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PREDICTIONS FOR THE TOP QUARK MASS
AND FOR THE GENERALIZED CABIBBO ANGLE

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A B S T R A C T

The next "onia" states made of $(t\bar{t})$ pairs are predicted at ~ 40 GeV ; the generalized Cabibbo angles $\theta_2 \simeq 7 \cdot 10^{-3}$ and $\theta_3 \sim 7 \cdot 10^{-2}$. The transition $t \rightarrow d$ is thus expected to be inhibited by about two orders of magnitude with respect to the standard Cabibbo transition $u \rightarrow s$. The basis for these predictions is a simple ansatz for the mass matrix of quarks and leptons.

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Following the discovery at CERN of neutral currents in weak interactions five years ago, the striking sequence of experimental results revealing the existence of the J/ψ , the τ lepton, the charmed particles, and more recently the T and the T' mesons, has perhaps brought to maturity a problem which is both fundamental and fascinating : the spectrum of quarks and leptons.

In fact a large number of papers has recently appeared trying to "decode" the mass pattern of quarks and leptons either on the basis of the gauge theories of the weak and electromagnetic interactions ^{1),2)} or following a purely empirical approach ³⁾. In this letter I shall follow a somewhat intermediate course.

There is an almost universal consensus among the high energy physicists that the fundamental building blocks of matter are the ones appearing on the Table.

Generation	lepton type	Q	Y_W	I_{3W}	quark type	Q	Y_W	I_{3W}
1	ν_e	0	-1	$\frac{1}{2}$	u	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{2}$
	e	-1	-1	$-\frac{1}{2}$	d	$-\frac{1}{3}$	$\frac{1}{3}$	$-\frac{1}{2}$
2	ν_μ	0	-1	$\frac{1}{2}$	c	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{2}$
	μ	-1	-1	$-\frac{1}{2}$	s	$-\frac{1}{3}$	$\frac{1}{3}$	$-\frac{1}{2}$
3	ν_τ	0	-1	$\frac{1}{2}$	t	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{2}$
	τ	-1	-1	$-\frac{1}{2}$	b	$-\frac{1}{3}$	$\frac{1}{3}$	$-\frac{1}{2}$

The fundamental (left-handed) building blocks of matter as of 1978 : (Q = charge, Y_W = weak hypercharge, I_{3W} = third component of the weak isospin).

Whether there should be other generations beyond the third, is an obviously open experimental question, which has been inviting some recent theoretical speculations ^{3),4)}.

Looking now at the experimental information (with the caveat that the determination of the masses of quarks carries considerable theoretical uncertainties ⁵⁾) three basic facts are worth remarking :

- (i) the neutrinos ν_e, ν_μ and ν_τ , the only fermions without charge, have all a mass which is consistent with zero ;
- (ii) the charged fermions, leptons and quarks, have a well-ordered mass pattern which makes it possible to assign them to "generations" ⁶⁾ with only small mixings at the weak interaction level :

$$\begin{aligned} m_e &< m_\mu < m_\tau \\ m_u &< m_c < m_t \\ m_d &< m_s < m_b \end{aligned} \quad ; \quad (1)$$

- (iii) for the first generation leptons and quarks there seems to be a basic SU(2) symmetry (within a few MeV)

$$\begin{aligned} m_e &\simeq m_{\nu_e} \\ m_u &\simeq m_d \end{aligned} \quad (2)$$

It is thus tempting to speculate that the electromagnetic interaction, with its proportionality to the charge Q ,

$$Q = I_{3W} + \frac{Y_W}{2} \quad (3)$$

acts as a "trigger" or a "catalyzer" for the mass symmetry breaking mechanism. This mechanism lifts the fundamental symmetry among quarks and leptons of different generations, which seems to be at the very root of the group $SU(2)_W \otimes U(1)$ ⁷⁾ governing the weak interactions and their probable unification with electromagnetism.

