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 Υ^* RESONANCES AND KAONIC ATOMS

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ABSTRACT

We study the role of the Υ_0^* and Υ_1^* resonances in kaonic ^{12}C and ^{32}S . Non-local effects due to propagation of the Υ_0^* are investigated together with the influence of possible changes in the Υ_0^* position and width. Most recent parametrizations of the $\bar{\text{K}}\text{N}$ amplitude will be used. It is shown that the kaonic widths and shifts suggest systematically a repulsive shift and an increased width of the Υ_0^* inside a nucleus.

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1. INTRODUCTION

Early studies [1] to explain the energy shifts and widths deduced from kaonic X-ray data [2] have led to the conclusion that, instead of a repulsive real part of the K^- -nucleus optical potential suggested by the K^-N scattering length, attraction is needed to fit the data. Since then it has become evident from various investigations [3-7] that kaonic atoms provide a probe of the subthreshold behaviour of the K^-N interaction. This amplitude is strongly energy dependent, basically because of the presence of the Y_0^* resonance [i.e. the $\Lambda(1405)$ s-wave resonance in the isospin $I = 0$ channel]. Owing to the dominance of the $\Lambda(1405)$, the K^-p interaction dominates strongly over the K^-n interaction, so that the nuclear physics part of the kaonic atom problem requires mainly a precise knowledge of proton surface distributions. Once these distributions are given (as determined, for example, by electron scattering experiments) it is relevant to ask whether and to what extent the kaonic widths and shifts are sensitive to the dynamics of the $\Lambda(1405)$ inside the nucleus. This position has been taken, in particular, in Refs. 6 and 7. In Ref. 7, indications were found for a moderately repulsive shift of the $\Lambda(1405)$ which might be of dynamical origin. On the other hand, these conclusions were based on a local approximation of the K^- nucleus optical potential. It was then pointed out by Thies [6] that the resonant behaviour of the elementary amplitude leads to non-localities in the optical potential because of resonance propagation, in a way similar to π -nucleus scattering close to the 3.3 resonance [8,9]. Such effects are best dealt with in a coupled-channel treatment [5,6], which shows that the non-localities are not too large [6a] for K^- -nucleus interactions not far from K^-N threshold. Nevertheless, they have to be taken into account once it comes to the discussion of many-body corrections to Y_0^* propagation.

Within the context of this paper, we shall point out that moderate non-locality corrections to the local s-wave K^- -nucleus optical potential can be approximated by an additional term of the $\vec{\nabla}\rho\vec{\nabla}$ type and are therefore easy to incorporate into standard routines to solve the Klein-Gordon equation.

Apart from the non-locality aspect, the purpose of this work is to investigate the sensitivity of the results to the subthreshold structure of the $\bar{K}N$ amplitude. In fact the parametrization used in Ref. 7, following older work of Kim [10], is somewhat outdated in the light of the most recent analysis of Martin [11] using dispersion relation constraints. Most of our present results will be based on the Martin amplitude.

Furthermore, we shall include the Y_1^* resonance [i.e. the $\Sigma(1385)$ p-wave, isospin $I = 1$ resonance] in our considerations. Although the $\bar{K}N$ p-wave scattering volumes at threshold are known to be quite small [1], the subthreshold extrapolation of the p-wave amplitude will reflect the presence of the Y_1^* , and it will be shown that this effect is not negligible.

2. ELEMENTARY $\bar{K}N$ AMPLITUDE AND Y^* RESONANCES

Various parametrizations of the s-wave $\bar{K}N$ amplitude below threshold have appeared in the literature. Most of the earlier treatments have been based on a K-matrix coupled-channels analysis [10,12-14] (the channels being $\bar{K}N$, $\pi\Sigma$, and $\pi\Lambda$), although only the amplitude of Kim [10] reproduces the $\pi\Sigma$ mass spectrum in the neighbourhood of the $\Lambda(1405)$, while the zero-range parametrizations of Refs. 12 and 13 are not satisfactory in that respect [14]. Probably the best amplitude at present available is the one of Martin [11] which combines the coupled-channel approach with strong dispersion relation constraints. This amplitude is shown in Fig. 1; we shall make use of it in almost all our calculations.

For later purposes, we wish to isolate the Y_0^* resonance part and write the s-wave $\bar{K}p$ amplitude in the c.m. system in the form:

$$f_{\bar{K}p}^{\ell=0}(s) = \frac{g_0^2/4\pi}{M_0^* - \frac{i}{2}\Gamma_0 - \sqrt{s}} + b_p(s) , \quad (1)$$

where $\sqrt{s}$ is the $\bar{K}N$ c.m. energy (at threshold, $s_0 = M + m_K$, with m_K the K^- mass), and

$$\frac{g_0^2}{4\pi} = 0.21 , \quad M_0^* = 1410 \text{ MeV} , \quad \Gamma_0 = 30 \text{ MeV} . \quad (2)$$

The background part $b_p(s)$ is a smooth function of energy and is adjusted numerically so as to reproduce the Martin amplitude at all relevant values of the c.m. energy $\sqrt{s}$. For simplicity, we disregard the energy dependence of Γ_0 .

