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***g*-2 Experiments as a Test of Special Relativity**

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An analysis of *g*-2 and lifetime measurements for electrons and muons at various velocities is used to compare the transformation of time, electromagnetic field, and mass in moving coordinate systems and to check the Thomas precession. The data are in good agreement with the special theory of relativity.

The gyromagnetic ratios *g* of the electron and muon are among the most accurately known physical quantities, and consistent results have been obtained¹⁻⁷ for various particle velocities; $\gamma \equiv (1 - v^2/c^2)^{-1/2} = 1-1.2$ for the electron, and 1.2-30 for the muon. It is possible, therefore, that the results imply an accurate verification of special relativity, and this has indeed been claimed by Newman *et al.*¹ As special relativity enters at many points in the interpretation of the experiments, a careful analysis is needed in order to establish the level at which the theory is tested.

The quantity measured in all *g*-2 experiments is the difference between the spin precession frequency f_s and the orbit frequency f_c for a particle turning in a uniform field $\vec{B}$:

$$f_a = f_s - f_c = (a/2\pi)(e/mc)B. \quad (1)$$

With knowledge of e/mc , the value of $a \equiv \frac{1}{2}(g-2)$ can be deduced. Equation (1) is independent of

the velocity of the particle (or of γ), and is equally valid in Newtonian mechanics or in relativity. It is therefore *a priori* not surprising to find that the observed precession frequency is independent of γ ; to turn this observation into a test of relativity therefore requires a sophisticated argument. Newman *et al.* remark that, in relativity theory, Eq. (1) results from the happy cancellation of several γ -dependent terms. These terms represent physically separable effects, and they conclude that the equality of these various γ 's is verified to an accuracy of 5×10^{-9} , while the values of f_a are equal only to 3×10^{-6} . We wish to point out that this conclusion is very much dependent upon the model adopted for the breakdown of special relativity, and to present one of the possible alternative assumptions which lead to somewhat different results.

If the γ factors in relativity theory may be of different types, one could in principle distinguish

between four such quantities, γ_t , γ_E , γ_M , and γ_T , for the transformation of time, electromagnetic fields, and mass, respectively, and for determining the Thomas precession of rest-frame coordinates relative to the laboratory $f_T = (1 - \gamma_T)f_c$, for a particle rotating at orbital frequency f_c . In Newtonian mechanics all these γ 's are 1, while in relativity all are equal to $(1 - v^2/c^2)^{-1/2}$.

If we choose a variant of the theory that gives consistent results when calculations are made using different frames of reference, then this requirement leads to relations between the various γ 's listed above. For example, consider an electron moving with velocity βc in a plane perpendicular to a magnetic field $\vec{B}$, and calculate its transverse acceleration: In laboratory coordinates it is $Be\beta/\gamma_M m$, while in the rest frame of the electron it is $eE^*/m = \beta\gamma_E eB/m$. By converting the latter to laboratory time scale using γ_t^2 , the requirement of consistency gives

$$\gamma_M \gamma_E = \gamma_t^2. \quad (2)$$

No doubt other self-consistency relations could be formulated, but Eq. (2) already illustrates the difference between the model we are using and that considered in Ref. 1. In fact, Newman *et al.* assume that $\gamma_t = \gamma_E = \gamma_T$ (these are called kinematic quantities) and that only γ_M can be different.

We now analyze the $g-2$ experiments, using the four γ factors listed above. The equation of spin motion is given as the sum of three frequencies:

$$f_a = f_L + f_T - f_c, \quad (3)$$

where f_L is the spin-precession frequency in the muon rest frame, corrected to the laboratory scale of time by the factor γ_t^{-1} , f_T is the Thomas frequency, and f_c the rotation frequency of the momentum vector. In the general case

$$\begin{aligned} f_a &= \frac{1}{2\pi} \left(\frac{g}{2} \frac{eB}{mc} \frac{\gamma_E}{\gamma_t} \right) + \frac{1}{2\pi} (1 - \gamma_T) \left(\frac{eB/mc}{\gamma_M} \right) \\ &\quad - \frac{1}{2\pi} \left(\frac{eB/mc}{\gamma_M} \right) \\ &= \frac{1}{2\pi} \left(\frac{eB}{mc} \right) \left(\frac{g}{2} \frac{\gamma_E}{\gamma_t} - \frac{\gamma_T}{\gamma_M} \right). \end{aligned} \quad (4)$$

