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AMBIGUITIES IN THE INFRA-RED REGULARIZATION OF QCD

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ABSTRACT

The next-to-leading corrections of leading log asymptotic freedom are determined in the known infra-red (IR) regularization schemes. On- and off-shell calculation leads to different answers; the n dimensional regularization of IR and mass singularities agrees with the on-shell results. The non-log corrections in Drell-Yan processes are important for $Q^2 \lesssim 10^3 \text{ GeV}^2$.

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Last year a number of authors¹⁾ have shown that the mass singularities appearing in perturbative QCD are factorizable and can be absorbed in the parton momentum distributions. Phenomenology so far was mostly concerned with the leading log approximation and it is still an unsettled question as to the importance of the next-to-leading terms in time-like processes. Calculation of their structure has been attacked under various assumptions and was found to give different answers.

In this paper we present the first results of a careful and systematic investigation concerning this question which will lead us to the conclusion that the applied IR regularization procedure²⁾ is of predominant importance. We determine the lowest order QCD corrections to deep inelastic lepton-hadron scattering and massive lepton-pair production and calculate under specific mass assumptions the non-log correction terms of the constituent cross-sections denoted by $d\hat{\sigma}^{DY}$, $d\hat{\sigma}^{DI}$.

The integrated cross-sections in deep inelastic lepton-hadron scattering and Drell-Yan pair production are defined by

$$\nu W_2 = \alpha_s e_q^2 \int_x^1 \frac{dy}{y} u(y) d\hat{\sigma}^{DI}(z \equiv \frac{x}{y}) \quad (1)$$

$$s \frac{d^2 \hat{\sigma}}{dQ^2} = \frac{4\pi\alpha_s^2}{9} e_q^2 \int_0^1 \frac{dx_1}{x_1} \frac{dx_2}{x_2} \left[u(x_1)u(x_2) + (1 \leftrightarrow 2) \right] d\hat{\sigma}^{DY}(z \equiv \frac{s}{x_1 x_2}) \quad (2)$$

The considered diagrams are shown in Fig. 1 where we define our notation. We first concentrate on those with an initial state gluon since only mass singularities appear. Straightforward calculation (in Feynman gauge) for any on- and off-shell mass assignments leads to the result

$$d\hat{\sigma}_{g,i}^{DY} = \frac{1}{4\pi} \left[\{z^2 + (1-z)^2\} \ln \frac{-(1-z)Q^2}{z^2 M_i^2(z)} + \frac{1}{2} + z - \frac{3}{2} z^2 - R_i(z) \right] \quad (3)$$

$$d\hat{\sigma}_g^{DI} = \frac{1}{8\pi} \sum_{i=1}^2 \left[\{z^2 + (1-z)^2\} \ln \frac{-Q^2}{z^2 M_i^2(z)} - 1 + 6z - 6z^2 - R_i(z) \right] \quad (4)$$

where

$$M_i^2(z) = z^{-1}(1-z)^{-1} \left\{ z(1-z)p^2 - z p_i^2 - (1-z)m_0^2 \right\} \quad (5)$$

$$R_i(z) = \left\{ M_i^2(z) \right\}^{-1} \left\{ 4m_0 m_i - m_0^2 + p^2 - p_i^2 \right\} \quad (6)$$

In these expressions we have explicitly kept all those mass terms which survive in the Bj limit after having performed all integrations. Note that all mass dependence coming from the quark-lines which are connected with the photon drops out. Formation of the Drell-Yan α_s correction term $d\hat{\sigma}_{gi}^{DY} - d\hat{\sigma}_g^{DI}$ makes obvious that all mass dependence disappears^{*}).

More problems arise from the diagrams with the gluon in the final state since in addition to the mass singularities also IR singularities appear. Using the mass assignment of Fig. 2 the constituent cross-sections take the following structure

$$d\hat{\sigma}_q^{DY} = \frac{4}{3} \frac{1}{2\pi} \sum_{i=1}^2 \left[\frac{1+z^2}{1-z} \ln \frac{-(1-z)Q^2}{z^2 M_i^2(z)} - 1 + z + R_i(z) \right] + a \delta(1-z) \quad (7)$$

$$d\hat{\sigma}_{q,i}^{DI} = \frac{4}{3} \frac{1}{2\pi} \left[\frac{1+z^2}{1-z} \ln \frac{-Q^2}{z^2 M_i^2(z)} + 2 + 3z - \frac{3/2}{1-z} + R_i(z) \right] + b \delta(1-z) \quad (8)$$

where $M_i^2(z)$ and $A_i(z)$ are given by Eqs. (5), (6) with $p^2 \neq p_i^2$ in the former expression. These formulas are composed of the hard gluon contribution in the Bj limit plus the elastic scattering term (parton model + virtual gluon + soft gluon radiation). The coefficients of the δ function a, b change with different mass assignments and IR cut-off choice. In the difference $d\hat{\sigma}_q^{DY} - 2d\hat{\sigma}_{qi}^{DI}$ any mass dependence in the hard gluon part drops out, but it persists in $(a-2b)\delta(1-z)$ which, as we know already, is gauge⁴⁾ and renormalization independent^{**}). We therefore emphasize that this

*) For particular mass choices we recover the expressions given in the literature.

