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BOOTSTRAP THEORY OF QUARKS

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ABSTRACT

Combinatorial topology reveals particle quark structure to be a consequence of S-matrix causality and unitarity. The two "basic quarks" correspond to symmetrical and unsymmetrical triangles; out of these, particle faces are built belonging both to hadrons and to weakly-interacting particles. Hadron quarks must themselves be built from at least three basic quarks and carry topological quantum numbers that include baryon number, four primary flavours with exact up-down symmetry and three colours. There is an infinite number of secondary flavours with successively-higher thresholds, a guess for the multiplicity of the next flavour group being 32.

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The event structure of physical (macroscopic) space time is effectively pointlike, with causal connections between events corresponding to physical particles. This fact, when combined with the quantum superposition principle, leads through the requirement of S-matrix unitarity to the association of analytic connected-part singularities with graphs (on-shell Landau graphs, not Feynman graphs) whose vertices correspond to connected parts (events) and whose arcs correspond to particles ⁽¹⁾. Assuming connected parts to be "built" from their singularities, we then face a problem of graphical self consistency-- each vertex in a Landau graph being "built" from an infinite sum of graphs. This graphical bootstrap problem can be approached through combinatorial topology, exploiting the fact that any graph can be embedded in some 2-dimensional manifold (surface). The general topological bootstrap approach will be described in detail elsewhere ⁽²⁾, but we here list a few essential ingredients. (A) Each connected part M_{fi} , connecting final and initial channels is decomposed into an infinite sum of topological components,

$$M_{fi} = \sum_Y M_{fi}^Y, \quad (1)$$

each component being associated with a 2-dimensional pseudomanifold (closed pseudosurface) capable of embedding its singularities ⁽³⁾. Each of these manifolds is represented by a polyhedron each of whose faces corresponds to one of the ingoing or outgoing particles. Faces with an odd number of edges are fermions; those with an even number are bosons. Since the closed surface is completely covered by particle faces, and the total number of face edges must be even, the total number of fermions among ingoing and outgoing particles

is even. A general basis for convergence of the topological expansion is described at the end of this paper.

(B) A Landau graph product of connected parts corresponds to addition of polyhedra by identification of corresponding face pairs, each pair belonging to an intermediate particle emitted by one polyhedron and absorbed by another. The full graph then corresponds to a new polyhedron, covered by faces belonging to external particles-- the intermediate-particle faces having disappeared.

(C) Because the complexity of a sum of a 2-dimensional pseudomanifolds (as measured, e. g., by genus and cross-cap number) ⁽⁴⁾ is greater than or equal to the sum of the complexities of ingredient pseudomanifolds, Landau (Cutkosky) discontinuity formulas are nonlinear only for spherical components of the topological expansion. That is to say, if we substitute the topological expansion (1) into any discontinuity formula

$$\text{disc } M_{fi} = M_{jk} \oplus M_{lm} \oplus M_{np} \dots, \quad (2)$$

and associate each term on the right with a corresponding polyhedron on the left, to achieve

$$\text{disc } M_{fi}^Y = \sum_{Y'Y''Y'''} M_{jk}^{Y'} \oplus M_{lm}^{Y''} \oplus M_{np}^{Y'''} \dots \quad (3)$$

then for a given surface complexity on the left, the same complexity can appear no more than once on the right, except for the sphere. Furthermore the other complexities on the right must be less than that on the left, except in the case of the sphere. The nonlinear bootstrap problem thus becomes concentrated into the spherical (planar) components of the topological expansion, all higher terms being calculable in succession through linear relations once the

spherical components are known⁽²⁾.

Breathtaking insight into particle properties flows from the foregoing, including potential understanding of the difference between strong and weak interactions as corresponding, respectively, to orientable and nonorientable surfaces. Generally speaking a hierarchy of broken symmetries is automatic-- the simpler and more important surfaces having the most symmetry. The striking occurrence of "threeness" in patterns of internal quantum numbers derives from the fact that the simplex for 2-dimensional manifolds is the triangle. It is plausible that all particle properties, quantitative as well as qualitative, flow without ambiguity from unitarity, as illuminated by combinatorial topology, but in this paper we concentrate on the single issue of hadron quark content-- baryon number, flavor and color.

What is the topological significance of the quark concept? Any particle face of a polyhedron is a partial polyhedron with a single boundary (perimeter) that corresponds to a polygon, each of whose edges (sides) carries a direction. The polyhedron is achieved by identifying face edges in pairs. Unitarity will be seen to demand that each particle face be divided into triangles whose edges carry directions, each particle type (e. g. ,proton) being associated with a unique collection of adjacent triangles that cover the particle face. There are only two different triangles, the symmetrical and the unsymmetrical, as shown in Fig.1. These we shall characterize as " basic quarks". It will develop that hadron faces have the special characteristic of being subdividable into more complex "hadron quarks", each built from at least three basic quarks.