The $\bar{K}^-n$ s-wave interaction is only smoothly dependent on energy (see Fig. 1) and is comparatively weak. In practice we shall again use $f_{\bar{K}^-n}^{\ell=0}$ as determined by Martin.

The $\bar{K}^-N$ p-wave interaction is dominated by the $\Sigma(1385)$. It is much less important than the s-wave, and we may use a simple parametrization for the forward scattering p-wave amplitude:

$$f_{\bar{K}^-p}^{\ell=1}(s) = \frac{1}{3} \frac{g_1^2}{4\pi m_K^2} \frac{q^2}{M_1^* - \frac{i}{2} \Gamma_1 - \sqrt{s}} + q^2 C_p, \quad (3a)$$

$$f_{\bar{K}^-n}^{\ell=1}(s) = \frac{2}{3} \frac{g_1^2}{4\pi m_K^2} \frac{q^2}{M_1^* - \frac{i}{2} \Gamma_1 - \sqrt{s}} + q^2 C_n. \quad (3b)$$

Here q is the $\bar{K}N$ c.m. momentum, and the $\Sigma(1385)$ parameters are:

$$\begin{aligned} M_1^* &= 1384 \text{ MeV} , \\ \Gamma_1 &= 39 \text{ MeV} , \\ \frac{g_1^2}{4\pi} &= 0.43 , \end{aligned} \quad (4)$$

where the $\bar{K}N Y_1^*$ coupling constant has been adjusted according to Ref. 15. The background parameters $C_{p,n}$ are again expected to vary smoothly with energy [16]. We take them to be constant and fixed by the p-wave scattering volumes at threshold [11]:

$$\begin{aligned} C_p &= 0.06 \text{ fm}^3 \\ C_n &= 0.14 \text{ fm}^3 . \end{aligned} \quad (5)$$

Neither the scattering volumes nor the coupling strength $g_1^2/4\pi$ are determined to a degree of accuracy that would justify going beyond the simple ansatz of Eqs. (3).

Given the elementary $\bar{K}^-N$ interaction, we can now proceed to construct the $\bar{K}^-$ -nucleus optical potential.

3. THE K^- -NUCLEUS OPTICAL POTENTIAL

3.1 s-wave interactions

The resonant part of the s-wave optical potential arises from the coupling of the kaon to a Y_0^* -hole state as shown in Fig. 2. We shall first assume that

- i) the nucleon holes are described by a standard single-particle shell model;
- ii) the Y_0^* , once created, does not interact with surrounding nucleons. This assumption will be modified later on.

The s-wave optical potential in momentum space can then be written as

$$2\mu(\vec{q}'|V^{(s)}|\vec{q}) = -4\pi\{U_p(\vec{q},\vec{q}') + U_n(\vec{q},\vec{q}')\}, \quad (6)$$

where μ is the kaon-nucleus reduced mass, and $\vec{q}, \vec{q}'$ are incoming and outgoing kaon momenta, as in Fig. 2.

The proton part U_p contains a resonant and a background piece, according to Eq. (2). The resonant part of U_p , describing the process of Fig. 2, is:

$$U_p^{\text{res}}(\vec{q},\vec{q}') = \frac{g_0^2}{4\pi} \int \frac{d^3p}{(2\pi)^3} \sum_{\alpha} \frac{Q_{\alpha}^*(\vec{p},\vec{q}') Q_{\alpha}(\vec{p},\vec{q})}{M_{\alpha}^* - \frac{i}{2}\Gamma_0 - \sqrt{s_{\alpha}}}, \quad (7)$$

where

$$\sqrt{s_{\alpha}} = (M_{\alpha}^{*2} - \vec{p}^2)^{1/2} \approx M_{\alpha}^* - \frac{\vec{p}^2}{2M_{\alpha}^*}, \quad (8)$$

with

$$M_{\alpha}^* = M + m_K + \epsilon_{\alpha},$$

is the effective c.m. energy of the K^-p system inside the nucleus, where the proton carries its single-particle energy ϵ_{α} , and the sum goes over all occupied proton states α . The momentum of the Y_0^* is denoted by $\vec{p}$. The quantity $Q_{\alpha}(\vec{p} - \vec{q})$ is the Fourier transform of the Y_0^* -hole transition density,

$$Q_{\alpha}(\vec{p},\vec{q}) = \int d^3r \psi_{\Lambda}^*(\vec{p},\vec{r}) e^{i\vec{q}\vec{r}} \psi_{\alpha}(\vec{r}), \quad (9a)$$

where $\psi_{\alpha}(\vec{r})$ is a bound proton single-particle wave function. If the Y_0^* wave function $\psi_{\Lambda}(\vec{p},\vec{r})$ is given by a plane wave, then

$$Q_{\alpha}(\vec{p},\vec{q}) = \psi_{\alpha}(\vec{p} - \vec{q}) \quad (9b)$$

in terms of the bound proton momentum distribution $\psi_{\alpha}(\vec{k})$.

