion experiment, which was sensitive to charmed particle lifetimes in the range $10^{-14} - 10^{-12}$ seconds, can be reconciled with the beam dump "cross-sections" if the typical lifetime is actually either $\gtrsim 10^{-12}$ or $\lesssim 5 \times 10^{-16}$ seconds. Direct experimental evidence for the lifetimes of charmed particles is presently scarce; there is some tentative indication for each of the above possibilities ^{60),61)} (!). (In any event, it seems likely the lifetime is not much longer than 10^{-12} seconds, otherwise charmed particle tracks would probably have been detected in bubble chamber experiments ^{59),60)}.) Supporting evidence for the CERN beam dump results comes from the preliminary

results of a Caltech-Stanford collaboration at FNAL, who find a qualitatively comparable level of single (unpaired) prompt muon production from 400 GeV/c incident protons⁶²⁾. Fritzsche and Streng⁶³⁾ have suggested a particular form of $\hat{s}$ threshold enhancement for charm production to explain the size of these cross-sections. It should be noted that the charm cross-sections extracted from the beam dump experiments should be divided by four if linear A dependence, rather than $A^{2/3}$ behaviour, is assumed (the emulsion upper limit would be similarly lowered), while they should be increased ($\sim$ factor 2 ?) if there is substantial rescattering of charmed particles within the target nucleus^{59),60)}.

Taking the above points into account, and after reviewing the experimental situation outlined in Table I, we feel that a charm production cross-section at the level of $\sim 20 \mu\text{b}$, for pp collisions at $\sqrt{s} = 27.4 \text{ GeV}$, may still be in accord with almost all present data, suitably interpreted, including the beam dump experiments. Our result at this energy is $8.1 \mu\text{b} [= 0.14(q\bar{q}) + 0.16(gg) + 5.5(qc) + 2.3(gc)]$, which is compatible with the above rate given the uncertainties indicated in Tables II and III. Obviously, however, this "universal agreement" is not particularly natural ! There remains considerable freedom for speculation on sources other than associated charm production for the (non-electromagnetic) prompt single lepton fluxes observed.

(iv) The aim of this paper has been to present a rather detailed calculation of the pp and $\bar{p}p$ total cross-sections for naked heavy flavour production in the framework of perturbative QCD. Attention has been paid to dependence of the numerical results on particular assumptions and parameter choices made in the course of the calculation. With the exception of Figs. 6 and 7 and the associated discussion in the text of Section 3, we have avoided considering momentum and angular distributions of the produced heavy quarks/particles. Much can be done in this direction (of special interest are p_T distributions - see below) using the matrix elements of Eqs. (2.7), (2.12), (2.14) and (2.15) [rather than the integrated $\hat{\sigma}(\hat{s})$]. Some calculations have already appeared^{42),57),64)}.

From the viewpoint of experimental searches for charmed states in hadronic interactions, Figs. 4 and 8 indicate that use of the highest possible beam energies should be a major consideration. Also, since all four of our basic subprocesses can fairly easily produce heavy quarks with moderately large transverse momentum (flavour excitation almost always),

charmed particle searches may be more successful in this kinematic region where the background is presumably less. A detailed discussion of how associated charm production in this region can be reconstructed from $e\mu$ and eK correlations is given in Ref. 65).