I have no definite proposal to make regarding the specific mechanism for such a symmetry breaking ⁸⁾ ; rather I shall simply assume that the mass matrix is proportional to the lowest non-trivial powers of the charge operator Q , and write M as

$$M = Q G_1(N^2) + Q^2 G_2(N^2) \quad (4)$$

where G_1 and G_2 are 3×3 Hermitian matrices acting on the three-dimensional "generation" space, and N^2 is any operator distinguishing between

quarks and leptons (for example, N = baryon number). The equality of the particle and antiparticle masses requires (in the basis where the charge conjugation operator is represented simply by transposition),

$$\begin{aligned} G_1^T &= -G_1 \\ G_2^T &= G_2 \end{aligned} \quad (5)$$

Thus G_1 is given by a linear combination, with real coefficients depending on the operator N^2 (which distinguishes between quarks and leptons, see the Table) of the "odd" Gell-Mann λ matrices λ_2, λ_5 and λ_7 ; whereas G_2 is expressed by a linear combination of $\lambda_0, \lambda_1, \lambda_3, \lambda_4, \lambda_6$ and λ_8 . Furthermore by an orthogonal transformation we can bring G_2 to a diagonal form, with G_1 still given by a linear combination of the "odd" λ matrices. Redefining the phases of the states we can thus write

$$G_1 = \begin{vmatrix} 0 & \rho & \sigma \\ \rho & 0 & -i\tau \\ \sigma & i\tau & 0 \end{vmatrix} \quad (6)$$

and

$$G_2 = \begin{vmatrix} \alpha & 0 & 0 \\ 0 & \beta & 0 \\ 0 & 0 & \gamma \end{vmatrix} \quad (7)$$

with $\alpha, \beta, \gamma, \rho, \sigma$ and τ real functions of the operator N^2 . The following are immediate consequences of the ansatz (4) :

- (a) neutrinos ($Q=0$) are massless ;
- (b) no CP violation appears in the leptonic sector ;
- (c) the following mass relation holds

$$m_u + m_c + m_t = 4(m_d + m_s + m_b) \quad (8)$$

- (d) the quark sector shows a CP violation of the same strength as the Cabibbo angle.

This last point obviously follows from the fact that were it not the first term in (4) there would be no Cabibbo angle as well as no CP violation. Thus in our simple ansatz Cabibbo mixing and CP violation have the same origin.

The smallness of the generalized Cabibbo angles ⁹⁾ suggests that $G_1 \ll G_2$, implying that a mass matrix containing only the second term in (4) must be able to describe pretty well the real world. This turns out to be the case, in fact keeping only G_2 and imposing the $SU(2)$ symmetry implied by (2) [i.e., setting $\alpha(N^2) = 0$] we obtain that :

- (a') neutrinos are massless ;
- (b') there is no CP violation ;
- (c') the electron is massless ;
- (d') the u,d quarks are massless ;
- (e')

$$m_c = 4 m_s \quad (9)$$

$$m_t = 4 m_b \quad (10).$$

As for the "experimental" quark masses I shall take the quark-geometrodynamic determination ¹⁰⁾ :

$$\begin{aligned} m_u &\simeq m_d = .10 \text{ GeV} \\ m_s &= .47 \text{ GeV} \\ m_c &= 1.86 \text{ GeV} \\ m_b &= 5.2 \text{ GeV} . \end{aligned} \quad (11)$$

The relation (9) is very well satisfied, from (11) one has in fact

$$\frac{m_c}{m_s} = 3.96 ;$$

and from (10) the top-quark mass is predicted to be

$$m_t \simeq 21 \text{ GeV} .$$

The residual term in the mass matrix should then :

- (i) give a small mass to the electron ;
 - (ii) give a mass to the u, d quark, while preserving $SU(2)_I$ symmetry ;
 - (iii) give the quark mixings in terms of the quark masses and eventually some other parameter ;
 - (iv) introduce a small CP violation ;
- and so it does.