In Eq. (7), we have disregarded a kinematic factor, taking into account the transformation from the K^-N c.m. system to the K^- -nucleus reference frame, which we shall supply later on.

The resonant optical potential of Eq. (7) is clearly non-local because of the propagation of the Y_0^* with a kinetic energy $\vec{p}^2/2M_\alpha^*$ [see Eq. (8)]. On the other hand, Thies [6a] has shown that the non-locality effects are quite small since large kaon off-shell momenta $\vec{q}$ and $\vec{q}'$ are not very likely to develop. We are therefore keeping the local part of Eq. (7) and adding non-local corrections linear in $\vec{q} \cdot \vec{q}'$.

The local approximation is obtained upon assuming $\vec{q}, \vec{q}'$ to be small and taking the closure sum over Y_0^* states, in which case we obtain

$$U_p^{\text{res}}(\vec{q}, \vec{q}') \cong \frac{g_0^2}{4\pi} \sum_{\alpha} \frac{F_{\alpha}(\vec{q} - \vec{q}')}{M_0^* - \frac{i}{2} \Gamma_0 - \langle \sqrt{s_{\alpha}} \rangle}, \quad (10)$$

where $\langle \sqrt{s_{\alpha}} \rangle$ is a suitably selected closure energy,

$$\langle \sqrt{s_{\alpha}} \rangle \cong M_{\alpha}^* - \langle p^2 \rangle_{\alpha} / 2M_{\alpha}^* ;$$

we shall choose $\langle p^2 \rangle_{\alpha}$ to be the mean square momentum in shell α . Furthermore, F_{α} is the proton form factor for each individual shell α :

$$F_{\alpha}(\vec{k}) = \int d^3r e^{-i\vec{k} \cdot \vec{r}} \rho_{\alpha}(\vec{r}), \quad (11a)$$

$$\rho_{\alpha}(\vec{r}) = |\psi_{\alpha}(\vec{r})|^2. \quad (11b)$$

In the next approximation, we shall keep terms of order proportional to $\vec{q} \cdot \vec{q}'$, but assume the momentum transfer $|\vec{q} - \vec{q}'|$ to be small compared with $\langle p^2 \rangle_{\alpha}^{1/2}$. Then we obtain a non-local correction

$$\Delta U_p^{\text{res}}(\vec{q}, \vec{q}') = \frac{g_0^2}{4\pi} \sum_{\alpha} \frac{(\vec{q} \cdot \vec{q}') F_{\alpha}(\vec{q} - \vec{q}')}{2M_{\alpha}^* |M_0^* - \frac{i}{2} \Gamma_0 - \langle \sqrt{s_{\alpha}} \rangle|^2}. \quad (12)$$

Clearly, Eq. (12) has the physically obvious feature that the non-locality is largest right at the position of the Y_0^* resonance. Also, ΔU_p^{res} transforms into a Kisslinger-type potential in r -space and is therefore very simple from a

practical point of view. The quality of the approximations involved in Eq. (12) will be checked against the results of the full coupled-channel calculation by Thies [6a]. Adding Eqs. (10) and (12) and non-resonant background terms, the optical potential of Eq. (6), supplemented by appropriate kinematical transformation factors, can now be written in r-space as

$$2\mu V^{(s)}(\vec{r}) = -4\pi\{U_p(\vec{r}) + U_n(\vec{r})\}, \quad (13)$$

where

$$U_p(\vec{r}) = \frac{g_0^2}{4\pi} \sum_{\alpha} \left\{ \frac{\eta \rho_{\alpha}(\vec{r})}{M_0^* - \frac{i}{2} \Gamma_0 - \langle \sqrt{s_{\alpha}} \rangle} - \frac{\vec{\nabla} \rho_{\alpha}(\vec{r}) \vec{\nabla}}{2\eta M_{\alpha}^* |M_0^* - \frac{i}{2} \Gamma_0 - \langle \sqrt{s_{\alpha}} \rangle|^2} \right\} + \eta \sum_{\alpha} b_p(\langle \sqrt{s_{\alpha}} \rangle) \rho_{\alpha}(\vec{r}). \quad (14)$$

