Diffractive production of the A_2 and its implications

E. L. Berger

High Energy Physics Division, Argonne National Laboratory, Argonne, Illinois 60439

J. T. Donohue

Laboratoire de Physique Théorique, Université de Bordeaux-I, Gradignan, France

G. Mennessier*

CERN, Geneva, Switzerland

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We study diffractive production of the A_2 resonance in the context of a unitarity-corrected Deck production amplitude for $\pi p \rightarrow (\rho\pi)p$. After projecting the D -wave $\rho\pi$ production amplitudes from the Deck model with π and ρ exchange terms, we include $\rho\pi$ final-state rescattering corrections. This corrected Deck amplitude fails to describe the A_2 data in both shape and absolute cross section. To reproduce the data, we require a dominant *direct* spin-flip coupling of the Pomeron at the πA_2 production vertex, in contradiction with the s -channel helicity and Gribov-Morrison phenomenological rules for diffraction.

I. INTRODUCTION

While hadronic diffraction has yet to be understood in any profound sense, various phenomenological "rules" have been devised in attempts to summarize experimental observations.¹ One of these rules concerns the s -channel helicity properties of diffraction. Evidence suggests that *elastic* scattering approximately preserves s -channel helicity at the nucleon vertex in $\pi N \rightarrow \pi N$ and at the meson vertex in $\gamma p \rightarrow \rho p$. The differential cross section for both processes also show strong forward peaks. In *inelastic* diffractive reactions, the situation is mixed. Strong and roughly energy-independent forward peaks are observed at high energy in $\pi p \rightarrow (\rho\pi)p$ and in $Kp \rightarrow (\rho K)p$. However, the low-mass $(\rho\pi)$ system is produced with approximate t -channel helicity conservation,² whereas the (ρK) system appears to obey approximate s -channel helicity conservation.³ The systematics of helicity conservation in these and other meson diffraction dissociation reactions have been shown to be well represented by the Deck model.⁴

The Gribov-Morrison rule is a second phenomenological ansatz.⁵ It suggests that in the diffractive reaction $A p \rightarrow A^* p$, the relationship $\Delta P = (-1)^{\Delta J}$ should hold for the parity P and spin J difference between A and A^* .

Production of the A_2 meson in $\pi p \rightarrow A_2 p$ provides a special opportunity for examining diffraction in a mode in which spin flip occurs and in which the Gribov-Morrison rule fails. The coupling of the natural-parity Pomeron exchange at the A_2 vertex demands that the A_2 be produced with t -channel helicity ≥ 1 . The differential cross section must vanish in the forward direction if the

target (p) helicity does not flip in the center-of-mass system. This result holds more generally for the process $0^- p \rightarrow M p$, where meson M has spin parity $J^P = 1^-, 2^+, \dots$. It is therefore perfectly possible to have quasi-two-body reactions whose differential cross sections remain roughly energy independent (i.e., are diffractive) but have forward dips. High-energy data² on the reaction $\pi p \rightarrow A_2 p$ show that it is a clear example of this type of "atypical" diffraction.

In a Regge-pole-exchange context, the phenomenological description of $\pi p \rightarrow A_2 p$ requires knowledge of (or provides a measure of) the strength of the πA_2 -Pomeron spin-flip coupling. Analyzing the data in this spirit, Irving⁶ showed recently that the high-energy A_2 data require a Pomeron flip coupling whose strength is at least as large as one-half that specified by f dominance of the Pomeron. By 40 GeV/c, his A_2 cross section is predominantly diffractive, with only a negligible Regge piece at that momentum.

Rather than admit a direct, spin-flip Pomeron coupling, it is interesting to inquire whether diffractive A_2 production can be explained via the Deck mechanism.⁷ In the Deck effect, shown in Fig. 1, the incident pion first dissociates into a pion and a ρ , one of which subsequently scatters diffractively from the nucleon. The $\rho\pi$ system so produced is nonresonant, to be sure, and the $J^P = 2^+$ component of the Deck amplitude does not provide a narrow peak in the A_2 region. However, because the $J^P = 2^+$ $\rho\pi$ amplitude is known to resonate at the A_2 , final-state interaction (or unitarity) corrections⁸ to the $J^P = 2^+$ Deck amplitude are necessary. Since the $J^P = 2^+$ $\rho\pi \rightarrow \rho\pi$ amplitude can be approximated fairly well at low masses by a simple Breit-Wigner resonance form, these

FIG. 1. Sketched are (a) the π exchange and (b) the ρ exchange Deck diagrams for the process $\pi p \rightarrow (\rho\pi)p$. Symbol P denotes the Pomeron.

unitarity corrections may be imposed without much ambiguity. The shape of the $\rho\pi$ mass distribution and the absolute normalization of the cross section are predictable. A possible virtue of this Deck approach is that it provides in principle an explicit prediction for the form and strength of the inelastic π - A_2 -Pomeron spin-flip coupling in terms of the assumed s -channel helicity-conserving elastic couplings of the $\pi\rho$ and $\rho\rho$ off-shell processes. If successful, the Deck approach would also preserve the Gribov-Morrison rule, in that A_2 production which disagrees with the rule could be regarded as a second-order correction.⁹

In this article we report a detailed investigation of diffractive A_2 production in the context of the Deck model corrected by final-state interactions or, equivalently, unitarity. To discuss A_2 production in the context of the Deck model, we must first secure the $J^P = 2^+$ $\rho\pi$ partial-wave amplitudes from the full π and ρ exchange amplitudes shown in Fig. 1. These partial-wave amplitudes are functions of the mass M of the $\rho\pi$ system and of the momentum transfer t_1 at which M is produced. The partial-wave analysis is described in Sec. II. A simple analytic form for the $J^P = 2^+$ wave may be derived from the π -exchange Deck graph alone. However, to include the effects of the ρ -exchange term and of possible pion form factors, we also provide a complete numerical treatment of the

partial-wave decomposition. It is essential to examine the partial-wave structure in detail, as we do, because the $J^P = 2^+$ partial wave is not the dominant wave in the π -exchange Deck amplitude. It is *a priori* possible that form factors and/or the ρ exchange term could produce substantial deviations in form and magnitude from the simple analytic form. We show this is not the case.