In the framework of the Kobayashi-Maskawa phenomenology we compute [on the assumption that $\sigma/\rho \ll 1$ ⁽¹¹⁾] (see Eq. (6))

$$\begin{aligned}
 \theta_1 &= \frac{1}{2} \left(\frac{m_d}{m_s} \right)^{1/2} \\
 \theta_2 &= \frac{1}{2} \left(\frac{m_d}{m_b} \right)^{1/2} \left(\frac{m_s}{m_b} \right) \\
 \theta_3 &= \frac{1}{2} \left(\frac{m_d}{m_b} \right)^{1/2} \\
 \delta &\simeq \left(\frac{\sigma}{\rho} \right) \frac{m_b^{3/2}}{m_s m_d^{1/2}} ;
 \end{aligned}
 \tag{12}$$

which imply, using the values (11) for the quark masses,

$$\begin{aligned}
 \theta_{\text{Cabibbo}} &\simeq .23 \\
 \theta_2 &\sim 7 \cdot 10^{-3} \\
 \theta_3 &\sim 7 \cdot 10^{-2} .
 \end{aligned}
 \tag{12'}$$

Thus we are able to compute correctly the Cabibbo angle and obtain a very small value for the angle θ_2 (inhibiting the transition $t \rightarrow d$). As for the CP violating phase δ we cannot calculate it, depending on the undetermined ratio σ/ρ , however, it is reasonable to expect it to be quite large due to the large value (~ 35) of the coefficient multiplying σ/ρ . Taking $\delta = O(1)$ we estimate ⁽¹²⁾ the CP violating parameter of K physics as :

$$|\epsilon| \simeq 1.5 \cdot 10^{-3},$$

and the neutron-electric dipole moment as ($m_W \simeq 60$ GeV)

$$\left| \frac{D}{e} \right|_n \simeq 10^{-31} \text{ cm.}$$

We may then conclude that our ansatz (4) provides a consistent picture of quark and lepton masses and mixings. In particular it predicts the values of the top-quark mass and of the generalized Cabibbo angles.

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REFERENCES

- 1) See, e.g. : S. Weinberg - "The Problem of Mass", Harvard preprint HUTP/77/A057 ;
S. Glashow - "A Question of Flavour", Harvard preprint HUTP/78/015.
- 2) For a nice review of these and related problems, see : H. Harari - Physics Reports 42C, 238 (1978).
- 3) J.D. Bjorken - "Some Speculations on the Pattern of Quark and Lepton Masses", SLAC Preprint SLAC-PUB-2195 (1978).
- 4) L. Maiani, G. Parisi and R. Petronzio - Nuclear Phys. B136, 115 (1978).
- 5) They mainly concern the u, d quarks, due to the PCAC extrapolation. It is perfectly fair to state that the question whether $m_{u,d} \sim \text{few MeV}$ or $m_{u,d} \sim 100 \text{ MeV}$ has not, so far, been settled.
- 6) This term has been coined by H. Harari, Ref. 2).
- 7) A. Salam - Proceedings 8th Nobel Symposium, Stockholm (1968) (Ed. N. Svartholm, Almqvist and Wiksells, Stockholm, 1968) p. 367 ;
S. Weinberg - Phys.Rev.Letters 19, 1264 (1967).
- 8) Even though I have in mind the very interesting ideas of S. Coleman and E. Weinberg - Phys.Rev. D7, 7 (1973). In this paper the possibility is investigated that radiative corrections may have a very important rôle in spontaneous symmetry breaking.
- 9) M. Kobayashi and K. Maskawa - Progr.Theor.Phys. (Kyoto) 49, 652 (1973).
- 10) G. Preparata - "Quark Geometrodynamics : A New Approach to Hadrons and their Interactions", CERN Preprint TH. 2517 (1978), to be published in the Proceedings of the 1977 Erice Summer School (A. Zichichi, Ed.).
- 11) This assumption comes from the idea that mixings between non-neighbouring generations should be much smaller than those between neighbouring generations.
- 12) J. Ellis, M.K. Gaillard and D.V. Nanopoulos - Nuclear Phys. B109, 213 (1976).