Here $\rho_{\alpha}(\vec{r})$ is the proton density distribution and*)

$$\eta = 1 + \frac{m_K}{N}. \quad (15)$$

Furthermore, the smooth energy dependence in the background term justifies the local approximation for the third term of Eq. (14). The sum is again over all bound proton states α . Similarly, the neutron part of Eq. (13) becomes

$$U_n(\vec{r}) = \eta \sum_{\beta} f_{K^-n}^{\ell=0}(\langle \sqrt{s_{\beta}} \rangle) \rho_{\beta}(\vec{r}), \quad (16)$$

where the sum runs over all occupied neutron orbits β . Here the local density approximation is again justified since the K^-n amplitude is almost independent of energy. Because of the smallness of f_{K^-n} , a precise knowledge of the neutron single particle densities $\rho_{\beta}(\vec{r})$ is not required. In practice it turns out to be sufficient to use the scattering length approximation

$$U_n(\vec{r}) = \eta a_n \rho_n(\vec{r}), \quad (17)$$

where $\rho_n = \sum_{\beta} \rho_{\beta}$ and $a_n = (0.35 + i 0.66)$ fm for the Martin amplitude.

*) In the second non-local term of Eq. (14), the transformation factor η^{-1} is valid for small momentum transfer $|\vec{q} - \vec{q}'|$, in accordance with the approximations used to derive this term.

3.2 p-wave interactions

To Eq. (14) we shall add a p-wave optical potential of the Kisslinger type which includes the Y_1^* resonance plus non-resonant background as in Eqs. (3) to (5):

$$2\mu V^{(p)}(\vec{r}) = \frac{4\pi}{\eta} \vec{\nabla} \left\{ \frac{1}{3} \frac{g_1^2}{4\pi m_K^2} \sum_{\alpha} \frac{\rho_{\alpha}(\vec{r})}{M_1^* - \frac{i}{2} \Gamma_1 - \langle \sqrt{s}_{\alpha} \rangle} + C_p \rho_p(\vec{r}) + \right. \\ \left. + \frac{2}{3} \frac{g_1^2}{4\pi m_K^2} \sum_{\beta} \frac{\rho_{\beta}(\vec{r})}{M_1^* - \frac{i}{2} \Gamma_1 - \langle \sqrt{s}_{\beta} \rangle} + C_n \rho_n(\vec{r}) \right\} \vec{\nabla}, \quad (18)$$

where the sums over α and β are again over proton and neutron orbits, respectively.

Since $V^{(p)}$ is only a correction to $V^{(s)}$, no attempt is made to include higher order non-locality effects due to the propagation of the Y_1^* . Nevertheless we note that the scattering length approximation should not be used for the p-wave part because of the rapid energy variation of the Y_1^* part of the amplitude.

4. MANY-BODY CORRECTIONS TO Y_0^* PROPAGATION

In Eq. (7), the $\Lambda(1405)$ resonance was assumed not to interact with the surrounding nucleons. This is certainly an oversimplification, and in this section we would like to discuss self-energy corrections to the Y_0^* propagator. More specifically, we would like to study the $\Lambda(1405)$ Green's function

$$G_{\Lambda}(s) = \frac{1}{M_0^* - \frac{i}{2} \Gamma_0 + \Sigma_{\Lambda}(s) - \sqrt{s}}, \quad (19)$$

and discuss contributions to the (generally complex) self-energy Σ_{Λ} in a perturbation theory framework.

Important contributions to Σ_{Λ} are shown in Figs. 3. Here Fig. 3a corresponds to a Pauli correction to the Y_0^* mass, while Fig. 3b includes a higher order rescattering process; Figure 3c involves the coupling to the $\pi\Sigma$ channel, with rescattering of the pion.

4.1 Pauli correlations

The self-energy operator corresponding to the diagram of Fig. 3a can be written

$$\Sigma_{\Lambda}^{\text{Pauli}}(s) = - \sum_{\alpha \in F} \int \frac{d^3 q}{(2\pi)^3} \frac{\delta H^+ |\vec{q}; N_{\alpha}\rangle \langle N_{\alpha}; \vec{q} | \delta H}{(\sqrt{s} - E_{\alpha})^2 - \vec{q}^2 - m_K^2 + i\delta}, \quad (20)$$

where the sum goes over all occupied bound nucleon states N_{α} and $|\vec{q}|$ denotes the intermediate kaon of momentum $\vec{q}$. Furthermore,

$$E_{\alpha} = M + \varepsilon_{\alpha}$$

where ε_{α} is a nucleon single-particle energy. The $\text{KN}\Lambda$ vertex operator is given by

$$\delta H = g_0 \psi_{\Lambda}^+ \{ \psi(p) \phi(K^-) + \psi(n) \phi(K^0) \} + \text{h.c.} \quad (21)$$

in terms of the corresponding field operators, respectively. The coupling constant g_0 is the same as in Eq. (2).