In Sec. III, we provide our treatment of the unitarization of the $J^P = 2^+$ amplitude. Our approach follows that devised in Ref. 8 for general diffractive processes, except that minor technical differences arise in obtaining a dispersion relation for the D wave. Here also, the $J^P = 2^+$ S matrix contains the well known A_2 resonance rather than the enigmatic A_1 discussed in Ref. 8. Thus, we can parametrize the $J^P = 2^+$ S matrix directly in terms of the A_2 phase shift. In Sec. III, we provide only a one-channel ($\rho\pi$) treatment of A_2 production. Because of D -wave threshold suppression factors, the threshold for $K^*\bar{K}$ is too high to affect our A_2 substantially. The branching ratios for the pseudoscalar decay modes $\eta\pi$ and $K\bar{K}$ are also not significant enough to affect our results.

The principal conclusion of Sec. III is that the unitarized Deck amplitude is capable of reproducing neither the form of the A_2 mass distribution, nor the magnitude of the production cross section. It provides a mass distribution with a dip near the A_2 position, rather than the peak observed experimentally. Second, in absolute size it is more than an order of magnitude too small. The data require the introduction of a *direct* spin-flip coupling of the Pomeron at the πA_2 vertex. The interference of this direct amplitude with the small unitarized Deck term has a small influence on the observed spectra. Because the magnitude of the unitarized Deck term is so small in this A_2 case, the data determine the magnitude of the dominant π - A_2 -Pomeron coupling unambiguously. The Gribov-Morrison rule fails even in first order. Our conclusion in the A_2 situation may be contrasted with our previous result⁸ for the A_1 . In diffractive A_1 production, the direct Pomeron coupling strength was found to be negligible in comparison with the size of the unitarized Deck term.⁸ Our conclusions are summarized in Sec. IV. A brief Appendix is included on the data we use.

II. SPIN-PARITY ANALYSIS OF THE DECK PRODUCTION AMPLITUDE

A. General comments

In the reaction $\pi p \rightarrow A_2 p$, the $J^P = 2^+$ A_2 meson may be produced with t -channel helicities $\lambda = \pm 2$,

± 1 , and 0. The proportions depend on dynamics. In this section, we concentrate on the $J^P = 2^+ \rho\pi$ partial waves in the process $\pi p \rightarrow (\rho\pi)p$. Beginning with π and ρ exchange Deck amplitudes for $\pi p \rightarrow (\rho\pi)p$, we deduce the magnitude and form for the different components of the $J^P = 2^+ \rho\pi$ waves.

In the $J^P = 2^+$ state, the $\rho\pi$ system is necessarily in an $l = 2$ partial wave. We may decompose the $\rho\pi$ states $|J, \lambda\rangle$ of definite J^P and t -channel helicity λ in terms of $\rho\pi$ states with orbital angular momentum l and projection m_l , and ρ spin projection m_ρ , $A_{l, m_l}^{m_\rho}$, in the following way:

$$|2, 2\rangle = (\frac{2}{3})^{1/2} A_{2,2}^0 - (\frac{1}{3})^{1/2} A_{2,1}^1, \quad (2.1)$$

$$|2, 1\rangle = (\frac{1}{3})^{1/2} A_{2,2}^{-1} + (\frac{1}{6})^{1/2} A_{2,1}^0 - (\frac{1}{2})^{1/2} A_{2,0}^1, \quad (2.2)$$

$$|2, 0\rangle = (\frac{1}{2})^{1/2} A_{2,1}^{-1} - (\frac{1}{2})^{1/2} A_{2,-1}^1. \quad (2.3)$$

The state $|2, 0\rangle$ cannot be produced by natural-parity exchange and is thus excluded from diffractive production.

The π and ρ exchange Deck amplitudes provide definite predictions for the partial amplitudes $A_{l, m_l}^{m_\rho}$. The dominant $l = 2$ state is $A_{2,1}^0$. This result is derived analytically, given certain simplifying approximations, in Sec. II B. The reliability of the simplifying approximations is difficult to judge in analytical fashion. Thus, in Sec. II C, we present the results of a full numerical analysis of the partial-wave decomposition of the π and ρ exchange Deck amplitudes.

Because $A_{2,1}^0$ is dominant, the state of principal importance according to the Deck model is that with $|\lambda| = 1$, Eq. (2.2).

B. Partial-wave structure of the Deck amplitudes

1. Pion exchange

We discussed the spin structure of the π and ρ exchange Deck amplitudes in some detail previously,⁴ and we refer to Ref. 4 for a discussion of some points summarized here. Briefly, the full π exchange Deck amplitude corresponding to Fig. 1(a) produces a $(\rho\pi)$ system in which the ρ has t -channel helicity zero, with the t -channel defined in the ρ rest frame. This amplitude is

$$A_{\lambda_p=0}^{\lambda_\pi} = -2g_{\rho\pi^+\pi^-} |\vec{p}_2^0| \frac{\exp[b_\pi(t_2 - m_\pi^2)]}{m_\pi^2 - t_2} \times i s_{\pi\rho} \sigma_{\pi\rho} \exp\left(\frac{B_\pi t_1}{2}\right). \quad (2.4)$$

Here $|\vec{p}_2^0|$ is the magnitude of the initial π momentum evaluated in the ρ rest frame at $t_2 = 0$; $|\vec{p}_2^0| = (m_\rho^2 - m_\pi^2)/(2m_\rho)$. The elastic scattering slope $B_\pi \approx 8 \text{ GeV}^{-2}$, $\sigma_{\pi\rho} \approx 24 \text{ mb}$, and we use form-factor slope $b_\pi = 1 \text{ GeV}^{-2}$. The coupling constant $g_{\rho\pi^+\pi^-} = 4\pi$ (2.4).