**) We have explicitly checked this property in ϕ^3 theory in six dimensions which is renormalizable, asymptotically free and without IR divergences.

quantity depends on the method of IR regularization which itself is directly correlated with the mass assignment. For each particular choice the actual change in (a-2b) can be traced down to the vertex function (although all contributions change) which analytically continued from the space-like to the time-like region produces an additional constant term due to the on-shell or off-shell IR regularization.

The above statements grew out of several different calculation procedures. In the off-shell case we followed, as one possibility, Ref. 4) and kept in all diagrams only one and the same loop internal mass m non-vanishing. The same results were obtained by assuming that all loop-internal masses vanish but the (non-loop) off-shell masses are non-zero and purely imaginary. The explicit forms for vertex + self-energy as well as the soft-gluon part were given in Ref. 3)*). We find

$$a = \frac{4}{3} \frac{1}{2\pi} (-1 - \pi^2) \quad , \quad b = \frac{4}{3} \frac{1}{2\pi} \left(-1 - \frac{1}{3}\pi^2\right) \quad (9)$$

Similar calculation for on-shell quarks has been performed as in QED in Refs. 5) and 6); we have in addition carried out partial integration in the hard-gluon part in order to cancel explicitly the $\log^2$ terms of the soft and vertex pieces. One finds

$$a = \frac{4}{3} \frac{1}{2\pi} \left(-4 - \frac{1}{3}\pi^2\right) \quad , \quad b = \frac{4}{3} \frac{1}{2\pi} \left(-\frac{5}{2} - \frac{\pi^2}{3}\right) \quad (10)$$

There is a slight change if, as in Ref. 6), the calculation is done for massive photons which, however, leaves (a-2b) unchanged. Inspection of this quantity for off- and on-shell mass assignment reveals a discrepancy of $(2\pi/3)$. It arises from different asymptotic space-like behaviour for off- and on-shell mass assignment which is $2 \cdot \ln^2(q^2/-p^2)$ in the former case and $\ln^2(q^2/-p^2)$ in the latter. Whilst analytically continuing to the time-like region we therefore pick up a factor $2\pi^2$ for off-shell quarks or π^2 if the quarks are on-shell. No analytic continuation would have left $(a-2b) = \frac{4}{3} \frac{1}{\pi} \left(\frac{1}{2} + \frac{\pi^2}{6}\right)$ for either choice of IR regularization.

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*) With both prescriptions we obtain the results given in Ref. 3) with the changes $-\delta(1-z)$ in Eq.19 and omission of (-1) in Eq. 27. The DY results are obtained from these equations by analytical continuation. In Ref. 3) the term $(\pi^2/3 - 1)$ is changed to $(\pi^2/3 + 1)$.

In order to gain more insight we study the scalar part of the vertex function and measure the off-shellness by $\mu^2 = m^2 - p_1^2$. Along the space-like region the vertex function depends quadratically and linearly on $\ln q^2$. We have formally determined the subsequent constant term which depends on the kinematical variable q^2 and the chosen masses. For our purposes we are only interested in its real part which is given by

$$\varphi = (\pi^2 - 2\pi\alpha) \theta(q^2 - \lambda^2 (\lambda^2 - \mu^2)^{1/2}) - \tilde{\pi}^2 + 2\alpha^2 \quad \text{with} \quad \alpha \equiv \arctan\left(\frac{\lambda \sqrt{q^2}}{\lambda^2 - \mu^2}\right) \quad (11)$$

We thus have the possibility to see explicitly the difference arising from the constant terms for on-shell and off-shell limits; $\mu^2 \ll \lambda^2$ gives $\varphi = -\frac{\pi^2}{2}$ and $\lambda^2 \ll \mu^2$ gives $\varphi = -\pi^2$ for any large q^2 .

