Ref.TH.2638-CERN

MODEL INDEPENDENT ANALYSIS OF THE WEAK INTERACTION
IN e^- -N ELASTIC SCATTERING EXPERIMENTS

E.Ch. Christova

Institute of Nuclear Research and Nuclear Energy,
Bulgarian Academy of Sciences, Sofia

S.T. Petcov

CERN - Geneva
and

Institute of Nuclear Research and Nuclear Energy,
Bulgarian Academy of Sciences, Sofia

ABSTRACT

We discuss what model independent information about the parity non-conserving electron-quark neutral current interaction can be obtained in the experiments on elastic scattering of longitudinally polarized electrons on unpolarized nucleons. Two possible forms of the neutron electric form-factor $G_E^{yn}(q^2)$ are used in our analysis. The interpretation of the P odd asymmetry, measured in these experiments, in terms of electron-quark neutral current coupling constants depends on the form chosen for $G_E^{yn}(q^2)$. We note also that the polarized e^- -p elastic scattering experiments might be used to study the q^2 dependence of $G_E^{yn}(q^2)$.

In recent polarization deep inelastic e^- -D scattering experiments at SLAC¹⁾ parity violation effects have been observed and the existence of weak e^- -N interaction has been established. Of particular importance now is to determine the form of the relevant electron-quark neutral current couplings which are believed to generate effectively the observed interaction. The possibility to solve this problem in a model independent way in the experiments on parity violation effects in deep inelastic e^- -N scattering and in atomic transitions has been examined in Ref. 2).

The parity non-conserving electron-quark neutral current couplings can be studied experimentally in particular, by measuring the P odd asymmetry arising in elastic scattering of longitudinally polarized electrons on unpolarized nucleons:

$$e^- + N \rightarrow e^- + N \quad (1)$$

The predictions of various gauge models for the magnitude of the asymmetry in this process have been analyzed recently³⁾. In this note we discuss from a model independent point of view what can be learned about the electron-quark neutral current interaction in polarized e^- -N elastic scattering experiments. High precision experiments with relatively low energy electrons ($100\text{MeV} \lesssim E \lesssim 800\text{MeV}$) in which P odd asymmetries $\sim (10^{-5} - 10^{-7})$ will be measured are in preparation at present^{4),5)}. At high energies such experiments are possible at SLAC.

We assume that at presently accessible energies the parity violation effects in the e^- -N interaction can be described by the following effective electron-quark current $\times$ current Lagrangian⁶⁾:

$$\begin{aligned} \mathcal{L}_{e-q}^{PV} = & -\frac{G}{\sqrt{2}} \left\{ \bar{e} \gamma_\mu \gamma_5 e \left[\frac{1}{2} \tilde{\alpha} (\bar{u} \gamma_\mu u - \bar{d} \gamma_\mu d) + \right. \right. \\ & + \frac{1}{2} \tilde{\gamma} (\bar{u} \gamma_\mu u + \bar{d} \gamma_\mu d) \left. \right] + \bar{e} \gamma_\mu e \left[\frac{1}{2} \tilde{\beta} (\bar{u} \gamma_\mu \gamma_5 u - \right. \\ & \left. \left. - \bar{d} \gamma_\mu \gamma_5 d) + \frac{1}{2} \tilde{\delta} (\bar{u} \gamma_\mu \gamma_5 u + \bar{d} \gamma_\mu \gamma_5 d) + \dots \right] \right\} \end{aligned} \quad (2)$$

where the dots denote isoscalar axial S quark and heavy-quark currents. The four real coupling constants $\tilde{\alpha}$, $\tilde{\beta}$, $\tilde{\gamma}$, $\tilde{\delta}$, are to be determined experimentally.

As shown in Ref. 2), measurements of the P odd asymmetry in deep inelastic e^- -N scattering give information about the combinations $(3\tilde{\alpha} + \tilde{\gamma})$ and $(3\tilde{\beta} + \tilde{\delta})$, while experiments on parity violating transitions

in bismuth measure $(-0.075\tilde{\alpha} + \tilde{\gamma})$. The parameters $\tilde{\alpha}$, $\tilde{\beta}$ and $\tilde{\gamma}$ can be determined, in principle by combining the results of the experiments on parity non-conservation in hydrogen and deuterium atoms⁷⁾. It seems impossible, however, to extract unique information about the axial constant $\tilde{\delta}$ in the elastic scattering or atomic physics parity violation experiments⁸⁾.