The amplitude defined by Eq. (2.4) generates a $(\rho\pi)$ system having several partial waves and helicities. However, for values of the mass M of the $\rho\pi$ system which are not too far above the threshold $(m_\rho + m_\pi)$, the rotation from the boosted t -channel rest frame of the $(\rho\pi)$ system to that of ρ is not large. Thus, Eq. (2.4) also implies $m_\rho \approx 0$ in the $(\rho\pi)$ t -channel rest frame. To first approximation, all states $A_{l, m_l}^{m_\rho}$ with $m_\rho \neq 0$ are excluded, and only $A_{2,2}^0$ and $A_{2,1}^0$ survive in Eqs. (2.1)–(2.3). To deduce the (l, m_l) content of Eq. (2.4), we first reexpress the amplitude in terms of the set of variables $(s, M, t_1, \theta, \phi)$, where θ and ϕ are the t -channel polar and azimuthal angles of the ρ 's momentum vector in the $(\rho\pi)$ rest frame. To leading order in $\sqrt{-t_1}$ and dropping terms which are down by a power of s , we deduce that

$$\frac{s_{\pi\rho}}{(m_\pi^2 - t_2)} = \frac{s}{M^2 - m_\pi^2} \left(1 - \frac{(2\sqrt{-t_1}/M) q_\rho \sin\theta \cos\phi}{(M - E_\rho) - q_\rho \cos\theta} \right). \quad (2.5)$$

Here $E_\rho = (M^2 + m_\rho^2 - m_\pi^2)/(2M)$ and $q_\rho = (E_\rho^2 - m_\rho^2)^{1/2}$.

Note that because the ϕ dependence in Eq. (2.5) is linear in $\cos\phi$, only the $m_l = 0$ and $|m_l| = 1$ projections survive. Moreover, for all $l \geq 1$, $|m_l| = 1$. For the remainder of this analytical treatment of the partial-wave decomposition, we omit the form-factor term in Eq. (2.4) (i.e., $b_\pi \equiv 0$).

Expanding the denominator of the second term in Eq. (2.5) in a series of Legendre polynomials, we derive

$$\frac{s_{\pi\rho}}{m_\pi^2 - t_2} = \frac{s}{M^2 - m_\pi^2} \left[1 - \frac{2\sqrt{-t_1}}{M} \int_0^\infty (2l+1) Q_l(\gamma_\pi) \times \sin\theta P_l(z) \cos\phi \right], \quad (2.6)$$

where $\gamma_\pi = (M - E_\rho)/q_\rho$ and $z = \cos\theta$. The functions Q_n are Legendre functions of the second kind, defined by

$$Q_n(t) = \frac{1}{2} \int_{-1}^{+1} \frac{1}{t-z} P_n(z) dz, \quad (2.7)$$

$$Q_1(\gamma_\pi) = 0.5\gamma_\pi \ln\left(\frac{\gamma_\pi+1}{\gamma_\pi-1}\right) - 1, \quad (2.8a)$$

$$Q_3(\gamma_\pi) = 0.5(5\gamma_\pi^2 - 3)Q_1(\gamma_\pi) - \frac{5}{8}. \quad (2.8b)$$

Equation (2.6) provides the entire partial-wave decomposition of our pion-exchange Deck amplitude, in the limiting case specified above. In particular, the dominant S-wave portion has the well known form $s/(M^2 - m_\pi^2)$. The net D-wave component of Eq. (2.6), receives contributions from both the $l = 1$ and $l = 3$ terms of the series. To express our final answer, we use the normalized spherical harmonic with $l = 2$ and $m_l = +1$:

$$Y_{2,1}(\theta, \phi) = -\left(\frac{15}{8\pi}\right)^{1/2} \sin\theta \cos\theta e^{i\phi}, \quad (2.9)$$

$$Y_{2,-1}(\theta, \phi) = -Y_{2,1}^*(\theta, \phi). \quad (2.10)$$

Collecting terms, we find that $|2, 1\rangle = (\frac{1}{6})^{1/2} A_{2,1}^0$, with

$$A_{2,1}^0 = T_{2,1}^{\text{Deck}}(s, M, t_1) Y_{2,1}(\theta, \phi), \quad (2.11)$$

and

$$T_{2,1}^{\text{Deck}}(s, M, t_1) = -2g_{\rho\pi^+\pi^-} |\vec{p}_2| \sigma_{\pi p} \frac{is}{M^2 - m_\pi^2} \left(\frac{\sqrt{-t_1}}{M}\right) \left(\frac{24\pi}{5}\right)^{1/2} [Q_1(\gamma_\pi) - Q_3(\gamma_\pi)] \exp\left(\frac{B_\pi t_1}{2}\right). \quad (2.12)$$

The complete M dependence of Eq. (2.12) is contained in the factor

$$\frac{1}{M(M^2 - m_\pi^2)} [Q_1(\gamma_\pi) - Q_3(\gamma_\pi)].$$

As $M \rightarrow (m_\rho + m_\pi)$, $\gamma_\pi^{-1} \rightarrow 0$ and $Q_1 \rightarrow \gamma_\pi^{-2}$. For large M , $\gamma_\pi \rightarrow 1$;

$$Q_1(\gamma_\pi) \rightarrow \frac{1}{2} \ln(M^2/m_\pi^2), \quad (2.13)$$

and $[Q_1 - Q_3] \rightarrow \text{constant}$ as $M \rightarrow \infty$.

Our normalization is specified as follows:

$$\frac{d^2\sigma}{dM dt_1} = \frac{1}{0.3893 \cdot 16s^2} \left(\frac{1}{2\pi}\right)^4 \int d\Omega |A_{\lambda_\rho=0}|^2, \quad (2.14)$$

with $A_{\lambda_\rho=0}$ provided in Eq. (2.4). If $\sigma_{\pi p}$ in Eq. (2.4) is expressed in mb units, Eq. (2.14) yields a differential cross section in units of mb/GeV³.