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Experiment	$\sqrt{s}$ GeV	Quoted result
i) $nN \rightarrow K^+ \pi^- + X$ ²¹⁾	19 average 24 max	$\sigma_{\bar{D}^0} < 12 \mu b$ ($x_F > 0.3$?)
ii) pN : emulsion experiment ²²⁾	24	σ (associated production of charm) $< 1.5 \mu b$
iii) $\pi^- N \rightarrow \mu^+ \mu^- \nu^0 + X$ ²³⁾	21	$\sigma_{D\bar{D}} < 10.4 \mu b$
iv) $pN \rightarrow K^\pm \pi^\mp + X$ ²⁴⁾	27	$\left. \frac{d\sigma}{dy} \right _{y=-0.4} < \begin{matrix} 23 \mu b & D^0 \\ 18 \mu b & \bar{D}^0 \end{matrix}$
v) $nN \rightarrow \mu^\pm e^\mp + X$ ²⁵⁾	24	σ (associated production of charm) $< 15 \mu b$
vi) $nN \rightarrow K^\pm \pi^\mp + X$ ²⁶⁾	22 average	$\sigma_{D^0} < 14 \mu b$ ($x_F > 0$)
vii) $pp \rightarrow \mu^\pm e^\mp + X$ ²⁷⁾	55 average	$\sigma_{D\bar{D}} < 76 \mu b$
viii) $pp \rightarrow \mu^\pm e^\mp + X$ ²⁸⁾	56 average	$\sigma_{D\bar{D}} = 70 \pm 36 \mu b$

TABLE I - A selection of experimental upper limits on charmed particle production in high energy hadronic interactions. Also shown is a single recent positive result based on muon-electron coincidences at the CERN-ISR. The values listed are those quoted in the original publications ²¹⁾⁻²⁸⁾, to which the reader is referred for assumptions made concerning distributional shapes, branching ratios, etc. The sole exception is that, for experiments i) and iv), we have converted a result for a cross-section times branching ratio into an absolute cross-section by using a (conservative) value for B.R. $(\bar{D}^0 \rightarrow K^{-(+)} \pi^{+(-)})$ of 1.6% ²⁹⁾. Note, in this regard, that experiment vi) used the median value 2.2% ²⁹⁾ for its result given above.

Change of parameter, etc. Standard choice in brackets, where appropriate.	$\sqrt{s}$ GeV	$\sigma/\sigma(\text{standard choice})$					
		$m_c = 1.87 \text{ GeV}$			$m_c = 5.4 \text{ GeV}$		
		pp	$\bar{p}p$	'gg $\rightarrow c\bar{c}$ '	pp	$\bar{p}p$	'gg $\rightarrow c\bar{c}$ '
$\Lambda = 600 \text{ MeV}$ (300 MeV).	19.4	1.1	1.3	0.67	0.08	0.57	0.0002
	62.4	1.7	1.7	1.6	0.77	0.96	0.21
	800	2.6	2.6	2.6	1.9	1.9	1.9
$Q^2 = \hat{s}$ throughout. ($Q^2 = \frac{1}{2}\hat{s}$ for $gg \rightarrow c\bar{c}$).	19.4	0.85	0.99	0.42	1.0	1.0	0.09
	62.4	0.75	0.78	0.72	0.94	0.99	0.52
	800	0.92	0.92	0.91	0.89	0.89	0.88
Initial sea and glue distributions. $n_s = 5(10)$, $n_g = 3(10)$ in Eq. (2.5).	19.4	6.4	1.5	19	130	1.0	1.2×10^6
	62.4	1.3	1.3	1.4	4.4	1.3	21
	800	0.40	0.40	0.40	0.65	0.66	0.66
Exact scaling model. Eq. (2.5) gives the parton distributions for <u>all</u> Q^2 .	19.4	6.8	5.9	11	820	43	2.7×10^6
	62.4	3.0	3.2	3.0	23	15	84
	800	1.4	1.4	1.4	2.6	2.7	2.6
$\alpha = 0.25$	19.4	1.7	1.4	2.6	200	11	6.7×10^5
	62.4	0.75	0.79	0.75	5.7	3.7	21
	800	0.36	0.36	0.36	0.65	0.65	0.65
$\sigma(\text{standard choice}) \text{ cm}^2$							
	19.4	4.9×10^{-32}	5.1×10^{-31}	1.2×10^{-32}	3.1×10^{-39}	1.1×10^{-34}	1.3×10^{-45}
	62.4	7.1×10^{-30}	8.2×10^{-30}	6.3×10^{-30}	2.9×10^{-33}	2.9×10^{-32}	3.8×10^{-34}
	800	3.1×10^{-28}	3.1×10^{-28}	3.0×10^{-28}	9.9×10^{-30}	1.0×10^{-29}	9.6×10^{-30}

TABLE II - Effects on predicted cross-sections for flavour creation (Fig. 1) due to various changes of parameters, etc. For each of the different modifications indicated, all the other quantities were given their standard values. The resulting cross-sections are shown as a ratio of the original predictions. The absolute values of the latter are given in the final rows.