As is widely known the interpretation of the results obtained on bismuth is accompanied by some uncertainties and the atomic experiments with hydrogen and deuterium are extremely difficult. Therefore one of our aims is to establish whether it is possible to determine all four coupling constants in Eq. (2) in experiments on e^-N scattering only. Before going into detailed discussion let us note that in the standard $SU_2^W \otimes U_1$ gauge theory⁹⁾ which successfully describes the existing neutral current data ($\sin^2\theta_w \sim 0.25$) we have:

$$\begin{aligned}\tilde{\alpha}^{W-S} &= -(1 - 2\sin^2\theta_w), \quad \tilde{\gamma} = \frac{2}{3}\sin^2\theta_w, \\ \tilde{\beta}^{W-S} &= -(1 - 4\sin^2\theta_w), \quad \tilde{\delta} = 0.\end{aligned}\tag{3}$$

The standard definition of the P odd asymmetry for the case of elastic scattering of electrons with longitudinal polarization λ on unpolarized nucleons is:

$$A_{e^-N}(E, \theta) = \frac{(\frac{d\sigma_{e^-N}}{d\Omega})_{\lambda=1} - (\frac{d\sigma_{e^-N}}{d\Omega})_{\lambda=-1}}{(\frac{d\sigma_{e^-N}}{d\Omega})_{\lambda=1} + (\frac{d\sigma_{e^-N}}{d\Omega})_{\lambda=-1}}\tag{4}$$

where $(\frac{d\sigma_{e^-N}}{d\Omega})_{\lambda}$ denotes the elastic scattering cross-section for electrons with initial energy E scattered on angle θ . The leading contribution to the asymmetry is given by the interference of the weak amplitude due to the interaction (2) and the relevant electromagnetic amplitude. Straightforward calculations show that^{3), 10)}

$$A_{eN} = -\frac{G}{\sqrt{2}} \frac{q^2}{2\tau\alpha} \frac{2(\frac{E}{M}-\tau)tg^2\frac{\theta}{2} G_{AN} G_M^{\gamma N} + 2\tau tg^2\frac{\theta}{2} G_M^{\gamma N} G_M^{\gamma N} + \frac{G_E^{\gamma N} G_E^{\gamma N} + \tau G_M^{\gamma N} G_M^{\gamma N}}{1+\tau}}{2\tau tg^2\frac{\theta}{2} (G_M^{\gamma N})^2 + \frac{(G_E^{\gamma N})^2 + \tau (G_M^{\gamma N})^2}{1+\tau}} \quad (5)$$

Here $G_E^{\gamma N}(q^2)$ and $G_M^{\gamma N}(q^2)$ are the electric and magnetic form factors of the nucleon N, $G_E^{\gamma N}(q^2)$ and $G_M^{\gamma N}(q^2)$ are the analogous weak vector form-factors arising in the matrix element of the hadron vector neutral current between the states of the nucleon N, $G_{AN}(q^2)$ is the weak axial form-factor - the coefficient of the $\gamma_\mu \gamma_5$ term in the matrix element of the hadron axial-vector neutral current, q is the momentum transferred to the nucleon and $\tau = q^2/4M^2$.

Using the isospin structure of the electromagnetic and neutral vector quark currents with fractionally charged quarks assumed we obtain the standard relations¹¹⁾:

$$G_{M,E}^{\gamma p(n)}(q^2) = (\pm) \frac{1}{2} \alpha [G_{M,E}^{\gamma p}(q^2) - G_{M,E}^{\gamma n}(q^2)] + \frac{3}{2} \tilde{\gamma} [G_{M,E}^{\gamma p}(q^2) + G_{M,E}^{\gamma n}(q^2)] \quad (6)$$

