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BACKWARD DISPERSION RELATIONS AND PION-⁴He
PHASE - SHIFT ANALYSES

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A B S T R A C T

The usefulness of backward dispersion relations in nuclear physics is discussed. As an example, pion-⁴He elastic scattering is considered. It is shown how the use of backward dispersion relations can help in reducing ambiguities of phase-shift analyses. In particular, the solution with the S wave changing sign twice between 150 and 260 MeV and with the real part of the backward scattering amplitude becoming positive at ~140 MeV and exhibiting resonating behaviour has been suggested as the most probable one.

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1. INTRODUCTION

Since the forward dispersion relations for particle-nucleus scattering have been written down ¹⁾, the use of analyticity in nuclear physics has proven to be fruitful. However, until now analyticity has been exploited in its simplest forms only : as forward dispersion relations and extrapolations in the angular variable at fixed energy [see reviews 2) and 3)]. In this note we propose to extend the use of backward dispersion relations ⁴⁾⁻⁶⁾ to nuclear physics.

The backward dispersion relations connect the real part of the s channel scattering amplitude with the integrals over the s channel and t channel imaginary parts, the u channel being excluded. In particle physics the backward dispersion relations have been used not only for testing the consistency of different phase-shift analyses with the hypothesis of analyticity, and for the determination of several coupling constants, but also for rather ambitious purposes like extrapolations of the πN scattering amplitudes into the region of $\pi\pi \rightarrow \bar{N}N$ scattering and obtaining information about the low energy $\pi\pi$ interaction.

In nuclear physics the presence of anomalous cuts would make the meaning of such extrapolations obscure. However, one may hope to benefit from backward dispersion relations in connection with ambiguity problems in phase-shift analysis or in attempts to determine some particle-nucleus coupling constants.

2. $\pi^4\text{He}$ BACKWARD DISPERSION RELATION

We begin by noting some kinematic relations for backward $\pi^4\text{He}$ scattering. The Mandelstam variables s and t are related to the s channel variables, the scattering angle θ and the c.m. momentum k , by

$$s = M^2 + \mu^2 + 2k^2 + 2[(k^2 + M^2)(k^2 + \mu^2)]^{1/2}, \quad (1)$$

$$t = -2k^2(1 - \cos\theta), \quad (2)$$

where μ and M are the pion and helium masses, respectively. For the special case of backward scattering, (2) becomes simply $t = -4k^2$. Backward scattering amplitude can obviously be specified in terms of the single variable $v = k^2$, and it is convenient to write backward dispersion relations in terms of this variable.

If we consider s as a function of v we see from (1) that it can be defined on a two-sheeted v plane, with a branch cut along the segment $-M^2 \leq v \leq u^2$. It is easy to verify that this cut corresponds to values of s on the circle $|s| = M^2 - u^2$. Let us define the first sheet so that $|s| \geq M^2 - u^2$ and the second sheet so that $|s| \leq M^2 - u^2$ on the first sheet, while $|u| \geq M^2 - u^2$ on the second.

If it is assumed, as usual, that the invariant amplitude $A(s, t)$ has only those singularities required by unitarity in each channel, then the backward scattering amplitude on the first sheet of the v plane has cuts for $v \geq v_a$ (due to the s channel unitarity cuts for $s \geq s_a$) and for $v \leq v_0$ (due to the t channel unitarity cuts for $t \geq t_0$). There are no singularities on the first sheet due to u channel process, since $|u| \leq M^2 - u^2$ on this sheet. Then the backward dispersion relation becomes

$$A(v) = \frac{1}{\pi} \int_{v_a}^{\infty} \frac{\text{Im} A(v')}{v' - v} dv' + \frac{1}{\pi} \int_{-\infty}^{v_0} \frac{\text{Im} A(v')}{v' - v} dv'. \quad (3)$$

There are no poles, v_a is the beginning of the s channel unphysical cut ($\pi^4\text{He} \rightarrow p^3\text{H}$), v_0 is the beginning of the t channel cut (${}^4\text{He}^4\text{He} \rightarrow 2\pi$), the t channel anomalous cut starts at $t > 9\mu^2$.

3. APPLICATION TO PHASE-SHIFT ANALYSES

In analyzing the dispersion relation (3), it is convenient to define the discrepancy function

$$\Delta(v) = \text{Re} f(v, \cos \theta = -1) - \frac{1}{\pi \sqrt{s}} P \int_0^W \frac{\sqrt{s'} \text{Im} f(v', \cos \theta = -1)}{v' - v} dv' \quad (4)$$

where

$$\left. \frac{d\sigma}{d\Omega} (\pi^4\text{He} \rightarrow \pi^4\text{He}) \right|_{\theta=\pi} = |f(v, \cos \theta = -1)|^2, \quad (5)$$

$$f(v, \cos\theta = -1) = \frac{1}{\sqrt{v}} \sum_{l=0}^L (2l+1) \frac{\eta_l \exp(2i\delta_l) - 1}{2i} (-1)^l \quad (6)$$

and W is the highest value of v for which phase-shift analyses are available. This discrepancy function can be constructed numerically in the range $0 < v < W$ and can then be analyzed in terms of both the high energy contributions to the dispersion integral (for $v > W$) and the contributions from the left-hand cut.