Equations (2.11) and (2.12) would provide our final answer for the states $|J=2, \lambda\rangle$ if we could ignore the pion form factor $\exp[b_\pi(t_2 - m_\pi^2)]$ in Eq. (2.4), and if we could ignore the ρ exchange Deck term. Neither assumption is obviously justified, especially in the case of $l=2$. The presence of the form factor changes the proportion of the different l states. It does not contribute to the population of states with $|\lambda| \neq 1$, however, since t_2 is independent of ϕ . The ρ exchange amplitude contributes in principle to all λ states. To evaluate the effects of the pion form factor and of ρ exchange, we resort to a numerical evaluation of the partial-wave decomposition, as reported in Sec. II C.

2. ρ exchange

Vector-exchange Deck amplitudes were derived in Ref. 4. Here we recall the results necessary for our present analysis of $\pi p \rightarrow (\rho\pi)p$. As discussed in Ref. 4, the basic ρ exchange amplitudes are expressed most conveniently in an s -channel helicity basis. The Deck amplitudes for producing a $(\rho\pi)$ system in which the ρ has s -channel helicity ± 1 are

$$A_1 = ig_{\rho\pi^+\pi^-} \frac{\sqrt{2}p_{2x}\sigma_{\rho p}}{(m_\rho^2 - u_2)} s_{\rho p} \exp\left(\frac{B t_1}{2}\right) \times \exp[b_\rho(u_2 - m_\rho^2)] \exp(i\tilde{\Phi}),$$

$$A_{-1} = -ig_{\rho\pi^+\pi^-} \frac{\sqrt{2}p_{2x}\sigma_{\rho p}}{(m_\rho^2 - u_2)} s_{\rho p} \exp\left(\frac{B t_1}{2}\right) \times \exp[b_\rho(u_2 - m_\rho^2)] \exp(-i\tilde{\Phi}).$$

Here $\tilde{\Phi}$ is the azimuthal angle of the final ρ in the reference frame in which the virtual- ρ momentum and the target-proton momentum are collinear and define the z axis, and in which the vector $(\vec{p}_2 + \vec{q}_3)$ has no y but a positive x component; $\vec{p}_2$ and $\vec{q}_3$ are the three-vector momenta of the initial-state and final-state pions, respectively.

The amplitude for s -channel helicity-zero ρ scattering is

$$A_0 = ig_{\rho\pi^+\pi^-} \xi' \frac{(-u_2)(p_{2x} + q_{3x})}{\nu(m_\rho^2 - u_2)m_\rho} \sigma_{\rho p} s_{\rho p} \times \exp\left(\frac{B t_1}{2}\right) \exp[b_\rho(u_2 - m_\rho^2)].$$

Here ν is the energy of the virtual ρ in the chosen reference frame. We set $B_\rho = B_\pi = 8$ (GeV/c)⁻² and $\sigma_{\rho p} = \sigma_{\pi p} = 24$ mb. In the analysis of Ref. 4, we determined that the choices $b_\rho = 2$ (GeV/c)⁻² and $\xi' = -1$ lead to a good representation of the experimental helicity systematics of meson diffractive dissociation in the dominant $J^P = 1^+$ amplitude. We continue to use the same values in our present study of the $J^P = 2^+$ state.

C. Numerical partial-wave analysis

The technical details of our method for partial-wave analysis of the Deck amplitudes may be found in the Appendix of Ref. 4. We present partial-wave amplitudes $A_\lambda^{J^P}(s, M, t_1) = |J, \lambda\rangle$; J is the total spin of the $\rho\pi$ system, and λ is its projection along the t -channel z axis. The partial waves were initially obtained in the $\rho\pi$ s -channel system of axes for the ρ exchange Deck terms and in the $\rho\pi$ t -channel system for the π exchange term. Appropriate rotations about the normal to the $\rho\pi$ production plane were then made to obtain the t -channel amplitudes for ρ exchange. The numerical computations were performed for $p_{\text{lab}} = 40$ GeV/c, but our results depend very little on the overall energy of the reaction.

In the preceding section, an analytic expression was derived for the contribution of a simple π -exchange Deck amplitude to the $J=2$, $L=2$, $\lambda=1$ $\rho\pi\pi$ partial wave. In the derivation, we made certain simplifying assumptions, such as neglecting Wigner rotations between axes defined in different rest frames, and working only to lowest order in t_1 . For phenomenological reasons, one would like to study the effects of a form factor at the $\pi\rho\pi$ vertex, as well as the contribution of vector-meson exchange. The latter is essential for understanding the spin systematics⁴ of the reaction $Kp \rightarrow K\rho p$, and must therefore be present as well in $\pi p \rightarrow \pi\rho p$. Although an analytic expression cannot be obtained when such complications are included, a numerical treatment is easily carried out, and we present the results of our analysis here.

Our main interest is in the $J=l=2$, $\lambda=1$ partial wave which is to be used subsequently in the unitarization procedure. The amplitude obtained by our numerical partial-wave analysis is displayed in Fig. 2(a) as a function of $M_{\rho\pi}$ for $t_1 = -0.1$ GeV², and in Fig. 2(b) as a function of t_1 for $M_{\rho\pi} = 1.3$ GeV. The contributions of the π and transverse and longitudinal vector-meson exchange terms are all shown. We observe that the latter two nearly cancel at $M_{\rho\pi} = 1.3$ GeV. Hence at this mass, the total contribution of vector-meson exchange turns out to be quite small compared to π exchange in the $J=2$, $\lambda=1$ partial wave. (However, at high $M_{\rho\pi}$ masses, the vector-exchange contribution increases relative to that of π exchange. The increase is a defect of our model in which elementary vector exchange is used.) Although the exact value of the numerical result is slightly different from the approximate

analytic expression obtained previously, there is an overall similarity, and we conclude that a minor modification of our analytic form brings it into quantitative agreement with our numerical result. Over the mass range $1.0 < M_{\rho\pi} < 2.0$ GeV, a fair representation of the M dependence of our numerical results is obtained by multiplying the analytic amplitude in Eq. (2.12) by the factor

$$f(M) = \left(0.96 + \frac{0.76}{M^2} - \frac{1.12}{M^4} \right).$$

We remark that the relative sign of the longitudinal vector-meson exchange was determined phenomenologically in Ref. 4. While the opposite choice would considerably increase the Deck contribution to the A_2 wave, the effect on the S wave would be a 50% reduction of the cross section.