Unitarity precludes quarks, either basic or complex, from individually corresponding to a particle face. A key aspect of unitarity, lying at the heart of the particle concept, is pole-residue factorizability. Physically, factorizability means that each particle possesses an identity independent of the channels in which it appears as a resonance (or bound state). Topologically such independence forbids divisibility of any triangulated particle face into two or more other triangulated particle faces. (An important aspect of the bootstrap problem, as we see below, is to find a set of particle triangulation patterns that consistently satisfies this requirement.) Because quarks, by definition, are discs from which particle faces are built, quarks cannot themselves be particle faces.

What is the special topological characteristic of a hadron face? If strong interactions correspond to orientable surfaces, including spheres, then hadron faces must be orientable discs (2-cells) and carry a corresponding conserved quantum number. To cover an orientable surface the total number of clockwise edges must equal the total number of counterclockwise edges. For each hadron face, we therefore define a conserved quantum number q as the excess of clockwise edges over counterclockwise edges on the perimeter. Constituent discs, such as basic quarks or hadron quarks, within the hadron face may also be assigned a value of q , which is seen to be additive. (Our two-basic quarks have $q = \frac{1}{3}$ and $q = \frac{2}{3}$ respectively.) It will turn out that baryon number is equal to $q/3$.

The quantum number q is not conserved for the nonorientable faces that we expect to correspond to leptons, heavy vector bosons, etc...Such faces cannot appear on orientable surfaces; hence the

associated particles cannot participate in strong interactions. (They nevertheless may be built from basic quarks). The converse is not true. Orientable faces may appear on nonorientable polyhedra, so hadrons may participate in weak interactions.

The nonlinear discontinuity formulas for the spherical components of the topological expansion correspond to unitarity in a Hilbert space of discs (or hemispheres)-- combinations of adjacent particle faces which collectively form a single disc. We shall call these channel discs. This idea has been described in Ref. (5) using the term "ordered S-matrix" and employing graphs dual to the polyhedra. Ref. (2) discusses in detail the special case of particle faces with 2-gon (lune) perimeters which belong to mesons and for which the dual graphs are the celebrated Harari-Rosner duality diagrams. (The idea of using combinatorial topology to analyze unitarity arose from Harari-Rosner diagrams). Ref. (5) introduced the notion that all particle faces (even meson 2-gons) should be divided into triangles, but only symmetrical triangles ($q = \frac{1}{2} + 3$) were employed and ^{to the} vertices of these triangles indices were attached. References (6) and (7), still using dual graphs, explore the idea in depth. This scheme succeeded in illuminating a number of physically-important questions and drew attention to a basic requirement: the complete contractability of the sphere. We explain below the meaning of topological contraction and make heavy use of the notion. However, indices that are not simply shorthand for the triangulation pattern have led to difficulties in achieving full satisfaction of unitarity. In this paper, we avoid such indices by employing unsymmetrical ($q = \frac{1}{2} + 1$) as well as symmetrical ($q = \frac{1}{2} + 3$) triangles. We then find it possible to deal more completely than

in Ref. (5) with pole-residue factorization.

Recall the demand that no particle face be divisible into two or more other particle faces. In the scheme of Ref. (5), this requirement was not met since all hadron faces were composed of symmetrical triangles--each of which could be interpreted as a baryon. How can such a topological demand conform to the fact that a spherical channel whose resonances are single particles, is a disc built from adjacent particle faces. The answer is provided by the notion of disc contraction, a principle central to the rules invented by Harari and Rosner for meson lunes. That is, two meson lunes, side by side with matching flavor indices, may contract into a single lune, when interpreted as a resonance. Three appropriately matched lunes, by two successive contractions, also become a single lune, etc.. There is thus no way to see in the topological representation of a meson by a lune, that any particular multimeson channel has a favored status.

The Harari-Rosner scheme is not purely topological, since it involves indices. It also is based on lunes rather than triangles, which are more general. We propose to generalize and reformulate in purely topological language the Harari-Rosner procedure. From this generalization comes an understanding of hadron quarks-- including their flavor and color.

Our contraction rule is simply that when two or more hadron faces are adjoined to form a channel disc, the resonance-particle face is to be obtained by erasing all pairs of adjacent conjugate subdiscs*. Conjugation of a triangulated disc means "turning it over" and

* The full disc must of course not be erased even if it is self conjugate-- built of two conjugate half discs.

looking at it from the opposite side. It is easy to show that the triangulated face for an antiparticle is the conjugate of that for the corresponding particle. So if indeed, all internal quantum numbers are represented by triangles with oriented edges, then a conjugate disc pair carries zero quantum numbers and its erasure does not disturb the overall balance. Such erasure, furthermore, if the two members of the conjugate pair are adjacent, does not disturb the relative topology of remaining subdiscs.