	V_s GeV	$\sigma/\sigma(\text{standard choice})$			
		$m_c = 1.87 \text{ GeV}$		$m_c = 5.4 \text{ GeV}$	
		pp (or $\bar{p}p$)	'gc $\rightarrow$ gc'	pp (or $\bar{p}p$)	'gc $\rightarrow$ gc'
Change of parameter, etc. Standard choice in brackets, where appropriate.					
$\Lambda = 600 \text{ MeV (300 MeV).}$	19.4 62.4 800	3.1 3.1 3.3	3.1 3.2 3.4	2.1 2.0 2.0	2.0 2.0 2.0
$Q^2 = \hat{s}$ throughout. $\left[\begin{array}{l} Q^2 = 0.5 [Q_o^2 + \frac{(\hat{s}-m_c^2)^2}{\hat{s}}] \text{ for } qc \rightarrow qc \\ Q^2 = 0.5 [\hat{s} - m_c^2] \text{ for } gc \rightarrow gc \end{array} \right]$	19.4 62.4 800	0.92 1.0 1.1	0.81 1.1 1.1	1.2 1.2 1.2	0.61 1.2 1.2
Initial sea and glue distributions. $n_s = 5(10), n_g = 3(10)$ in Eq. (2.5).	19.4 62.4 800	3.7 1.2 0.46	11 1.4 0.45	69 3.3 0.55	7100 8.8 0.54
$Q_o^2 = m_c^2$ $(Q_o^2 = 1.8 [m_c/1.87]^2 \simeq 0.51 m_c^2)$	19.4 62.4 800	0.14 0.20 0.27	0.08 0.17 0.27	0.03 0.16 0.28	0.002 0.10 0.27
$\hat{s} - m_c^2 = x_1 x_2 s$ $(\hat{s} = x_1 x_2 s)$ (see end of Section 2)	19.4 62.4 800	2.7 1.6 1.1	4.8 1.8 1.1	25 2.4 1.2	170 3.7 1.2
Heavy quark sea put equal to ordinary sea, i.e. $c(x, Q^2)$ and $\bar{c}(x, Q^2)$ replaced by $\bar{u}(x, Q^2) \equiv \bar{d}, s, \bar{s}, u_{\text{sea}}, d_{\text{sea}}$ for $Q^2 > Q_o^2$.	19.4 62.4 800	8.7 4.3 2.2	15 4.9 2.2	100 11 3.5	340 18 3.6
$\sigma(\text{standard choice}) \text{ cm}^2$					
	19.4 62.4 800	1.8×10^{-30} 1.0×10^{-28} 5.3×10^{-27}	2.9×10^{-31} 6.0×10^{-29} 4.4×10^{-27}	9.1×10^{-36} 1.1×10^{-31} 6.3×10^{-29}	3.2×10^{-38} 2.2×10^{-32} 4.8×10^{-29}

TABLE III - Effects on predicted cross-sections for flavour excitation (Fig. 2) due to various changes of parameters, etc. See caption to Table II.

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FIGURE CAPTIONS

Figure 1 Lowest order QCD diagrams for the "flavour creation" processes :
 (a) $q\bar{q} \rightarrow c\bar{c}$;
 (b) $gg \rightarrow c\bar{c}$.

Figure 2 Lowest order QCD diagrams for the "flavour excitation" processes :
 (a) $qc \rightarrow qc$;
 (b) $gc \rightarrow gc$.