The axial form-factor $G_{AN}(q^2)$ is a linear combination of isovector $G_{AN}^{I=1}(q^2)$ and isoscalar $G_{AN}^{I=0}(q^2)$ form-factors:

$$G_{AN}(q^2) = \beta G_{AN}^{I=1}(q^2) + G_{AN}^{I=0}(q^2) \quad (7)$$

The isovector form-factor $G_{AN}^{I=1}(q^2)$ is related through an isospin transformation to the axial isovector form-factor $G_A(q^2)$ measured in the β decay and quasi-elastic reaction $\nu_\mu + n \rightarrow \mu^- + p$:

$$G_{Ap}^{I=1}(q^2) = - G_{An}^{I=1}(q^2) = \frac{1}{2} G_A(q^2) \quad (8)$$

We assume further, in accord with the existing rather accurate data^{12),13)} that

$$G_E^{\sigma p}(q^2) = \frac{G_M^{\sigma p}(q^2)}{1+\mu_p} = \frac{G_M^{\sigma n}(q^2)}{\mu_n} = \left(1 + \frac{q^2}{0.71 \text{ GeV}^2}\right)^{-2} \quad (9)$$

$$G_A(q^2) = g_A \left(1 + \frac{q^2}{0.90 \text{ GeV}^2}\right)^{-2}, \quad g_A = 1.24, \quad (10)$$

where $\mu_p = 1.79$ and $\mu_n = -1.91$ are the proton and neutron anomalous magnetic moments.

For the axial isoscalar and neutron electric form-factors there is considerable uncertainty. No experimental information about the q^2 dependence of $G_{AN}^{I=0}(q^2)$ exists. We choose it to be identical to that of $G_{AN}^{I=1}(q^2)$ in analogy with the same q^2 dependence exhibited by the isoscalar and isovector magnetic form-factors. It was argued recently in Ref. 8) that the value of $G_{AN}^{I=0}(q^2)$ at $q^2 = 0$ is not directly related to the constant $\tilde{\sigma}$ and should be considered as an independent parameter in a phenomenological analysis:

$$G_{AN}^{I=0}(0) = C_a \quad (11)$$

Further, the uncertainties in the data on $G_E^{Yn}(q^2)$ taken at relatively high values of q^2 are so large that even the case $G_E^{Yn}(q^2) = 0$ is not excluded^{14),12)}. In order to show explicitly the dependence of the

conclusions about $\tilde{\alpha}, \tilde{\beta}, \tilde{\gamma}$ and c_a to be drawn in the discussed experiments, on the form chosen for $G_E^{yn}(q^2)$ we shall consider two possibilities:

$$G_E^{\pi n}(q^2) = 0 \quad (A)$$

$$G_E^{\pi n}(q^2) = -\tau G_M^{\pi n}(q^2) \quad (B)$$

Taking into account Eqs. (5)-(11) we get two different expressions for the asymmetry $A_{e-N}(E, \theta)$ corresponding to the two forms of $G_E^{yn}(q^2)$:

$$A_{e-p(n)}(E, \theta) = -\frac{1}{2} \frac{G}{\sqrt{2}} \frac{q^2}{2t\alpha} \left[(1.6c_a \pm \tilde{\beta}) \left\{ \begin{matrix} A_a^{p(n)}(E, \theta) \\ B_a^{p(n)}(E, \theta) \end{matrix} \right\} + \right. \\ \left. + \tilde{\alpha} \left\{ \begin{matrix} A_{\tilde{\alpha}}^{p(n)}(E, \theta) \\ B_{\tilde{\alpha}}^{p(n)}(E, \theta) \end{matrix} \right\} + \tilde{\gamma} \left\{ \begin{matrix} A_{\tilde{\gamma}}^{p(n)}(E, \theta) \\ B_{\tilde{\gamma}}^{p(n)}(E, \theta) \end{matrix} \right\} \right] \quad \begin{matrix} \text{- case (A)} \\ \text{- case (B)} \end{matrix}$$

Here $A_{a, \tilde{\alpha}, \tilde{\gamma}}^N(E, \theta)$ and $B_{a, \tilde{\alpha}, \tilde{\gamma}}^N(E, \theta)$ are kinematic coefficients fixing the relative weight with which the parameters $(1.6c_a \pm \tilde{\beta})$, $\tilde{\alpha}$, and $\tilde{\gamma}$ enter the asymmetry.