The on- or off-shell calculations presented so far are non-straight-forward and cumbersome. We point to the n dimensional regularization method⁷⁾ to cancel the IR, UV and some of the mass divergences, which turns out to be a most powerful instrument, in particular in diagrams with increasing complexity, to determine the sought cross-sections in the Bj limit. All masses here are set to zero (which facilitates phase space calculation) and the only remaining mass parameters are due to the kinematical variables and the dimensionality of the coupling constant denoted by M^{4-n} . One finds

$$d\sigma_q^{DY} = \frac{4}{3} \frac{1}{\pi} \left[\frac{1+z^2}{1-z} \left(\frac{2}{n-4} + \gamma \right) + \frac{1+z^2}{1-z} \ln \frac{-(1-z)q^2}{z M^2} - 1 - z + \frac{2}{1-z} \right] + a \delta(1-z) \quad (12)$$

$$d\sigma_q^{DI} = \frac{4}{3} \frac{1}{2\pi} \left[\frac{1+z^2}{1-z} \left(\frac{2}{n-4} + \gamma \right) + \frac{1+z^2}{1-z} \ln \frac{-(1-z)q^2}{z M^2} + 2 + z + \frac{1/2}{1-z} \right] + b \delta(1-z) \quad (13)$$

where the constants due to the vertex + self-energy + soft-gluon contributions are given by

$$a = \frac{4}{3} \frac{1}{2\pi} \left(-5 + \frac{2\pi^2}{3} \right), \quad b = \frac{4}{3} \frac{1}{2\pi} \left(-3 - \frac{\pi^2}{3} \right) \quad (14)$$

The explicit forms of the vertex + self-energy and of the soft-gluon parts arising from Drell-Yan processes are

$$V^{DY} = \frac{4}{3} \frac{1}{2\pi} \left[-\frac{8}{(n-4)^2} + \frac{2-4\gamma}{n-4} - 5 + \gamma - \gamma^2 + \frac{7\pi^2}{6} - \frac{4 \ln q^2/M^2}{n-4} + (1-2\gamma) \ln \frac{q^2}{M^2} - \ln^2 \frac{q^2}{M^2} \right] \quad (15)$$

$$R-S_{\text{soft}}^{DY} = \frac{4}{3} \frac{1}{2\pi} \left[\frac{8}{(n-4)^2} + \frac{4+4\gamma}{n-4} + \gamma^2 + 2\gamma - \frac{\pi^2}{2} - \frac{4 \ln q^2/M^2}{n-4} - \frac{8 \ln M^2/\Delta^2}{n-4} - (2\gamma+2) \ln \frac{q^2}{\Delta^2} - (4+4\gamma) \ln \frac{M^2}{\Delta^2} + \ln^2 \frac{q^2}{M^2} + 4 \ln^2 \frac{M^2}{\Delta^2} \right] \quad (16)$$

and analogously for deep inelastic lepton-hadron scattering we obtain

$$V^{DI} = \frac{4}{3} \frac{1}{2\pi} \left[-\frac{8}{(n-4)^2} + \frac{2-4\gamma}{n-4} - 5 + \gamma - \gamma^2 + \frac{\pi^2}{6} - \frac{4 \ln \frac{q^2}{H^2}}{n-4} + (1-2\gamma) \ln \frac{q^2}{H^2} - \ln^2 \frac{q^2}{H^2} \right] \quad (17)$$

$$R-S_{\text{soft}}^{DI} = \frac{4}{3} \frac{1}{2\pi} \left[\frac{8}{(n-4)^2} + \frac{4+4\gamma}{n-4} + 2 + \frac{\gamma}{2} + \gamma^2 - \frac{\pi^2}{2} - \frac{4 \ln \frac{H^2}{\Delta^2}}{n-4} - (\frac{1}{2}+2\gamma) \ln \frac{M^2}{\Delta^2} + \ln^2 \frac{H^2}{\Delta^2} \right] \quad (18)$$

The cut-off parameter Δ^2 is cancelled after partial integration by the integrated hard gluon part. One notices the appearance of the $(n-4)^2$ or 1 poles which replace the $\ln^2$ and $\ln$ terms in the earlier discussed calculation procedure. The double poles completely cancel in the sum: vertex + soft + hard whereas the single poles disappear in $d\sigma_q^{DY} - 2d\sigma_q^{DI}$ after partial integration of the hard gluon part has been applied. We point to the dependence on the regulator mass M^2 (which scales q^2). γ stands for the Euler constant. We emphasize that this calculation procedure leads to the earlier results for (a-2b) found for the on-shell mass assignment.

The above findings lead us to conclude:

- i) the proposed methods of IR regularization in calculating integrated cross-sections provide us with non-unique answers; the mass limits are essential^{*)}

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^{*)} We have also investigate this problem in ϕ^3 in six dimensions and found for all mass choices the same answer.

- ii) the appearing non-log terms are in all cases of importance and dominate the leading log results to a substantial degree for $Q^2 \lesssim 10^3 \text{ GeV}^2$ 8)
- iii) application of the technique of n dimensional regularization is in this kind of problem most useful and prevents unnecessary technicalities.

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Figure captions

- Fig. 1 : Initial gluon graphs for lepton pair production (a), and for deep inelastic lepton-hadron scattering (b).
- Fig. 2 : Final and virtual gluon graphs for lepton pair production (a), and for deep inelastic lepton-hadron scattering (b).

fig.1

fig.2