Figure 3 Drell-Yan predictions using non-scaling structure functions for proton + (platinum) nucleon $\rightarrow \mu^+\mu^- + X$ at 400 GeV/c and zero muon pair rapidity. The predictions are plotted as a ratio of the experimental fit ⁴⁶⁾

$$\left. \frac{d\sigma}{dMdy} \right|_{y=0} = 2.6 \times 10^{-33} e^{-0.986M} \text{ cm}^2 \text{ GeV}^{-1}$$

Notional data points are shown to indicate the statistical quality of the data determining the fit. n_s and n_g are the sea and glue parameters in the initial, $Q^2 = 1.8 \text{ GeV}^2$, parton distributions of Eq. (2.5). The dotted area is the T region where the continuum is not experimentally well-determined.

Figure 4 Associated production of heavy flavoured states via the flavour creation mechanisms $q\bar{q} \rightarrow c\bar{c}$ and $gg \rightarrow c\bar{c}$. Cross-section predictions for pp and $\bar{p}p$ interactions, and for two different choices of the heavy quark mass, are shown as a function of the centre-of-mass energy. The $gg \rightarrow c\bar{c}$ contribution alone (the same for both pp and $\bar{p}p$) is also displayed.

Figure 5 As Fig. 4, except that the cross-section predictions are plotted as a function of the heavy quark mass for three different choices of the centre-of-mass energy.

Figure 6 x_F distributions of pairs of charmed ($m_c = 1.87 \text{ GeV}$) quarks and antiquarks produced via flavour creation in pp interactions at the three energies shown. The $gg \rightarrow c\bar{c}$ contribution alone is also displayed. The distributions have been integrated over $\hat{s}$, and are plotted only for positive x_F , the full curves being symmetric about $x_F = 0$.

Figure 7 $d\sigma/d\hat{s}^{\frac{1}{2}}$ for pairs of charmed ($m_c = 1.87$ GeV) quarks and anti-quarks produced via flavour creation in pp interactions at the three energies shown. $\hat{s}^{\frac{1}{2}}$ is the invariant mass of the heavy quark pair. The distributions have been integrated over x_F (or x_L). The $q\bar{q} \rightarrow c\bar{c}$ contribution alone is also displayed.

Figure 8 Associated production of heavy flavoured states via the flavour excitation mechanisms $qc \rightarrow qc$ and $gc \rightarrow gc$. Cross-section predictions for pp (or $\bar{p}p$) interactions, for two different choices of the heavy quark mass, are shown as a function of the centre-of-mass energy. The $gc \rightarrow gc$ contribution alone is also displayed.

Figure 9 As Fig. 8, except that the cross-section predictions are plotted as a function of the heavy quark mass for three different choices of the centre-of-mass energy.

Figure 10 Individual subprocess cross-sections : $q\bar{q} \rightarrow c\bar{c}$ (solid curve), $g\bar{g} \rightarrow c\bar{c}$ (dash-dot curve), $qc \rightarrow qc$ (dashed curves), $gc \rightarrow gc$ (dotted curves). The long-dashed and sparsely-dotted curves correspond to $Q_0^2 = 1.8(m_c/1.87)^2 \simeq 0.51 m_c^2$, while the short-dashed and densely-dotted curves are for $Q_0^2 = m_c^2$. Note that the quantity plotted, $\hat{s}/\alpha^2(Q^2)$, is a function of $\hat{s}^{\frac{1}{2}}/m_c$ (and, for $qc \rightarrow qc$ and $gc \rightarrow gc$, Q_0^2/m_c^2) only.

Figure 11 Example of a graph, (a), which contributes to both (b) a flavour creation, and (c) a flavour excitation, amplitude.

(a)

(b)

FIG. 1

(a)

(b)

FIG. 2

FIG. 3

FIG. 4

FIG. 5

FIG. 6

FIG. 7

FIG. 8

FIG. 9

FIG. 10

FIG. 11