In order to obtain a simple estimate of this Fock term, let us assume that the $\Lambda(1405)$ is represented by a plane wave normalized to unity inside the nuclear volume V , so that

$$\langle \Lambda(\vec{p}) | \delta H | N_{\alpha}; \vec{q} \rangle = \frac{g_0}{\sqrt{V}} \psi_{\alpha}(\vec{p} - \vec{q}), \quad (22)$$

where $\psi_{\alpha}(\vec{k})$ is the bound nucleon momentum distribution^{*)}. We then obtain between Y_0^* states of different momentum $\vec{p}$ and $\vec{p}'$,

$$\langle \Lambda(\vec{p}') | \Sigma_{\Lambda}^{\text{Pauli}}(s) | \Lambda(\vec{p}) \rangle = - \frac{g_0^2}{V} \frac{1}{2} \sum_{\alpha \in F} \int \frac{d^3 q}{(2\pi)^3} \frac{\psi_{\alpha}^*(\vec{p}' - \vec{q}) \psi_{\alpha}(\vec{p} - \vec{q})}{(\sqrt{s} - E_{\alpha})^2 - \vec{q}^2 - m_K^2 + i\delta} \quad (23)$$

(the factor 1/2 is a spin average). Suppose now that the $\Lambda(1405)$ is at rest ($\vec{p} = \vec{p}' = 0$) and that we neglect ε_{α} with respect to m_K , i.e. $\sqrt{s} = M + m_K$, then

$$\langle \Sigma_{\Lambda}^{\text{Pauli}} \rangle \approx \frac{g_0^2}{V} \frac{1}{2} \sum_{\alpha} \int \frac{d^3 q}{(2\pi)^3} |\psi_{\alpha}(\vec{q})|^2 / \vec{q}^2 = g_0^2 \sum_{\alpha} \frac{n_{\alpha}}{2} \langle \frac{1}{p^2} \rangle_{\alpha}, \quad (24)$$

where n_{α} is the number of nucleons per unit volume in each individual shell α .

The shift of the Y_0^* mass due to $\Sigma_{\Lambda}^{\text{Pauli}}$ is a sizeable effect; we have a repulsive shift that is typically of the order of 30 MeV.

*) For simplicity we have omitted the Kronecker symbol for Λ and nucleon spins. The matrix elements of Eq. (22) vanish unless Λ and N_{α} have the same spin projection.

4.2 Rescattering effects

The repulsion from Pauli correlations will be at least partly removed by higher order kaon rescattering processes as in Fig. 3b, which will lead to an attractive real part of the corresponding (complex) self-energy and damping due to absorption of the kaon in the intermediate state. The size of the attractive real part is hard to estimate. Since there is no restriction here on the intermediate nucleon-particle momenta, it will strongly depend on KNY_0^* form factors; nothing is known about the cut-off mass associated with such vertex factors.

We note that Thies [6b], in a self-consistent coupled-channel calculation of Pauli effects, finds in fact that these are strongly damped by the reabsorption of the kaon in Fig. 3a.

4.3 Coupling to $\pi\Sigma$ channel and pion rescattering

The coupling of the Y_0^* to the free $\pi\Sigma$ system is already incorporated in terms of the free width Γ_0 of the Y_0^* . However, the decay pion interacts strongly with surrounding nucleons. This effect, shown in Fig. 3c, has been estimated by Eisenberg [17] for nuclear matter. It was assumed there that the pion rescattering goes mainly through the excitation of the Δ resonance. It is then found that the Y_0^* mass shift due to such mechanisms is repulsive and of the order of 10 MeV at densities corresponding to the outer parts of the nuclear surface. Associated with this shift is an increase of the Y_0^* width by about 20-30% over its free space value.

Not included in Ref. 17 are non-mesonic decays of the Y_0^* via the channel $Y_0^*N \rightarrow \Sigma N$ which also appears in Fig. 3c, e.g. through the exchange of virtual π and ρ mesons. Such processes also enhance the Y_0^* width in a similar way to that where the $\Delta N \rightarrow NN$ decay adds to the Δ isobar width in pion-nucleus scattering [8,9].

In conclusion so far, we expect that self-energy interactions of the Y_0^* inside the nucleus give rise to a mass shift $\Delta M_0^* = \text{Re } \Sigma_\Lambda$ and to an effective decay width which is larger than the free width by an amount $\Delta\Gamma = 2|\text{Im } \Sigma_\Lambda|$. Quantitative estimates of Σ_Λ are difficult to perform (except for the lowest order Pauli correction) because of the strong dependence on unknown cut-off factors associated with KNY_0^* and $\pi\Sigma Y_0^*$ vertices appearing in Figs. 3b-c.