In addition to the possibility of using the Deck model to estimate A_2 production, one may investigate certain other questions which involve the higher partial waves. We address ourselves here to the following list: Are the P - and D -wave projections of the Deck amplitude in agreement with data? What is the spin and helicity structure of the principal amplitudes? Is there any appreciable mass-slope correlation, particularly for $\lambda=1$ amplitudes? Finally we investigate whether the D -wave $J^{PC}=1^{++}$ amplitudes are approximately consistent with the S-wave $J^P=1^+$ amplitudes, as factorization would require.

The first main result is that the helicity structure of the low partial waves is much simpler if t -channel rather than s -channel axes are used. Indeed, we find that even with vector-meson exchange, the partial-wave cross sections with t -channel helicity ≥ 2 are negligible compared to those with $\lambda=0$ or 1. To some extent this is caused by the dominance of the π -exchange graph, which, as our analysis of Sec. II B.1 shows, has only $\lambda=0$ and 1, provided one neglects the Wigner rotations. Our model s -channel amplitudes, however, display rather complicated behavior as function of t_1 . In analyses of 3π systems, results are generally presented in terms of t -channel cross sections.² We shall therefore confine our attention here to those with $\lambda=0$ and 1.

In Figs. 3–6, we show the partial-wave cross sections plotted vs t_1 for $M_{\rho\pi} = 1.1, 1.3$, and 1.5 GeV, and plotted vs M for $t_1 = -0.1$ GeV². In the $\lambda=0$ sector, throughout the mass range 1.1 – 1.5 GeV, the hierarchy $S > P > D$ is respected in the model, whereas in the $\lambda=1$ sector, one finds D waves approximately as strong as P waves, with S waves slightly greater. Among the P waves with $\lambda=0$, the $J^P=2^-$ cross section is about 5 times greater than the $J^P=0^-$ cross section (not shown).

FIG. 2. The amplitude with $J=2$, $L=2$, and t -channel helicity $\lambda=1$ obtained from our numerical partial-wave analysis of the Deck amplitude is plotted in (a) vs M at $|t| = 0.1$ GeV², and in (b) vs t at $M = 1.3$ GeV. The normalization is such that $d\sigma^{J\lambda}/dMdt$ is equal to $q_\rho |A^{J\lambda}|^2 / [0.3893 \times 16 \times (2\pi)^4]$. The contributions are shown from the π -, and from the transverse and longitudinal ρ -exchange Deck amplitudes.

FIG. 3. The S-wave partial cross sections obtained from our numerical partial-wave analysis. Shown in (a) for $t = -0.1 \text{ GeV}^2$ are the M dependences of the t -channel helicity $\lambda=0$ and 1 partial cross sections obtained from the π -exchange graph alone (dashed line), and after the ρ exchange term is included (solid line). In (b), we show the t dependences of the $\lambda=0$ (upper set) and $\lambda=1$ (lower set) S-wave cross sections for three values of mass M .

Among the D waves, the largest wave has $J^P=3^+$, while the $2^+ \lambda=1$ is somewhat smaller than $J=3^+ \lambda=1$, but appreciably larger than $J=1^+ \lambda=1$. The general trend of $\lambda=0$ dominance in the model is reminiscent of the Gribov-Morrison selection rule, which suggests that partial waves which cannot be produced at $t'_1=0$ via natural parity

exchange are not important. The diffractive production of A_2 shows that this rule is not exact, and our unitarization procedure fails to generate the necessary cross-section using the Deck-model input (cf. Sec. III).

Evidence for a fairly strong mass-slope correlation is clear in Figs. 3(b)–6(b). In the $\lambda=0$

FIG. 4. The P-wave partial cross sections shown vs M in (a), and vs t in (b) for different indicated values of J and t -channel helicity λ .

FIG. 5. The D -wave partial cross sections with t -channel helicity $\lambda = 0$ are plotted vs M in part (a), and vs t in part (b) for the different indicated values of J .

sector, this manifests itself by a decrease of the logarithmic slope as $M_{\rho\pi}$ increases. For the $\lambda = 1$ cross sections, the effect is quite important, even though it is not so visibly striking because of the kinematic factor $(-t_1')$. If one parametrizes the t_1 dependence by $-t_1' e^{bt_1}$, then values of b decreasing from 16 GeV^{-2} at $M_{\rho\pi} = 1.1 \text{ GeV}$ to 10 GeV^{-2} at $M_{\rho\pi} = 1.5 \text{ GeV}$ are found for the various

partial waves. In the data somewhat lower values of b ($\sim 8 \text{ GeV}^{-2}$) are observed,² although the experimental range in t_1 may not be sufficiently great to detect a faster falloff. It should be remarked that the simple $(-t_1' e^{bt_1})$ provides only an approximate fit to the Deck cross sections.

An important question related to the possible existence of the A_1 concerns the D -wave 1^+ state.

FIG. 6. The D -wave partial cross sections with t -channel helicity $\lambda = 1$ are plotted vs M in part (a) and vs t in part (b) for the indicated values of spin J .

Although the decay of the B into $\omega\pi$ is known to be a mixture of S and D waves, the partial-wave analyses of 3π production generally find little or no $1^+ D$ wave under the dominant S wave. The $\pi+\rho$ Deck effect calculation suggests that there is very little D wave under the 1^+ (see Fig. 5), but the unitarization procedure of Basdevant and Berger⁸ will preserve this result only if the genuine A_1 resonance has a very small D -wave decay amplitude. It should be noted that for a bona fide resonance, the amplitude should satisfy a factorization condition $A_{\lambda}^{J'l} = A_{\lambda}^J D^l$, where D^l denotes the decay amplitudes for different l values. A simple consequence for the 1^+ state would then be that the ratio $A_{\lambda}^{12}/A_{\lambda}^{10}$ should be independent of λ . In our model, it clearly is not, as a glance at Figs. 3, 5, and 6 shows. Nevertheless, it is approximately true, in the sense that for both $\lambda=0$ and $\lambda=1$ the ratios are quite small. The factorization hypothesis is not grossly violated in the model owing to the smallness of the D waves. However, to assume it *a priori* might introduce serious biases into 3π programs.