If condition (2) is used,

$$f_a = \frac{1}{2\pi} \left(\frac{eB}{mc} \right) \left(\frac{g}{2} \frac{\gamma_t}{\gamma_M} - \frac{\gamma_T}{\gamma_M} \right). \quad (5)$$

If it is further assumed that the Thomas precession follows directly from the coordinate transformations so that $\gamma_T = \gamma_t$, then

$$f_a = (a/2\pi)(eB/mc)(\gamma_t/\gamma_M), \quad (6)$$

with $a \equiv \frac{1}{2}(g-2)$. Thus, with the additional assumption $\gamma_T = \gamma_t$, the accuracy in determining γ_t/γ_M is the same as the accuracy in f_a .

In Ref. 1 the electron $g-2$ result of Van Dyck, Schwinger, and Dehmelt,² which has a precision of 0.17 part per million (ppm) and is obtained at a velocity $\beta \sim 5 \times 10^{-5}$, is compared with that of Wesley and Rich³ which has a precision of 3 ppm and is obtained at a velocity $\beta \sim 0.57$. In order to give some point to the comparison between these experiments it is necessary to assume that $\gamma_t/\gamma_M \rightarrow 1$ as $\gamma_t \rightarrow 1$ and so we write γ_t/γ_M as a linear function of γ_t :

$$\gamma_t/\gamma_M = 1 + A(\gamma_t - 1). \quad (7)$$

The coefficient A can then be found from

$$(f_a)_1 - (f_a)_2 = (a/2\pi)(eB/mc)A(\gamma_{t1} - \gamma_{t2}). \quad (8)$$

Comparing the two electron experiments^{2,3} quoted above, we obtain $A < 2.5 \times 10^{-5}$ while, by comparing the lowest-energy result² with the reported⁴ agreement (to <1%) at $\gamma \sim 22\,000$, we find a limit $A < 7.7 \times 10^{-7}$, which is an order of magnitude better. A comparison between the most recent muon $g-2$ result⁷ and that of a previous muon experiment⁶ yields $A < 1.5 \times 10^{-5}$. However, some theoretical models for departures from special relativity^{8,9} predict effects proportional to γ^2 . If such higher-order terms are important, then the electron experiment⁴ at $\gamma \sim 22\,000$ will be by far the most significant.

So far we have only discussed the extent to which the various γ factors in relativity must be equal to each other. But in the muon $g-2$ experiments^{7,10,11} carried out at CERN two γ factors were measured directly and the other two can be deduced with the aid of Eqs. (2) and (4).

Firstly, the orbit frequency $f_c = eB/2\pi mc\gamma_M$ of the muons was found to be 6.8069(9) MHz, and the magnetic field was measured in terms of the proton NMR frequency $f_p = 62.783\,06(3)$ MHz. This allowed the muon-spin precession frequency at rest in the same field $(g/2)(eB/2\pi mc) = \lambda f_p$ to be calculated from the known¹² ratio of the muon to proton NMR frequencies,

$$\lambda = 3.183\,347(8).$$

The ratio $g/2$ is known to be 1.001 162(5) to more than sufficient accuracy from the first (nonrelativistic) $g-2$ experiment.⁵ Hence

$$\gamma_M = (2/g)\lambda(f_p/f_c) = 29.327(4).$$

Secondly, the decay lifetime of the muon in flight in the ring magnet was measured¹⁰ and compared with the lifetime at rest, giving

$$\gamma_t = 29.320(26).$$

Relation (2), which results from our assumed model, can now be used to calculate

$$\gamma_E = \gamma_t^2 / \gamma_M = 29.313(52).$$

Finally, Eq. (4) can be rewritten

$$f_a = \lambda f_p (\gamma_E / \gamma_t) - \gamma_T f_c. \quad (9)$$

The $g-2$ frequency f_a was measured as 0.232 750 2(16) MHz, and all other terms in Eq. (9) are known except γ_T . This gives

$$\gamma_T = 29.320(26).$$

In this way the transformations of mass, time, electromagnetic fields, and rotating coordinates are separately determined at a velocity of $0.9994c$. It will be seen that all γ values are consistent within their standard errors, the largest of which is less than 0.2%. We would like to emphasize that limits on special relativity produced as we have done here, and as was done in Ref. 1, cannot be considered universal but only valid within the framework of the model adopted.

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