We shall consider first scattering on protons. We have:

$$A_a^p(E, \theta) = 2 \frac{(1+\mu_p)(1+\tilde{\tau})(\frac{E}{M}-\tilde{\tau})tq^2\frac{\theta}{2}}{1+\tau(1+\mu_p)^2(1+2tq^2\frac{\theta}{2})(1+\tilde{\tau})} \frac{G_a(q^2)}{G_E^{\pi p}(q^2)} \quad (12)$$

$$A_{\tilde{\alpha}}^p(E, \theta) = 1 - \frac{\mu_n}{1+\mu_p} + \frac{\mu_n}{1+\mu_p} \frac{1}{1+\tau(1+\mu_p)^2(1+2tq^2\frac{\theta}{2})(1+\tilde{\tau})} \quad (13)$$

$$A_{\tilde{\gamma}}^P(E, \theta) = 3 \left(1 + \frac{\mu_n}{1+\mu_p} - \frac{\mu_n}{1+\mu_p} \frac{1}{1 + \tau (1+\mu_p)^2 (1 + 2tg^2 \frac{\theta}{2} (1+\tau))} \right) \quad (14)$$

$$B_a^P(E, \theta) = A_a^P(E, \theta) \quad (15)$$

$$B_{\tilde{\gamma}}^P(E, \theta) = A_{\tilde{\gamma}}^P(E, \theta) + \frac{\tau \mu_n}{1 + \tau (1+\mu_p)^2 (1 + 2tg^2 \frac{\theta}{2} (1+\tau))} \quad (16)$$

$$B_{\tilde{\gamma}}^P(E, \theta) = A_{\tilde{\gamma}}^P(E, \theta) - 3 \frac{\tau \mu_n}{1 + \tau (1+\mu_p)^2 (1 + 2tg^2 \frac{\theta}{2} (1+\tau))} \quad (17)$$

It follows from Eqs. (16) and (17), that for any fixed value of the energy E there are regions of relatively small and/or large scattering angles in which $B_{\tilde{\alpha}, \tilde{\gamma}}^P(E, \theta) \simeq A_{\tilde{\alpha}, \tilde{\gamma}}^P(E, \theta)$. This is seen on Fig. 1 where the dependence of the coefficients $A_{a, \tilde{\alpha}, \tilde{\gamma}}^P$ and $B_{a, \tilde{\alpha}, \tilde{\gamma}}^P$ on θ in the region $0^\circ \leq \theta \leq 120^\circ$ is shown for $E = 20; 0.6$ and 0.15 GeV. At $E = 20$ GeV the difference between $A_{a, \tilde{\alpha}, \tilde{\gamma}}^P$ and $B_{a, \tilde{\alpha}, \tilde{\gamma}}^P$ becomes negligible for $\theta \gtrsim 60^\circ$ and $\theta \lesssim 2^\circ$ and therefore unique interpretation of the data taken only in these regions is possible. The regions where $A_{a, \tilde{\alpha}, \tilde{\gamma}}^P$ and $B_{a, \tilde{\alpha}, \tilde{\gamma}}^P$ differ considerably are smaller at lower energies. At $E \sim 150$ MeV, for example, practically $A_{a, \tilde{\alpha}, \tilde{\gamma}}^P = B_{a, \tilde{\alpha}, \tilde{\gamma}}^P$ for all values of θ . Measurements at small scattering angles ($\theta \lesssim 20^\circ$) in this energetic region would give information about the combination $(\tilde{\alpha} + 3\tilde{\gamma})$, the contribution of the axial term $(1.6c_a + \tilde{\beta})$ being strongly suppressed kinematically. At large

scattering angles ($\theta \gtrsim 60^\circ$) the multipliers in front of $(1.6c_a + \tilde{\beta})$ in the expression for the asymmetry are of the same order as those in front of $\tilde{\alpha}$ and $\tilde{\gamma}$. So measurements of the asymmetry A_{e-p} at $E \sim 150$ MeV will provide information about the two combinations $(\tilde{\alpha} + 3\tilde{\gamma})$ and $(1.6c_a + \tilde{\beta})$.