III. UNITARIZATION AND RESULTS

In Sec. II, we deduced an explicit analytic form for the M dependence of $J^{PC}=2^{++} \rho\pi$ partial wave from a simplified Deck model for $\pi p \rightarrow (\rho\pi)p$. Subsequently, we verified that this simple analytic form is a reasonable approximation to the full $J^{PC}=2^{++}$ amplitude which we obtained from a detailed numerical analysis of the complete Deck amplitude, including pion form factors and a ρ exchange contribution. We showed that a decent representation of the M dependence of the $J^{PC}=2^{++}$ amplitude is obtained by multiplying the analytic result in Eq. (2.12) by the modest correction factor

$$f(M) = \left(0.96 + \frac{0.76}{M^2} - \frac{1.12}{M^4} \right). \quad (3.1)$$

The $J^{PC}=2^{++}$ Deck amplitude does not display the resonant A_2 line shape, of course. Because this resonance is known to be produced diffractively in the $J^{PC}=2^{++}$ wave, another representation must be sought for the M dependence of the full $J^{PC}=2^{++}$ amplitude. Within the context of the Deck model, it is conceivable that unitarity (final-state rescattering) effects, when properly implemented, would modify the $J^{PC}=2^{++}$ amplitude sufficiently to permit good fit to the diffractive A_2 data. If successful, such a scheme would permit various dynamical simplifications, including retention of the hypothesis of s -channel helicity conservation for Pomeron couplings. We show in this section that unitarity is not the answer in this case.

Rather, proper implementation of unitarity constraints forces us to conclude that the data require a dominant direct coupling of the Pomeron to the πA_2 system. This coupling is necessarily of a spin-flip character and, thus, the Pomeron manifestly does not satisfy s -channel helicity conservation.

A. Unitarity

To implement unitarity, we devise a dispersion relation for the M^2 dependence of the $J^{PC}=2^{++}$ amplitude. The Deck amplitude provides the discontinuity across the left-hand cut $M^2 < (m_\rho - m_\pi)^2$, and unitarity provides the right-hand discontinuity for $M^2 > (m_\rho + m_\pi)^2$. From this information the fully "corrected" Deck amplitude is derived.

We begin with the Deck amplitude $A(M^2)$, where

$$A(M^2) = f(M) T_{2,1}^{\text{Deck}}(s, M, t_1). \quad (3.2)$$

The function $f(M)$ is provided in Eq. (3.1), and $T_{2,1}^{\text{Deck}}$ is given in Eq. (2.12).

To take the threshold behavior into account, we define reduced amplitudes $\tilde{A}$, having the same asymptotic behavior as A :

$$\tilde{A}(M^2, s, t_1) = A(M^2, s, t_1) (M^2 + W_s^2) / (4q^2 M^2). \quad (3.3)$$

The parameter W_s reflects some arbitrariness in this subtraction procedure.

We assume that the full amplitude $\tilde{A}^{\text{tot}}(M^2, s, t_1)$, which we abbreviate $\tilde{A}^{\text{tot}}(M^2)$ is analytic in the whole M^2 plane, except for the usual right- and left-hand cuts. We retain only the two-particle $\pi\rho$ intermediate states in the unitarity relation, treating the vector meson as stable, so that the right-hand cut begins at the threshold $M_R^2 = (m_\rho + m_\pi)^2$. The discontinuity formula, along the right-hand cut, can then be written as⁸

$$(\tilde{A}^{+\text{tot}} - \tilde{A}^{-\text{tot}}) / 2i = g\bar{p}\tilde{A}^{-\text{tot}}. \quad (3.4)$$

Here $\tilde{A}^{+\text{tot}}$ and $\tilde{A}^{-\text{tot}}$ are the values of $\tilde{A}^{\text{tot}}(M^2)$ above and below the cut, g is the strong-interaction t -matrix for scattering in the $J^P=2^{++} \rho\pi$ state, and $\bar{p}$ is a phase-space factor. The Eq. (3.4) can be re-expressed as

$$\tilde{A}^{+\text{tot}} = S\tilde{A}^{-\text{tot}}, \quad (3.5)$$

$$S = 1 + 2i\bar{p}g = \exp(2i\delta). \quad (3.6)$$

Here $S(M^2)$ is the ordinary partial-wave S matrix, and $\delta(M^2)$ the phase shift describing the D -wave elastic $\rho\pi$ scattering, i.e., the A_2 phase shift. We presume that $\delta(M^2)$ is "known."

Equation (3.5) is the standard Watson's theorem. It specifies that apart from a global "production phase," the amplitude $\tilde{A}^{\text{tot}}(M^2)$ has the same phase $\delta(M^2)$ as the elastic A_2 amplitude.