4.4 Lorentz-Lorenz correction

In addition to self-energy processes, vertex corrections due to short-range baryon-baryon correlations have to be taken into account for the resonant part of the K-nucleus optical potential [16]. Such corrections can also be incorporated approximately in terms of an effective repulsive shift of the Y_0^* mass. To show this, let us consider the case of nuclear matter at density $\rho = \rho_p + \rho_n$; the lowest order resonant optical potential is given by

$$2\mu V_{\text{res}} = - \frac{g_0^2 \rho_p}{M_0^* - \frac{i}{2} \Gamma_0 - \sqrt{s}}, \quad (25)$$

whereas the incorporation of short-range repulsive correlations (other than kaon exchange) between Y_0^* and the nucleon lead to a modification

$$2\mu \tilde{V}_{\text{res}} = \frac{2\mu V_{\text{res}}}{1 - \lambda^2 2\mu V_{\text{res}}}, \quad (26)$$

where λ is a measure of the distance over which the baryon-baryon repulsive core acts. This equation is derived under the assumption that the correlation can be represented by a local correlation function of the form $1 - g(r)$, with $g(r) = \exp(-r/\lambda)$ in the limit of small kaon momentum, $q \ll \lambda^{-1}$. More generally, we have $\lambda^2 = \int d^3r [g(r)/4\pi r]$.

Clearly, Eq. (25) can be written

$$2\mu \tilde{V}_{\text{res}} = - \frac{g_0^2 \rho_p}{M_0^* - \frac{i}{2} \Gamma_0 - \sqrt{s} + \Delta_{\text{LL}}}, \quad (27)$$

with an effective mass shift due to the Lorentz-Lorenz correction:

$$\Delta_{\text{LL}} = \lambda^2 g_0^2 \rho_p. \quad (28)$$

This is not a large shift at the surface densities relevant to our problem. For $\lambda = 0.7$ fm and ρ_p equal to 1/8 times nuclear matter density, we have a Δ_{LL} of about 5 MeV.

Summing up the whole set of many-body correlations, we expect a moderately repulsive shift of the Y_0^* mass, together with an increase of Y_0^* width. Clearly, a precise calculation of all such effects is extremely complicated and subject to

uncertainties in the microscopic input information. Nevertheless, on the basis of the preceding discussion, we shall incorporate such corrections phenomenologically in terms of an average complex shift ΔM^* of the Y_0^* position. That is, in the optical potential of Eq. (14) we shall replace

$$\begin{aligned} M_0^* &\rightarrow M_0^* + \text{Re } \Delta M^* \\ \frac{i}{2} \Gamma_0 &\rightarrow \frac{i}{2} \Gamma_0 + i |\text{Im } \Delta M^*| \end{aligned} \quad (29)$$

and study the sensitivity of kaonic widths and shifts with respect to variations of ΔM^* .

No attempt will be made to apply corresponding corrections to the propagation of the $Y_1^*(1385)$ because of the relative smallness of the leading terms.

5. RESULTS AND DISCUSSION

Strong interaction widths and shifts for kaonic ^{12}C and ^{32}S have been obtained by solving the Klein-Gordon equation with the optical potential of Eqs. (13) to (18). The proton single-particle densities $\rho_\alpha(r)$ and the shell model parameters ϵ_α and $\langle p^2 \rangle_\alpha$ are taken from a combined analysis of elastic electron scattering and quasi-elastic data [18,19] and are listed in Table 1. Neutron distributions are assumed to be identical to those for protons. Because of the weakness of the K^-n s-wave interaction, this is not a critical assumption.

As explained in the previous section, a complex energy shift $\Delta M^* = \text{Re } \Delta M^* - (i/2) \Delta \Gamma$ is added to the free Y_0^* mass and width, $M_0^* - (i/2) \Gamma_0$, to account phenomenologically for interactions of the Y_0^* with the surrounding nuclear many-body system. All results will be discussed as a function of $\text{Re } \Delta M^*$ and $\Delta \Gamma$.

Calculations have been carried out selectively at three different levels of approximation, including

- i) only the local s-wave potential [omitting the $\vec{\nabla} \rho \vec{\nabla}$ term in Eq. (14)]
- ii) local s-wave plus p-wave potential [Eq. (18)]
- iii) s-wave and p-wave potential incorporating non-locality effects through the $\vec{\nabla} \rho \vec{\nabla}$ term of Eq. (14).