It seems much more difficult to extract a unique information about the electron-quark neutral current coupling constants by studying the e^-n scattering. In this case we have:

$$A_a^n(E, \theta) = 2 \frac{(1+\epsilon)(\frac{E}{M} - \tilde{\epsilon}) t_f^2 \frac{\theta}{2}}{\mu_n \tau (1 + 2 t_f^2 \frac{\theta}{2} (1+\epsilon))} \frac{G_A(q^2)}{G_E^{\gamma p}(q^2)} \quad (18)$$

$$A_{\tilde{\alpha}}^n = 1 - \frac{1+\mu_p}{\mu_n} \approx 2,46 \quad (19)$$

$$A_{\tilde{\gamma}}^n = 3 \left(1 + \frac{1+\mu_p}{\mu_n} \right) \approx -1,38 \quad (20)$$

$$B_a^n(E, \theta) = A_a^n(E, \theta) \frac{1 + 2 t_f^2 \frac{\theta}{2} (1+\epsilon)}{(1+\epsilon) (1 + 2 t_f^2 \frac{\theta}{2})} \quad (21)$$

$$B_{\tilde{\alpha}}^n(E, \theta) = A_{\tilde{\alpha}}^n(E, \theta) + \frac{1 + \tau(1 + \mu_p)}{\mu_n(1 + \tau)(1 + 2tg^2 \frac{\theta}{2})} \quad (22)$$

$$B_{\tilde{\gamma}}^n(E, \theta) = A_{\tilde{\gamma}}^n - 3 \frac{1 + \tau(1 + \mu_p)}{\mu_n(1 + \tau)(1 + 2tg^2 \frac{\theta}{2})} \quad (23)$$

The dependence of $A_{\tilde{\alpha}, \tilde{\gamma}}^n$ and $B_{\tilde{\alpha}, \tilde{\gamma}}^n$ on θ for $E = 20; 0.6$ and 0.15 GeV is shown in Fig. 2. It is clear from Fig. 2 that in the whole kinematic interval considered: 1) the interpretation of the asymmetry A_{e-n} in terms of the parameters $\tilde{\alpha}, \tilde{\gamma}$ and $(1.6c_a - \tilde{\beta})$ strongly depends on the assumed form of $G_E^{yn}(q^2)$ and 2) the kinematic factors in front of $\tilde{\alpha}, \tilde{\gamma}$ and $(1.6c_a - \tilde{\beta})$ are of the same order.

It follows from our analysis that the polarized e^-N elastic scattering experiments would be more effective in providing information about the electron-quark neutral current couplings if more accurate measurements of the neutron electric form-factor $G_E^{yn}(q^2)$ will be performed.

In conclusion we would like to make the following remark. In principle, the high precision experiments we are discussing might be used to determine the form of $G_E^{yn}(q^2)$ in a definite q^2 region. In particular, in the case of scattering off protons, in the region where

$$2\left(\frac{E}{M} - \tau\right)tg^2 \frac{\theta}{2} (1 + \mu_p) \ll 1 \quad (24)$$

$$2\tau \frac{q^2}{2} (1+\mu_p)^2 \ll 1 \quad (25)$$

the expression for the asymmetry A_{e-p} takes a very simple form:

$$A_{e-p}(q^2) \approx -\frac{G}{\sqrt{2}} \frac{q^2}{2\pi\alpha} \frac{G_E^{\delta P}(q^2)G_E^{\delta V}(q^2) + \tau G_M^{\delta P}(q^2)G_M^{\delta V}(q^2)}{(G_E^{\delta P}(q^2))^2 + \tau (G_M^{\delta P}(q^2))^2} \quad (26)$$