We now require two further conditions: (1) All polyhedra are completely contractable (when regarded as a whole, without attention to the interface boundaries). Both the Harari-Rosner scheme and its generalization in Ref. 5 have this property, which is to be expected if conserved internal quantum numbers reside in the triangulation pattern. (2) The particle face resulting from all possible contractions within a channel disc is unique; i. e. it does not depend on the order in which contractions are made. Again, Harari- Rosner contractions and their generalization in Ref.(5) obey this requirement. If there exists no general topological theorem to guarantee condition (2), we must add this condition to those imposed by unitarity on particle-face triangulations.

The final demand responds to the starting point of our consideration of contraction: We require that for two or more particles to "resonate", i.e. for their adjoined faces to constitute a channel disc, contractions must erase the original interface edges. A natural way to accomplish all foregoing requirements for spherical polyhedra built from hadron faces is as follows:

(1) Divide a hadron face with n edges on its perimeter into n triangular

zones, each of which contributes one edge to the perimeter. See Fig.2. For the moment we do not consider edge directions. Each of these triangular subdiscs, even if triangulated further, will be called a "hadron quark".

(2) Require that, for adjacent hadron faces to constitute a spherical channel disc, the triangulation patterns ("quark content") of the hadron quark pair adjacent to each interface edge must be conjugate. I. e. , hadron quark must be adjacent to hadron antiquark. Complete contractibility for any polyhedron is thereby assured.

What kind of triangulation interior to a hadron quark is possible? In general there are two ways to divide a single triangle into 3 triangles, as shown in Fig.3, which we shall describe as triangulation of the first and the second-kinds respectively. (The creation of hadron quarks, as we discuss later, is an application of first-kind triangulation.) Now maintenance of complete contractability for the polyhedron means that first-kind triangulation within any particle face can immediately be contracted away unless the face includes on its perimeter two successive edges joined at a vertex of valence 2 (a trivial vertex). Since creation of hadron quarks removed all such vertices from the hadron face perimeter, the only meaningful triangulation of a hadron quark is the second kind. Second-kind triangulation (Fig. 3b) singles out one of the 3 hadron-quark edges. Again because of complete polyhedron contractibility, the only meaningful possibility is when the special hadron-quark edge lies on the particle-face perimeter. Thus the simplest structure for a hadron quark is that of Fig.4, where the face-perimeter edge is shown heavy and given a direction. The intermediate interior edge is

assigned the unique direction that does not allow contraction of the structure. There remain four edges, each of which can be assigned two possible directions; thus 16 different hadron quarks exist of the simplest type*.

We now show that with respect to mesons the 16 hadron quarks correspond to four flavors. A meson disc starts with the triangulated structure of Fig. 5(a), where each of the indices, i, j, k corresponds to four possible direction combinations. Contraction to Fig. 5(b) eliminates (sums over) the index j , so the meson face is composed of lunar quark ($q=2$) and antiquark ($q=-2$). Each lunar quark (Fig. 6) is characterized by a four-valued index that we identify with flavor, since it is easily seen that the rules for flavor contraction when mesons are combined are exactly those of the Harari-Rosner scheme.

We choose to number the four flavors as shown in Fig. 7. Under reversal of all edge directions, followed by conjugation, an operation that leaves quark number unchanged, flavor 1 becomes interchanged with flavor 2, while flavors 3 and 4 remain unaltered. One therefore expects to associate flavors 1 and 2 with "up" and "down" and to find exact up-down symmetry in all orientable orders of the topological expansion. Because flavors 1, 2, 3 correspond to unsymmetrical triangles adjacent to the particle-face perimeter, while flavor 4 corresponds to a symmetrical triangle, one is led to conjecture that flavor 3 is strangeness while flavor 4 is

* Four of the sixteen can be contracted to basic quarks, but because the intermediate edge removed by the contraction has a unique direction, the physical content of the uncontracted and contracted patterns is the same. When erased intermediate edges can assume a variety of directions, contraction corresponds physically to summing over all directions.

charm. An optimistic guess, currently under scrutiny, is that SU_3 flavor symmetry is a characteristic of the sphere, broken by higher components of the topological expansion, while there is no SU_4 flavor symmetry at any level.

The next simplest set of flavors, achieved by second-kind triangulation, corresponds to "lunar quarks" with the general structure shown in Fig. 8. Here there are 5 internal edges and only one pair constrained by the contraction rule, so the index i takes 16 possible values. Together with the reflected quark (the internal lune placed on the right side instead of the left), the total is 32. We characterize the flavors of Fig. 7 as primary and those of Fig. 8 as secondary. It is not known whether unitarity in fact permits secondary flavors, but we have no reason at this point to suppose the possibilities exhausted by primary flavors. Our best guess is that the number of flavors is indefinitely large. From a dynamical standpoint, primary flavors are compatible with a purely meson bootstrap, while secondary flavors involve extra uncorrelated baryon-antibaryon pairs. One therefore expects secondary flavors to have a high threshold, which increases with the complexity of the associated lunar quark.