In order to test the usefulness of backward dispersion relations, we have chosen the phases of the $\pi^4\text{He}$ scattering amplitude obtained in Ref. 7) where, contrary to another extensive $\pi^4\text{He}$ phase-shift analysis ⁸⁾, the results are presented in tabular form.

The nuclear scattering amplitude is parametrized in Ref. 7) as follows :

$$f(t) = f(0) \exp\left(-\frac{1}{6} R_s'^2 t\right) \prod_{j=1}^2 (1 - t/t_j) , \quad (7)$$

where

$$f(0) = \frac{\kappa}{4\pi} \sigma_{\text{tot}} [i + \rho] , \quad (8)$$

$$t_j = 2k^2 (1 - z_j) . \quad (9)$$

Using the values of σ_{tot} and of parameters ρ , R_s' and z_j as given in Table 4 of Ref. 7) we have calculated at $T=51, 60, 68, 75, 110, 150, 180, 220$ and 260 MeV four possible sets of $f(v, \cos\theta = -1)$ which correspond to different choices of signs of $\text{Im}(1 - z_j)$ at energies higher than 110 MeV. We have added a threshold point and a point at $T=24$ MeV from Fig. 11 of Ref. 7).

The smoothed curves illustrating this parametrization are shown in Figs. 1 and 2 and the resulting discrepancy functions in Fig. 3.

What is the cause of these discrepancies ? They come from two different sources : neglect of the s channel high energy contribution and omission of the t channel integral. The s channel unphysical cut should

have a minor influence only, because its effect in dispersion relation calculations is known to be small ¹⁾.

As already mentioned, the $\pi^4\text{He}$ scattering amplitude has no real poles. Therefore one might expect no significant structure in the discrepancy functions and one should be able to parametrize them in a simple form. We have chosen the following parametrization :

$$\Delta(\nu) = \frac{a_1}{\nu - a_2} + \sum_{n=0}^N b_n \nu^n . \quad (10)$$

The pole should represent the contribution from the nearby left-hand t channel unphysical cut and the sum parametrizes the high energy s channel integral.

We are able to describe discrepancies in the first three cases only (see Fig. 3) : Δ_{II} with $N=2$, Δ_{I} with $N=3$ and Δ_{III} with $N=4$. Case IV is not consistent with the simple expression (10). It is interesting to note that for Δ_{II} one gets $a_2 = -0.129 \text{ GeV}^2$, what is rather close to the position of the ω meson, while fits for Δ_{I} and Δ_{III} require a pole at $a_2 = -0.098 \text{ GeV}^2$ and $a_2 = -0.069 \text{ GeV}^2$, respectively.

4. CONCLUSIONS

The analysis based on the backward dispersion relation excludes case IV and suggests case II as the most probable one. This should be compared with the conclusions of the authors of Ref. 7). They favour case I, from continuity arguments, but do not exclude any other case.

Case II is interesting from the physical point of view. The s wave changes sign twice between 150 and 260 MeV (Fig. 4) and the real part of the backward scattering amplitude becomes positive at ~ 140 MeV exhibiting a resonating behaviour (Fig. 2).

We have shown how backward dispersion relation can help to reduce ambiguities in the $\pi^4\text{He}$ phase-shift analysis. The relation (4) can be directly used in other spinless cases, e.g., $\pi^{12}\text{C}$ or $\pi^{16}\text{O}$ scattering. In other reactions (e.g., $\pi^3\text{He}$, $N^4\text{He}$ scattering) complications due to spin and possible pole terms (e.g., $N^3\text{He}$ scattering) should be taken into account.

Of course, any a priori physical information which could be incorporated into the parametrization of the discrepancy function would strengthen the reliability of conclusions based on the use of backward dispersion relations.

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FIGURE CAPTIONS

Figure 1 : The imaginary part of the $\pi^4\text{He}$ backward scattering amplitude as a function of ν evaluated on the basis of parametrization (7). Curve I corresponds to the choice $\text{Im}(1-z_1) > 0$ at all energies and to $\text{Im}(1-z_2) < 0$ above 110 MeV. Curve II corresponds to the choice $\text{Im}(1-z_1) > 0$ at all energies and to $\text{Im}(1-z_2) > 0$ above 110 MeV. Curve III corresponds to the choice $\text{Im}(1-z_1) < 0$ at all energies and to $\text{Im}(1-z_2) < 0$ above 110 MeV. Curve IV corresponds to the choice $\text{Im}(1-z_1) < 0$ at all energies and to $\text{Im}(1-z_2) > 0$ above 110 MeV.

Figure 2 : The real part of the $\pi^4\text{He}$ backward scattering amplitude as a function of ν . Other conventions are the same as in Fig. 1.

Figure 3 : The discrepancy function of the $\pi^4\text{He}$ backward scattering amplitude as a function of ν . Other conventions are the same as in Fig. 1.

Figure 4 : Results of the phase-shift analysis ⁷⁾ for the first three partial waves only, case II. (1:24 MeV; 2:51 MeV; 3:60 MeV; 4:68 MeV; 5:75 MeV; 6:110 MeV; 7:150 MeV; 8:180 MeV; 9:220 MeV; 10:260 MeV).

FIG.1

FIG.2

FIG.3

FIG.4