We construct the full (reduced) amplitude $\tilde{A}^{\text{tot}}$ from knowledge of its singularity structure. The discontinuity across the right-hand cut was described just above. For the left-hand cut, we assume that the singularities of $\tilde{A}^{\text{tot}}$ are provided by the Deck production model $\tilde{A}$. The construction of amplitudes having these analytical properties is well known. One first constructs an analytic function $D(M^2)$, which possesses only the right-hand cut discontinuity, and is invertible, i.e., which vanishes nowhere. This function $D(M^2)$ is provided by the "Omnès-Mushkelishvili" formula:

$$\tilde{A}^{\text{tot}}(M^2) = \tilde{A}^u(M^2) + D(M^2)P(M^2), \quad (3.8)$$

$$\tilde{A}^u(M^2) = \exp[i\delta(M^2)] \left(\tilde{A}(M^2) \cos\delta(M^2) + \frac{\exp[\gamma(M^2)]}{\pi} P \int_{M_R^2}^{\infty} ds' \frac{\tilde{A}(s') \sin\delta(s') \exp[-\gamma(s')]}{(s' - M^2)} \right). \quad (3.9)$$

In Eq. (3.8), the polynomial $P(M^2)$ results from the possible contribution of the circle at infinity in the Cauchy integral. We denote $\tilde{A}^u$ the *unitarized* reduced amplitude. The final amplitudes are obtained by multiplying the reduced ones by the threshold factor

$$A^u(M^2) = \tilde{A}^u(M^2) (4q^2 M^2) / (M^2 + W_s^2)^2. \quad (3.10)$$

According to the discussion of Ref. 8, the amplitude A^u represents the unitarized Deck amplitude, corrected by rescattering, while the polynomial term represents mainly a "direct" production, plus the contribution of small neglected singularities located far away on the left-hand cut. The direct production amplitude is illustrated in Fig. 7.

In the present case, where the phase of the A_2 is assumed to go from $\delta=0$ at threshold to $\delta=\pi$ at high energy, the function

$$D(M^2) \underset{M^2 \rightarrow \infty}{\sim} 1/M^2.$$

We restrict the polynomial $P(M^2)$ to be a constant in order to have an acceptable high- M^2 behavior for the total amplitude A^{tot} .

FIG. 7. Diagram illustrating a direct coupling of the Pomeron to the πA_2 vertex.

$$\begin{aligned} D(M^2) &= \exp \frac{M^2}{\pi} \int_{M_R^2}^{\infty} \frac{ds' \delta(s')}{s'(s' - M^2 - i\epsilon)} \\ &= \exp[\gamma(M^2) + i\delta(M^2)] \\ \gamma(M^2) &= \frac{M^2}{\pi} P \int_{M_R^2}^{\infty} \frac{ds' \delta(s')}{s'(s' - M^2)}. \end{aligned} \quad (3.7)$$

One may then write a dispersion relation for either the function $D^{-1}\tilde{A}^{\text{tot}}$, which has only a left-hand cut given by $D^{-1} \times \text{disc } \tilde{A}$, or the function $D^{-1}(\tilde{A}^{\text{tot}} - \tilde{A})$ which has only a right-hand cut given by $-\tilde{A} \text{ disc } D^{-1}$. We choose to write the Cauchy integral for $D^{-1}(\tilde{A}^{\text{tot}} - \tilde{A})$. We derive

B. Model for $\rho\pi$ elastic scattering

We approximate the $J^P = 2^+ S$ matrix by a single elastic $\rho\pi$ channel. We parametrize it using a K -matrix formula:

$$\begin{aligned} \exp(2i\delta) &= S = (1 + iKq^5) / (1 - iKq^5), \\ K(M^2) &= \left(\frac{g^2}{W_R^2 - M^2} + a \right) \left(\frac{W_R^2 + \bar{s}}{M^2 + \bar{s}} \right)^5, \end{aligned} \quad (3.11)$$

where W_R , g , a describe the mass, coupling constant, and background of the A_2 . The cutoff $\bar{s}$ is introduced to guarantee that $\delta(M^2) \rightarrow 180^\circ$ for $M \gg W_R$. [q is the modulus of the π or ρ momentum in the $(\rho\pi)$ rest frame.]

Other representations for $\delta(M^2)$ may be devised. However, Eq. (3.11) is both convenient and sufficiently general. The parameters of the K matrix are chosen to describe an A_2 resonance located near $W_R = 1.32$ GeV, with full width $\Gamma \approx 120$ MeV ($g = 3.3$ GeV). Values of $\delta(M^2)$ are shown in Fig. 8 for three choices of the parameter set. For all three curves, $g = 3.3$ GeV and $a = 0$. Otherwise, the solid curve is obtained with $W_R = 1.325$ GeV, and $\bar{s} = 5$ GeV². The short-dashed curve results from $W_R = 1.33$ GeV and $\bar{s} = 10$ GeV². Finally, the dot-dashed curve is obtained with $W_R = 1.32$ GeV and $\bar{s} = 3$ GeV². These different parametrizations for the $J^{PC} = 2^{++} \rho\pi$ elastic amplitude have a modest influence on the final results, as we discuss presently.

C. Results

With $\delta(M^2)$ provided by Eq. (3.11), we evaluated $\tilde{A}^u(M^2)$, as expressed in Eq. (3.9). In Fig. 9, the results are compared with our pure Deck amplitude. The results in Fig. 9 are presented as a

FIG. 8. The M dependence of the $A_2 \rho\pi$ phase shift in our model. Shown are three curves obtained with parameters described in the text.

cross section $d\sigma/dM$ integrated from $|t_1| = 0.17$ GeV^2 to $|t_1| = 0.33$ GeV^2 , in order to facilitate comparison with the data summarized in the Appendix. The full t_1 dependence of our model amplitude is provided in Eq. (2.12), with a mean $B_r = 8.5$ GeV^{-2} , in agreement with data. In this approximation we ignore the mass-slope correlation provided by the model. The latter has no

FIG. 9. Displayed as a function of mass are different representations of the $J=2, l=2, \lambda_1=1$ partial cross section as obtained from our unitarized Deck model. Presented for comparison is the basic Deck result ($\times\times\times$), before unitarization. The four unitarized-model curves are obtained for different choices of parameters described in the text. For the solid curve, $W_R = 1.325$ GeV , $g = 3.3$ GeV , $\bar{s} = 5$ GeV^2 , and $W_s = 0$. For the short-dashed curve, $W_R = 1.33$ GeV , $g = 3.3$ GeV , $\bar{s} = 10$ GeV^2 , and $W_s = 0$. For the dot-dashed curve, $W_R = 1.32$ GeV , $g = 3.3$ GeV , $\bar{s} = 3$ GeV^2 , and $W_s = 0$. The long-dashed curve is obtained with $W_R = 1.325$ GeV , $g = 3.3$ GeV , $\bar{s} = 5$ GeV^2 , and $W_s = 1.35$ GeV^2 .

influence on our conclusions.