Figures 4 and 5 show calculations of the shift and lower width for the 3d-2p transition in kaonic ^{12}C . Figure 4 has been obtained with the $\bar{K}N$ amplitude used in Ref. 7, adjusted to the Kim amplitude [10], while Fig. 5 is evaluated with the more recent amplitude of Martin [11]. Clearly, the differences between different forms of the $\bar{K}N$ subthreshold amplitude are quite appreciable. A comparison of curves A and B of either figure shows that the effect of incorporating the Y_1^* p-wave resonance is appreciable. Even more pronounced is the effect of the non-locality due to propagation of the Y_0^* resonance (see curve C). Figure 4 shows that the non-local effects approximated by the simple $\vec{V}_\rho \vec{V}$ term of Eq. (14) are comparable in their relative size to those calculated in the full coupled-channel scheme of Ref. 6a. Quantitative differences to Ref. 6a seem to be more strongly related to the use of realistic (rather than harmonic oscillator) single-proton distributions in our work than to the approximations involved in Eq. (14).

Note that in both Figs. 4 and 5, at all different levels of approximation, the variation with the mass shift parameter $\text{Re } \Delta M^*$ shows a systematic tendency towards better agreement with data in the case of a repulsive shift,

$$\text{Re } \Delta M^* \approx 20-35 \text{ MeV} .$$

Adding a width $\Delta\Gamma$ of about 10 MeV does not change this conclusion substantially, as shown in Fig. 6 for the 3d-2p transition in ^{12}C . Here the Martin amplitude has been used throughout in our calculations. Figures 7 and 8 for ^{32}S show a qualitatively similar tendency to that in ^{12}C : non-localities and the inclusion of the Y_1^* are quite important. The variation with the complex mass shift ΔM^* again shows reasonable agreement with data for $\text{Re } \Delta M^* = 20-35 \text{ MeV}$. An increase of $\Delta\Gamma/\Gamma_0 \sim 40\%$ over the free Y_0^* width would be acceptable. We can therefore conclude that, with respect to variation of the shifts and widths with ΔM^* , a similar trend is observed in two different nuclei for a variety of model assumptions.

Note, in particular, that in all cases, a value $\Delta M^* = 0$ or $\text{Re } \Delta M^* < 0$ (i.e. an attractive shift of the Y_0^*) would lead to disagreement with data.

For completeness we present in Table 2 the calculated upper level widths Γ_{up} for various different models. Note that while Γ_{up} remains quite model-independent

for ^{12}C , variations are larger for ^{32}S . Not very much can be deduced from Γ_{up} alone, except that those sets of parameters which describe ϵ and the lower level widths will yield also reasonably consistent results for Γ_{up} .

6. CONCLUSIONS

In summary, we make the following concluding remarks:

Kaonic widths and shifts are sensitive to both the elementary $\bar{K}N$ input parameters as well as to many-body correlations and non-locality effects associated with the propagation of the Y_0^* resonance.

We have employed the $\bar{K}N$ amplitude of Martin [11] as a starting point, since it represents the most recent and complete analysis by combining a K-matrix coupled-channel scheme with forward dispersion relations for $K^\pm p$ and $K^\pm n$, with strong constraints resulting for the subthreshold extrapolation of $\bar{K}N$ amplitudes.

We have taken realistic density distributions for various proton shells as determined by electromagnetic probes. The sensitivity to finer details of the neutron surface is minor because of the dominance of the resonant $\bar{K}p$ interaction and the corresponding relative weakness of the $\bar{K}n$ interaction.

Given accurate $\bar{K}N$ input data and precise nuclear distributions, our aim has been to study the sensitivity of kaonic shifts and widths to properties of the Y_0^* resonance in the presence of surrounding nucleons. Shell effects have been studied selectively for the bound protons. The kinematic shift due to nucleon binding moves the effective $\bar{K}$ -proton centre-of-mass energy right into the Y_0^* resonance region, as shown earlier in Ref. 4. According to Section 4, there is room for an additional dynamical shift of the Y_0 mass and an increase of its width because of coupling to many-body degrees of freedom. For a discussion of such features, it is necessary to go beyond simple local approximations of the lowest order optical potential, as pointed out in Ref. 6. Comparison with the coupled-channel calculation in Ref. 6a shows that a reasonable approximation to the non-local resonant s-wave optical potential can be obtained by adding a $\vec{V}_\rho \vec{V}$ term to the lowest order closure result. The contribution of p-wave $\bar{K}N$ interactions, although negligible at $\bar{K}N$ threshold because of the small scattering volumes, becomes

non-negligible below threshold because of the Y_1^* resonance which is located only about 20 MeV below the Y_0^* position. Modifications of the shifts and widths due to the presence of the Y_1^* are typically of the same order of magnitude as non-localities due to Y_0^* propagation, at least for positive Y_0^* mass shifts $\text{Re } \Delta M^*$.

If compared to experimental data for ^{12}C and ^{32}S , all models (local s-wave, non-local s-wave, with or without Y_1^* p-wave resonance) suggest a trend towards a repulsive ($\sim 20\text{--}30$ MeV) shift of the Y_0^* mass, possibly accompanied by an increased width, in addition to the kinematical upward shift due to nucleon binding.