Using Eqs. (6) and (9) we can rewrite it as

$$A_{e-p}(q^2) \approx -\frac{1}{2} \frac{G}{\sqrt{2}} \frac{q^2}{2\pi\alpha} \left\{ (\tilde{\alpha} + 3\tilde{\gamma}) + (-\tilde{\alpha} + 3\tilde{\gamma}) \times \right. \\ \left. \times \frac{\mu_n \tau}{1 + \tau(1+\mu_p)^2} \left[1 + \mu_p + \frac{G_E^{\delta n}(q^2)}{\mu_n \tau G_E^{\delta P}(q^2)} \right] \right\} \quad (27)$$

So, if the combinations $(\tilde{\alpha} + 3\tilde{\gamma})$ and $(-\tilde{\alpha} + 3\tilde{\gamma})$ can be determined experimentally with high precision, measurements of the P odd asymmetry, even in the case of e^-p elastic scattering, may be used to study the q^2 dependence of $G_E^{\gamma n}(q^2)$ in a certain kinematic region. This region is determined, in fact, by the inequality (24) (the restriction (25) is not a necessary one) ensuring that the uncertainty due to the axial form-factor $G_{Ap}(q^2)$ will not contribute. If the Weinberg-Salam theory is correct, which is widely assumed at present, then the region where the contribution of $G_{Ap}(q^2)$ can be neglected will be probably larger than the one defined by (24) and will extend up to $q^2 \sim 5 \text{ GeV}^2$ because $(\beta^{w-s} + 1.6c_a^{w-s}) \ll 1^8$, and we expect that $G_{Ap}(q^2) \ll 1$.

ACKNOWLEDGEMENT

We are grateful to S.M. Bilenky for stimulating discussions. One of us (S.T.P.) wishes to thank J. Ellis for useful comments.

REFERENCES

- 1) C. Prescott et al., SLAC-PUB-2148, July 1978.
- 2) J.D. Bjorken, SLAC-PUB-2146, June 1978.
- 3) R.N. Cahn and F.J. Gilman, Phys. Rev. D17 (1978) 1313;
E. Hoffman and E. Reya, University of Mainz preprint, MZ-TH 78/5 (1978);
E. Reya and K. Schilcher, Phys. Rev. D10 (1974) 952.
- 4) E.W. Otten et al., University of Mainz proposal and Proc. of the
Topical Meeting on Intermediate Energy Physics: Experiments and
Experimental Methods, Zuoz, Switzerland, April 1976 (SIN, Villigen,
1976), Vol. I, p. 62.
- 5) V.W. Hughes et al., Yale Univ. Proposal for MIT-BATES, Sept. 1977.
- 6) J.J. Sakurai, Proc. of the "Neutrino '77" Conf., USSR, 1977.
- 7) See, for example, B. Derman and W. Marciano, Rockefeller University
preprint COO-2232B-164 (1978).
- 8) L. Wolfenstein, Carnegie-Mellon University preprint, COO-3066-119,
(1978).
- 9) S. Weinberg, Phys. Rev. Lett. 19 (1967) 1264;
A. Salam, Proc. of the 8th Nobel Symposium (J. Wiley, N.Y. 1968).
- 10) F.J. Gilman and T. Tsao, SLAC-PUB-2139, June 1978.
- 11) For a more extended discussion of the relations between different
form-factors, see, e.g., J.D. Bjorken, Proc. of the SLAC Summer
Institute on Particle Physics, SLAC Report 198, November 1976.
- 12) M. Gourdin, Phys. Reports 110 (1974) 29.
- 13) D.H. Perkins, Proc. of the 1975 Intern. Conf. on Lepton and Photon
Interactions at High Energies, Stanford, California (1975).
- 14) S.I. Bilenkaja et al., JINR preprint, E-I-6647, Dubna (1972).

Figure captions

- Fig. 1 : The kinematic coefficients $A_{a,\tilde{\alpha},\tilde{\gamma}}^P$ and $B_{a,\tilde{\alpha},\tilde{\gamma}}^P$ as functions of θ for $E = 20; 0.6$ and 0.15 GeV.
- Fig. 2 : The kinematic coefficients $A_{a,\tilde{\alpha},\tilde{\gamma}}^n$ and $B_{a,\tilde{\alpha},\tilde{\gamma}}^n$ as functions of θ for $E = 20; 0.6$ and 0.15 GeV.

FIG.1

FIG. 2