Fig. (2) shows that the structure of all hadrons except mesons is characterized by the triangular hadron quark rather than the lunar quark because there is no possibility of erasing part of the hadron quark boundary by contraction. The triangular hadron quark content, as shown in Fig. (9), involves two 4-valued indices-- which we refer to as outer (i) and inner (j). The outer index may always be interpreted as flavor because this index alone determines which mesons may be adjoined to the perimeter edge. The contraction which

eliminated the inner index from meson quark content means that in effect each meson carries all values of the inner index. Thus any meson whose flavor index matches the outer index of a triangular hadron quark, may be adjoined to a particle-face perimeter edge belonging to that quark.

One may guess that the inner index has a connection with the concept that in quark models and quantum chromodynamics has been called "color". Accordingly we shall tentatively refer to the inner index as "topological color". Our statement that all values of the inner index are contained in each meson might be expressed by saying that any meson is a "color singlet". The inner index will be reflected in the multiplet structure of hadrons other than mesons, but there is a sense in which all hadrons might be described as color singlets. This sense relates to the zero triality rule that hadron faces must have q values that are multiples of 3 (baryon number $=q/3$). This rule has not been justified here on a unitarity-basis, leaving an important loose end. But suppose we adopt a major component of the scheme proposed in Ref. (5) by the following 2-step rule* for edge directions on the hadron faces of Fig. (2):

- (1) Combine n symmetrical triangles so as to achieve a polygon of degree $n+2$ with all vertices on the perimeter**.
- (2) Divide each symmetrical triangle into 3 hadron quarks by first-kind triangulation, matching edge directions so that contractions erase the internal edges of the symmetrical triangles.

* The rule is self consistent but it must be shown that other self-consistent rules do not exist.

** The meson case is exceptional. Here 2 symmetrical triangles are combined to achieve a 2-gon with an internal vertex.

Now within each symmetrical triangle, before contraction, the inner indices are constrained to a total of $2^3 = 8$ different combinations (see Fig. 10). It is plausible that two of these combinations, yielding the central vertices of Fig. 11, should be eliminated on the unitarity grounds discussed in Refs. (5), (6), (7), that they leave certain particle faces on one polyhedron with an overall symmetry that prevents unique identification with a corresponding hadron face on some other polyhedron. The remaining six inner index combinations may then be put into correspondence with the permutations of three "differently-colored" hadron quarks, if we now define hadron quark color as the q value $+3, +1, -1$. Thus before contraction to achieve the final hadron face covered with hadron quarks and antiquarks, one sees that quarks always appear in nested combinations of 3 different colors (Ditto for antiquarks).

By using standard analyticity considerations in complex energy-momentum one should now be able to develop an explicit hadron spectrum for the sphere. Hopefully, hadron spins will emerge along with hadron masses.

We close with the remark that spherical terms dominate the topological expansion primarily because all intermediate channels contributing to spherical discontinuities correspond to single resonances. discontinuities of more complex topological components always involve uncorrelated multiparticle channels-- not associated with discs-- which are suppressed by phase-space factors. We do not of course, appeal to the existence of a large number of flavors and colors to justify our a priori assumption of effective convergence for the topological expansion. Internal quantum numbers are not

assumed but are generated by the requirements of self consistency. It is nevertheless well known that topological convergence is assisted by the existence of many internal quantum numbers, and we are happy to accept such help.

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FIGURE CAPTION

1. The two "basic quarks": symmetrical (a) and unsymmetrical (b) triangles.
2. Division of hadron faces with n-gon perimeters into n hadron quarks.
3. Triangulation of the first (a) and second (b) kinds.
4. Simplest structure for a hadron quark. The face-perimeter edge is shown heavy.
5. Partial contraction of triangular hadron quark-antiquark pair, building a meson face, into a lunar quark-antiquark pair. Topological color disappears through the contraction.
6. A primary flavor lune.
7. The four primary flavors.
8. A secondary flavor lune.
9. Topological -color (j) and flavor (i) structure of a triangular hadron quark.
10. Topological color structure within a symmetrical triangle.
11. Symmetrical topological- color structure disallowed by unitarity.

(a)

(b)

Fig. 1

n =

2

3

4

5

Fig. 2

(a)

(b)

Fig. 3

Fig. 4

Fig. 5

Fig. 6

Fig. 7

Fig. 8

Fig. 9

Fig. 10

Fig. 11