The purely phenomenological treatment of self-energy effects in terms of a complex mass shift parameter is of course not ultimately satisfactory. We note that our results with constant Y_0^* mass shift are consistent with those of Thies [6a]; on the other hand, a local density ansatz for the Y_0^* potential of the form $W_0\rho(r)$ would bring us nowhere close to experiment if W_0 is repulsive*). Such a local density approximation would, however, be hard to justify, especially for the $\pi\Sigma$ piece of Fig. 3c.

Clearly, a calculation of all the self-energies of Fig. 3 for finite nuclei is highly desirable and is in progress [20]. For the Pauli term (Fig. 3a), this is relatively simple and can be done straightforwardly in the coupled-channel scheme [6b], together with a self-consistent inclusion of binding and absorption corrections for the kaon (partly contained in Fig. 3b). But consideration of the coupling to $\pi\Sigma$ many-body channels is certainly necessary in order to arrive at a more complete picture.

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*) M. Thies, private communication.

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Table 1

Single particle energies and root mean square momenta for various shells in ^{12}C and ^{32}S used in the optical potential of Eqs. (14) and (18)

	$-\varepsilon_{\alpha}$ (MeV)				$\langle p^2 \rangle_{\alpha}^{\frac{1}{2}}$ (MeV/c)			
α	1s	1p	1d	2s	1s	1p	1d	2s
^{12}C	38	17.5	-	-	154	185	-	-
^{32}S	50	24	10.8	8.8	124	161	191	191

Table 2

Upper level widths of kaonic X-ray transitions in ^{12}C and ^{32}S calculated with local s-wave optical potential [column (1)], local s-wave plus p-wave potential [column (2)] and full potential of Eqs. (14) and (18) [column (3)], as function of Y_0^* mass shift $\text{Re } \Delta M^*$. Introduction of an increased Y_0^* width with $\Delta\Gamma/\Gamma_0 = 0.4$ lowers these numbers by between 5% and 20%.

	Γ_{3d} (eV)			Γ_{4f} (eV)		
ΔM^* (MeV)	^{12}C			^{32}S		
	(1)	(2)	(3)	(1)	(2)	(3)
-20	0.82	0.84	0.96	3.11	3.21	4.25
-10	1.11	1.13	1.29	4.46	4.52	5.97
0	1.23	1.25	1.36	5.62	5.66	6.53
+10	1.22	1.21	1.25	5.36	5.46	5.77
+20	1.20	1.13	1.16	4.77	4.92	5.05
+30	1.14	1.05	1.08	4.46	4.50	4.61
Exp.	0.98 ± 0.19			3.25 ± 0.41		

Figure captions

- Fig. 1 : K^-p and K^-n s-wave subthreshold amplitudes as determined by Martin [Ref. 11].
- Fig. 2 : Resonant K^- -nucleus interaction through excitation of a Y_0^* resonance together with a proton hole. The proton shell is denoted by α .
- Fig. 3 : Contributions to the Y_0^* self-energy:
- a) Pauli correction (Fock term);
 - b) Kaon rescattering through Y_0^* intermediate state;
 - c) Modification of $\pi\Sigma$ decay channel due to interaction of π through nucleon- or Δ -isobar-hole excitation.
- Fig. 4 : Shift and lower width for kaonic ^{12}C calculated with the optical potentials of Eqs. (14) to (16) and (18). The s-wave input parameters are according to Ref. 7 (fit to the Kim amplitude, see text).
- Curve A: local s-wave potential only [Eq. (10) plus background terms].
- Curve B: local s-wave, plus p-wave [Eq. (18)] potential.
- Curve C: as curve B, with non-local s-wave correction according to Eq. (14).
- The sequence of points on each curve corresponds to variation of the Y_0^* mass shift $\text{Re } \Delta M^*$ [see Eq. (29)], where the numbers in parentheses (same for all curves) indicate values of $\text{Re } \Delta M^*$ in MeV. The free width of the Y_0^* has been used here (i.e. $\Delta\Gamma = 0$). Experimental data are taken from Ref. 2.
- Fig. 5 : As Fig. 4, but using the s-wave K^-N amplitude of Martin (see Fig. 1) as input.
- Fig. 6 : Effect of increasing the Y_0^* width by 40% over the free width (right curve). The full optical potential (local and non-local s-wave plus p-wave) has been used here, with the Martin amplitude of Fig. 1 as input. Notations are otherwise the same as in Fig. 4.

Fig. 7 : Same as Fig. 5, for kaonic ^{32}S .

Fig. 8 : Same as Fig. 6, for kaonic ^{32}S .

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5

Fig. 6

Fig. 7

Fig. 8