Most apparent in Fig. 9 is the fact that the unitarized Deck amplitude vanishes between $M = 1.3$ and 1.4 GeV . The precise position of the zero depends somewhat on the parameter set. This zero is provided by the $\cos\delta$ term in Eq. (3.9). It is a simple consequence of unitarity, as discussed in detail in Ref. 8. Owing to this zero, the resulting unitary amplitude is considerably altered from the broad featureless form of the pure Deck (nonunitary) result. We note that there is indeed a peak in the unitarized cross section $d\sigma/dM$, a feature most notably of the short-dashed curve. However, the position of this peak, its structure, and its full width are not those observed in the A_2 data. The sharpening of the Deck term, and its enhancement below the true resonance location are desirable characteristics of A_1 phenomenology. For the $J^P = 2^+ A_2$ wave, it is evident that the unitarized Deck amplitude fails to yield the correct Breit-Wigner-type form for the $\rho\pi$ mass distribution. The unitarized $J^P = 2^+$ Deck model fails in another important respect. In absolute normalization, the peak value of the short-dashed curve in Fig. 9 is a factor of 30 below the peak value of the A_2 data. Integrating over the A_2 mass region, we find a factor of 80 discrepancy in normalization. This may be contrasted with the $A_1 J^P = 1^+$ situation, where the Deck model provides an absolute cross section in excellent agreement with data. In no sense can the unitarized Deck model be considered even an acceptable local average over the A_2 region.

We conclude that whereas diffractive A_1 production is in large measure a Deck process, diffractive A_2 production is decidedly not. The A_2 data can be fitted only by reliance on the direct production term $D(M^2)P(M^2)$ in Eq. (3.8). The factor $D(M^2)$ provides the desired A_2 Breit-Wigner line shape. The diffractive character of this term is assured by choosing $P(M^2, s, t_1) \propto is$. As remarked above, and as discussed in detail in Ref. 8, the term $D(M^2)P(M^2)$ represents a direct coupling of the A_2 resonance to the Pomeron, illustrated in Fig. 7. The amplitudes $\tilde{A}^u(M^2)$ and $D(M^2)P(M^2)$ in Eq. (3.18) will certainly interfere, and the sign of the interference changes when $\cos\delta$ passes through zero. However, because the magnitude of $\tilde{A}^u(M^2)$ is so small, the interference is not important in the A_2 case.¹⁰ Essentially the entire cross-section shown in Fig. 10 is provided by the direct term alone.

We take $P(M^2, s, t_1)$ to be independent of M^2 in obtaining the results presented in Fig. 10. The t_1 dependence of P was chosen to be $\sqrt{-t_1} \exp(bt_1)$ with $b = 8.5$ GeV^{-2} , independent of M . The normalization of P was arranged to provide the ob-

FIG. 10. Our fit to the A_2 mass distribution. For the solid curve, $W_R = 1.325$ GeV, $g = 3.3$ GeV, $\bar{s} = 5$ GeV², $a = 0$, and $W_s = 0$. The dashed curve results from changing W_s^2 to 1.35 GeV².

served cross section at $M = 1.315$ GeV. We note that our three sets of phase shifts provide nearly identical results for $W_s^2 = 0$. Thus, the A_2 phase shift cannot be deduced with much precision from these data; there is a 10° difference between sets at $M = 1.3$ GeV. Results are more sensitive to the choice of subtraction point W_s^2 .

IV. CONCLUSIONS

We set out to ascertain whether the Deck model, when corrected by final-state interactions, can account for diffractive production of the A_2 . If successful, the approach would have permitted us to view the observed diffractive spin-flip coupling to the A_2 as a unitarity correction, and we would have been able to retain the concept that Pomeron couplings obey s -channel helicity conservation. Our conclusion is that the unitarized Deck model cannot describe diffractive production of the A_2 . The data can be fitted only if a dominant direct spin-flip Pomeron coupling is admitted. s -channel helicity conservation cannot be regarded

as an exact property of Pomeron couplings to mesons. The strength of the Pomeron flip coupling and its relationship to other couplings can best be obtained through standard Regge-model analysis, as done, e.g., in Ref. 6. We note here that diffractive production of the A_1 is very different from that of the A_2 in spite of the fact that the energy dependences are similar² in the two cases. In almost perfect contrast to the A_2 , diffractive production of the A_1 was found to be essentially completely through final-state interactions, with only a negligible direct coupling.⁸

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APPENDIX: A_2 DATA AT 40 GeV/c

We present a brief discussion of our procedure for handling the data from Ref. 2, shown in our Fig. 10. The mass distribution was presented experimentally in units of events/50 MeV, with the t selection $0.17 < |t| < 0.33$ (GeV/c)². In addition, it was reported that the parametrization

$$\frac{d\sigma}{dt} = a |t| \exp(bt) \quad (A1)$$

describes the t dependence of the data, with $b = 8.6 \pm 1.2$ (GeV/c)⁻². For $1.2 < M < 1.4$ GeV, integration over the full t range was reported to yield $\sigma(A_2) = 18 \pm 2$ μ b. From these figures we obtained the data points in our Fig. 10, as follows.

Integrating Eq. (A1), we derive $a \approx 18(8.6)^2 = 1.33$ mb (GeV/c)⁻⁴ for $1.2 < M < 1.4$ GeV. In the restricted interval of $0.17 < |t| < 0.33$ (GeV/c)², we then find

$$\sigma = 6.2 \pm 0.6 \text{ mb},$$

again for $1.2 < M < 1.4$ GeV.

Summing the events shown in Fig. 4(a) of Ref. 2, we find a total of 934 events in the same interval of M and t . We deduce a conversion factor of ~ 6.64 mb/event which we use to translate Fig. 4(a) of Ref. 2 into our Fig. 10.

*Present address: Laboratoire de Physique Mathématique, U. S. T. L. Place E. Bataillon, 34060 Montpellier, France.

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